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Socio-Economic Determinants of Food Consumption and Production in Rural Sierra Leone: Application of an Agricultural Household Model with Several Commodities

by

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I. Introduction

The nutritional well-being of households, particularly those with low incomes, has become an important consideration for governments of developing countries. However, policy planners rarely have much indication how different policies will affect household food consumption and thereby nutritional well-being. This is especially so for rural households (household-firms) that produce the foods which they consume. For such households, a change in price, by affecting the profits from home production, will shift the budget constraint. Consequently, there will be an extra income (or "profit") effect which does not occur in traditional demand theory. This additional income effect permits consumption to respond positively to own price. Whether this will be the case, indeed whether the "profit" effect may be ignored in predicting consumption behavior, is an empirical question.

This paper reports the consumption responses of five food items, nonfoods and labor supply to prices, full income, and certain demographic variables for rural households in Sierra Leone when their profits are held fixed and then allowed to vary. To obtain these responses, a household-firm model is specified and estimated. On the demand side, a system of demand equations is used, the Quadratic Expenditure System, which allows for a quadratic relationship between full income and commodity expenditure. Demographic variables are explicitly incorporated into the model allowing for a richer specification than can be gotten by use of per capita variables.

The production side of the household-firm model is estimated econometrically in a way in which allows for zero production of individual outputs. The paper than integrates the results from the demand and production sides. For most crops, the own price effects on consumption remain negative when profits are allowed to vary. Cross price elasticities are both positive and sizeable. The effects of prices and other variables on caloric availability to the household are computed. Elasticities of caloric availability with respect to total expenditure are found to be sizeable, varying little by expenditure group. Price elasticities of calorie availability are generally positive; however, an important exception occurs for price of the staple food, rice. This exception has several important policy implications which are also explored.

The data are from a cross-section survey of households in rural Sierra Leone. Price variation for all commodities exists by region, permitting estimation of complete systems of commodity demand and of output supply and variable input demands.

II. Household-Firm Models

2.1 Review of Previous Work

Economic models of household-firm behavior are not new. Recent papers have been written by Nakajima (1969) and Jorgenson and Lau (1969). An extension to macro sector modeling was provided by Lau and Yotopoulos (1974). All household-firm models have a common structure of maximizing a utility function subject to three constraints: a production function, a time constraint, and a budget constraint. Certain models (e.g., Nakajima's subsistence model) hypothesize that some markets do not exist and others (e.g., Jorgenson and Lau) explore intra-household distribution by using a social welfare function approach. For our purposes, we shall assume that households are semi-subsistence households. That is, markets do exist; households produce and consume goods and buy or sell the difference.

Only a very few attempts have been made to estimate household-firm models, and all assumed that markets existed for each good. With one exception, all attempts have been highly aggregated; using one agricultural commodity, one non-agricultural commodity, and leisure. All previous attempts have used demand systems which impose extremely restrictive assumptions concerning the effect of full income on expenditures. Lau, Lin, and Yotopoulos (1978) estimated a three-commodity Linear Logarithmic Expenditure System, which imposes unitary elasticities of expenditure with respect to full income. The production side of their model was specified by aggregating all outputs into a single (agricultural) output. A profit function and input demand function was estimated using a Cobb-Douglas production function (Lau, Lin, and Yotopoulos, 1976). Their data were averages grouped by farm size and by region for each of two years, over households in Taiwan. Prices varied by region and over time. Barnum and Squire (1979) and Singh and Squire (1978) use a three-commodity Linear Expenditure System, which imposes linear Engel curves, for the demand side of their household-firm models. They estimate, directly, a Cobb-Douglas production function for a single agricultural commodity on the production side. Their data are from a cross-section of households in Malaysia, the only price variation being in wages (using an LES variation in one price allows identification of all price elasticities, though not of all LES parameters). Ahn, Singh, and Squire (1981) use six commodities (including four foods plus leisure and non-foods), but also use a Linear Expenditure System. The production side of their model is specified using a linear programming approach. Their data are from a cross-section of households in Korea, again only showing price variation for labor.

2.2 Derivation of Model

Our unit of analysis is the household. A household utility function is assumed with arguments being household consumption of various goods and of leisure. Goods may be either produced or bought or sold in the market, while labor may be bought or sold in the market. Goods are produced using labor, land, and fixed capital. Land is assumed fixed in total amount but must be distributed between uses. A time constraint exists equating household leisure plus labor time to total time available. Finally, a budget constraint exists equating the value of net product transactions plus exogenous income plus the value of net labor transactions to zero. Product prices and wages are taken exogenously by the household, markets are assumed to be perfectly competitive, and family and hired labor are assumed perfect substitutes.

Formally, let the household maximize:

$$U = U(\overline{L}, X^c), \text{ where } \begin{aligned} \overline{L} &\equiv \text{leisure} \\ X^c &\equiv (X_1^c, X_2^c, \ldots, X_{N-1}^c) \\ X_i^c &\equiv \text{good } i \text{ consumed} \end{aligned}$$

subject to:

$$G(X, L_T, D, K) = 0$$

$$X_i^c = X_i - S_i \qquad i=1,\ldots,N-1$$

$$S_N = L_H - L_T$$

$$\overline{L} = T - L_H$$

$$\sum_{i=1}^{N} p_i S_i + A = 0$$

where:

$G(\cdot) \equiv$ implicit production function

$X \equiv (X_1, \ldots, X_{N-1})$

$X_i \equiv$ production of good i

$L_T \equiv$ labor demanded (including hired labor)

$D \equiv$ land area

$K \equiv$ fixed capital

$S_i \equiv$ net sales of good i (purchase if negative), i=1,...,N, with labor as Nth good

A = exogenous income

T = total time available to the household to allocate between work and leisure

L_H = household labor supply

p_i = price of good i, i=1,...,N

Assume the utility function to be well-behaved: twice differentiable, increasing in its arguments, and strictly quasi-concave. Assume the implicit production function is also well-behaved: twice differentiable, increasing in outputs, decreasing in inputs, and strictly quasi-convex.

After substituting the time constraint into the budget constraint, we can set up the Lagrangian as:

$$(2.1)\quad W = U(\bar{L},X^c) + \lambda\Big(\sum_{i=1}^{N-1} p_i(X_i-X_i^c) + p_N(T-\bar{L}-L_T) + A\Big) + \mu G(X,L_T,D,K)$$

Assuming interior solutions, our first order conditions are:

$$\begin{aligned}
&\partial W/\partial X_i^c = \partial U/\partial X_i^c - \lambda p_i = 0 && i=1,\ldots,N-1\\
&\partial W/\partial \bar{L} = \partial U/\partial \bar{L} - \lambda p_N = 0 &&\\
(2.2)\quad &\partial W/\partial X_i = \lambda p_i + \mu\partial G/\partial X_i = 0 && i=1,\ldots,N-1\\
&\partial W/\partial L_T = -\lambda p_N + \mu\partial G/\partial L_T = 0 &&\\
&\partial W/\partial \lambda = \sum_{1}^{N-1} p_i(X_i - X_i^c) + p_N(T-\bar{L}-L_T) + A = 0 &&\\
&\partial W/\partial \mu = G(X,L_T,D,K) = 0 &&
\end{aligned}$$

These may be expressed in the conventional way of equating marginal rates of substitution to the ratio of prices to the marginal rates of transformation in production:

$$(2.3)\quad \begin{aligned}
&\frac{\partial U}{\partial X_i^c} \Big/ \frac{\partial U}{\partial X_j^c} = \frac{p_i}{p_j} = \frac{\partial G}{\partial X_i} \Big/ \frac{\partial G}{\partial X_j} = \frac{-\partial X_j}{\partial X_i}, \quad i\neq j=1,\ldots,N-1\\
&\frac{\partial U}{\partial \bar{L}} \Big/ \frac{\partial U}{\partial X_i^c} = \frac{p_N}{p_i} = \frac{-\partial G}{\partial L_T} \Big/ \frac{\partial G}{\partial X_i} = \frac{\partial X_i}{\partial L_T}, \quad i=1,\ldots,N-1
\end{aligned}$$

Graphically, for outputs, the household produces on its transformation function between two goods at the point at which the slope of the transformation curve equals relative market prices. Consumption is at the point of tangency between the same market possibilities line and a household indifference curve. Net marketed surpluses are measured by the usual trade triangles. In this case, C-B of good j is sold and B-A of good i purchased. Between outputs and labor, the same situation holds.

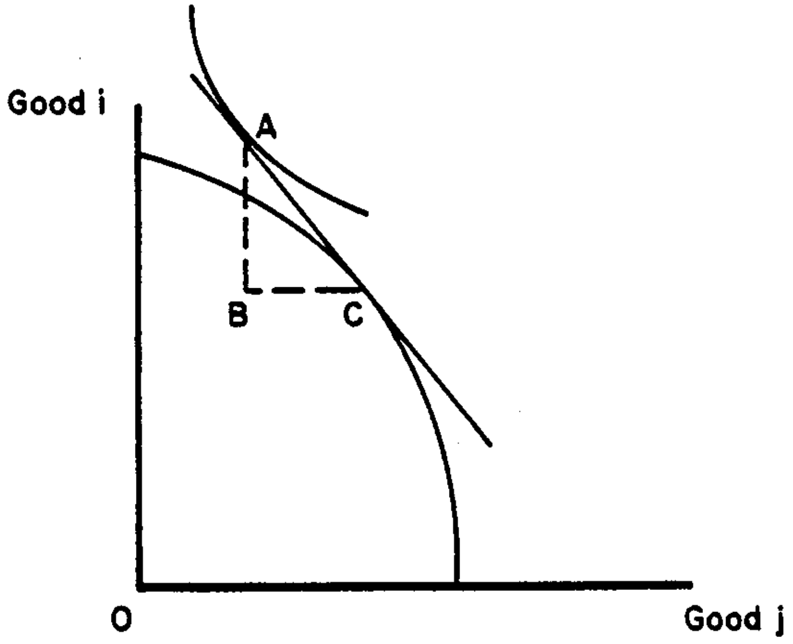

Figure 1

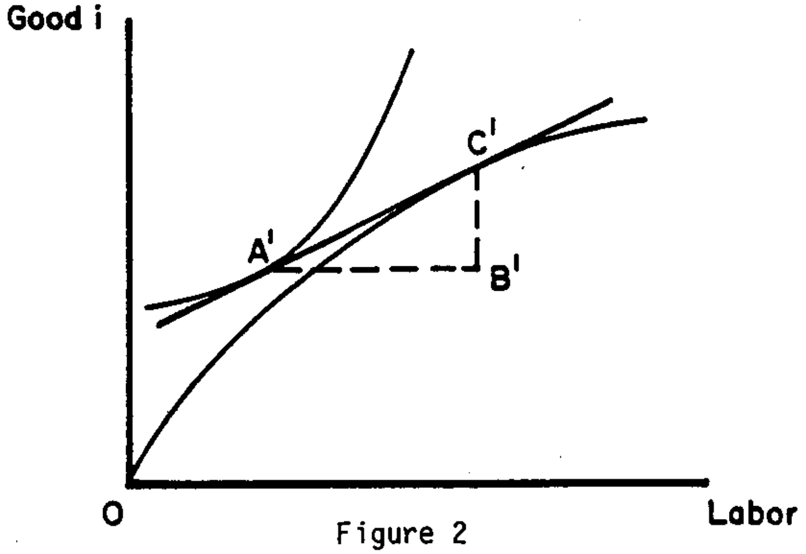

Figure 2

In the case pictured, C'-B' of good i is sold and A'-B' of labor is hired.

An extremely important property of this model is that it is recursive. The household behaves as though its production decisions are made first and subsequently used in allocating potential full income between consumption of goods and of leisure. This result is wholly dependent on the existence of markets for goods and labor, and on perfect substitution between hired and family labor. Intuitively, this allows the family to separate its decisions on goods demanded and household goods supplied, the difference being bought (or sold). This can be seen graphically in Figures 1 and 2. More formally, in the first order conditions the partial derivatives with respect to outputs yield N-1 equations in N+1 unknowns (N-1 good outputs, total labor demanded, and the ratio of two multipliers). Two more equations are added by the partial derivative with respect to total labor demanded and with respect to the multiplier of the implicit production function. This system of N+1 equations in N+1 unknowns can be solved in terms of all prices, the wage rate, fixed land and capital, as the result of the quasi-convexity of the implicit production function, first order conditions, and the implicit function theorem. Such solutions may then be substituted into the budget constraint. With the partial derivatives with respect to leisure and consumption of goods, this yields an additional N+1 equations in N+1 unknowns (N-1 good consumptions, leisure, and a multiplier), which may also be solved in terms of prices, the wage rate, and nonearned income, as the second order conditions are met.

Thus, conditional on the production decisions, the problem becomes the traditional goods-leisure choice problem. This implies that the usual constraints of economic theory apply: zero homogeneity of demand with respect to prices, wage rate and unearned income, and symmetry and negative semi-definiteness of the Slutsky substitution matrix. Likewise, on the production side: the profit function (the profits equation after input demands and output supplies have been solved for in terms of prices of outputs and of variable inputs, and in terms of quantities of fixed inputs) is homogeneous of degree one in all prices and convex in prices.

When we later look at comparative static changes due to changes in one price, we can separate this movement into three parts. If we hold profits constant and let the household find a new optimum consumption bundle, that movement can be separated into the traditional substitution and income effects. If we allow profits to vary, a new optimum consumption and production bundle will be found. This part of the comparative static change we call the "profit effect." Rearranging the budget constraint, we have:

$$(2.4) \quad A + \left(\sum_{1}^{N-1} p_i X_i - p_N L_T \right) + p_N T = \sum_{1}^{N-1} p_i X_i^c + p_N \overline{L}$$

Define $\pi = \sum_{1}^{N-1} p_i X_i - p_N L_T$ which can be interpreted as short-run profits. The left-hand side of equation 2.4 is full income. It is distributed over consumption of goods and of leisure. When prices change, one effect on consumption will be caused by changing π, the profits component of full income in the budget constraint.

III. The Demand Side: Systems of Demand Equations

Taking advantage of the recursiveness property of the household-firm model, we can separately specify the consumption-leisure and production components of the model. For the consumption-leisure choice problem, we use a system of demand equations.

Systems of demand equations relate an exhaustive set of expenditures to all prices and total expenditure (or income). Two broad approaches are used in specifying functional form. First, one can specify a particular functional form. This can be done either for the direct or indirect utility function (the direct utility function after demand for goods has been solved for in terms of goods' prices and total expenditure) in which case one works forward to derive the demand functions; or for the demand functions, in which case one derives a class of direct or indirect utility functions giving rise to that function. Second, one can approximate either an unknown direct or indirect utility function or an unknown demand function, at a point. In the former case, the demand functions are derived from the approximated utility function. Both approaches generally impose three restrictions on the demand function: adding up of expenditures to total expenditure, zero degree homogeneity in prices and total expenditure, and symmetry of the Slutsky substitution matrix. Negative semi-definiteness of the substitution matrix is usually not imposed, but tested with the data upon estimation.

As a general rule, approximating functions, when taken to the second degree of approximation as most have been thus far (e.g., translog or generalized Leontief), involve independent parameters to be estimated increasing as a multiple of the square of the number of commodities in the system. Since many of the resultant demand systems are highly non-linear in parameters, the cost of estimating such systems is high. To decrease the number of parameters to be estimated, additional constraints need to be placed on the system. Some specific functional forms have the number of parameters increasing as a multiple of the number of commodities included. This is achieved at the price of restrictions on the type of behavior admitted by that form, particularly restrictions concerning the effect on consumption of one good of a change in the price of another good.

The linear expenditure system is one example of a demand system having the number of parameters being a multiple of the number of commodities. For our purposes, it has a severe limitation: it restricts Engel curves to be linear. Of lesser concern is the fact that it allows for no Hicks-Allen complementarity. An alternative system also parsimonious in parameters but which does not suffer from the first defect is the quadratic expenditure system. Howe, Pollak, and Wales (1979) have shown that any quadratic expenditure system (QES) consistent with Engel aggregation (summing up of expenditures), zero homogeneity in prices and total expenditure, and symmetry of the substitution matrix is generated by an indirect utility function of the form $V(p,y) = -g(p)/(y-f(p)) - a(p)/g(p)$, where $g(\cdot)$, $a(\cdot)$ and $f(\cdot)$ are functions homogeneous of degree one and y = total expenditure. For our household-firm model, full income $A + \pi + p_N T$, replaces total expenditure in the indirect utility function. Hence, to use the indirect utility function in deriving demand curves in the household-firm model, we need only Roy's identity:

$$X_i^c = \frac{-\partial V/\partial p_i \big|_{d(\pi+A+p_N T) = 0}}{\partial V/\partial(A+\pi+p_N T)} \quad \text{and} \quad \overline{L} = \frac{-\partial V/\partial p_N \big|_{d(\pi+A+p_N T) = 0}}{\partial V/\partial(A+\pi+p_N T)}$$

Setting $g(p) = \prod_{k=1}^{N} p_k^{a_k}$, $f(p) = \sum_{k=1}^{N} p_k C_k$ and $a(p) = -\prod_{k=1}^{N} p_k^{(2a_k-d_k)}$ we obtain the indirect utility function:

$$(3.1) \quad V = -\prod_{k=1}^{N} p_k^{a_k}/(A + p_N T + \pi - \sum_{k=1}^{N} p_k C_k) + \prod_{k=1}^{N} p_k^{(a_k-d_k)}$$

$\sum_{k=1}^{N} a_k = \sum_{k=1}^{N} d_k = 1$, where leisure is treated as the Nth good. The C_k, d_k, and a_k are parameters to be determined from the data. The resulting expenditure equation is:

$$(3.2) \quad p_i X_i^c = p_i C_i + a_i(p_N T + \pi + A - \sum_{k=1}^{N} p_k C_k) - (a_i - d_i) \prod_{k=1}^{N} p_k^{-d_k}$$

$$(p_N T + \pi + A - \sum_{k=1}^{N} p_k C_k)^2 \qquad i=1,\ldots,N$$

with $\overline{L}$ replacing X_i^c for i = N.

This has as a special case the linear expenditure system provided $a_i = d_i$, $\forall_i$.[1/]

IV. Incorporating Demographic Variables Into the Model

Since our unit of analysis is the household rather than the individual, we must decide how to incorporate household characteristics such as size and age distribution into our analysis.[2/] The method we use is translation. It subtracts commodity-specific indices from quantities in the direct utility function, $U(X) = U(X_1 - V_1,\ldots,X_N - V_N)$, where V_i is a function of household characteristics. One possible interpretation of the V_i's is as committed quantities of goods; however, there is no reason for the V_i's to be positive. Using this specification, the effects of demographic variables on quantities consumed come through income effects. This may be seen by writing the indirect utility function associated with this specification, $V(p,\ p_N T + \pi + A - \sum_{k=1}^{N} p_k V_k)$. That is everywhere full income appears one subtracts from it the sum of values of these commodity indices.

