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A Dual Approach to Estimation With Constant Prices

by

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Giannini Foundation of Agricultural Economics

A Dual Approach to Estimation With Constant Prices

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Abstract

In a recent paper, Mundlak assumes that the price-taking, risk-neutral and profit-maximizing entrepreneur makes his decisions on the basis of a planning model that maximizes expected profit using expected prices. In the same paper, the author asserts that when there is no sample price variation across competitive firms, it is impossible to estimate the supply and factor demand functions from cross-section data using a dual approach. In a famous paper, titled "To Dual or not to Dual," Pope asserted a similar opinion. This paper shows that, using Mundlak's assumption about planning decisions based upon expected profit, it is possible to use a dual estimator to estimate supply and factor demand functions. This objective is achieved by using Mundlak's assumption about the individuality of the firm's expectation process. A two-phase procedure is suggested to obtain consistent estimates of the expected quantities and prices which are then used, in phase II, in a nonlinear seemingly unrelated equations problem to obtain efficient estimates of the technological parameters.

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A Dual Approach to Estimation With Constant Prices

For many years, econometricians have denied the possibility of using a dual approach for estimating economic relations when the measured prices are the same across sample units or when they are collinear. As recently as 1996, Mundlak (p. 433) wrote the following: "*In passing we note that the original problem of identifying the production function as posed by Marschak and Andrews assumed no price variation across competitive firms. In that case, it is impossible to estimate the supply and factor demand functions from cross-section data of firms and therefore (the dual estimator)* $\hat{\gamma}_p$ *cannot be computed. Thus, a major claimed virtue of dual functions---that prices are more exogenous than quantities--- cannot be attained. Therefore, for the dual estimator to be operational, the sample should contain observations on agents operating in different markets*."

The debate whether the primal approach is "superior" to the dual approach (in the sense of being able to deliver the desired estimates) has seen several participants. For example, in a paper by the provocative title "*To Dual or Not to Dual*", Pope recorded the following case among "duality failings." Discussing a cost function and the goal of recovering the underlying technology, Pope (p. 348) wrote: "*Suppose* $P_1 = P_2 = P$*, then the cost function is linear in* P_1*: rank conditions (of the Jacobian) are not satisfied, and no unique finite solution for* P_1 *and* P_2 *is guaranteed. Indeed, one can only identify the composite* $x_1 + x_2$ *from the cost function via Hotelling's Theorem and technology cannot be recovered… Are there reasonable economic situations where such occurs? It would seem so. Intraseasonal prices of an input (water, fertilizer, etc.) may be very co-linear and yet these dated inputs would enter the technology other than as a sum.*"

This quotation is in the spirit of Mundlak's assertion: the lack of price variability either across sample units and/or across different prices prevents the use of a dual approach to estimate the desired economic relations which could then be used for policy analysis and in the recovery of the underlying technology.

Our paper contributes the following results. The above indictments of duality are unwarranted. We will present a theoretical and empirical framework for price-taking, profit-maximizing and risk-neutral firms operating with the same technology and in the same market environment. This means that the input and output prices observed by the econometrician will appear to be the same for all sample units. Nevertheless, the theoretical and empirical approach presented here (which follows closely Mundlak's assumptions) allows the use of duality in the estimation of output supply and input demand functions. A concomitant result is the demonstration that efficient (in the sense of utilizing all the available information) estimates of the technological and economic parameters of a production and profit system require the joint estimation of primal and dual relations. Hence, the question of whether primal methods are superior to dual methods is put to rest. The specification of the estimable model proposed in this paper assumes the form of a nonlinear errors-in-variables problem for which we provide a novel and general solution.

Production and Profit Environments

In this paper we postulate a static context. Following Mundlak, we assume that the profit-maximizing firms of our sample make their output and input decisions on the basis of expected quantities and prices and the entrepreneur is risk neutral. That is to say, a planning process can be based only upon expected information.

The process of expectation formation is characteristic of every firm (this assumption was advanced by Mundlak). Such a process is known to the firm's entrepreneur but is unknown to the econometrician. The individuality of the expectation process allows for a variability of input and output decisions among the sample firms even in the presence of a unique technology and measured output and input prices that appear to be the same for all sample firms.

Let the expected production function $f^e(\cdot)$ for a generic firm have values

(1) $$y \leq f^e(\mathbf{x})$$

where y is the level of output for any strictly positive $(J \times 1)$ vector $\mathbf{x}$ of input quantities. After the expected profit-maximization process has been carried out, the input vector $\mathbf{x}$ will become the vector of expected input quantities $\mathbf{x}^e$ and the scalar variable y will become the expected output supply y^e that will satisfy the firm's planning target. The expected production function $f^e(\cdot)$ is assumed to be twice continuously differentiable, quasi-concave, and non-decreasing in its arguments.

We postulate that the profit-maximizing risk-neutral firm solves the following problem:

(2) $$\pi^e(p^e, \mathbf{w}^e) \stackrel{\text{def}}{=} \max_{\mathbf{x}, y} \{ p^e y - \mathbf{w}'^e \mathbf{x} \mid y \leq f^e(\mathbf{x}) \},$$

where $\pi^e(\cdot)$ is the expected profit function, p^e is the expected output price, $\mathbf{w}^e$ is a $(J \times 1)$ vector of expected input prices and "$'$" is the transpose operator.

Assuming an interior solution, first order necessary conditions for problem (2) are given by

(3) $$\frac{\partial \pi}{\partial \mathbf{x}} = p^e \mathbf{f}_{\mathbf{x}}^e(\mathbf{x}) - \mathbf{w}^e = \mathbf{0}.$$

The solution of equations (3), gives the expected profit-maximizing input demand functions $\mathbf{d}^e(\cdot)$, with values

(4) $$\mathbf{x}^e = \mathbf{d}^e(p^e, \mathbf{w}^e),$$

which in turn, using the production function (1), give the expected output supply function

(5) $$y^e = g^e(p^e, \mathbf{w}^e).$$

In case equations (3) have no analytical solution, the input demand and output supply functions exist via the duality principle.

The above theoretical development corresponds precisely to the textbook discussion of the profit-maximizing behavior of a price-taking, risk-neutral firm. The econometric representation of that setting requires the specification of the error structure associated with the observation of the firm's environment and decisions. We regard the expected quantities and prices as non-random information since the expected quantities reflect the entrepreneur's profit-maximizing decisions in which the expected prices are fixed parameters resulting from the expectation process of the individual entrepreneur.

Observation of quantities and prices implies measurement errors. Mundlak (p. 432) writes: "As $\mathbf{w}^e$ is unobservable, the econometrician uses $\mathbf{w}$ which may be the observed input price vector or his own calculated expected input price vector." The additive error structure of input prices is therefore stated as $\mathbf{w} = \mathbf{w}^e + \boldsymbol{\nu}$. Mundlak (p. 432) calls $\boldsymbol{\nu}$ "the optimization error, but we note that in part the error is due to the econometrician's failure to read the firm's decision correctly rather than the failure of the firm to reach the optimum." Similarly, we postulate measurement errors on all the other sample information. Thus, the observed output, output price and input quantities bear an additive relation with their expected counterparts such as: $y = y^e + \varepsilon_0$, $p = p^e + \nu_0$, and $\mathbf{x} = \mathbf{x}^e + \boldsymbol{\varepsilon}$. The commission of measurement errors in the observation of any quantity is hardly deniable. Hence, we identify such errors with any type of sample information.