An alternative specification due to Barten (1964) is scaling. For this, one writes the direct utility function as $U(X_i/I_i,\ldots,X_N/I_N)$, where the I_i are functions of the demographic variables and have the interpretation of commodity-specific consumer equivalence scales (for the QES the I_i must be positive). The resulting indirect utility function is of the form $V(p_1 I_1,\ldots,p_N I_N,\ p_N T + \pi + A)$; hence, the influence of demographic variables will be felt through prices. The scaling specification was tried but discarded for reasons to be discussed later (see footnote 12).

Another variable dependent on household characteristics is total time available to the household. Using a linearly homogeneous specification for the translation parameters, we write $V_i = \sum_{r=1}^{K} \sigma_{ir}\eta_r$, where η_r, r = 1,...,K are household characteristics and the σ_{ir}'s are parameters.[3/] Likewise, for total time, we may write $T = \sum_{r=1}^{q} \gamma_r m_r$, where m_r, r = 1,...,L are household characteristics (some possibly identical to the η_r's) and the γ's are parameters. The resulting expenditure equation of the QES is:

$$(4.1)\quad p_i X_i^c = p_i C_i + p_i \sum_{r=1}^{K} \sigma_{ir}\eta_r + a_i\Big(p_N \sum_{r=1}^{q} \gamma_r m_r + \pi + A - \sum_{k=1}^{N} p_k\big(C_k + \sum_{r=1}^{K} \sigma_{kr}\eta_r\big)\Big)$$

$$- (a_i - d_i) \prod_{k=1}^{N} p_k^{-d_k} \Big(p_N \sum_{r=1}^{q} \gamma_r m_r + \pi + A - \sum_{k=1}^{N} p_k\big(C_k + \sum_{r=1}^{K} \sigma_{kr}\eta_r\big)\Big)^2$$

Since leisure is not directly observed, we subtract from both sides of the leisure expenditure equation the value of time available to the household. The left-hand side becomes the negative of the value of household labor, which we do observe. Thus, the leisure equation becomes:

$$(4.2)\quad -p_N L_H = p_N C_N + p_N \sum_{r=1}^{K} \sigma_{Nr}\eta_r - p_N \sum_{r=1}^{q} \gamma_r m_r + a_i\Big(p_N \sum_{r=1}^{q} \gamma_r m_r + \pi + A$$

$$- \sum_{k=1}^{N} p_k\big(C_k + \sum_{r=1}^{K} \sigma_{kr}\eta_r\big)\Big) - (a_i - d_i) \prod_{k=1}^{N} p_k^{-d_k} \Big(p_N \sum_{r=1}^{q} \gamma_r m_r$$

$$+ \pi + A - \sum_{k=1}^{N} p_k\big(C_k + \sum_{r=1}^{K} \sigma_{kr}\eta_r\big)\Big)^2$$

This device avoids the need to impose values for T, such as a male having exactly 16 hours per day available for work and leisure. With N commodities, K translation demographic variables, and q demographic variables for total time, this system has at most (3 + K)N - 2 + q parameters to estimate. If some of the η_r's and m_r's are identical, there will be fewer parameters since in that case we may combine parameters as $p_N \sum_{r=1}^{q'} m_r(\gamma_r - \sigma_{Nr})$, where $q' \equiv$ the number of common m's and η's. Clearly, only the difference $\gamma_r - \sigma_{Nr}$ is identified, not both parameters separately.

V. The Production Side

Specifying the production block of the household-firm model will involve a set of factor demand and output supply equations plus a short-run profits function. Let us initially specify an implicit production function of the form $G(X,L_T,D,K)$. We could stop at this point, making this function operational (or its associated short-run profit function) using a flexible form such as the translog. However, we must be conscious of our parameter usage, particularly since we are not primarily interested in the production side. The usual way to achieve parsimony in parameters is by using assumptions on the nature of the production function. Two general possibilities suggest themselves. At one extreme, we could assume nonjointness, that is the existence of individual production functions for each output. With fixed land and capital, this would insure dependency of each output in whose production function land and capital appeared, on all the output prices. However, assuming production functions to differ would entail at least (N-1)M parameters, where N-1 is the number of outputs and M the number of inputs. More importantly, there are inadequacies in our data for using this approach (see page 69). Alternatively, we could assume some form of separability. One logical possibility would be to assume outputs as a group to be separable from inputs as a group. That is, $G(X,L_T,D,K) = H(X)-F(L_T,D,K)$. We could further assume almost homogeneity of degree $\frac{1}{s}$, that is, $H(\lambda X) = F(\lambda^s L_T,\lambda^s D,\lambda^s K)$ (see Lau, 1978).

Among the possible functional forms to use for inputs, one appealing, though restrictive, form is the Cobb-Douglas (CD). Its strength, for our purposes, is its requiring only M+1 parameters. For outputs, we might think of the counterpart to the constant elasticity of substitution function, the constant elasticity of transformation (CET) introduced by Powell and Gruen (1968). That function, of the form $H(X) = (\Sigma\delta_i X_i^\rho)^{1/\rho}$, (summation is from 1 to N-1) where $\delta_i > 0$ and $\rho > 1$ to insure convexity, entails only N parameters. Consequently, a CET-CD system would require N+M+1 parameters which must surely be pushing the lower bound of parameters in any reasonable system. Writing the CD function for inputs as $F(L_T,D,K) = A_o L_T^{\beta_{L_T}} D^{\beta_D} K^{\beta_K}$, we have:

$$(5.1)\qquad (\sum_i \delta_i X_i^\rho)^{1/\rho} = A_o L_T^{\beta_{L_T}} D^{\beta_D} K^{\beta_K}$$

This production system requires one of two normalizations; either $A_o=1$ or $\sum_i \delta_i=1$. This can be seen since we can write the left-hand side as $(\sum_i \delta_i)^{1/\rho}\,(\sum_i \delta_i^* X_i^\rho)^{1/\rho}$ where $\delta_i^*= \delta_i/\sum_i \delta_i$ and $\sum_i \delta_i^*=1$. In this case, A_o and $(\sum_i \delta_i)^{1/\rho}$ are not distinguishable, so one would estimate $A_o^*=A_o/(\sum_i \delta_i)^{1/\rho}$ when using the normalization $\sum_i \delta_i^*=1$. Alternatively, we can leave the δ_i's as they are and set $A_o=1$, which is what we do.

The parameter ρ can be transformed into $\frac{1}{\rho-1}$, the elasticity of transformation between outputs. For this production function, the elasticity of transformation parameter is constant, hence the name CET. Moreover, it is the same for all pairs of outputs. Indeed, one generalization of this functional form would be to write it as a multilevel CET (Mundlak and Razin, 1971) to capture differing transformation elasticities between outputs from different groups.

The δ_i parameters have their meaning in the marginal rate of transformation. It is easily seen that $\frac{-\partial X_i}{\partial X_j} = \frac{\delta_j}{\delta_i}\left(\frac{X_j}{X_i}\right)^{\rho-1}$. On the input side, the β parameters have the usual meaning for a Cobb-Douglas specification, that is, the percent change in all outputs due to an infinitesimal proportionate change in the particular input. The sum of the β's is the degree of almost homogeneity (Lau, 1978).

Maximizing profits subject to (5.1) (normalizing $A_0=1$) and to D and K being fixed, we arrive at the output supply and labor demand equations.

$$(5.2)\quad p_iX_i = \beta_L^{\beta_L/1-\beta_L}\ \delta_i^{-1/(\rho-1)}\ p_i^{\rho/(\rho-1)}$$

$$(\sum_k \delta_k^{-1/(\rho-1)}\ p_k^{\rho/(\rho-1)})^{(\rho\beta_L-1)/\rho(1-\beta_L)}$$

$$(D^{\beta_D}K^{\beta_K})^{1/(1-\beta_L)}\ p_N^{(-\beta_L/(1-\beta_L))}$$

$$i=1,\dots,N-1$$

$$p_NL_T = (D^{\beta_D}K^{\beta_K})^{(\frac{1}{1-\beta_L})}\ \beta_L^{(\frac{1}{1-\beta_L})}\ p_N^{(-\beta_L/(1-\beta_L))}$$

$$(\sum_i \delta_i^{\frac{-1}{\rho-1}}\ p_i^{\rho/(\rho-1)})^{(\frac{\rho-1}{\rho(1-\beta_L)})}$$

One advantage of the CET-CD specification apparent from equation 5.2 is that the supply of any output is a function of all output prices. If we alternatively specified a separate production function for nonfood, and it did not include land as an input, then nonfood supply would be a function of only nonfood price, wage, and nonfood capital. This is a result of assuming labor can be freely sold and purchased, so that labor supply to the firm is not fixed.

VI. Data Preparation and Sample Characteristics

6.1 Data Preparation

The data are from a cross-section survey of households in rural Sierra Leone taken during the 1974-75 cropping year (May-April). Sierra Leone was divided into eight geographical regions chosen to conform with agro-climatic zones, and those were used to stratify the sample. Within these regions, three enumeration areas were randomly picked and households sampled within these. Households were visited twice in each week to obtain information on production, sales, and labor use, among other variables. Half the households were visited twice during one week per month to obtain market purchase information.

Estimates of quantities apparently consumed out of home production were derived by subtracting sales, wages in kind paid out (and seed use for rice, the major crop) from production, and adding wages in kind received. These were adjusted for processing to avoid double-counting, and for storage losses.[4/] These quantities consumed out of own production were multiplied by regional farm gate sales prices to transform into values. Values of foods purchased were then added to calculate total value of foods consumed. These were then aggregated into five groups, which with nonfoods and labor are the seven commodities used in the study. A listing appears in Table VI.1.

Values of production were derived by multiplying quantities produced by farm gate sales price, and then added into the appropriate groups. Production of raw products was used; processed product production was not added in order to avoid double-counting. For example, only estimates of fresh and not dried fish production were used.

Household labor supplied data were formed by summing hours worked for agricultural and nonagricultural enterprises and for labor sold out (see Table VI.1). Labor supply includes such activities as work by women and children on vegetable production, for which women generally take responsibility, as well as the cleaning of rice. Excluded are household (home labor) activities such as food preparation, child care, and ceremonies. Units are in terms of male equivalents with weights 1 for males over 15, .75 for females over 15, and .5 for children aged 10-15. The weights are derived from relative wage rates in the sample as reported by Spencer and Byerlee (1977). Household labor demand, also measured in male equivalents, includes work on all agricultural and nonagricultural activities in the household exclusive of processing agricultural products. Both family and hired labor are included.

Prices used in estimating the demand system were formed by the eight geographical regions. Annual sales prices were formed using the larger sample of 328 households for which reliable production and labor use data were available. Value of regional sales was divided by sales quantity for each of 195 commodities. Likewise, regional purchase prices were formed for 113 commodities. A concordance between commodities purchased and sold was established and a commodity price for each region was then formed by taking a weighted average of sales

Table VI.1

Components of Commodities

Commodity-Subgroup	No.	Components
Rice	1	
Root crops and other cereals	2	
Root crops		Cassava (including gari, foofoo and cassava bread), Yam, Water Yam, Chinese Yam, Cocoyam, Sweet potato, Ginger, Unspecified
Other cereals		Benniseed, Fundi, Millet, Maize (shelled), Sorghum, Agidi,[1] Biscuits (Natco)[1]
Oils and Fats	3	Palm oil, Palm kernel oil, Palm kernels,[2] Groundnut oil,[1] Coconut oil, Cocoa butter, Margarine,[1] Cooking oil,[1] Unspecified[1]
Fish and animal products	4	
Fish		Bonga (fresh), Bonga (dried),[1] Other saltwater (fresh), Other saltwater (dried),[1] Frozen fish,[1] Freshwater (fresh),[1] Tinned fish[1]
Animal products		Beef, Pork,[1] Goats and sheep (dressed), Poultry (dressed), Dear (dressed), Wild bird (dressed), Bush meat (dressed), Cow milk, Milk (tinned),[1] Eggs, Honey bee output, Unspecified[1]
Miscellaneous foods	5	
Legumes		Groundnuts (shelled), Blackeyed bean (shelled), Broadbean (shelled), Pigeon pea (shelled), Soybean (shelled), Green bean (in shell), Unspecified (shelled)
Vegetables		Onions, Okra, Peppers and Chillies, Cabbage, Eggplant, Greens, Jakato, Pumpkin, Tomato, Tomato paste,[1] Watermelon, Cucumber, Egusi, Other
Fruits		Orange, Lemon, Pineapple, Banana, Plantain, Avocado, Pawpaw, Mango, Guava, Breadfruit, Coconut, Unspecified
Salt and other condiments		Salt,[1] Sugar,[1] Maggicubes,[1] Unspecified[1]
Kolanut		
Nonalcoholic beverages		Coffee, Tea,[1] Soft drinks (bottled),[1] Ginger beer (local)[1]
Alcoholic beverages		Palm wine, Raffia wine, Beer (Star and Heineken),[1] Omole,[1] Gin (local), Liquor (Rum, etc.)[1]
Nonfoods	6	Clothing, Cloth, Fuel and light, Metal work, Woodwork, Other household and personal goods, Transport, Services and ceremonial, Education, Local saving, Tobacco products, Miscellaneous
Household labor	7	All farm and nonfarm production and marketing activities (for labor demand, work on processed agricultural products excluded), Labor sold out.[1] Excludes household activities such as food preparation, child care and ceremonies

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[1] Commodity is not included in production figures for use in estimating system of output supplies and labor demand either because it is only purchased or because it is a more processed form of a commodity already counted.

[2] Not included in consumption data but included in production data.

and purchase prices with region-specific weights being the share of total expenditure for a commodity coming from either purchases or home production. Commodities were then aggregated into six groups with regional values consumed being used as weights to form arithmetically weighted prices.[5/] Wage is in terms of male equivalents.

Farm sales prices for the 128 foods were aggregated into the same groups as were the weighted sales and purchase prices. In this case, the weights were the proportion of value of regional sales for the group represented by each of its component foods. These were the prices used in estimating the system of output supplies and labor demands. There is room for disagreement as to whether these weighted sales prices or the weighted "consumption" prices used in the QES estimation ought to have been used on the production side. On the one hand, the household-firm model does not distinguish between the two prices; indeed, it assumes they are equal. From this point of view, we should use the same set of prices for each component of the model. However, looking at the dichotomous nature of the model, we first maximize short-run profits subject to a production function. If this were to be done as a separate study, sales prices are the appropriate ones to use.

Data on household characteristics were available for total size and age composition by 0-5 years, 5-10 years, 11-15 years, 16-65 years, and over 65 years. In addition, data on number of wives, years of English and Arabic education by the household head, age of household head, ethnic group (there are three major ones in our sample), and region lived in are available. Since ethnic groups tend to live in contiguous areas, this information is also regional in character (though not identical to the eight survey regions).

Land is measured as total land area cropped, in acres. It includes land in perennial as well as annual crops. It is a simple sum of acres. No weighting to reflect different qualities (for example, of swamp and of upland lands) was made because no such data were available.[6/]

Capital is measured as the value of its flow. For variable capital, this represents no problem. However, variable capital for our sample is minuscule, mostly rice seed. Only a very little fertilizer is used and a little machinery hired, and these were added into the total. However, since there are some values for variable capital, which is a flow, it was necessary to convert the stock of fixed capital into the equivalent flow in order to add the two.[7/]

6.2 Sample Characteristics

As one can see by the average total expenditure shares, reported in Table VI.2, rice is the major staple with cassava (included in "root crops and other cereals") the main substitute. Rice tends to be eaten with a sauce and boiled cassava with a stew, both cooked with palm oil. Both sauce and stew are made with vegetables (onions, peppers, tomatoes, and leafy greens) and some meats. Sauces tend to include dried fish and stews fresh fish.

Sample characteristics of the variables which enter into the demand system are shown in Table VI.3. The sample is divided into three expenditure groups and for each group simple averages are computed for each characteristic. These groups are total expenditure under 350 Leones, between 350 and 750 Leones, and greater than 750 Leones. To get an idea of how poor these households are, the annual per capita expenditures in 1974-75 U.S. dollars are $54, $88, and $136, respectively, for the low, middle, and high expenditure groups. For the capital city, Freetown (which was sampled for a migration component of this study), when divided into three groups, the average income of the middle group is $153. Hence, even our "high" expenditure households are quite poor, both compared to urban Sierra Leone as well as compared to other countries.

Production characteristics of the sample of 138 households are shown in Table VI.4. For reporting average values, the sample is divided into the ten households in Enumeration Area 13 (EA 13) and the remainder. The former are mostly commercial fishermen who also grow and sell a large amount of vegetables to the Freetown market. In their production characteristics, they are quite different from the rest of the households (this is not so true of their consumption characteristics). The fishing households cultivate much less land than the other households (an average of 1.6 rather than 6.8 acres), but have considerably more capital in the form of boats and the like. Prices are also different, with the price of fish and animal products being considerably lower in EA 13.

Table VI.5 presents the quantities of production, total consumption, and the difference, net marketed surplus, by expenditure group. Except for rice, the high expenditure group tends to sell more or buy more than do lower expenditure groups. The only groups for which net purchases from the market are made are nonfoods, labor for middle and high expenditure groups, and fish and animal products for low and middle expenditure groups. We have to remember, however, that these are net figures. A household may hire labor during peak season and sell labor in the slack season. The figures reported here combine these two transactions.

Table VI.2

Actual Average Total Expenditure Shares
By Expenditure Group

Commodity	Expenditure Group			
	Low	Middle	High	Mean
Rice	.25	.24	.24	.24
Root crops and other cereals	.05	.06	.14	.10
Oils and fats	.08	.07	.11	.10
Fish and animal products	.13	.12	.11	.12
Miscellaneous foods	.12	.13	.09	.11
Nonfoods	.38	.37	.30	.33

Table VI.3

Mean Values of Consumption Data By Expenditure Group[1]

Variable	Low	Middle	High	Mean
	Expenditure Group			
Expenditures[2]				
Rice	58.2	125.2	262.9	146.7
Root crops & other cereals	10.7	32.4	147.4	61.3
Oils and fats	19.2	37.2	122.8	58.1
Fish and animal products	30.6	61.9	118.3	69.5
Miscellaneous foods	28.0	65.8	99.0	64.1
Nonfoods	90.0	190.1	324.0	199.9
Value of Household Labor	306.4	361.8	530.1	396.5
Prices[3]				
Rice	.25	.23	.27	.25
Root crops & other cereals	.36	.66	.63	.55
Oils and fats	.73	.62	.66	.67
Fish and animal products	.62	.60	.39	.54
Miscellaneous foods	.56	.58	.60	.58
Nonfoods	.62	.64	.75	.66
Household labor	.08	.08	.09	.08
Household characteristics[4]				
Total size	4.8	6.4	8.7	6.7
Members under 10 years	1.2	2.1	2.7	2.0
Members, 11-15 years	.5	.7	1.1	.8
Males over 15 years	1.7	1.8	2.6	2.1
Females over 15 years	1.4	1.8	2.3	1.8
Proportion Limba or Temne	.45	.29	.44	.39
Proportion northern	.43	.25	.40	.36
Number of households	44	51	43	138

[1] Households in the low expenditure group are those with total expenditure less than 350 Leones. Households in the middle expenditure group are those with total expenditure between 350 and 750 Leones. Households in the high expenditure group are those with total expenditure greater than 750 Leones.