The empirical nonlinear generalized additive error model of production and profit can now be stated using the theoretical relations (1), (3), (4), (5) and the error structure specified above. Symmetry considerations and the necessity to estimate the expected output price dictate that the output supply function should appear in inverse form. Hence, the measurable primal and dual relations of the production and profit system appear as follows:

Primal

(6)	$y = f^e(\mathbf{x}^e) + \varepsilon_0$	production function
(7)	$\mathbf{x} = \mathbf{x}^e + \boldsymbol{\varepsilon}$	
(8)	$\mathbf{w} = p^e \mathbf{f}_{\mathbf{x}}^e(\mathbf{x}^e) + \boldsymbol{\nu}$	input price functions
(9)	$p = p^e + \nu_0$	

Dual

(10) $\mathbf{x} = \mathbf{d}^e(p^e, \mathbf{w}^e) + \boldsymbol{\varepsilon}$ input derived demand functions

(11) $\mathbf{w} = \mathbf{w}^e + \boldsymbol{\nu}$

(12) $p = s^e(y^e, \mathbf{w}^e) + \nu_0$ inverse output supply function

(13) $y = y^e + \varepsilon_0$.

The symmetry of the production and profit system (6)-(13) can be emphasized in a number of ways. The primal relations are composed by a quantity equation (production function) and J price equations (input price functions). The dual relations are composed by J quantity equations (input derived demand functions) and a price equation (inverse output supply function). It is interesting to notice that the connection between the primal and the dual systems is given by the additive error equation (9) of the output price. This feature can be clearly seen by considering the estimation of the primal relations only. In this case, equations (6) through (9) constitute a complete system. Imagine now of estimating only the dual relations. For this purpose, equations (10) through (13) need the addition of equation (9). In reality, the additive error equation of the output price is not, per se, a primal relation but it is interesting to notice that it alone must be included in the separate estimation of either the primal or dual systems.

The system of primal and dual relations (6)-(13) constitutes a nonlinear errors-in-variables model. The traditional approach to estimate this type of models is to replace the unobserved expected components of quantities and prices with their measurable counterparts. That is, for example, $\mathbf{x}^e = \mathbf{x} - \boldsymbol{\varepsilon}$. In general, this step induces undesirable properties on the estimator such as under identification of the model and biased and inconsistent estimates of the parameters. The estimation approach proposed in this paper avoids this replacement step and suggests a two-phase procedure: In phase I the expected quantities and prices will be estimated jointly with the technological and economic

parameters by a nonlinear least-squares algorithm. In phase II, the estimated expected quantities and prices will be used as instrumental variables in a nonlinear seemingly unrelated (NSUR) equations model to be estimated by three-stage least squares. The estimates so obtained are consistent if we assume the conditions stated by Davidson and MacKinnon (ch. 5) in their theorem 5.1.

It should be apparent by now, that observed constant or collinear prices do not prevent the estimation of dual relations. This is due to the fact that the observed constant or collinear prices are only a proxy for the "true" expected prices adopted by the decision makers. These observed prices, then, are decomposed, firm by firm, into an estimated expected portion of the observed prices and the complementary residual. Hence, the estimated expected prices, which now vary across sample units and inputs, are used as instrumental variables in phase II of the estimation methodology to obtain efficient estimates of the parameters using only dual relations.

Estimation of the Nonlinear GAE Model with Constant and Collinear Prices

We assume a sample of cross-section data on N profit-maximizing firms, $i = 1,...,N$. The vector of error terms $\mathbf{e}_i' = (\boldsymbol{\varepsilon}_i', v_{0i}, \mathbf{v}_i', \varepsilon_{0i})$ appearing in equations (6)-(13) is assumed to be distributed according to a multivariate normal density with zero mean vector and variance matrix Σ. Thus we assume independence of the disturbances across firms and contemporaneous correlation of them within a firm.

Let $\boldsymbol{\beta} = (\boldsymbol{\beta}_y, \boldsymbol{\beta}_{\mathbf{w}}, \boldsymbol{\beta}_{\mathbf{x}}, \boldsymbol{\beta}_p)$ be the vector of technological and economic parameters to be estimated. In phase I, the nonlinear least-squares estimation problem consists in minimizing the residual sum of squares

(14) $$\min_{\boldsymbol{\beta}, y_i^e, x_{ij}^e, w_{ij}^e, p_i^e, \mathbf{e}_i} \sum_{i=1}^{N} \mathbf{e}_i' \mathbf{e}_i$$

with respect to the residuals and all the parameters, including the expected quantities and prices for each firm, subject to primal and dual equations of the production and profit system and the error structure postulated in section 2, that is,

(15) $$y_i = f^e(\mathbf{x}_i^e, \boldsymbol{\beta}_y) + \varepsilon_{0i}$$

(16) $$\mathbf{x}_i = \mathbf{x}_i^e + \boldsymbol{\varepsilon}_i$$

(17) $$\mathbf{w} = p_i^e \mathbf{f}_{\mathbf{x}}^e(\mathbf{x}_i^e, \boldsymbol{\beta}_{\mathbf{w}}) + \boldsymbol{\nu}_i$$

(18) $$p = p_i^e + \nu_{0i}$$

(19) $$\mathbf{x}_i = \mathbf{d}^e(p_i^e, \mathbf{w}_i^e, \boldsymbol{\beta}_{\mathbf{x}}) + \boldsymbol{\varepsilon}_i$$

(20) $$\mathbf{w} = \mathbf{w}_i^e + \boldsymbol{\nu}_i$$

(21) $$p = s^e(y_i^e, \mathbf{w}_i^e, \boldsymbol{\beta}_p) + \nu_{0i}$$

(22) $$y_i = y_i^e + \varepsilon_{0i}$$

(23) $$\sum_{i=1}^{N} y_i^e \varepsilon_{0i} = 0$$

(24) $$\sum_{i=1}^{N} w_{ij}^e \nu_{ij} = 0, \quad j = 1, \ldots, J$$

(25) $$\sum_{i=1}^{N} p_i^e \nu_{0i} = 0$$

(26) $$\sum_{i=1}^{N} x_{ij}^e \varepsilon_{ij} = 0, \quad j = 1, \ldots, J\,.$$

We have removed the observation index from the measured output and input prices in order to signify that all sample units appear to face the same prices, as measured by the econometrician. We can take the input prices to be the same for all inputs in order to simulate the condition of collinearity discussed by Pope. The vectors of technological and economic parameters $\boldsymbol{\beta}_y, \boldsymbol{\beta}_{\mathbf{w}}, \boldsymbol{\beta}_{\mathbf{x}}, \boldsymbol{\beta}_p$ characterize the functions referred to by their

subscript and enter, in general, in a nonlinear fashion. This nonlinearity is another item of contention in the literature. Pope (p. 349), in fact, also wrote: "*Hence, it seems that duality works poorly when the objective function is nonlinear in parameters*." Our answer is different. For example, the Cobb-Douglas profit function is certainly nonlinear in the parameters of the corresponding technology, but no problem arises in the estimation of the output supply function and input demand functions using Hotelling lemma and exclusively dual relations, as demonstrated in the empirical application further on.

Constraints (23)-(26) guarantee the orthogonality (non correlation) of the residuals and the corresponding estimated expected quantities and prices, exactly as is dictated by the definition of an instrumental variable, a role that they play in phase II. We assume that a unique solution of the phase I problem exists and can be found using a nonlinear optimization package such as, for example, GAMS (see Brooke *et al.*).

With the estimates of the expected quantities and prices obtained from phase I, a traditional NSUR problem can be stated and estimated in phase II using conventional econometric packages such as SHAZAM (Whistler *et al.*). Let $\hat{\Sigma}$ be the covariance matrix estimated in phase I. For clarity, the phase II nonlinear least-squares problem can be stated as

(27) $$\min_{\boldsymbol{\beta},\mathbf{e}_i} \sum_{i=1}^{N} \mathbf{e}_i' \hat{\Sigma}^{-1} \mathbf{e}_i$$

subject to

(28) $$y_i = f^e(\hat{\mathbf{x}}_i^e, \boldsymbol{\beta}_y) + \varepsilon_{0i}$$

(29) $$\mathbf{w} = \hat{p}_i^e \mathbf{f}_{\mathbf{x}}^e(\hat{\mathbf{x}}_i^e, \boldsymbol{\beta}_{\mathbf{w}}) + \boldsymbol{\nu}_i$$