[2] In Leones. One Leone = U.S. $1.1 in 1974/75.

[3] Weighted average of sales and purchase prices. In Leones per kilogram for foods and per hour of male equivalent for labor.

[4] In numbers.

Table VI.4

Mean Values of Production-Related Data, EA 13 and Other Households

Variable	EA 13	Non-EA 13
Value of Production[1]		
Rice	62.7	283.5
Root crops & other cereals	27.9	64.4
Oils and fats	20.6	104.2
Fish and animal products	733.5	23.0
Miscellaneous foods	331.8	53.3
Nonfoods	82.8	25.0
Value of Labor demand	954.7	367.5
Prices[2]		
Rice	.19	.22
Root crops & other cereals	.25	.14
Oils and fats	.37	.41
Fish and animal products	.17	.52
Miscellaneous foods	.15	.29
Nonfoods	2.23	1.25
Labor	.15	.08
Household Characteristics		
Cultivated land[3]	1.6	6.8
Capital[4]	214.3	35.1

[1] In Leones. Valued by weighted sales prices.

[2] Weighted sales prices. In Leones per kilogram for foods and per hour of male equivalent for labor.

[3] In acres.

[4] Annual flow in Leones.

Table VI.5

Quantities[1] Produced, Consumed, and Marketed by Expenditure Group

Commodity	Expenditure Group	Produced	Consumed	Marketed
Rice	Low	902.8	232.8	670.0
	Middle	1,164.3	544.3	620.0
	High	1,622.2	973.7	648.5
	Mean	1,227.5	586.8	640.7
Root crops and other cereals	Low	69.0	29.7	39.3
	Middle	335.8	49.1	286.7
	High	744.6	194.9	549.7
	Mean	422.1	111.5	310.6
Oils and fats	Low	85.5	26.3	59.2
	Middle	242.0	60.0	182.0
	High	447.2	186.1	261.1
	Mean	242.2	86.7	155.5
Fish and animal products	Low	18.0	49.4	-31.4
	Middle	48.3	103.2	-54.9
	High	508.7	303.3	205.4
	Mean	151.5	128.7	22.8
Miscellaneous foods	Low	93.0	50.0	43.0
	Middle	191.3	113.4	77.9
	High	515.3	165.0	350.3
	Mean	262.3	110.5	151.8
Nonfoods	Low	10.8	145.2	-134.4
	Middle	19.4	297.0	-277.6
	High	33.9	432.0	-398.1
	Mean	22.1	302.9	-280.8
Labor[2]	Low	3,963.8	3,800.3	163.5
	Middle	4,286.7	4,425.1	-138.4
	High	5,687.8	6,141.4	-453.6
	Mean	4,670.2	4,829.7	-159.5

[1] In kilograms for foods, hours for labor.

[2] Produced and Consumed correspond to supply and demand.

Finally, and not surprisingly, households specialize in production more than in consumption. Using our commodity definitions, we have 3 households which do not produce rice, 19 which have no production of root crops and other cereals, 24 for oils and fats, 35 for fish and animal products, 12 for miscellaneous foods, and 59 for nonfoods. The relatively large number of zero outputs gives rise to statistical problems which will be considered in Appendix 3.

VII. Results of QES Estimation

7.1 Final QES Specification

For the QES specification of prices and of the deterministic part of full income, $(A + \pi)$ is dictated by the commodity classification; however, specification of the translation parameters and of household total time is not. Not all of the potential household characteristic variables could be included in the QES estimation because too many parameters would be involved (remember, each demographic variable has N parameters associated with it in a system of N-1 commodities plus leisure). In order to choose which characteristic variables should enter the system, single-equation demand regressions were run using all of the potential variables. Function form was chosen to mimic the QES and the equations were estimated in share form.[8] All possible subsets of independent variables were examined and ranked by adjusted $R^2(\bar{R}^2)$. In general, equations with maximum $\bar{R}^2$ included the relevant price and expenditure variables. When this was not so, equations having the highest $\bar{R}^2$ and including these variables were chosen. From this exercise, several household characteristic variables did well in the sense of being included in the chosen equations for several commodities. Moreover, some variables had coefficients which were fairly consistently close in magnitude; hence, they could be combined. The final set of chosen demographic variables for translation parameters consisted of household size, children under 10, and either an ethnic dummy set to one if the household was Temne or Limba (Mende is the other major group), or a regional dummy set to one if the household lived in the northern region.[9] For total time available to the household, the variables chosen were persons over 10, females over 15, and children aged 11-15. Since adding a child under 10 also increases household size by one, the total effect of adding a child under 10 on the translation parameters will be the sum of the children under 10 and household size coefficients. The children under 10 coefficient may be interpreted as being the differential effect of children under 10 from persons over 10.

From equation 4.1 or 4.2, we can see that the household characteristic variables are multiplied by prices when they enter the QES. An identification problem arises from our choice of demographic variables because wage times household size equals wage times persons over 10 plus wage times persons under 10. Hence, one of these variables must be dropped to avoid perfect multicollinearity. We drop the household size variable and rewrite equation 4.1.

$$
\begin{aligned}
(7.1)\quad p_i X_i^C = {} & p_i C_i + p_i \sum_{r=1}^{3} \sigma_{ir}\eta_r + a_i(p_7 m_1(\gamma_1 - \sigma_{71}) + p_7 \sum_{r=2}^{3} \gamma_r m_r + \pi \\
& + A - \sum_{k=1}^{6} p_k(C_k + \sum_{r=1}^{3} \sigma_{kr}\eta_r) - p_7(C_7 + (\sigma_{72} + \sigma_{71})\eta_2 + \sigma_{73}\eta_3)) \\
& - (a_i - d_i) \prod_{k=1}^{7} p_k^{-d_k} (p_7 m_1(\gamma_1 - \sigma_{71}) + p_7 \sum_{r=2}^{3} \gamma_r m_r + \pi + A \\
& - \sum_{k=1}^{6} p_k(C_k + \sum_{r=1}^{3} \sigma_{kr}\eta_r) - p_7(C_7 + (\sigma_{72} + \sigma_{71})\eta_2 + \sigma_{73}\eta_3))^2
\end{aligned}
$$

where we have used the fact that N=7, K=q=3. It is apparent from equation 6 that the coefficient of wage times persons over 10 $(\gamma_1 - \sigma_{71})$ is identified, but not its components. Likewise, for the coefficient of wage times children under 10 $(\sigma_{72} + \sigma_{71})$.[10/] In consequence, total time, $T = \sum_{r=1}^{3} \gamma_r m_r$, is not identified. For the major questions in which we are interested, this is not troublesome.

The final QES specifications which we estimate have seven commodities, three translation demographic variables, and three total time demographic variables. The number of parameters is 42.[11/] These systems in their expenditure form were estimated using the Davidon-Fletcher-Powell algorithm as available on the GQOPT package of numerical optimization routines.[12/] For details of the specification of the disturbances and of estimation procedures, see Appendices 1 and 2.

7.2 Statistical Tests

Use of the ethnic group dummy resulted in a lower log-likelihood value (see Table A.2.2), -3577.1 as against -3487.4 for the estimating using the regional dummy. Regularity conditions were tested by computing eigenvalues of the Slutsky substitution matrix.[13/] For the system using the regional dummy regularity conditions held at 113 out of 138 sample points[14] as against none when using the ethnic group dummy.

The reason for this failure using the ethnic dummy was a small negative (i.e., -.2) compensated own price elasticity for labor supply. The other compensated own elasticities were of the expected signs and somewhat higher in absolute value than those of the system using the regional dummy. For these two reasons, the regional dummy variable seems preferable and results from that estimation will be used in the ensuing discussion.[15]

Using the regional dummy, 22 out of 42 parameters have the absolute value of their coefficients greater than 1.96 times their standard errors, 26 have absolute values of coefficients more than 1.65 times their standard errors, and 30 have standard errors less than their coefficient's absolute value.

A series of Wald tests were run on different hypotheses and are reported in Table VII.1. First, we test $H_0{:}a_i = d_i$, $\forall_i = 1,\dots,6$.[16] The value of the statistic is 19.0 which is asymptotically distributed as a chi-square variable with six degrees of freedom. This is significant at somewhat less than the .005 level; hence, we can reject the hypothesis that we should have estimated a linear expenditure system. It may be that for individual commodities the hypothesis that $a_i = d_i$ is not rejected. In fact, that is true for miscellaneous foods and for nonfoods. The standardized normal statistics for testing $a_i = d_i$ are 1.2 and 0.1, respectively. The statistic for fish and animal products is 1.6, corresponding to a probability value of roughly .15. Miscellaneous foods and nonfoods are more highly aggregated commodities; hence, linear expenditure curves for them are not implausible. The coefficients on household size, which is the effect of a unit change in persons over 10 on the commodity-specific translation parameters, are jointly significant as are the coefficients for children under 10. Hence, children under 10 affect the translation parameters in a way different from household members over 10. Since the total effect of children under 10 on translation parameters is the sum of their coefficients plus household size coefficients, it is interesting to test whether the sum of these is jointly significantly different from zero. As can be seen, the statistic is 100.1 which with six degrees of freedom is highly significant. The price coefficients, the C_i's, are jointly significant as are the regional coefficients. This means that the price coefficients for southern households (for whom the dummy is zero) are significant and significantly different from the price coefficients for northern households. Since the price coefficients for the latter are the sum of the

Table VII.1

Chi-Square Statistics from Wald Tests[1]

Test of	Statistic	Degrees of Freedom
1. LES as special case of QES	19.0	6
2. Household size coefficients	29.1	6
3. Children under 10 years coefficients	70.1	7
4. Equality with opposite signs of household size and children under 10 coefficients	100.1	6
5. Price coefficients	38.9	7
6. Ethnic group dummy coefficients	50.1	7
7. Equality with opposite signs of price and ethnic group dummy coefficients	18.1	7

[1] From QES with regional dummy.

southern price coefficients and the dummy coefficients, we test whether this sum is jointly significantly different from zero, which it turns out to be between the .025 and the .01 levels.

That the demographic variables should turn out to be statistically significant is not surprising when one looks at the demand systems literature (see, for example, Pollak and Wales, 1980, 1981). However, Kelley (1981) has recently suggested that for developing countries household expenditures are relatively invariant to household size and age distribution, a result contrary to what we observe.

7.3 Expenditure Shares, Price Elasticities, and Demographic Effects

Shares of marginal expenditure, price elasticities of demand, and marginal effects of household characteristic variables are functions, using the QES, not only of parameters but also of data. Hence, one has to choose at which sample points to evaluate these. We have chosen to divide the sample into three groups based on total expenditure for this purpose. The dividing lines chosen are less than 350 Leones annual expenditure, between 350 and 750 Leones inclusive, and greater than 750 Leones.[17/] The sample sizes for these groups are 44, 51, and 43, respectively. The main justification for such a division is that many observers are concerned with responses of people in different income groups, particularly the lower ones.

One can see from Table VI.3 that the lower expenditure group faces relatively lower prices for root crops and other cereals and for nonfoods, but higher prices for oils and fats and fish and animal products.[18/] Household size tends to be smaller for the lower expenditure group as does the proportion of family members under 10 years.

Shares of marginal total expenditure[19/] are reported in Table VII.2. They generally seem to be plausible. The share for rice declines with higher total expenditure as one would expect, although the .02 share for high expenditure households seems a little low. The low share for root crops and other cereals is not surprising, though one would not have expected the marginal share to rise with expenditure.[20/] For all expenditure groups, the marginal share is less than the estimated average share. Thus, the estimated average share for root crops and other cereals is declining for each of the representative low, middle, and high expenditure households. The fact that both marginal and average shares are higher for a high expenditure household than for a middle or low expenditure household is due to the Engel curve shifting upward (and shifting slope) when it

Table VII.2

Shares of Marginal Total Expenditure[1] by Expenditure Group

Commodity	Low	Middle	High	Mean
	Expenditure Group			
Rice	.22	.16	.02	.13
Root crops and other cereals	.03	.06	.12	.07
Oils and fats	.13	.20	.36	.23
Fish and animal products	.13	.11	.07	.11
Miscellaneous foods	.09	.07	.04	.07
Nonfood	.40	.40	.39	.39

[1] Partial derivative of commodity expenditure with respect to total income divided by partial derivative of total expenditure with respect to total income. Evaluated at expenditure group means using QES with regional dummy.

is evaluated at higher expenditure groups. This is possible since prices and household characteristics (Engel curve shifters) are different for different expenditure groups (see Table VI.3). In addition, the marginal share for root crops and other cereals is not negative at our mean evaluation points. This is interesting because many observers have hypothesized that cassava may be an inferior good for higher income groups in West Africa. This may still be the case, however, since the group root crops and other cereals contains expenditures on sorghum roughly equal to those on cassava, sorghum may not be an inferior good.

Uncompensated price elasticities of demand are reported in Table VII.3. For rice, the own price elasticity declines in absolute value with expenditure group. Part, but not all, of this is due to an income effect declining with expenditure group.[21/] This is certainly not surprising. Root crops seem not to be price responsive. The higher expenditure group is slightly more responsive to price, partly due to an increasing income effect. The relative unresponsiveness of total household labor supplied to wage rate changes (-.06 to .28) is not really surprising since this is measuring total supply, not its allocation between uses. The negative sign for the low expenditure group is due to the income effect (see below) and gives some slight evidence for a backward bending supply curve.

The cross-price effects with respect to rice price are negative except for fish and miscellaneous foods. This is not surprising due to the large budget share of rice leading to a relatively large income effect. However, with respect to nonfood price, only the effects for root crops and other cereals and oils and fats are negative. This is somewhat surprising since one would expect substitution effects between food commodities and rice to be larger than between food commodities and nonfood. This does not seem to be the case for our sample. Another cross-price effect of some interest is between rice and root crops. One can see that root crop demand is more responsive to changes in price of rice than rice demand is to changes in price of root crops. Since rice represents a larger budget share, its income effect is likely to be greater.

Income compensated price elasticities of demand are reported in Table VII.4. At the sample average and for all three expenditure group averages, the substitution matrix was negative semi-definite.

As with the uncompensated elasticities, there is a tendency for price responsiveness of rice to decline with total expenditure.[22/] All goods are

Table VII.3

Uncompensated Quantity Elasticities with Respect to Price[1] by Expenditure Group

With Respect to Price of	For Expenditure Group	OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low		-1.26	-.16	-.23	.02	.03	-.01	.01E-1
	Middle		-.78	-.13	-.31	.02	.02	-.02	.04E-1
	High		-.45	-.12	-.38	.05	.07	-.04	.01
	Mean		-.74	-.10	-.29	.03	.03	-.03	.01
Root crops and Other cereals	Low		-.02	-.15	-.02	-.02	-.02	-.02	.01
	Middle		-.02	-.26	-.04	-.02	-.01	-.02	.01
	High		-.01	-.31	-.02	-.01	-.01	-.01	.04
	Mean		-.01	-.22	-.02	-.02	-.01	-.02	.01
Oils and Fats	Low		.04	.04	-.82	.05	.03	.05	-.02
	Middle		.01E-1	.04E-1	-1.10	.02E-1	.01E-1	.04E-1	-.02E-1
	High		-.01E-1	.05E-1	-1.25	.02E-1	.01E-1	.01	-.03E-1
	Mean		.04E-1	.01	-.97	.01	.01	.01	-.01
Fish and Animal Products	Low		.02	-.08	-.12	-1.29	.01	-.01	.01E-1
	Middle		.03	-.06	-.15	-.92	.01	-.01	.03E-1
	High		.06	-.05	-.15	-.81	.04	-.03E-1	-.04E-1
	Mean		.04	-.04	-.12	-.95	.02	-.01	.01E-3
Miscellaneous Foods	Low		.01	-.06	-.10	-.03E-1	-.99	-.01	.04E-1
	Middle		.01	-.06	-.14	-.03E-1	-.60	-.02	.01
	High		.04	-.04	-.14	.02	-.63	-.02	.01
	Mean		.02	-.04	-.11	.03E-1	-.71	-.02	.01
Nonfoods	Low		.10	-.16	-.21	.06	.06	-1.17	-.01
	Middle		.07	-.16	-.36	.02	.03	-.90	.01
	High		.14	-.12	-.38	.07	.08	-1.05	-.04E-1
	Mean		.09	-.11	-.30	.04	.05	-1.01	-.04E-1
Labor	Low		1.30	.72	1.81	1.38	1.03	1.39	-.06
	Middle		.56	.48	1.53	.71	.44	.74	.09
	High		.20	.31	1.16	.43	.31	.65	.28
	Mean		.47	.34	1.25	.67	.47	.78	.14

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[1] Calculated at mean for each expenditure group. Uses QES with regional dummy.

Table VII.4

Income Compensated Quantity Elasticities with Respect to Price[1] by Expenditure Group

With Respect to Price of	For Expenditure Group	OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low		-1.05	.02E-1	.14	.26	.20	.23	-.08
	Middle		-.68	.01	.12	.17	.11	.15	-.09
	High		-.44	-.01	.01	.13	.11	.12	-.09
	Mean		-.65	.01	.09	.17	.13	.16	-.10
Root crops and Other cereals	Low		.03E-2	-.13	.02	.03E-1	.01E-1	.05E-1	-.01E-1
	Middle		.03E-2	-.23	.05	.01	.04E-1	.01	-.01
	High		-.01	-.27	.11	.02	.01	.04	-.02
	Mean		.03E-1	-.20	.05	.01	.01	.18	-.01
Oils and Fats	Low		.04	.04	-.81	.05	.04	.05	-.02
	Middle		.04	.05	-.95	.05	.03	.06	-.03
	High		.01	.09	-.93	.07	.04	.13	-.08
	Mean		.04	.05	-.84	.06	.04	.08	-.04
Fish and Animal Products	Low		.13	.01	.08	-1.17	.10	.12	-.04
	Middle		.08	.02	.09	-.84	.06	.08	-.05
	High		.06	.01	.07	-.76	.06	.08	-.06
	Mean		.08	.01	.07	-.88	.07	.09	-.05
Miscellaneous Foods	Low		.09	.04E-1	.05	.09	-.92	.08	-.03
	Middle		.06	.01	.06	.06	-.56	.06	-.03
	High		.04	.05E-1	.03	.05	-.61	.05	-.04
	Mean		.06	.01	.04	.06	-.67	.06	-.04
Nonfoods	Low		.36	.04	.24	.35	.27	-.87	-.11
	Middle		.23	.07	.30	.25	.16	-.64	-.15
	High		.15	.08	.35	.23	.18	-.75	-.19
	Mean		.22	.06	.28	.26	.19	-.73	-.16
Labor	Low		.42	.04	.28	.42	.32	.38	.28
	Middle		.27	.07	.34	.30	.19	.28	.36
	High		.18	.08	.36	.26	.21	.32	.49
	Mean		.26	.06	.30	.31	.23	.31	.40

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[1] Calculated at mean values for each group. Uses QES with regional dummy.

Hicks-Allen substitutes except for root crops and rice at high expenditure levels. This last is unlikely; however, the magnitude is small, -.01. Perhaps, then, it should be interpreted as suggesting independence. Also note that the substitution effects with respect to wage are small so that the uncompensated wage effects are largely income in nature due to changes in wage rate changing nominal full income as well as real income. Also, the response of household labor supply to wage rates, while small, does increase with expenditure group, which is the main reason why the uncompensated labor supply elasticities increase with expenditure group. We must remember, though, that we are observing points on different (shifted) curves, not points along one curve. This is why it is not so disconcerting that the negative sign for labor supply occurs for the low expenditure group. Had we observed points along a curve, we would have expected the supply elasticity to decline with higher wage (expenditure group).

Changes in expenditure due to a marginal change in household composition variables are shown in Table VII.5. These changes are evaluated at the sample average except for the regional dummy variable which is set to one for northern households and to zero for southern households. Changes in the older three age groups affect demand in two ways. Given a level of full income, the commodity composition of demand changes. Second, full income changes since persons of these ages can work or take leisure. Children under ten years in our sample do not work; hence, they do not affect full income, but they do change the commodity composition of goods and leisure demanded. For the higher age groups, the sign of marginal expenditures is predominately positive. This is not surprising since any negative composition effect is outweighed by the effect due to increasing full income. One can see that the largest marginal expenditures are for rice, nonfoods, and oils and fats (except for changes in children under 10). Total expenditures increase for increases in each age-sex group. Also, region makes no real reference.[23/] For males over 15, the value of household labor supply is also affected importantly. That labor supply responds positively to changes in all age groups indicates that if adult household size decreases, say because of migration, that the remaining members do not work so hard as to make it up, as Sen (1966) postulated. Indeed, our results imply that per person total labor supply decreases, not increases, as the number of male adults decreases, though it does increase when female adults decrease in number.

The discrepancy between changes in the value of household labor due to a marginal change in males over 15 and those due to a change in females over 15 is

Table VII.5

Change in Expenditure by Commodity Due to Marginal Change in Age-Group Variables by Region[1]

Commodity	Region	Age Group	Under 10	11-15	Males over 15	Females over 15
Rice	North		10.1	6.8	17.6	9.2
	South		9.7	7.0	18.4	9.5
Root crops and other cereals	North		4.3	-2.5	3.7	-1.2
	South		4.5	-2.7	3.4	-1.3
Oils and fats	North		-5.9	8.7	28.9	13.2
	South		-5.4	8.4	28.0	12.8
Fish and animal products	North		-1.8	2.0	10.9	4.0
	South		-1.9	2.1	11.1	4.1
Miscellaneous foods	North		10.1	-2.5	3.0	-1.2
	South		10.0	-2.4	3.2	-1.2
Nonfoods	North		8.7	5.6	39.2	13.0
	South		8.7	5.6	39.1	13.0
Household labor	North		25.5	18.1	103.3	37.0
	South		25.6	18.0	103.2	37.0

[1] Calculated at sample averages except for regional dummy variable.

partly due to certain home activities, such as child care, being excluded from the labor supply variable. That changes in expenditures on goods is greatest for changes in males over 15 is probably reflective of changes in full income being largest for changes in males over 15. This may be due in part to our assumption concerning the relative efficiency units of male adult, female adult, and children aged 11-15 labor.

As persons under 10 do not affect total household time, hence full income, the change in the value of household labor due to changes in persons under 10 is the negative of the marginal change in expenditure on leisure. Hence, for our sample, the value of leisure decreases due to an increase in persons under 10 years. This means that total expenditure on commodities increases. This increase may represent an attempt to mitigate any reduction in child quality. In addition, it is possible that an exogenous increase in children under 10 may not result in less savings as hypothesized in much of the population literature (in our data, local forms of savings are included in the nonfoods category).

In each age bracket, the marginal changes in goods expenditure less the change in value of labor supplied equals zero, since the sum of total expenditures minus the value of labor supplied always equals the "profits" part of full income, which is constant.

7.4 Some Implications

Clearly, there are many interesting results in these tables to which we cannot do justice in this paper. Of significance for food consumption modeling is the proposition that household characteristic variables are important determinants, in addition to prices and full income, of household food consumption. This result is in the mainstream of the demand systems literature, but contrary to the recent assertions of Kelley (1981).

Of significance for development efforts is the general proposition that food demand is reasonably responsive to price (except for root crops and other cereals). That food demand is price responsive is consistent with the results of Lau, Lin, and Yotopoulos. For their agricultural commodity, they found an own price elasticity of -.72 (profits constant), evaluated at the sample mean (they did not evaluate elasticities at different household expenditure levels). Barnum and Squire and Ahn, Singh, and Squire found very small own price elasticities for rice in Malaysia and Korea, -.04 and -.18, respectively. Part of the difference between those estimates and the ones reported here may be due to the households in the Malaysian and Korean samples having much larger per capita incomes than households in the Sierra Leone sample. That higher income households have lower own price elasticities for staples is confirmed in our results for rice. In addition, having greater commodity disaggregation may be partly responsible for the relatively higher magnitude of our elasticities.

Price as an important short-run allocator of food consumption and hence caloric consumption has been stressed in recent years by such people as Mellor (1975) and Timmer (1978). Mellor has focused on the real income effect of price, which is supported here. However, we find own price substitution effects also to be important, contrary to previous expectations. Partly, this is due to the commodity disaggregation we have used (five food groups including two of staples). Our results also supply information important to the nutritional planner. For example, the negative uncompensated effects on root crops with respect to rice price mean that decreases in rice consumption due to increases in rice price are not likely to be compensated by increases in cassava consumption, rather the opposite. Of course, for households producing these foods, changes in prices will result in changed profits, hence full income, and may reverse some of the foregoing results. For example, an increase in rice price will lead to higher nominal full income, which will be distributed over consumption of all goods and of leisure. It is possible that rice consumption might actually increase when such "profit" effects are allowed (see Lau, Lin, and Yotopoulos). Even if not, it may be that consumption of other foods increases enough to offset lowered consumption of rice so that total caloric availability to the household increases when rice price increases. These possibilities suggest the need to include in analysis estimates of the production side of this household-firm model, so that the jointness of the consumption and production decisions may be utilized. Only then can we know if estimating solely the consumption-leisure component of a household-firm model suffices to explain the behavior of rural households.

VIII. CET-CD Estimates

8.1 Final Specification

The production side of a household-firm model for which several commodities are specified can best be estimated by econometric means when using data exhibiting price variation for all outputs and variable inputs. Such data allows one to estimate profit functions or systems of output supply and input demand functions. At the household level, such a specification is likely to avoid problems of endogeneity of independent variables (e.g., outputs). Alternatively, estimating a multi-product production function directly requires availability of instrumental variables to avoid such problems (see Mundlak, 1963). Even when using duality theory to specify the production side, severe econometric problems may be encountered when the data are from the household level and several commodities are specified. This is due to the possibility that household specialization in production is so complete that some commodities are not produced by some households; the data are censored (see Appendix 3).

For estimating the system of output supply and input demand equations, variable selection is largely specified by choice of outputs, inputs, and production function. It bears repeating here that land is not adjusted for quality as labor and capital flows are (see footnote 6). There is some room for adding generality after the outputs, inputs, and production function have been specified, providing one hypothesizes parameters of the production function to be a function of other variables. In production function analysis, this has a time-honored tradition when using cross-section, time series data (see Mundlak, 1961) as firm and time effects. This amounts to using shift dummies corresponding to firm or time when estimating the production function. More recently, Mundlak (1980) has made slope parameter functions of certain variables.

As an example, we might hypothesize that the parameters differ between EA 13 and non-EA 13 households. Fitting completely different production functions for each region would reduce both sample size and price variation. If one can assume that the overall functions are the same but that certain parameters differ by region, then advantage may be taken of pooling the regions in estimation. Suppose one lets the shift parameters of the CET-CD production function vary by region. As we saw in Section V, this function requires normalization by either the δ_i parameters summing to one or the shift parameter being unity. We have chosen the latter method. However, let $A_0=a_0+a_1D$, where D=dummy variable and a_0 and a_1 are parameters. Dividing both sides of equation 5.1 by A_0 gives the normalization which we use of the shift dummy equaling one. Now, however, the δ_is are each divided by A_0^ρ and the new coefficient will take on different values for each region. The coefficients thus derived $\delta_i/(a_0+a_1D)^\rho$ are a bit cumbersome. A simpler way to achieve this result and to maintain the normalization that $A_0=1$ is to make each δ_i depend linearly on the dummy variable $\delta_0=\delta_{i0} + \delta_{i1}D$. This introduces N-1 new parameters rather than just one, where N-1 is the number of outputs. However, it presumably allows somewhat more flexibility. In principle, all the coefficients might be allowed to vary with region. However, to keep matters simpler, only the equivalent to a shift dummy was permitted.

8.2 Parameter Estimates and Statistical Tests

The system of output supplies and labor demand was estimated in quantity form.[24/] Numerical maximum likelihood techniques were used (specification of the disturbances, which allows for zero production of some outputs, is described in Appendix 3). Parameter estimates and their asymptotic standard errors (computed from the inverse information matrix) are given in Table A.3.1. The first sixteen parameters correspond to the production function coefficients in equation 5.2. The last seven parameters are the standard errors from the likelihood function in equation A.3.2. Nine out of sixteen production function parameters have absolute values of coefficients greater than their standard errors, with four having this ratio greater than two. For the δ_i parameters, we use the one-tailed test since they are constrained to be positive. One parameter (for rice) is significant at a probability level less than .1 (corresponding to a standard normal statistic of greater than 1.29) and two have probability levels of roughly .11. For the $\delta_{i0} + \delta_{i1}$ parameters (which correspond to EA 13 households),[25/] two have coefficient absolute values greater than 1.29 their standard errors. Wald test statistics of the joint significance of the δ_i parameters are low, as is seen in Table A.3.2.

The coefficient ρ is 4.25, corresponding to an elasticity of transformation between outputs of .31. The production function is almost homogeneous of degree .78, significantly less than one. The estimate of the coefficient for land, .07, is low. This is very different from the usual single agricultural output Cobb-Douglas results in which land has the largest coefficient. Two reasons suggest themselves for this. First, some of our outputs such as fishing and animal products, oils and fats, and nonfoods are not going to be much affected directly by land cultivated by the household. Capital and labor are far more important inputs for these activities. Perhaps, had the production function specification been to allow separate functions for these activities, the land coefficient might have been higher for the remaining crop activities. Be that as it may, this was not possible due to the data inadequacies described in Appendix 3. Given the output detail and function specification used, these coefficients may not be unreasonable. A second potential reason is the absence of any quality adjustments in defining the land variable. This misspecification affects all coefficients. Had the model been linear in parameters and if increasing size of farm was associated with lower quality land, then land's estimated coefficient would be lower than the true value. Whether this result applies, given that the model is highly nonlinear in parameters, is not clear.

8.3 Output Elasticities with Respect to Prices and Final Inputs

Price elasticities of quantity of output supply and labor demand are given in Table VIII.1 for EA 13 households, the remaining households, and the sample average. The elasticities are evaluated at average values for these three groups. The formula used is discussed in Appendix 4. All the output elasticities are less than .5. In general, the more important the activity to the group of households, the more price-responsive it is. For EA 13 households, fish and animal products and miscellaneous foods (remember vegetable production is important for these households) have own-price elasticities of .45 and .35, respectively. For non-EA 13 households, rice is the most price-responsive, having an elasticity of .36. For these households, root crops and other cereals, oils and fats, and miscellaneous foods have own-price elasticities ranging from .09 to .14. Labor is much more elastic than outputs for all households, being -1.37 and -1.17 for EA 13 and non-EA 13 households, respectively. One possible reason for these low price elasticity estimates may be a downward bias due to omission of land quality. If higher quality land is associated with more output and lower prices, then the price effects on output should be biased downwards. By the same reasoning, wage effects on output are probably biased upwards in absolute value.

For oils and fats (which include palm kernels), a cash crop, the own-price elasticity of .13 for non-EA 13 households, is at first glance surprisingly low. However, it should be remembered that exogenous variables are averaged over households of which only some are major producers of oils and fats. This may bring price responsiveness down. More importantly, the stock of oil palm trees of bearing age is fixed so the major response to price can come only by varying labor, that is, by varying the amount of fruit picked and processed.[26] At the sample means, own-price responsiveness tends to be low. The largest elasticities are for miscellaneous foods, .15, and for rice, .11.

Cross-price elasticities of outputs tend to be low except with respect to wage rate. The latter is not surprising since labor demand is reasonably price-responsive. The cross-price elasticity with respect to wage can be written as the product of the own-price elasticity of labor demand and the output elasticity of labor, where the latter is written $\frac{E(L_T)}{E(X_i)} \frac{\partial E(X_i)}{\partial E(L_T)}$. Cross-price elasticities of labor demand are also not negligible. As with own-price output elasticities, the more important the activity corresponding to the price changing, the more responsive labor demand is. The signs of the output cross-elasticities are positive. That is, increasing price of output i leads to increased production of output j. As output price changes, there is a substitution effect, that is movement along a

Table VIII.1

Elasticities of Expected Quantities of Outputs Supplied and Labor Demanded with Respect to Price from CET-CD System in Quantity Form[1]

With Respect to Price of	Household Group	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Mis-cel-laneous Foods	Nonfoods	Labor
Rice	EA 13	.08	.02E-1	.03E-2	.06E-1	.06E-1	.03E-1	.03
	Non-EA 13	.36	.03	.04	.02	.05	.01	.53
	Mean	.11	.01	.03E-1	.01	.02	.06E-1	.14
Root Crops and Other Cereals	EA 13	.08E-1	.12	.01E-1	.03	.03	.01	.18
	Non-EA 13	.03	.09	.01	.07E-1	.01	.04E-1	.16
	Mean	.02	.10	.04E-1	.01	.03	.08E-1	.20
Oils and Fats	EA 13	.02E-2	.04E-2	.02	.01E-1	.01E-1	.04E-2	.06E-1
	Non-EA 13	.02	.08E-1	.13	.06E-1	.01	.03E-1	.14
	Mean	.02E-1	.02E-1	.02	.02E-1	.03E-1	.09E-2	.02
Fish and Animal Products	EA 13	.04	.05	.07E-1	.45	.14	.06	.83
	Non-EA 13	.03	.01	.02	.08	.02	.05E-1	.20
	Mean	.02	.02	.05E-1	.09	.03	.09E-1	.23
Miscellaneous Foods	EA 13	.01	.02	.02E-1	.05	.35	.02	.27
	Non-EA 13	.02	.06E-1	.09E-1	.05E-1	.14	.03E-1	.11
	Mean	.01	.01	.03E-1	.01	.15	.05E-1	.13
Nonfoods	EA 13	.03E-1	.04E-1	.05E-2	.01	.01	.13	.06
	Non-EA 13	.04E-1	.01E-1	.02E-1	.09E-2	.02E-1	.04	.02
	Mean	.03E-1	.02E-1	.05E-2	.02E-1	.04E-1	.04	.03
Labor	EA 13	-.14	-.20	-.03	-.54	-.54	-.23	-1.37
	Non-EA 13	-.47	-.15	-.21	-.12	-.23	-.07	-1.17
	Mean	-.17	-.14	-.03	-.13	-.24	-.07	-.75

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[1] Calculated at mean values for each household group using $\frac{P_j}{E(X_i)} \frac{\partial E(X_i)}{\partial p_j}$. Uses EA 13 - Non-EA 13 dummy.

Table VIII.2

Elasticities of Expected Quantities of Outputs Supplied and Labor Demanded with Respect to Fixed Inputs[1]

Commodity	Household Group	WRT	Land	Capital
Rice	EA 13		.03	.14
	Non-EA 13		.09	.49
	Mean		.04	.18
Root Crops and Other Cereals	EA 13		.04	.20
	Non-EA 13		.03	.15
	Mean		.03	.15
Oils and Fats	EA 13		.05E-1	.03
	Non-EA 13		.04	.21
	Mean		.07E-1	.04
Fish and Animal Products	EA 13		.11	.56
	Non-EA 13		.02	.13
	Mean		.02	.13
Miscellaneous Foods	EA 13		.11	.56
	Non-EA 13		.05	.24
	Mean		.05	.25
Nonfood	EA 13		.05	.24
	Non-EA 13		.01	.07
	Mean		.01	.07
Labor	EA 13		.10	.50
	Non-EA 13		.08	.42
	Mean		.05	.27

[1] Using CET-CD system with EA 13 – Non-EA 13 dummy in quantity form. Calculated at mean values for each household group using $\frac{Z_j}{E(X_i)} \frac{\partial E(X_i)}{\partial Z_j}$.

production transformation frontier. This should be negative. There is also an output effect, a shift of the transformation frontier due to changes in outputs other than i and j, and more importantly due to changes in labor demand. An increase in price i should increase labor demand as well as output i, shifting the transformation frontier between goods i and j outward. Whether the outward shift of the transformation frontier is sufficient to outweigh the substitution effect is an empirical question. For the CET-CD production function, it turns out that sign $(\frac{\partial E(X_i)}{\partial p_j})$ = sign $(\rho\beta_L - 1)$, which is positive for our estimates.

The price elasticities derived all assume that quantities, not prices, of land and of capital are fixed to the household. In the longer run, the reverse should be true, which should increase the price responsiveness of both outputs and labor. In the short run, a possibly interesting question is what are the expected output elasticities with respect to fixed inputs. These elasticities are presented in Table VIII.2. The elasticities with respect to capital are roughly five times greater than those with respect to land. Again, the magnitudes are largest for those activities which are more important. These are fish and miscellaneous food outputs for EA 13 households and rice for non-EA 13 households, and labor demand for both.

IX. Household-Firm Model Results

9.1 Profit Effects

Having estimated separately the demand system and production system components of our household-firm model, we can now examine the model in its entirety. Consumption demand may be written $X_i^c = f(p,\eta,p_N T(m)+\pi(p,z))$, where p=prices, η=household characteristic variables affecting taste, T=time available to the household, m=household characteristic variables determining T (some of which may be identical to some of those in η), z=fixed inputs, and π=profits. In Section VII, we examined the price elasticities holding profits constant. If we now allow profits to vary, we can write:

$$\frac{\partial X_i^c}{\partial p_j} = \frac{\partial X_i^c}{\partial p_j}\Big|_{d\pi=0} + \frac{\partial X_i^c}{\partial \pi}\frac{\partial \pi}{\partial p_j}$$

In elasticity form:

$$\frac{p_j}{X_i^c}\frac{\partial X_i^c}{\partial p_j} = \frac{p_j}{X_i^c}\frac{\partial X_i^c}{\partial p_j}\Big|_{d\pi=0} + \frac{p_j \partial X_i^c \partial \pi}{X_i^c \partial \pi \partial p_j} \quad . \tag{9.1}$$

The first term is simply the usual uncompensated elasticity of demand of good i with respect to price j. The second term is what we might call the "profit effect" in elasticity form. The term $\frac{\partial X_i^c}{\partial \pi}$ is easily found from the share of marginal full income reported in Table IX.1. The term $\frac{\partial \pi}{\partial p_j}$ is somewhat complicated to derive due to the existence of so many zero outputs in our data. Its derivation is discussed in Appendix 4.