(30) $$\mathbf{x}_i = \mathbf{d}^e(\hat{p}_i^e, \hat{\mathbf{w}}_i^e, \boldsymbol{\beta}_{\mathbf{x}}) + \boldsymbol{\varepsilon}_i$$

(31) $$p = s^e(\hat{y}_i^e, \hat{\mathbf{w}}_i^e, \boldsymbol{\beta}_p) + v_{0i}$$

where $(\hat{y}_i^e, \hat{p}_i^e, \hat{\mathbf{w}}_i^e, \hat{\mathbf{x}}_i^e)$ are the expected quantities and prices of the i-th firm estimated in phase I and assume the role of instrumental variables in phase II. The matrix $\hat{\Sigma}$ can be updated iteratively to convergence.

The model presented in this section encompasses both primal and dual relations. It is clear, therefore, that the estimates obtained from the estimation of such a model are efficient in the sense that they utilize all the available theoretical and sample information.

Estimation of a Model with Constant Prices Using Only Duality Relations

We now fulfill the main goal of the paper which consists in showing that duality does not fail even when all the price-taking, risk-neutral and profit-maximizing firms appear to face the same observed input and output prices.

Let the vector of parameters to be estimated in this case be specified as $\boldsymbol{\beta}_{-y,-\mathbf{w}} = (\boldsymbol{\beta}_{\mathbf{x}}, \boldsymbol{\beta}_p)$. Then, the phase I nonlinear least-squares estimation problem using only dual relations consists of the following specification:

(32) $$\min_{\boldsymbol{\beta}_{-y,-\mathbf{w}}, y_i^e, w_{ij}^e, p_i^e, \mathbf{e}_i} \sum_{i=1}^{N} \mathbf{e}_i' \mathbf{e}_i$$

subject to

(33) $$p = p_i^e + v_{0i}$$

(34) $$\mathbf{x}_i = \mathbf{d}^e(p_i^e, \mathbf{w}_i^e, \boldsymbol{\beta}_{\mathbf{x}}) + \boldsymbol{\varepsilon}_i$$

(35) $$\mathbf{w} = \mathbf{w}_i^e + \boldsymbol{\nu}_i$$

(36) $$p = s^e(y_i^e, \mathbf{w}_i^e, \boldsymbol{\beta}_p) + v_{0i}$$

(37) $$y_i = y_i^e + \varepsilon_{0i}$$

(38) $$\sum_{i=1}^{N} p_i^e \nu_{0i} = 0$$

(39) $$\sum_{i=1}^{N} w_{ij}^e \nu_{ij} = 0, \quad j = 1,\ldots,J$$

(40) $$\sum_{i=1}^{N} y_i^e \varepsilon_{0i} = 0 .$$

Equations (34) and (36) are the input demand and inverse output supply functions, respectively. Equations (33), (35) and (37) define the error structure of the corresponding quantities and prices whose expected portion enters in the dual relations.

With the estimates of the expected quantities and prices obtained from phase I, a traditional NSUR problem can be stated for the dual relations and estimated in phase II using conventional econometric packages. Let $\mathbf{u}_i = (\boldsymbol{\varepsilon}_i, \nu_{0i})$ be the vector of residuals of the dual relations and $\hat{\Sigma}_{-P}$ be the covariance matrix estimated in phase I but with errors associated with primal relations deleted. For clarity, the phase II nonlinear least-squares problem can be stated as

(41) $$\min_{\boldsymbol{\beta}_{-y,-\mathbf{w}}, \mathbf{u}_i} \sum_{i=1}^{N} \mathbf{u}_i' \hat{\Sigma}_{-P}^{-1} \mathbf{u}_i$$

subject to

(42) $$\mathbf{x}_i = \mathbf{d}^e(\hat{p}_i^e, \hat{\mathbf{w}}_i^e, \boldsymbol{\beta}_{\mathbf{x}}) + \boldsymbol{\varepsilon}_i$$