Table IX.2 reports the "profit effects" in elasticity form, the second term in equation 9.1, for low, middle, high, and mean expenditure households assuming proportional changes in sales and purchase prices.[27/] In most cases, the effects are larger, often much larger, for the lowest expenditure households, declining with higher expenditure. Two reasons exist for this tendency to decline. First, for some goods, marginal expenditures out of full income decline with higher expenditure. Second, mean consumption of all goods and mean labor supply increases with higher expenditure level. Indeed, even for root crops and oils and fats, for which marginal expenditures out of full income rise with total expenditure level, the profit effect, which is in an elasticity form, falls. Goods having higher marginal expenditures, such as oils and fats and nonfoods, tend to have larger profit effects. This factor is also responsible for many of the cross-profit effects being large. A change in full income generated by a changing price is distributed over all commodities according to the marginal expenditure out of full income.

The largest own profit effect, at the sample mean, is .27 for fish and animal products. Oils and fats has a profit effect of .24. The other own effects at the mean household level are all lower than .17.

For the low expenditure group, the largest own profit effect is .82 for rice, followed by .78 for fish and animal products, and then .63 for oils and fats. In addition to the reasons previously advanced, the profit effect for rice is large because the term $\frac{E(\pi)}{\partial p}$ rises substantially when computed for the low expenditure group.

The signs of the profit effects with respect to goods prices are positive except for household labor supply. This is due to the marginal expenditures out of full income being positive for all goods. The sign in household labor is the opposite of the sign on household leisure. Since leisure is a normal good for

Table IX.1

Shares of Marginal Full Income[1]
by Expenditure Group

Commodity	Expenditure Group			
	Low	Middle	High	Mean
Rice	.15	.11	.01	.09
Root crops & other cereals	.02	.04	.09	.05
Oils and fats	.09	.14	.26	.16
Fish and animal products	.09	.08	.05	.07
Miscellaneous foods	.06	.05	.03	.05
Nonfoods	.27	.27	.28	.28
Leisure	.31	.31	.29	.30

[1] Partial derivative of commodity expenditure with respect to total income. Evaluated at expenditure group means using QES with regional dummy.

Table IX.2

Profit Effects in Elasticity Form by Expendituге Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	.82	.63	1.44	.91	.66	.94	-.32
	Middle	.11	.15	.46	.16	.10	.18	-.11
	High	.05E-1	.08	.26	.05	.03	.11	-.07
	Mean	.08	.09	.35	.14	.09	.17	-.10
Root crops and other cereals	Low	.49	.38	.86	.54	.40	.56	-.19
	Middle	.14	.19	.56	.19	.12	.22	-.13
	High	.01	.16	.58	.12	.07	.24	-.15
	Mean	.12	.16	.55	.21	.14	.27	-.15
Oils and fats	Low	.36	.28	.63	.40	.29	.41	-.14
	Middle	.08	.11	.31	.11	.06	.13	-.07
	High	.03E-1	.04	.17	.03	.02	.07	-.04
	Mean	.05	.07	.24	.09	.06	.12	-.06
Fish and animal products	Low	.71	.54	1.24	.78	.58	.81	-.28
	Middle	.25	.35	1.05	.35	.22	.41	-.24
	High	.09E-1	.13	.45	.10	.06	.19	-.12
	Mean	.16	.21	.73	.27	.18	.36	-.20
Miscellaneous foods	Low	.23	.18	.40	.25	.19	.26	-.09
	Middle	.08	.11	.33	.11	.07	.13	-.08
	High	.05E-1	.06	.22	.05	.03	.09	-.06
	Mean	.06	.07	.25	.10	.06	.13	-.07
Nonfoods	Low	.12	.10	.22	.14	.10	.14	-.04
	Middle	.04	.06	.18	.06	.04	.08	-.04
	High	.02E-1	.04	.12	.02	.02	.06	-.04
	Mean	.04	.04	.14	.06	.04	.08	-.04
Labor	Low	-.56	-.43	-.99	-.62	-.46	-.64	.22
	Middle	-.13	-.19	-.56	-.19	-.11	-.22	.13
	High	-.08E-1	-.11	-.38	-.08	-.05	-.15	.10
	Mean	-.10	-.13	-.43	-.17	-.11	-.21	.12

[1] Calculated as $\frac{p_j}{X_i^c} \frac{\partial X_i^c}{\partial \pi} \frac{\partial E(\pi)}{\partial p_j}$ at expenditure group means, using parameter estimates from the CET-CD system in quantity form, and assuming proportional sales and purchase prices.

these households, labor supply is lowered as full income increases due to rising goods prices. With respect to wage rate, the signs for profit effects on goods are negative, for the same reason. Profits are reduced as wage increases so expenditures fall. Household labor, however, increases in this case.

9.2 Total Price Elasticities of Consumption and Effects of Fixed Inputs

Having derived the profit effects, we can add these to the uncompensated elasticities with respect to price, which hold profit constant, to arrive at the total price elasticities of quantities of goods demanded and of labor supplied. These are presented in Table IX.3. The own total price effects for commodities remain negative when profit effects are added except for root crops and other cereals at the low expenditure group. The fact that root crops and other cereals consumption responds positively to own price for low expenditure households is reflective of the lack of responsiveness of consumption to own price holding profits constant and of the higher profit effect for these households. In the other cases, the short-run responsiveness, holding profits constant, to own price is large enough to overwhelm the profit effect. However, the profit effect does have the interesting consequence that the total own price elasticities for several commodities such as rice, oils and fats, and fish and animal products no longer drop in absolute value with higher expenditure levels. Indeed, for rice, the total own price elasticity is as low for low expenditure households as for high expenditure households. For root crops and other cereals, the negative response of consumption to own price is greater for high than for middle expenditure households. As seen in Table VII.3, this is mostly a result of the uncompensated (profits constant) price elasticities being higher in absolute value for the high expenditure group. Secondarily, the profit effects are slightly higher for the middle than for the high expenditure group. For household labor supply, the response to wage is now positive at all expenditure levels, rising to almost .4 for high expenditure households and being roughly .25 at the sample mean. The fact that this still rises with the higher expenditure group is due to the classical demand substitution effects rising with expenditure as explained in Section VII.

In general, the total cross-price effects are positive. Negative classical demand income effects are reversed in sign by the profit effects. The exceptions are for root crops and other cereals and oils and fats consumption with respect to nonfoods price, and for those two commodities with respect to rice price for

Table IX.3

Total Quantity Elasticities with Respect to Price[1] by Expenditure Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	-.44	.47	1.21	.93	.69	.93	-.32
	Middle	-.67	.02	.15	.18	.12	.16	-.11
	High	-.44	-.04	-.12	.10	.10	.07	-.06
	Mean	-.66	-.01	.06	.17	.12	.14	-.09
Root crops and other cereals	Low	.47	.23	.84	.52	.38	.54	-.18
	Middle	.12	-.07	.52	.17	.11	.20	-.12
	High	.01E-1	-.15	.56	.11	.06	.23	-.14
	Mean	.11	-.06	.53	.19	.13	.25	-.14
Oils and fats	Low	.40	.32	-.19	.45	.32	.46	-.16
	Middle	.08	.11	-.79	.11	.06	.13	-.07
	High	.02E-1	.05	-1.08	.03	.02	.08	-.04
	Mean	.06	.08	-.73	.10	.07	.13	-.07
Fish and animal products	Low	.73	.46	1.12	-.51	.59	.80	-.28
	Middle	.28	.29	.90	-.57	.23	.40	-.24
	High	.07	.08	.30	-.71	.10	.19	-.12
	Mean	.20	.17	.61	-.68	.20	.35	-.20
Miscellaneous foods	Low	.24	.12	.30	.25	-.80	.25	-.09
	Middle	.09	.05	.19	.11	-.53	.11	-.07
	High	.05	.02	.08	.07	-.60	.07	-.05
	Mean	.08	.03	.14	.10	-.65	.11	-.06
Nonfoods	Low	.22	-.06	.01	.20	.16	-1.03	-.05
	Middle	.11	-.10	-.18	.08	.07	-.82	-.03
	High	.14	-.08	-.26	.09	.10	-.99	-.05
	Mean	.13	-.07	-.16	.10	.09	-.93	-.05
Labor	Low	.74	.29	.82	.76	.57	.75	.16
	Middle	.43	.29	.97	.52	.33	.52	.22
	High	.19	.20	.78	.35	.26	.50	.38
	Mean	.37	.21	.82	.50	.36	.57	.26

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[1] Sum of uncompensated quantity elasticities and profit effects in elasticity form. Assumes proportional sales and purchase prices.

the high expenditure group (and sample mean for root crops and other cereals). Some of the positive cross-price elasticities are of large magnitude; for example, oils and fats consumption with respect to the price of root crops and other cereals. However, in general, the cross-price responsiveness declines with higher expenditure, as the profit effects do, and is not large when evaluated at the sample mean. For labor supply, the cross-price effects are negative, due to the profit effect. The cross-effects with respect to wage rate are cut substantially from the effects when profits are held constant, but remain positive and non-negligible. Rises in the wage rate increase full income by increasing the value of time available to the household, but decrease full income by decreasing the profit component. Evidently, the former effect is the dominant one because the positive income effect, found by subtracting the income compensated from the uncompensated elasticities, is larger in absolute value than the negative profit effect.

The total own price elasticities reported here differ from the results of previous household-firm model studies in that they are generally negative in sign. Lau, Lin, and Yotopoulos report a total own price elasticity of .22. Barnum and Squire and Ahn, Singh, and Squire find total own price elasticities of .38 and .01, respectively.

Prices are not the only exogenous variables in our household-firm model in which we are interested. The effect of changes in household characteristic variables on consumption was examined in Section VII. Since these variables do not enter into the production side, those are the total effects. On the production side, we can look at changes in consumption due to the profit effect of changes in fixed inputs. In elasticity form, we have $\frac{Z_j \partial X_i^c}{X_i^c \partial \pi} \frac{\partial E(\pi)}{\partial Z_j}$, where Z_j is either total land acreage or value of capital flow. These elasticities are reported in Table IX.4. The elasticities, with respect to capital flow, are larger than those with respect to land because the term $\frac{Z \partial E(\pi)}{\partial Z}$ is larger for capital than for land. This is a reflection of expected quantity output being more responsive to changes in capital flow than to changes in land cropped. As is the case for the profit effects due to changes in prices, these profit effects are larger at lower expenditure levels, and for the same reasons. Also, they tend to be larger for commodities having larger marginal expenditures out of full income. The magnitudes of the elasticities are low, all being less than .05 at

Table IX.4

Quantity Elasticities with Respect to Fixed Inputs[1] by Expenditure Group

Commodity	Expenditure Group	With Respect To	
		Total Land Cultivated	Value of Capital Flow
Rice	Low	.08	.43
	Middle	.01	.06
	High	.01E-1	.04E-1
	Mean	.01	.04
Root crops and other cereals	Low	.06	.33
	Middle	.02	.08
	High	.01	.05
	Mean	.01	.06
Oils and fats	Low	.15	.76
	Middle	.04	.23
	High	.04	.19
	Mean	.04	.20
Fish and animal products	Low	.09	.48
	Middle	.02	.08
	High	.08E-1	.04
	Mean	.01	.07
Miscellaneous foods	Low	.07	.35
	Middle	.01	.05
	High	.04E-1	.02
	Mean	.01	.05
Nonfoods	Low	.09	.50
	Middle	.02	.09
	High	.01	.08
	Mean	.02	.10
Household labor	Low	-.03	-.17
	Middle	-.01	-.05
	High	-.01	-.05
	Mean	-.01	-.05

[1] Calculated as $\frac{Z_j}{X_i^c} \frac{\partial X_i^c}{\partial \pi} \frac{\partial E(\pi)}{\partial Z_j}$, where Z_j is either acres of total land cultivated or Leones of capital flow.

the sample mean, with respect to land, and .20 or less, with respect to capital. It should be remembered that these elasticities reflect an autonomous change in these variables. In the longer run, when capital and total land are varied endogenously, the elasticities of consumption, with respect to price of capital and to price of land, will not correspond to these short-run figures.

X. Determinants of Marketed Surplus and Household Caloric Availability

10.1 Marketed Surplus Price Elasticities

We now have the total price elasticities of consumption of commodities and of labor supply. There are many questions which can be explored using these. One such is what happens to quantities sold or bought on the market when price changes and households' full income changes as a result. The response to price of marketed surplus, which can be either positive or negative, is an important question to governments interested in supplies to urban areas and to other rural areas. There is a very large literature on this, both theoretical (for example, Krishna, 1962; Dixit, 1969) and empirical (e.g., Behrman, 1966; Medani, 1975). A review is provided by Newman (1977).

Behrman in his formulation assumed that $\frac{\partial X_i^c}{\partial p_i}$ and $\frac{\partial X_i^c}{\partial y}$ were both zero (based on insignificant regression parameters). Thus, price affected marketed surplus only by affecting production. His analysis used aggregative time series data for one commodity, but allowing relative prices to enter into the acreage response function. K. Bardhan (1970) and Haessel (1975) assumed that production was fixed; hence, price affected marketed surplus only by changing consumption. Bardhan estimated the relationship for a reduced form, that is by regressing marketed surplus directly on price and other variables, using village level, time series data. Medani derived a reduced form from a structural model, assuming production to be independent of price. Income was also assumed to be independent of price so that profit effects are not allowed when computing the response with marketed surplus and consumption to price. None of those studies examined marketed surplus of labor. Only Lau, Lin, and Yotopoulos and Barnum and Squire have computed the response of marketed surplus to both price and non-price variables while explicitly using the household-firm framework, which is implicit in earlier analyses. Both of those studies were highly aggregated, using one agricultural commodity, one non-agricultural commodity, and labor. Marketed surplus was considered only for the agricultural commodity in both cases.

Let MS_i=marketed surplus of commodity i. We have $MS_i=X_i-X_i^c$. Given our data construction, marketed surplus includes net sales plus wages paid in kind minus wages received in kind. Then:

$\frac{\partial MS_i}{\partial p_j} = \frac{\partial X_i}{\partial p_j} - \frac{\partial X_i^c}{\partial p_j}$ and in elasticity form:

$$(10.1)\quad \frac{p_j}{|MS_i|}\frac{\partial MS_i}{\partial p_j} = \frac{X_i}{|MS_i|}\frac{p_j}{X_i}\frac{\partial X_i}{\partial p_j} - \frac{X_i^c}{|MS_i|}\frac{p_j}{X_i^c}\frac{\partial X_i^c}{\partial p_j}$$

The elasticity of marketed surplus is then a weighted difference of output elasticities and of total price elasticities of quantities consumed. The weights are the ratio of quantity produced to surplus, for production, and quantity consumed to surplus, for consumption. Given our Tobit estimation of the production side, we use $\frac{\partial E(X_i)}{\partial p_j}$ in the first term (see Appendix 4). Also, the divisor is the absolute value of marketed surplus. This is used so that one can easily tell the sign of $\frac{\partial MS_i}{\partial p_j}$, that is, whether production increases more or less than consumption.

If the sign of the elasticity is positive and the net surplus is positive, then an increase in price will result in more being sold on the market. If the elasticity is positive and the household is a net purchaser (a negative surplus), then an increase in price will lead to less being purchased on the market. A negative elasticity and a positive surplus will lead to less being sold to the market and negative elasticity and a negative surplus means more will be purchased. We continue to assume proportional sales and purchase prices.

As Krishna pointed out, the magnitudes of the own price marketed surplus elasticities may be a good deal higher than the output elasticities if production is very much larger than surplus. Providing the total own price elasticities of consumption are negative, these will reinforce the effect of increasing production, further increasing the marketed surplus elasticity. Indeed, the only way in which this measure can be negative is for the total own price elasticity to be sufficiently positive and the ratio of consumption to marketed surplus be large enough so that their product outweighs the effect of increasing production. Given our total price elasticities, this will only be possible for root crops and other cereals for low expenditure households.

The matrix of marketed surplus price elasticities is shown in Table X.1. All the own price elasticities are positive and reasonably high. There is a tendency for the price responsiveness of marketed surplus to decline at higher expenditure levels. In large part, this is due to the absolute value of marketed surplus, part of the denominator, increasing with higher expenditure levels (see Table VI.5). The marketed surplus being low is the reason for the high magnitude of the own price elasticity for root crops and other cereals for low expenditure households. If absolute changes in kilograms marketed due to a one percent increase in price were shown, they would be roughly equal for the low and middle expenditure groups, rising for the high expenditure group. For household labor, the large values of the marketed surplus elasticity, with respect to wage rate, are also caused by the small values of marketed surplus in the denominator. This contrasts markedly with Rosenzweig's (1980) results. He separated male and female labor supply (which we combine using relative wages as weights) and found the former's marketed surplus responds negatively to wage. However, his equations are a reduced form, relating observed market labor hours to wage. His measure of labor supplied to the market seems to be a gross measure, that is a sum of actual hours worked over a year, rather than a net measure, total hours sold out less total hours of hired labor. It is possible that the gross measure might respond negatively to wage, if income effects are strong enough, while the net measure responds positively, provided hired labor responds negatively enough to wage changes.

The cross-price elasticities of marketed surplus tend to be negative because of the strong profit effect in the cross total price elasticity of demand. The latter term is generally positive and often large. Since it is subtracted, after being weighted appropriately, from a generally small positive cross-price effect on production, the difference will usually be negative. For example, an increasing price of root crops and other cereals will lead to a decrease in the marketed surplus of oils and fats. Consumption of oils and fats will increase because full income has increased, while production will be little affected. Also, a decrease in the marketed surplus of nonfoods will take place. However, since nonfoods are purchased on the market (the surplus is negative), the decrease in marketed surplus means that more will be purchased on the market.

Some positive cross-price elasticities exist. For example, the surplus for root crops and other cereals responds positively to all prices except for oils and fats and the wage rate. Also, the surplus for oils and fats responds positively to nonfoods price.