(43) $$p = s^e(\hat{y}_i^e, \hat{\mathbf{w}}_i^e, \boldsymbol{\beta}_p) + \nu_{0i}$$

where $(\hat{y}_i^e, \hat{p}_i^e, \hat{\mathbf{w}}_i^e)$ are the expected quantities and prices of the i-th firm estimated in phase I. This model demonstrates that, under Mundlak's and Pope's assumptions, it is possible to use duality relations exclusively in order to estimate the profit-maximizing behavior of price-taking firms which face prices that appear the same to the

econometrician. In reality, we know that this uniformity of prices reflects more the failure of our statistical reporting system rather than a true uniformity of prices faced by entrepreneurs in their individual planning processes.

An Application of the Duality Approach With Constant and Collinear Prices

The model and the estimation procedure described in previous sections have been applied to a sample of 84 California cooperative cotton ginning firms. These cooperative firms must process all the raw cotton delivered by the member farmers. Hence, the level of their output is exogenous and their economic decisions are made according to a cost-minimizing behavior. In order to adapt the sample information to a simulation of profit-maximizing behavior, therefore, the output variable was generated according to the following Cobb-Douglas model:

$$y_i = 3(x_{1i}^e)^{.3}(x_{2i}^e)^{.4}(x_{3i}^e)^{.2} + N(0,2) \tag{44}$$

with decreasing returns to scale equal to the sum of the production elasticities, where the expected inputs were chosen as $x_{1i}^e = x_{1i} - N(0,0.5)$, $x_{2i}^e = x_{2i} - N(0,0.5)$, $x_{3i}^e = x_{3i} - N(0,0.5)$ and the disturbance terms were drawn according to a normal random variable with zero mean and standard deviation as specified. A nonlinear model possesses a "natural" scale for its variables in the sense that it is not possible to choose any arbitrary scale (as in linear models) and hope to obtain a feasible solution. In other words, there is a "natural" range of scaling for the variables that will allow to achieve an optimal solution. The choice of the standard deviation of all the variables in equation (44) was made with this "natural" scale in mind.

There are three inputs: labor, energy and capital. Observed labor is defined as the annual labor hours of all employees. The observed wage rate for each gin was computed by dividing the labor bill by the quantity of labor. Observed energy expenditures include the annual bill for electricity, natural gas, and propane. British thermal unit (BTU) observed prices for each fuel were computed from each gin's utility rate schedules and then aggregated into a single BTU observed price for each gin using BTU observed quantities as weights for each energy source. The observed variable input energy was then computed by dividing energy expenditures by the aggregate energy price.

A gin's operation is a seasonal enterprise. The downtime is about nine months per year. The long down time allows for yearly adjustments in the ginning equipment and buildings. For this reason capital is treated as a variable input. Each component of the capital stock was measured using the perpetual inventory method and straight-line depreciation. The rental prices for buildings and ginning equipment was measured by the Christensen and Jorgenson formula. Observed expenditures for each component of the capital stock were computed as the product of each component of the capital stock and its corresponding rental rate and aggregated into total capital expenditures. The observed composite rental price for each gin was computed using an expenditure-weighted average of the gin's rental prices for buildings and equipment. The observed composite measure of the capital service flow is computed by dividing total yearly capital expenditure by the composite rental price.

Ginning cooperative firms receive the raw cotton from the field and their output consists of cleaned and baled cotton lint and cottonseeds in fixed proportions. These outputs, in turn, are proportional to the raw cotton input. Total output for each gin was then computed as a composite commodity by aggregating cotton lint and cottonseed using a proportionality coefficient.

The observed price of the composite output y is defined as $P \stackrel{\text{def}}{=} P_c + \phi P_s$, where P_c is the price per 500-pound bale of cotton lint, P_s is the price per ton of cotton seed, and ϕ is the ratio of tons of seeds per 500-pound bale of cotton lint. The ratio ϕ captures the difference, if any, between the picking and stripping methods of removing the raw cotton

from the plant. This ratio, however, is not under the control of the gins, as it reflects the choice of stripping technique employed by the cotton member-growers of the co-op.