Table X.1

Price Elasticities of Marketed Surplus[1] by Expenditure Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	.89	.66	-.32	-1.05	-.47	-1.00	-18.45
	Middle	.73	.05	-.04	-.23	-.09	-.17	-5.74
	High	.75	.04	.09	-.12	-.03	-.08	-1.31
	Mean	.71	.06	-.03	-.72	-.05	-.15	-4.42
Root crops and other cereals	Low	-.11	3.10	-.31	-.70	-.34	-.58	-7.53
	Middle	-.09	.37	-.17	-.23	-.09	-.21	-5.54
	High	.02	.39	-.40	-.10	.02	-.25	-3.09
	Mean	-.08	.46	-.29	-.73	-.04	-.27	-6.61
Oils and fats	Low	-.08	.06	.79	-.58	-.27	-.50	-7.09
	Middle	-.07	-.01	.29	-.19	-.07	-.14	-2.56
	High	-.02E-1	-.01	.78	-.04	-.07E-1	-.09	-.58
	Mean	-.05	-.02	.44	-.44	-.04	-.14	-2.35
Fish and animal products	Low	-.18	.04	-.41	2.15	-.56	-.86	-10.84
	Middle	-.22	.03	-.29	1.81	-.22	-.43	-10.56
	High	-.09	.02	-.21	1.33	-.01	-.21	-2.56
	Mean	-.16	.02	-.33	5.94	-.08	-.38	-8.80
Miscellaneous foods	Low	-.05	.11	-.09	-.32	1.97	-.27	-4.22
	Middle	-.06	.03	-.06	-.13	1.29	-.12	-3.77
	High	-.06	.03	-.05	-.07	.49	-.08	-1.36
	Mean	-.06	.04	-.08	-.34	.81	-.12	-3.44
Nonfoods	Low	-.07	.08	.03E-1	-.30	-.17	1.12	-1.59
	Middle	-.09	.02	.06	-.14	-.09	.88	-1.24
	High	-.21	.04	.19	-.12	-.04	1.08	-.80
	Mean	-.12	.04	.09	-.52	-.06	1.01	-1.85
Labor	Low	-1.22	-7.45	-1.49	-3.30	-2.37	-.83	27.41
	Middle	-.58	-.60	-.37	-2.02	-1.29	-.56	16.41
	High	-.42	-.54	-.58	-.93	-.44	-.55	8.57
	Mean	-.49	-.72	-.51	-5.82	-.78	-.62	17.18

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[1] Calculated as $\frac{X_i}{|MS_i|} \frac{p_j}{X_i} \frac{\partial E(X_i)}{\partial p_j} - \frac{X_i^c}{|MS_i|} \frac{p_j}{X_i^c} \frac{\partial X_i^c}{\partial p_j}$ and assuming proportional sales and purchase prices.

Some of the magnitudes of the cross-price elasticities are fairly large. Again, this is caused by the strong profit effect on consumption. The magnitudes do tend to fall with the higher expenditure groups, as they do for the own price elasticities. They are not negligible, however, so it is not wise to ignore them as most past studies have done.

10.2 Effects of Prices and Expenditure on Household Caloric Availability

This study is concerned ultimately with determinants of food consumption. This can be further translated into effects of prices and other variables in our model on availability to the household of different nutrients. Of greatest interest to development economists recently is caloric availability. Sukhatme's (1970) work indicating that sufficient caloric intake is usually accompanied by sufficient protein intake and caloric deficiencies with protein deficiencies is partly responsible for this emphasis on calories.

More germane to this study, UCLA's (1978) study of the nutritional situation in Sierra Leone, based on anthropometric data, found that chronic malnutrition (underweight for age) was the principal nutritional problem of children aged 0-5 years. The little evidence which exists for other groups, primarily pregnant and lactating women, also suggests that being underweight is the major problem. In view of these findings, only the impact on calories will be examined here, although one can in principal use our results to examine the impact of socioeconomic variables on many nutrients.

We want to calculate $\frac{\partial cal}{\partial p_j} = \sum_{i=1}^{5} \frac{\partial cal}{\partial X_i^c} \frac{\partial X_i^c}{\partial p_j}$, where cal=calories and 1-5 are our food groups. In elasticity form, we want $\frac{p_j}{cal} \frac{\partial cal}{\partial p_j} = \frac{1}{cal} \sum_{i=1}^{5} \frac{\partial cal}{\partial X_i^c} \frac{p_j \partial X_i^c}{\partial p_j}$. We calculate effects on calories of price changes both when profits are constant and when they are variable. The difference will point out clearly the effect of families producing the foods which they consume.

Elasticities of caloric availability with respect to total expenditure are reported in Table X.2 (see Appendix 5 for conversion factors from kilograms into calories). Total expenditure, as opposed to full income, is endogenous in our model, but the results will still be of interest. The magnitudes are around .85

Table X.2

Elasticities of Calorie Availability with Respect to Total Expenditure[a] by Expenditure Group

Low	Middle	High	Mean
.85	.83	.93	.86

[a] Calculated as $\frac{TEXP}{Cal} \sum_{1}^{5} \frac{\partial Cal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial TEXP}$ (see Table VII.2 for $\frac{\partial E(p_i X_i^c)}{\partial TEXP}$).

with little variation among expenditure groups. That the elasticity for the high expenditure group is slightly higher than for the low expenditure group results from the marginal total expenditure share of oils and fats, an important contributor of calories, rising with the expenditure group. This apparently offsets the declining total expenditure share on rice. The elasticity magnitudes we report compare to a range of .15 to .30 used by Reutlinger and Selowsky (1976). They believed .15 and .3 to be bounds on the calorie elasticity with respect to income.

Our estimates of the total expenditure elasticity of calorie availability are much closer to those of Pinstrup-Anderson and Caicedo (1978). They estimate Engel curves from cross-section household data in Colombia and find a calorie elasticity with respect to income of over .5 ranging to over .6 for low-income households.

Tables X.3 and X.4 report caloric elasticities with respect to prices with profits held constant and allowed to vary. With profits being held constant, increases of commodity prices result in decreased caloric availability, except with respect to nonfoods price at the low expenditure group. There is no general pattern of elasticities across expenditure group, but the absolute change in caloric availability often increases with expenditure group. For commodity prices, the largest response of caloric availability is for changes in the price of rice, the major staple. These range from -.58 to -.28.

Table X.3

Elasticities of Calorie Availability with Respect to Price, Profits Constant,[1] by Expenditure Group

With Respect to Price of:	Expenditure Group	Change in Kilocalories[2]	Elasticity
Rice	Low	-11.9	-.58
	Middle	-18.5	-.38
	High	-23.2	-.28
	Mean	-19.1	-.38
Root crops and other cereals	Low	-0.7	-.03
	Middle	-2.1	-.04
	High	-5.2	-.06
	Mean	-2.3	-.05
Oils and fats	Low	-1.5	-.07
	Middle	-6.0	-.12
	High	-20.9	-.25
	Mean	-7.4	-.15
Fish and animal products	Low	-3.9	-.19
	Middle	-4.0	-.08
	High	-6.9	-.08
	Mean	-4.2	-.08
Miscellaneous foods	Low	-1.5	-.07
	Middle	-4.4	-.09
	High	-6.3	-.08
	Mean	-4.2	-.08
Nonfoods	Low	0.2	.08E-1
	Middle	-1.1	-.02
	High	-1.9	-.02
	Mean	-0.9	-.02
Labor	Low	23.0	1.12
	Middle	28.0	.57
	High	36.5	.45
	Mean	28.1	.56

[1] Calculated as $\frac{p_j}{cal} \sum_i \frac{\partial cal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j} \Big|_{d\pi=0}$ at expenditure group means.

[2] Change in kilocalorie availability due to infinitesimal percentage change in price, $\frac{p_j}{100} \sum_i \frac{\partial kcal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j} \Big|_{d\pi=0}$.

Table X.4

Elasticities of Calorie Availability with Respect to Prices, Profits Variable,[1] by Expenditure Group

With Respect to Price of:	Expenditure Group	Changes in Kilocalories[2]	Elasticity
Rice	Low	3.9	.19
	Middle	-11.7	-.24
	High	-16.7	-.20
	Mean	-12.8	-.26
Root crops and other cereals	Low	8.8	.43
	Middle	6.4	.13
	High	8.6	.11
	Mean	7.5	.15
Oils and fats	Low	5.5	.27
	Middle	-1.4	-.03
	High	-16.9	-.21
	Mean	-3.0	-.06
Fish and animal products	Low	9.8	.48
	Middle	11.5	.23
	High	3.9	.05
	Mean	8.8	.18
Miscellaneous foods	Low	2.9	.14
	Middle	0.6	.01
	High	-0.8	-.01
	Mean	0.3	.07E-1
Nonfoods	Low	2.6	.12
	Middle	1.5	.03
	High	1.1	.01
	Mean	1.9	.04
Labor	Low	12.2	.59
	Middle	19.8	.40
	High	27.3	.33
	Mean	20.3	.41

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[1] Calculated as $\frac{p_j}{cal} \sum_i \frac{\partial cal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j}$ assuming proportional sales and purchase prices.

[2] Change in kilocalorie availability due to one percent change in price,

$$\frac{p_j}{100} \sum_i \frac{\partial kcal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j} \; .$$

For absolute changes in caloric availability, the largest annual change, of -19,000 calories, occurs for an average household when rice price changes. This change translates into a change of slightly under -52 calories per household per day, or roughly -8 calories per capita per day (using the mean household size of 6.5 persons).

When profits can vary, the situation changes substantially. Now most of the commodity price elasticities of calories are positive. Increasing price may result in decreased consumption of that good, but the expected increase in full income is distributed on increases in consumption of other foods, enough so as to increase total caloric availability. The exceptions to this are for rice and oils and fats prices at all but the low expenditure group, and for the price of miscellaneous foods at the high expenditure group. The magnitudes of the positive elasticities are not high for the sample mean, but some are sizable for the low expenditure group. Even absolute changes in calorie availability tend to decline as expenditure group rises except for changes in the prices of rice, oils and fats, and labor.

For all commodities, the positive effect of a change in price with profits variable is greatest for low expenditure households, reflecting the fact that for every commodity own-price profit effects are greatest among such households. For rice and for oils and fats, it is only for low expenditure households that the profit effect is large enough to dominate the negative own-price effects upon calorie availability with profits constant. This is partially because in the middle and high expenditure households the negative own-price effects--profits constant--are stronger for rice and for oils and fats than for other commodities.

While caloric availability increases for low expenditure households, with an increase in rice or in oils and fats prices, it decreases for middle and high expenditure households, and at the sample mean. For rice price, the elasticities for the two higher expenditure groups are still sizably negative, between -.2 and -.25. Hence, when profit effects are accounted for, rice price increases seem to lessen the discrepancy in calories available to the rural expenditure groups. They increase availability for very low expenditure households and decrease availability for higher expenditure households. From Table A.5.1, we see that the mean daily caloric availability per capita for high expenditure households is quite high (2,600 calories). Although some households in this group will have caloric availability lower than the mean, it may be that lower availability will still allow these households to have available sufficient calories for weight maintenance under "normal" activity levels.

XI. Policy Implications

These results have significant implications for the development process in Sierra Leone and for future modeling of this kind. First, we state the obvious: prices and full income affect household caloric availability, although the ability of the household to produce what it consumes mitigates this effect. Response by the household in its role as a firm does make a difference. Secondly, for the representative low expenditure household to have caloric availability even at the level of 1,900 calories per capita per day (see Table A.5.1) would require increases in income of a magnitude not likely to occur anytime soon. With prices and household characteristics constant, an average low expenditure household would need an increase in annual full income of about 270 Leones to reach the availability level of 1,900 calories per capita per day. This new level of full income would result in total expenditures of roughly 445 Leones. That figure is 88 percent higher than the existing expenditure level of the representative low expenditure household--237 Leones. Assuming, optimistically, an annual growth rate in total expenditures of 3 percent, it would take nearly 22 years for an average low expenditure family to reach this point. Of course, if family size grew along with total expenditure, which is likely, even longer would be needed.

The usual caution is needed here. Caloric availability at the household level says little about intake of individuals. For example, one of the variables in our model is household labor supply, of which one part is labor supplied by lactating women. If, with increasing household full income, lactating women spend more time at home breastfeeding infants, the caloric intake of infants may increase more than suggested by total household availability. As another example, food waste may be influenced by variables such as full income.

The price responsiveness, especially with respect to rice price, of food availability and ultimately of calorie availability implies that there is a trade-off to be made between long-run output growth and short-run nutritional status. A secularly rising price of rice may lead to increased output levels, and possibly to increased growth rates if technical change is endogenous, but will lower the caloric availability to many rural households. Very low expenditure households may enjoy some nutritional benefits from such a rise. Of course, this analysis is at the partial equilibrium level. At the general equilibrium level, nominal wages may rise due to a rise in rice price, and in the long run, households may invest in more capital (some embodying technical progress perhaps) and in more land. As shown in Table VI.5, more land or capital will increase quantities of food availability, hence of calorie availability. Whether this would offset the decreasing caloric availability due to increasing price will depend on how much capital and land increase. Likewise for effects of wage increases.

In the longer run, rice price may be lower than otherwise if production growth has been stimulated. Distributional impacts of technical change have long been debated. Questions of access to technology cannot be addressed by these research results. However, differential price effects of technical change may be addressed. Most producers in rural areas would seem to be helped nutritionally by rice price being lower than it otherwise might be, ceteris paribus. However, those lowest expenditure households who are nutritionally worst off (see Table A.5.1) may be hurt unless they participate in the technical change sufficiently so that the autonomous increase in full income due to the technical change is enough to offset the lowered caloric availability due to a rice price lower than otherwise. These effects of price changes due to technical change are somewhat different from those generally postulated in the literature. Distributional impacts have been limited to examining the impact on pure consumers and on pure producers. Hayami and Herdt (1974) examine the impact on each with producers selling a portion of the crop (rice) to the market. However, consumption out of home production is assumed to be completely price inelastic and since purchases are ignored, total consumption of rice is assumed price inelastic. This enables them to examine the impact only on cash income. In their model, a decline in rice price reduces cash income, hence welfare, but differentially depending on the proportion marketed. In our model, full income, not cash income, matters, and consumption of rice is affected by price changes, though the decomposition of changes on consumption of home produced versus changes in consumption of purchased rice is not identified. Nevertheless, the price impact on nutrition of technical change can now be positive for rural rice producing households, and is for representative households of all but the lowest expenditure group.

An oft-asked question is to what extent to promote exports of cash crops such as palm oil, coffee, and cocoa. Some have argued that increasing the production of cash crops at the expense of subsistence crops will have an adverse impact on nutritional status. Such persons argue that less orientation toward the market will result in better nutrition. In our household-firm model, marketed surplus is endogenous, being simultaneously determined with production and consumption. As an endogenous variable, it is affected by many exogenous variables. Hence, it stands to reason that one exogenous variable will affect marketed surplus and consumption differently than another, so that the relationship between marketed surplus and consumption may depend upon what exogenous variable(s) is changing. Hence, in principle, it need not be true that increased reliance on the market leads to worse nutritional status.

Looking at our results, if we examine oils and fats, of which palm products are the lion's share in value, an increase in own price results in decreased calorie availability for high and middle expenditure groups but increased availability for the low expenditure group. Marketed surplus increases for all groups. Hence, increased reliance on the market for oils and fats as a consequence of a rise in oils and fats price results in higher caloric availability for a typical low expenditure household, but lower caloric availability for typical middle and high expenditure households. However, if wages rise along with oils and fats price, this result may not hold up since marketed surplus may then decrease if the wage increase is large enough.

Alternatively, an increase in rice price decreases the marketed surplus of oils and fats for the low and middle expenditure groups. Such an increase in rice price will lead to increased calorie availability for the low expenditure group and decreased availability for the middle expenditure group. Hence, for an increase in rice price, lower reliance on the market for oils and fats is accompanied by lower calorie availability for a representative middle expenditure household. This is contrary to the relationship often hypothesized. However, for low expenditure households, decreased market reliance is associated with higher calorie availability when the source of the change is an increase in rice price. Note that the relationships for low and middle expenditure households between the direction of change of marketed surplus of oils and fats and of calorie availability are different when they are a result of changed rice price than when they result from a changed price of oils and fats. This is in accordance with the general proposition advanced earlier that the relation between two endogenous variables will depend upon what exogenous variable is changed.

When the expected relationship of greater market reliance coinciding with reduced caloric availability does occur, the sources of this relationship turn out to be the opposite of the sources which have heretofore been suggested. More, not less, is consumed of rice and root crops and other cereals when the price of oils and fats increases (Table IX.3). This is primarily due to the profit effect in increasing full income. As a result, less is marketed of these foods, while less, not more, is consumed of oils and fats. It is that reduction in consumption which is the source of lowered caloric availability.

When the price of rice increases, oils and fats consumption goes up and rice consumption decreases. For the low expenditure group, a reduction in reliance on the market for the oils and fats due to rice price changes results in the expected increase in caloric availability, but again for different reasons than commonly assumed. In this case, calorie availability increases because enough additional oils and fats, as well as other commodities, are consumed to offset the reduced consumption of rice.

The foregoing partial-equilibrium implications are examples of the wider variety of policy questions which may be addressed by using a household-firm model having more commodity detail than the heretofore used three commodity models. Our research has shown that such multi-commodity household-firm models can be estimated econometrically using cross-section household-level data. This can be done using functional forms allowing for a wide variety of behavior. Using more commodity detail also allows the results to be fitted into a policy relevant general equilibrium analysis (see Lau and Yotopoulos, 1974), which is an important potential future extension of these models, and which may modify some of the partial equilibrium analysis.

Appendix 1

Specification of the Error Terms

To specify the error structure of the household-firm model, we use the conventional approach of adding disturbances linearly to the reduced form. The question arises onto which form of the reduced form should errors be added. The choices are threefold: for the demand system, they are the quantity demanded equations, the expenditure equations, or the share equations. The choice will depend upon which form one expects the disturbances to have desirable statistical properties. For household t, let ε_t be an n vector error. Assume ε_t's to be iid $N(0,\Sigma)$ so that $\varepsilon = (\varepsilon_1', \varepsilon_2', \ldots, \varepsilon_T')' \sim N(0, I_T \otimes \Sigma)$. On which form of the reduced form is this most likely to hold? In particular, for which form are the ε_t's identically distributed? Pollak and Wales, in most of their work, believe the share equations are the proper ones to which to add this error structure. Using experience from estimating Engel curves, they feel the errors on expenditure equations have a heteroskedastic nature of the form $E(\varepsilon_{ti}\varepsilon_{tj}) = \sigma_{ij}y_t^2$, $y_t \equiv$ total expenditure. Hence, dividing each equation by y_t, resulting in share equations, is the appropriate solution. Alternatively, one might assume, as did Pollak and Wales (1969), that errors on the demand equations have structure $E(\varepsilon_{ti}\varepsilon_{tj}) = \sigma_{ij}\hat{X}_{it}^c\hat{X}_{jt}^c$ where the hats indicate non-stochastic portions. Defining

$$F_t = \begin{pmatrix} \hat{X}_{1t}^c & & 0 \\ & \ddots & \\ 0 & & \hat{X}_{nt}^c \end{pmatrix}$$

we have $\varepsilon_t \sim N(0, F_t\Sigma F_t)$. However the error structure is specified, residuals may be examined for the appropriateness of the specification, and if heteroskedasticity is suspected, statistical tests may be performed.