In order to conform the empirical information to the assumptions of the paper, we computed the average of the observed output price and of all the observed input prices and assigned these averages to each firm. Hence, all the sample units face the same observed prices.

On the basis of the Cobb-Douglas specification of equation (44), the system of equations that specify the production and profit environments of the price-taking firms is constituted of the following eight primal and dual relations:

Cobb-Douglas production function

(45) $$y_i = A\prod_{j=1}^{3}(x_{ij}^e)^{\alpha_j} + \varepsilon_{0i}$$

Input price functions

(46) $$w_k = p_i^e[\alpha_k A \prod_{j\neq k=1}^{3}(x_{ij}^e)^{\alpha_j}(x_{ik}^e)^{\alpha_k - 1}] + v_{ik}, \quad k = 1,2,3$$

Input demand functions

(47) $$x_{ij} = \alpha_j[A\prod_{k=1}^{3}\alpha_k^{\alpha_k}]^{1/(1-\eta)}(p_i^e)^{1/(1-\eta)}(w_{ij}^e)^{(\Sigma_{k\neq j}\alpha_k - 1)/(1-\eta)}\prod_{k\neq j=1}^{3}(w_{ik}^e)^{-\alpha_k/(1-\eta)} + \varepsilon_{ij}, \quad j = 1,2,3$$

Inverse output supply function

(48) $$p = [A\prod_{j=1}^{3}\alpha_j^{\alpha_j}]^{-1/\eta}(y_i^e)^{(1-\eta)/\eta}\prod_{j=1}^{3}(w_{ij}^e)^{\alpha_j/\eta} + v_{0i}$$

where $\eta = \Sigma_j \alpha_j, j = 1,2,3$. The observed prices in equations (46) and (48) do not carry a sample unit subscript to indicate that all the firms face the same prices as perceived by the econometrician. We must point out that with technologies (such as the Cobb-Douglas production function) admitting an explicit solution of the first-order necessary conditions,

either the primal constraints (45) and (46) or the dual constraints (47) and (48) are redundant in the phase I estimation problem, and thus can be dropped from the constraint set. They are not redundant, however, in the phase II NSUR estimation problem because of the error structure, as noted earlier.

The relations of the Cobb-Douglas system (45)-(48) were estimated under different but nested specifications using the two-phase procedure described in sections 3 and 4. The computer package GAMS (Brooke *et al.*) was used for the estimation of the nonlinear least-squares problem of phase I, and SHAZAM (Whistler *et al.*) for phase II. The first specification is the primal-dual model stated in equations (14)-(26) and the related NSUR equations (27)-(31). The second specification is the main objective of this paper and is represented by the duality relations as stated in equations (32)-(40) and the related NSUR equations (41)-(43).

(Table 1)

The results are reported in Table 1 with *t*-ratios of the estimates in parentheses. The values of the estimated Cobb-Douglas production elasticities vary substantially between the two models. The experimental returns-to-scale parameter was selected at 0.9, as shown in equation (44). In this particular data sample, the dual model overstates this value while the primal-dual model understates it. In both models, the returns-to-scale parameter (sum of the production elasticities) indicates that the sample firms of our simulated experiment operate under decreasing returns to scale. The *t*-ratios indicate that the primal-dual model produces more efficient estimates.

As the dual model is nested in the full primal-dual model, it was possible to test whether it could rationalize the sample firms' behavior with parsimony of

computational effort. The test is the likelihood ratio statistic reported in Table 1. It turns out that the dual model is rejected in favor of the full primal-dual model at any confidence level. The degrees of freedom were computed as the difference between the covariance parameters.

The profit-maximization hypothesis was tested using the Bayesian approach developed by Geweke. In this test, a large number of parameter samples is drawn from a suitable universe defined by the empirical estimates. The proportion of those samples that satisfy the conditions defining the given hypothesis is recorded. The higher the proportion of "successes", the higher the confidence that the hypothesis is "true". Given the small standard errors of the estimates, the profit-maximization hypothesis is accepted unanimously in the two models with a proportion of "successes" equal to one.