As is usual for complete systems $\sum_{k=1}^{N} \varepsilon_{tk} = 0, \forall t$, since the value of expenditures on goods and leisure adds to full income at all sample points. Hence, the full covariance matrix is singular and we drop one equation for estimation (see Barten, 1969).[28] Doing that, we can write the likelihood function as:

$$\text{(A.1.1)} \quad L = (2\pi)^{-(N-1)T/2}|\Sigma|^{-T/2} \prod_{t=1}^{T} ||F_t||^{-1} \exp(-1/2\ \varepsilon_t' F_t^{-1} \Sigma^{-1} F_t^{-1} \varepsilon_t) \qquad \underline{29/}$$

This function is non-linear in parameters. Barnett (1976) and Gallant and Holly (1980) have shown that under suitable regularity conditions maximum likelihood estimators are consistent and asymptotically efficient with the asymptotic distribution of $\sqrt{T}\,(\hat{B} - B)$ being $\lim_{T\to\infty} (\ell/T)^{-1}$, where $\ell \equiv$ information matrix.

One remaining question is the independence of π and ε_t. If π is assumed given, as is usually done for total expenditure in demand systems, then there is no problem. However, this sytem is derived from a household firm model; hence, if π has a stochastic component, it might be correlated with ε_t in which case there is an endogeneity problem and the demand system cannot be consistently estimated apart from the production system. If, however, the disturbances on the demand and production equations of the household-firm model are independent, then the system is block recursive[30/] and indeed separate maximum likelihood estimation is identical to maximum likelihood estimation of the larger system. We assume such independence, although the assumption is a testable one.[31/] The practical advantage in making the independence assumption is that separate estimation of the consumption and production sides of the model entails far fewer parameters being estimated for each side separately. This greatly reduces the cost and tractability of numerical maximum likelihood estimation, enabling us to estimate many more parameters in total. This means that we can include more commodity disaggregation and more demographic variables in our estimation, making the problem more interesting.

Appendix 2

Estimation of the Quadratic Expenditure System

Since there was question a priori whether the disturbances on the expenditure equations were identically distributed, we took squared residuals from these equations and regressed them on variables to which the variances were hypothesized to be proportionate. In particular, they were regressed on a constant and the square of fitted value (i.e., $\mathrm{var}(\varepsilon_{ti}) = (\hat{X}_{ti}^{c})^2\sigma_{ii}$), and a constant and the square of the observed part of total income ($\mathrm{Var}(\varepsilon_{ti}) = \pi^2\sigma_{ii}$). The results of the latter were mixed, in three out of six regressions the constant term being significant and not squared profits and vice versa. As can be seen from Table A.2.1, squared fitted values were very significant in five out of six regressions and significant at the .10 level in the sixth. Moreover, regression standard errors for the regression using squared fitted values were uniformly lower than for the regressions using squared profits. The error specification giving rise to this result is $\varepsilon_t \sim N(0,F_t\Sigma F_t)$ where F_t = diagonal $(|p_i\hat{X}_{ti}^{c}|)$. Alternatively, this amounts to weighting each equation for observation t and good i by $1/|p_i\hat{X}_{ti}^{c}|$. Clearly, then the function is not defined for $|p_i\hat{X}_{ti}^{c}| = 0$.

The error specification using absolute fitted values was used and maximum likelihood estimation tried. Unfortunately, the algorithms kept stopping at a point at which $|p_i\hat{X}_{ti}^{c}|$ was nearly zero for some i and some t, but which were clearly not local optima.[32/] Different starting values for parameters were tried, unsuccessfully. It was then decided to use for $p_i\hat{X}_{ti}^{c}$ the values from estimation of the expenditure form equations, and to treat these as constants.[33/] This is an extension to regressions non-linear in parameters of Amemiya's (1973) suggested two-step procedure for the linear regression case. He showed such two-step estimators to be consistent with a known distribution, but not asymptotically efficient. Halbert White (1980) has shown (theorem 2.4) that an unweighted non-linear least squares estimator is a strongly consistent estimator when error terms are not identically distributed, under some fairly weak regularity assumptions. What we have is a system of non-linear seemingly unrelated regressions. Since estimating such equations jointly affects only efficiency, not consistency (assuming no misspecification), White's result is applicable to our first-round estimators. In particular, our estimates of fitted values are consistent. That, in turn, means our second-stage estimates are consistent. These estimates are not unrestricted maximum likelihood estimates and so are presumably not asymptotically efficient. Conditional on the first-round estimates of fitted values,

Table A.2.1

Regression Coefficients and Standard Errors for Regression of Squared Unweighted and Weighted QES Residuals on Squared Fitted Values[1]

Commodity	Equation	Constant	Squared Fitted Value	R^2[2]
Rice	Unweighted	4,657.5	.78E-1	
		(2,130.8)	(.45E-1)	.02
	Weighted	.54	-.33E-5	
		(.11)	(.39E-5)	.01
Root crops and other cereals	Unweighted	7,032.8	.57	
		(4,478.3)	(.44E-1)	.55
	Weighted	2.0	.11E-4	
		(.96)	(.88E-4)	---
Oils and fats	Unweighted	1,928.3	.31	
		(875.2)	(.22E-1)	.58
	Weighted	9.3	-.22E-4	
		(2.51)	(.45E-4)	---
Fish and animal products	Unweighted	831.4	.24	
		(528.5)	(.59E-1)	.11
	Weighted	1.1	-.80E-4	
		(.29)	(.46E-4)	.02
Miscellaneous foods	Unweighted	1,428.4	.24	
		(594.2)	(.69E-1)	.08
	Weighted	1.9	-.12E-3	
		(.35)	(.61E-4)	.03
Nonfoods	Unweighted	5,107.1	.15	
		(2,580.8)	(.30E-1)	.15
	Weighted	.64	-.16E-5	
		(.21)	(.20E-5)	---

---

[1] Unweighted residuals are residuals from initial unweighted QES estimates, using regional dummy. Weighted residuals from the second stage QES estimates, which were weighted by fitted values from the initial estimates.

[2] -- indicates R^2 less than .005.

they are mle and $\sqrt{T}\ (\hat{B}-B)$ should be asymptotically distributed as $N(0, \lim_{T\to\infty} (\ell/T)^{-1})$, with the information matrix calculated treating F_t as being fixed.

The second-stage conditional maximum likelihood estimates were obtained with resulting parameters and their asymptotic standard errors shown in Table A.2.2. The heteroskedasticity problem has nearly disappeared. Table A.2.1 shows a significant constant term and insignificant coefficient for squared fitted values on four out of six regressions of squared weighted residuals on those variables. For one regression, both constant and squared fitted values are significant and for the other the constant term is significant and the squared fitted value term borderline.[34/]

Table A.2.2

Coefficients and Asymptotic Standard Errors of Quadratic Expenditure Systems

Type of dummy variable:	Regional		Ethnic Group	
Parameter	Coefficient[1]	Standard Error[2]	Coefficient[1]	Standard Error[2]
C_1	-189.1	79.0	-167.8	53.2
C_2	42.4	16.4	-180.8	19.0
C_3	-12.2	23.3	-128.4	41.3
C_4	9.3	21.9	10.9	15.5
C_5	6.8	13.9	10.7	29.1
C_6	-.4	54.5	-1,907.4	690.7
C_7	-1,522.3	500.8	-1,309.3	1,579.5
σ_{11}	7.3	15.0	8.7	10.7
σ_{12}	61.5	23.5	8.4	15.5
σ_{13}	214.0	73.1	102.1	52.2
σ_{21}	-9.8	5.6	40.2	3.8
σ_{22}	24.9	8.8	4.0	9.0
σ_{23}	-30.8	28.2	153.9	28.4
σ_{31}	-.6	5.0	-1.3	6.6
σ_{32}	11.4	8.4	6.9	7.5
σ_{33}	-47.1	19.9	19.6	14.7
σ_{41}	-3.7	2.9	-1.9	1.9
σ_{42}	11.0	4.3	1.5	2.9
σ_{43}	-4.2	19.9	-18.2	15.1
σ_{51}	-8.5	3.2	-5.1	2.8
σ_{52}	32.0	5.6	22.3	4.8
σ_{53}	20.8	20.2	-27.5	21.8
σ_{61}	-14.6	8.2	-27.2	22.6
σ_{62}	60.3	13.2	25.0	34.4
σ_{63}	-37.7	37.9	97.1	115.4
$\sigma_{72}+\sigma_{71}$	-20.5	103.9	-396.5	208.0
σ_{73}	-152.1	371.1	-2,129.3	993.4
$\gamma_1-\sigma_{71}$	1,846.6	143.3	2,174.4	158.9
γ_2	-1,437.3	152.5	-1,461.6	229.7
γ_3	-1,117.7	167.7	-1,628.5	251.8
a_1	.23162	.35E-1	.55362E-1	.20E-1
a_2	-.1405E-1	.11E-1	.13175	.42E-1
a_3	-.2803E-2	.36E-1	.420258E-1	.94E-2
a_4	.109989	.20E-1	.16796E-1	.90E-2
a_5	.7929E-1	.24E-1	-.2092E-2	.17E-1
a_6	.269242	.60E-1	1.0045	.58E-1
d_1	.23160	.35E-1	.55360E-1	.20E-1
d_2	-.1404E-1	.11E-1	.13170	.42E-1
d_3	-.2774E-2	.36E-1	.420263E-1	.94E-2
d_4	.109983	.20E-1	.16801E-1	.90E-2
d_5	.7928E-1	.24E-1	-.2086E-2	.17E-1
d_6	.269243	.60E-1	1.0044	.58E-1
Value of log-likelihood	-3,487.7		-3,577.1	

---

[1] Single subscripts refer to commodity number as given in Table A.1 and double to commodity and demographic variable numbers. Demographic variable numbers for the σ's are 1-household size, 2-under 10 years, 3-regional or ethnic group dummy=1 if northern or Limba-Temne household. For the γ's the numbers are 1-over 10 years, 2-11 to 15 years, 3-females over 15.

[2] From information matrix calculated from second derivatives of log-likelihood function.

Appendix 3

Tobit Estimation of the Output Supply and Input Demand Equations

For estimating the system of output supply and input demand equations, we begin with equation 5.2, derived from a Constant Elasticity of Transformation-Cobb-Douglas (CET-CD) multiple output production function. Following the discussion in Appendix 1, we add error terms which are distributed as $N(0,\Sigma)$ to these equations, which are in quantity form. If there were no other considerations, we could obtain our maximum likelihood estimates easily. However, we saw in Section VI that for several of our six goods many households have no production. In particular, for production of nonfoods, oils and fats, and fish and animal products, this is so. If it is physically possible for households to produce these goods, then the first-order conditions from the maximization of profits subject to the production function are the Kuhn-Tucker conditions.

$$(A.3.1)\quad p_i-\mu\frac{\partial G}{\partial X_i} \leq 0,\ X_i\left(p_i-\mu\frac{\partial G}{\partial X_i}\right) = 0 \qquad i=1,\ldots,N-1$$

$$-p_N-\mu\frac{\partial G}{\partial L_T} \leq 0,\ L_T\left(-p_N-\mu\frac{\partial G}{\partial L_T}\right) = 0$$

$$G\leq 0,\ \mu G=0$$

Assume no technical inefficiencies, so that $G=0$, and assume that labor is always demanded, which is true for our sample, so that $p_N+\mu\frac{\partial G}{\partial L_T} = 0$. Then, $\frac{p_i}{p_N} \leq \frac{-\partial G/\partial X_i}{\partial G/\partial L_T},\ \forall_i$. The right-hand side is the reciprocal of the marginal product of labor in producing good i. We have then that the value or marginal product of labor for good i is less than or equal to the price of labor. When this holds as an equality, the good is produced and when it is an inequality, the good is not produced.[35/]

Under this circumstance, randomness can be accounted for in two ways. First, one can append error terms to the Kuhn-Tucker first-order conditions. This was done for a system of demand equations by Wales and Woodland (1979). By assuming a joint distribution for the disturbances, one can derive a likelihood function for the observed outputs and labor inputs. Second, one can add error terms directly to the reduced form of output supply and input demand equations, as done for a demand system by Wales and Woodland (1978). This is akin to the

Tobit model $y_i^* = g_i(\beta) + \varepsilon_i$, $y_i = \max(0, y_i^*)$, where y_i^* is not observed but y_i is. Here $g_i(\cdot)$ is the value of output supply or labor demand from equation 5.2 and β = a vector of parameters.[36/] If $\varepsilon \sim N(0,\sigma^2)$, then $E(y) = E(y/y>0) \cdot P(y>0) + E(y/y=0) \cdot P(y=0)$, where $E(\cdot)$ is the expectations operator and $P(\cdot)$ is probability. Of course, $E(y/y=0)=0$ so $E(y) = E(y/y>0) \cdot P(y>0)$. $E(y/y>0) = g(\beta) + E(\varepsilon/y>0)$ and from Johnson and Kotz (1970), we have $E(\varepsilon/y>0) = E(\varepsilon/\varepsilon>-g(\beta)) = E(\varepsilon/\frac{\varepsilon}{\sigma}>\frac{-g(\beta)}{\sigma}) = \sigma f(g(\beta)/\sigma)/F(g(\beta)/\sigma)$, where $f(\cdot)$ is the standard normal density and $F(\cdot)$ is the standard normal distribution function. In particular, $E(\varepsilon/y>0) \neq 0$ so that regression using only observations with positive y's leads to inconsistent parameter estimates. This last implies that the mean of the disturbances using all observations on y, $E(\varepsilon/y>0) \cdot P(y>0)$ is also not zero, so these OLS parameter estimates are inconsistent also. For the linear in parameters model, Greene (1981) has shown $E(\hat{\beta}_{OLS}) = \beta F(\bar{X}\beta/\sigma)$, so that the lower the probability of a positive observation the greater is the bias. What is happening in this model is that the entire normal distribution of ε is not being observed. The lower tail in which $\varepsilon<-g(\beta)$, corresponding to y=0, is piled up at $-g(x,\beta)$, providing we observe y when it is equal to zero. This is so because we observe y, not y*. If y is not observed when it is zero, the distribution of ε is simply cut off, or truncated, at $\varepsilon=-g(\beta)$. The former situation (y observed), which we have in our data, is called censored data; the latter is called truncated data.

The foregoing applied to a single equation model. The output supply and input demand equations are a system but the same model is applicable. In this case, ε is an n+1 vector with covariance matrix Σ. Also, there exist cross-equation parameter restrictions, for instance, that ρ is the same in all equations. The system can be estimated consistently using maximum likelihood techniques. The probability density for each household involves evaluating multiple integrals, one for each good not produced. In our data, there are many households not producing one or two goods and a few households not producing as many as four goods. For these households, the corresponding density involves evaluating a quadruple integral. This is not only extremely messy to program, but expensive to compute as well. Indeed, in their two papers, Wales and Woodland used only three commodities, one of which was always consumed.

One way around this difficulty would be to aggregate to, say, three outputs plus labor. Since one output is always produced and labor always demanded, this would involve at most double integrals, which would still be expensive, but perhaps manageable. An alternative not involving more aggregation is to assume

Σ, the covariance matrix of ε, to have zeroes in certain places. If Σ were block diagonal, then the multivariate density would be a product of densities of the outputs (and input) corresponding to each block. This would reduce the dimension of the multiple integrals to be evaluated. In the extreme case of assuming independence between each of the error terms, the household density would be the product of 7-K normal densities and K standard normal distribution functions. If K outputs were not produced, only a single integral would have to be evaluated, but one for each of the normal distribution functions corresponding to the K outputs not produced. However, evaluating a single integral K times is a much less costly and less difficult procedure than evaluating a K- dimension integral once. Although one need not go so far as assuming independence between all of the error terms, to choose which error terms are correlated in such a way as to result in block diagonality for Σ would seem to involve as much arbitrariness as assuming complete independence. Since the latter results in a considerably simpler estimation procedure, it was chosen.

It should be noted that one reason why this would be an unreasonable assumption for a demand system does not hold for output supplies and input demands. As we have seen for the demand side, expenditures on goods plus value of household leisure equal total income, resulting in error terms summing to zero. Hence, the covariance matrix is singular, which it could not be if it were diagonal. However, this is not true for the values of output supply less value of input demand. On the other hand, one can argue that the probability of producing rice conditional on the household not producing any other commodity but demanding labor is not equal to the unconditional probability of producing rice. Clearly, in this case, the conditional probability is one, but the unconditional probability is not. Yet independence of the error terms implies these probabilities are equal. Still, assuming independence does make the computation problem manageable. Moreover, ignoring cross-equation restrictions, maximum likelihood estimates assuming independence retain their consistency even if the assumption is violated. Hence, the assumption remains attractive statistically. All that would be sacrificed is asymptotic efficiency. The likelihood function to be maximized is thus:

$$(\text{A.3.2}) \quad L = \prod_{t}\Big[\prod_{i\varepsilon P} \frac{1}{\omega_i} f\left(g_{ti}(\beta)/p_i\omega_i\right) \prod_{j\varepsilon NP} F\left(-g_{tj}(\beta)/p_j\omega_j\right)\Big]$$

where $\dfrac{g_{ti}(\beta)}{p_i}$ is the ith quantity of output (or of labor demand if i=7) equation for household t, $f(\cdot)$ is the standard normal density, $F(\cdot)$ the standard normal distribution function, ω_i is the standard error of the ith equation, P corresponds to goods produced, and NP to goods not produced.

To justify use of the multivariate Tobit model, one has to be convinced that there is positive probability of producing non-produced outputs. Looking at the data, many of the zero outputs are spread throughout all regions. That is, some households within an enumeration area will be producers and others not. In these cases, there is evidently no environmental reason why the particular good cannot be produced. There do exist some cases in which the zero observations are clustered geographically so that none of the particular output is produced by our sample of 138 in a particular enumeration area. This occurs for root crops and other cereals in EA 72, for oils and fats in EAs 52 and 53, for fish and animal products in EAs 32 and 72, and for nonfoods in EA 72. To get a better idea of whether there exist environmental constraints on production of those goods in these enumeration areas, we examined the larger sample of 328 households for which production data were considered reliable by Spencer and Byerlee. In all cases except oils and fats in EAs 52 and 53, and fish and animal products in EA 72, there was some production of the good in question. For EAs 52 and 53, the 1970/71 Agricultural Survey of Sierra Leone showed that oils and fats were indeed produced in the Bombali areas in question. For EA 72, the Agricultural Survey indicates that game was captured. Since fish and animal products includes game, it was concluded that it was possible to produce this "good" in the area in question.

Another potential problem in using the Tobit model is misspecification of the production function. Instead of separability of all outputs and all inputs in the implicit production function, it can be argued that there are separate production functions for some outputs, perhaps for nonfoods, oils and fats, and fish and animal products. As an example, one might assume nonfood production to be a function of nonfood labor and nonfood capital. With capital fixed, either a Cobb-Douglas or a CES function implies zero supply of output if there is no capital. Hence, if households have no nonfood capital, the probability of producing nonfood output is zero. This approach runs into severe data problems with our sample. For example, there are households reporting no capital or labor use for fishing and animal product activities, yet reporting positive outputs. Many households reporting zero production of nonfoods report positive labor use to produce nonfoods. When inputs are aggregated, as we have done, into total labor, total capital, and total land, there is a greater chance than for using disaggregated inputs that such errors cancel each other out.