Conclusion

This paper has shown that a duality approach can be utilized in the estimation of input demand and output supply functions of a sample of price-taking risk-neutral firms even when the observed prices are the same across sample units and across inputs. Therefore, the alleged failures of duality estimators often asserted in the literature are unfounded.

The process of solving this vintage problem relies upon plausible features of the underlying economic theory and upon the most general approach to measurement. The economic theory is founded upon the assumption that risk-neutral entrepreneurs make their planning decisions on the basis of expected quantities and prices. Unfortunately, entrepreneurs and their accountants do not record, in general, this expected information in a form readily accessible to the econometrician. When, at a later stage, the

econometrician intervenes, therefore, she must measure by reconstruction the level of quantities and prices that might have been used by the decision maker. In this process, she commits measurement errors on all the collected information.

The econometric specification of the resulting system of production and profit thus becomes a generalized nonlinear errors-in-variables problem that has been regarded as unyielding for a long time. By contrast, the approach to its solution advanced in this paper is rather simple. A two-phase algorithm generates consistent estimates of the desired parameters by first solving a nonlinear least-squares problem with respect to all the expected quantities and prices jointly estimated with the technological and economic parameters. The solution of this phase I problem is not a trivial endeavor. For example, with a Cobb-Douglas technology and with N = 84 observations and J = 3 inputs, the number of constraints of the phase I primal-dual model is equal to $(J+1)(3N+2)=1016$ while, with K = 4 Cobb-Douglas parameters, the overall number of decision variables (expected variable, errors and Cobb-Douglas parameters) to estimate is equal to $4(J+1)N+K=1348$. In phase II, the estimated expected quantities and prices are used as instrumental variables in a NSUR estimation model that produces the final and consistent estimates of the technological and economic parameters.

The dual estimation framework presented in this paper may be useful in the estimation of consumer demand functions when prices seem to be the same for all individual households.

References

Brooke, A., K., D. Kendrick and A. Meeraus. *GAMS, User Guide*. Boyd and Fraser Publishing Company. Danvers, MA, 1988.

Christensen, L.R. and D.W. Jorgenson. "The Measurement of U.S. Real Capital Input 1929-1967." *Rev. Income and Wealth* 15(1969):135-151.

Davidson, R. and J. G. MacKinnon. *Estimation and Inference in Econometrics*. Oxford University Press, New York, N.Y., 1993.

Fuller, W. A. "The Use of Indicator Variables in Computing Predictions." *J. Econometrics* 12(1980):231-243.

Geweke, J. "Exact Inference in the Inequality Constrained Normal Linear Regression Model." *J. Applied Econometrics* 1(1986):127-141.

Marschak, J., and W. H. Andrews Jr. "Random Simultaneous Equations and the Theory of Production." *Econometrica* 12(1944):143-206.

Mundlak, Y. "Production Function Estimation: Reviving the Primal." *Econometrica* 64(1996):431-438.

Pope, R. D. "To Dual or Not to Dual." *West. J. Agr. Econ.* 7(1982):337-51.

Whistler, D., K. White, S.D. Wong, and D. Bates. *SHAZAM, The Econometric Computer Program*, User's Reference Manual, Version 9. Northwest Econometrics Ltd. Vancouver, B.C. Canada, 2001.

Table 1. NSUR Estimates of Production and Profit Models with Constant Prices

Technological Parameters	Full Covariance Model	Dual Model
Efficiency, A	3.3507	2.5172
	(635.08)	(222.09)
Capital, α_K	0.3191	0.3555
	(296.52)	(91.741)
Labor, α_L	0.3225	0.3826
	(414.29)	(95.377)
Energy, α_E	0.1748	0.2019
	(986.28)	(48.045)
Returns to Scale, $\sum\alpha_i$	0.8164	0.9400
Loglikelihood	-65.911	-261.138
Likelihood ratio test		390.454
Degrees of freedom		26

t-ratios in parenthesis