Table A.3.1

Coefficients and Asymptotic Standard Errors of CET-CD System in Quantity Form[1]

Parameter[2]	Coefficient	Standard Error[3]
δ_{10}	.14E-5	.96E-6
δ_{11}	.26E-2	.13E-1
δ_{20}	.96E-5	.95E-5
δ_{21}	.29E-4	.92E-4
δ_{30}	.16E-2	.15E-2
δ_{31}	12.7	134.8
δ_{40}	.131223E-2	.15E-2
δ_{41}	-.131218E-2	.15E-2
δ_{50}	.7319E-3	.60E-3
δ_{51}	-.7307E-3	.60E-3
δ_{60}	90.8	107.7
δ_{61}	-78.8	108.5
c	4.25	.3
β_D	.69E-1	.3E-1
β_K	.36	.29E-1
β_L	.35	.17E-1
ω_1	1,008.4	63.1
ω_2	2,635.2	171.5
ω_3	512.7	34.7
ω_4	1,066.5	95.9
ω_5	504.0	32.4
ω_6	88.1	7.3
ω_7	2,924.2	184.4
Value of log-likelihood function	-6,071.0	

---

[1] Uses EA 13 - Non-EA 13 dummy variable.

[2] Single subscripts refer to commodity number listed in Figure 4.1. Double subscripts refer to commodity number and 1 for dummy coefficient, 0 if not.

[3] From information matrix calculated from second derivatives of log-likelihood function.

Table A.3.2

Chi-Square Statistics From Wald Tests Using Estimates From CET-CD System in Quantity Form

Test of	Statistics	Degrees of Freedom
1. CET parameters for non-EA 13 households, δ_{i0}	3.6	6
2. CET dummy parameters, δ_{i1}	2.2	6
3. CET parameters for EA 13 households, $\delta_{i0}+\delta_{i1}$	2.4	6
4. Degree of almost homogeneity, $\beta_D+\beta_K+\beta_L$	37.6	1

Appendix 4

Derivation of Formulas Used in Computing Supply Elasticities and Profit Effects

A complication arises when implementing equation 9.1 with our data. We have $\pi=E(\pi)+u$, where u is an error term with mean zero, independent of price and fixed inputs. Then $\frac{\partial\pi}{\partial p_j} = \frac{\partial E(\pi)}{\partial p_j}$. However, due to the censoring in our data, Hotelling's lemma no longer holds. We can write $\pi = \sum_{i=1}^{6} p_i X_i - p_7 L_7$. From Appendix 3, we know that when using our parameter estimates from the quantity form of the production system $E(p_i X_i) = F(\frac{g_i(\beta)}{p_i\omega_i})\, g_i(\beta) + p_i\omega_i\, f(\frac{g_i}{p_i\omega_i})$, and likewise for $E(p_7L_7)$. Hotelling's lemma asserts that $\sum_{i=1}^{6} \frac{\partial g_i(\beta)}{\partial p_j} - \frac{\partial g_7(\beta)}{\partial p_j} = \frac{g_j(\beta)}{p_j}$, which is in fact true of the CET-CD production function. Then, if the data were uncensored, so that the error terms had mean zero conditional on positive outputs, the lemma would apply. However:

$$\frac{\partial E(p_iX_i)}{\partial p_j} = F(\frac{g_i}{p_i\omega_i})\,\frac{\partial g_i}{\partial p_j} \quad \text{and} \quad \frac{\partial E(p_iX_i)}{\partial p_i} = F(\frac{g_i}{p_i\omega_i})\,\frac{\partial g_i}{\partial p_i} + \omega_i f(\frac{g_i}{p_i\omega_i})$$

Using this, we have:

$$\text{(A.4.1)} \quad \frac{\partial E(\pi)}{\partial p_j} = \sum_{i=1}^{6} F(\frac{g_i}{p_i\omega_i})\,\frac{\partial g_i}{\partial p_j} + \omega_j f(\frac{g_j}{p_j\omega_j}) - F(\frac{g_7}{p_7\omega_7})\,\frac{\partial g_7}{\partial p_j} \qquad j=1,\ldots,6$$

and

$$\frac{\partial E(\pi)}{\partial p_7} = \sum_{i=1}^{6} F(\frac{g_i}{p_i\omega_i})\,\frac{\partial g_i}{\partial p_7} - F(\frac{g_7}{p_7\omega_7})\,\frac{\partial g_7}{\partial p_7} - \omega_7 f(\frac{g_7}{p_7\omega_7}) \qquad j=7$$

Since we have estimates for the necessary parameters, $\frac{\partial E(\pi)}{\partial p_j}$ can be constructed from our data.

Similarly, for supply elasticities with respect to price, we want to compute $\frac{p_j}{E(X_i)}\,\frac{\partial E(X_i)}{\partial p_j}$. In light of the foregoing results, this equals:

$$\frac{p_j}{E(X_i)}\,(F\,(\frac{g_i(\beta)}{p_i\omega_i})\,\frac{\partial}{\partial p_j}\,(\frac{g_i(\beta)}{p_i}))$$

Appendix 5

Caloric Equivalents of Food Groups

Having determined the quantities available for consumption from home production and from market purchases, nutrient availabilities may be calculated by using conversion rates available from food composition tables. This was done by William Whelan using the FAO tables prepared for Africa (FAO, 1968). For this purpose, quantities purchased and available from home production were added without value weights for each of the 128 foods in our data. The nutritional composition of a food was thus assumed to be identical from either source. The conversion into nutrients accounted for the inedible portion of each food (using figures available from the food composition tables). What was derived, then, were the nutrients available from each food at the farm gate or retail level, taking out the inedible portion. Left in, however, is whatever part of the edible portion is wasted by the household before ingestion. This will vary greatly by household and by food. The FAO, in its calculations, assumes this to average 10 percent (FAO, 1973, pp. 87-88).

Table A.5.1 reports total caloric availability expressed per capita per day, and its sources by our five food groups for each of the expenditure groups. For this purpose, caloric availability by food was summed into the five food groups and then totaled. Not surprisingly, caloric availability increases dramatically with expenditure group, particularly between the low and middle groups. The sample mean of 2,109 cal/cap/day compares to an estimated availability of 2,090 cal/cap/day computed by FAO from food balance sheets for the entire country for a 1972-74 average and a 1975-77 average (FAO, 1980, p. A41). The availability calculated from food balance sheets covers urban as well as rural areas. It is formed by taking production, subtracting net exports, seed, feed, waste (storage and marketing), and net change of storage. The remaining figures are converted into units sold at retail level by further adjusting for processing. The FAO food balance sheet availability figures are comparable to ours, as is their caloric availability figure (which takes account of the inedible portion; FAO, 1972, p. 45).

To obtain the conversion from kilograms of our five food groups into calories, $\frac{\partial cal}{\partial X_i^c}$, we use the conversion factors available for each of our 128 foods from food composition tables. Within each food group, we obtain the calories available for each household from each food in the group, by multiplying those

Table A.5.1

Calorie Availability and Its Components by Food Group by Expenditure Group

Proportion of Calories from:	Expenditure Group			
	Low	Middle	High	Mean
Rice	.44	.45	.43	.44
Root crops & other cereals	.17	.17	.15	.16
Oils and fats	.12	.12	.20	.16
Fish and animal products	.17	.10	.10	.11
Miscellaneous foods	.11	.15	.11	.12
Total calories per cap per day	1,188	2,132	2,608	2,109

conversion ratios by the sum of consumption out of home production and consumption from purchases. These figures are then summed over households and over all foods in the group. The sums are then divided by the total quantity consumed of each of the five food groups, where quantity is defined as total value of consumption divided by group price. These group quantities are weighted sums of quantities in straight kilograms. The weights are the ratio of the sales or purchase price of an individual food (depending on whether it was purchased or not) to the consumption price of the group. This weight will, of course, vary among the eight agro-climatic regions to which prices correspond. The numerator, calorie availability, will also vary by household, because the components consumed within each food group vary. In other words, from a nutritional perspective, the aggregated commodity groups correspond to different commodities depending on the region and on the household. Heretofore, we have assumed that the commodities were identical for all households. For our previous economic analysis, this last assumption makes sense. Now, however, it does not. Since we want to apply the caloric conversions to low, middle, and high expenditure household groups separately, we calculate separate conversions for each group. The conversions may differ between groups for two reasons. First, the weights in calculating quantities for the denominator differ by region, particularly for root crops and other cereals. Second, the proportion of calories available for each food group from each of its components will differ by expenditure group. If we want to ask what would the effect of price changes be on caloric availability for a "typical" low expenditure household in our sample, it makes sense to use caloric conversions specific to that group.

Caloric conversion rates are reported in Table A.5.2. The magnitudes for rice and for oils and fats do not require explanation, but the rest do. Comparing these rates to rates available for disaggregated foods in food composition tables shows large differences. For root crops and other cereals, cassava was assumed to have 1,490 calories per kilogram and sorghum, 3,420. These are the two major components of this group, yet both their calorie conversion rates are substantially below the sample mean group rate of 7,506 calories per kilogram. The reason for this is as follows. The numerator in our calculation is the best estimate of actual calories available for our sample from the particular group. If we had divided this by the simple sum of kilograms consumed of the components of the roots crops and other cereals group (e.g., kilograms of cassava plus kilograms of sorghum, etc.), the conversion rate would look reasonable. It would

Table A.5.2

Calorie Conversion Rates of Food Groups[1] by Expenditure Group

Food	Expenditure Group			
	Low	Middle	High	Mean
Rice	3,759.1	3,848.6	3,664.6	3,743.3
Root crops and other cereals	8,679.4	10,270.6	5,956.1	7,505.6
Oils and fats	9,909.1	9,241.1	9,001.0	9,143.6
Fish and animal products	5,647.3	3,770.1	2,485.2	3,196.4
Miscellaneous foods	2,430.2	5,184.5	4,748.9	4,430.7

[1] In calories per kilogram of weighted quantity.

then be a weighted average of food composition conversion rates, with weights being the proportion of unweighted group quantities for each component. For root crops and other cereals, the dominant quantity weight is for cassava. Over 300 kilograms of cassava per household are consumed in our sample, while only about 50 kilograms of sorghum are consumed. However, in deriving weighted quantities, the large quantity of cassava, most of which comes from home production, is multiplied by the ratio of cassava sales price to group consumption price. This price ratio is generally very small. While the sorghum quantities are multiplied by ratios which are generally a little greater than one, those quantities are not large. The result is that the weighted quantity of root crops and other cereals is much smaller than the unweighted quantity. Hence, the large calorie conversion rate. Since the quantity units used in our model are weighted quantities, we use calorie conversion rates which are in terms of the same weighted quantities.

FOOTNOTES

[1] The original Howe, Pollak, Wales specification set $a(p) = -\lambda \prod_{k=1}^{N} p_k^{(2a_k - d_k)}$; hence, it had an extra parameter, λ. In that formulation, the QES becomes an LES if either $\lambda = 0$ or $a_i = d_i$, $\forall_i$. Since the important property (allowing quadratic Engel curves) of the QES is not affected by setting $\lambda = 1$, and since we reduce by one the number of parameters needing estimation, we do so.

[2] For a much more complete discussion of entering demographic variables into systems of demand equations, see Pollak and Wales (1980, 1981).

[3] In the QES, the C_i's may be interpreted as translation parameters in which case $V_i = C_i + \sum_{r=1}^{K} \sigma_{ir}\eta_r$; however, Pollak and Wales (1980) suggest that the C_i's are better described as being part of the untranslated demand system.

[4] Net changes in storage were assumed to be zero.

[5] In principle, we could have calculated separate prices for each household; however, that would have created serious statistical problems. Assume that every household in a region faced the same set of sales and purchase prices. Even with a common utility function, different households would buy and sell foods at different times during the year, due to differences in household characteristics and in full income. Since prices have a seasonal movement, calculating an average price for each household would result on those averages being different for each household, even though the households actually faced the same set of prices. Not only would there be spurious variation in such prices, but these prices would be endogenous to the household-firm model we use to explain household behavior. This is so since purchase and sales decisions are endogenous to the model. Hence, using these prices to estimate a system of demand equations would result in inconsistent parameter estimates. The same problem would occur if we used sales prices for net sellers and purchase prices for net purchasers. It is in order to avoid the problems of spurious variation and endogeneity of prices that we average prices across households. Region was chosen instead of enumeration area as the definition of market area because it was feared that the latter might be too small. Also, region is the area used by Byerlee and Spencer when they compute their prices.

[6] The rental markets are very thin and rental prices reflect a household's standing in the community as much as the economic value of the land (Spencer and Byerlee, 1977, pp. 21-24). For a very few households, data on the land variable were missing. Since these households had usable data for all other variables, they were not dropped. Byerlee and Spencer had classified households into many different farm types. From the production sample of 328 households, we computed average land-labor use ratios for each farm type. Knowing the farm type and the labor used for these households, we were able to estimate total land cropped.

[7] We let $K = \frac{rV}{1-(1+r)^{-n}}$ where K = annual service user cost, V = acquisition cost of capital, and n = expected life of capital in years. In a perfect market, the acquisition cost of the asset equals the discounted sum of its annual flows. Assuming the annual flows to be constant in real value, and assuming the flows start in year one, we obtain the equation for K. Spencer and Byerlee use a discount rate of .1 and expected lives that were different for different types of capita (1977, pp. 47-48). The types of capital included are farm tools, animal equipment (including fishing equipment), nonfarm equipment, livestock, and tree crops.

[8] The equation estimated was $\frac{p_i X_i^c}{y} = b_0 + b_1 \frac{y}{p_i} + \sum_{j=1}^{N} \gamma_j \frac{p_j}{y} + \sum_{k=1}^{K} \sigma_k \frac{p_i \eta_k}{y}$

where y = total expenditure, p_i = price of good i, η_k = household characteristic k. This equation is homogeneous of degree zero in prices and total expenditure and has a quadratic term in total expenditure. Subsequent to the estimation of these single equations, a data error was discovered. Seven households were mistakenly classified as Mende rather than Temne. Rerunning several of the regressions showed no major changes in coefficients except for the ethnic dummy coefficient (that is, the others were generally within one standard deviation of the estimates using the corrected data). The mistake was corrected before obtaining the systems estimates.

[9] The ethnic and regional dummies are closely correlated. The northern region is predominantly Limba and Temne and the southern region predominantly Mende.

[10] Note that the effect of the ethnic dummy variable, η_3, is to add σ_{k3} to the price of coefficient C_k.

[11] That is, (3 + 3)7 - 2 + 3 or 43 parameters, less one due to the identification problem. The ethnic and regional dummy variables were included separately.

[12] In light of some evidence that scaling outperforms translation when using the QES (Pollak and Wales, 1980, 1981; Barnes and Gillingham, 1981), estimation was first attempted of a QES with demographic variables entering through scaling. In the QES, this involves raising the I_i scaling parameters to the $-d_i$ power. As the d_i's are not integers, this requires the I_i's to be positive for the function to exist. The I_i's were specified as $I_i = \sum_{r=1}^{3} \sigma_{ir}\eta_r$; hence, they had to be constrained to be positive. Unfortunately, the DFP algorithm kept getting "stuck" on an edge of the function where it was undefined (i.e., where I_i was almost zero for some i and some observation) and was unable to converge to a local optimum. Much effort was spent trying to obtain convergence, including use of several starting values for parameters and use of alternative algorithms. Finally, the translation specification was chosen because it has no undefined region. Alternatively, we might have specified the I_i as $I_i = \prod_{r=1}^{2} \eta_r^{\sigma_{ir}} e^{\sigma_{i3}\eta_3}$, which is necessarily positive and always defined since the η_r's are positive. Since we are not so interested in comparing the translation and scaling specifications, this was not pursued.

[13] The substitution matrix was computed as $\partial X_i^c/\partial p_j|_{du=0} = \partial X_i^c/\partial p_j + \hat{X}_j^c \partial X_i^c/\partial(p_N T + \pi + A)$ where $\hat{X}_j^c$ represents fitted value so that the matrix will be symmetric as imposed by the QES. For i, j = 7 (i.e., labor), $X_{i,j}^c = -L_H$ (see equation 4.2).

[14] One might have improved this situation in several ways. First, using the QES, the demographic specification could have been changed by changing the variables used and/or the way in which they enter the system (i.e., use scaling). Alternatively, another system could be tried. Finally, it is conceivable that all or some of our sample simply do not behave as demand theory postulates.

[15] Since the question of which dummy variable to use is non-nested, comparing log-likelihood values does not permit statistical inference. We could have re-estimated the system using both variables, making the hypothesis a nested one. However, this would introduce several more parameters into an already very large system, and in addition, the two dummy variables are highly inter-correlated. Alternatively, we could have performed a non-nested test of the type suggested by Pesaran and Deaton (1978). However, in light of the non-negativity results, and due to the greatly added expense that such a test would entail, it was not performed.

[16] Which implies $a_7 = d_7$ since $\sum_{i=1}^{7} a_i = \sum_{i=1}^{7} d_i = 1$. In this case, the QES simplifies into a linear expenditure system.

[17] In 1974-75, one Leone = U.S. $1.1.

[18] A relatively large number of low-expenditure households are found in areas in which cassava constitutes a large proportion of "root crops and other cereals." A relatively large number of high-expenditure households are found in areas that produce fish.

[19] We can write $\partial p_i X_i^c / \partial(p_N T + \pi + A) = \partial p_i X_i^c / \partial(\sum_{j=1}^{N-1} p_j X_j^c)\ \partial(\sum_{j=1}^{N-1} p_j X_j^c)/\partial(p_N T + \pi + A)$ from which we solve for $\partial p_i X_i^c / \partial(\sum_{j=1}^{N-1} p_j X_j^c)$, the marginal total expenditure for good i.

[20] Middle and high expenditure households tend to be in areas for which the root crops and other cereals group contains a relatively high proportion of cereals.

[21] That one can obtain a share of marginal total expenditure falling with higher total expenditure, and a falling uncompensated price elasticity of demand for a staple using a Quadratic Expenditure System should put to rest fears by Timmer (1981) that this may not be possible.

[22] This is consistent with Timmer's (1981) hypothesis that poor households may substitute staples more easily than wealthier households.

[23] Differences due to expenditure group are larger, which is not surprising since household characteristic variables affect expenditure through an income effect when entered into the demand system by translation. The differential effects at different expenditure levels are available, but not reported here.

[24] The system was also estimated in value form. If the error terms appended to the quantity form are homoskedastic, then those added to the value form are not; and vice versa. Tests of homoskedasticity were performed for each form of the system estimates. For the value form, homoskedastic error terms were clearly rejected. For the quantity form, the evidence is mixed. In addition, the own price supply response can be negative when computed from estimates of the system in value form. This is so despite the fact that the CET-CD functional form constrains such responses to be positive. The reason is that the expected quantity of output is a function of the standard error as well as the mean of the uncensored distribution (see Appendix 4). $E(X_i) = F(\frac{g_i(\beta)}{p_i\omega_i})\,\frac{g_i(\beta)}{p_i} + \omega_i f(\frac{g_i(\beta)}{p_i\omega_i})$. If the standard errors of the error terms on the value form are a constant σ_i, then on the quantity form they are σ_i/p_i (corresponding to ω_i). As p_i increases, σ_i/p_i decreases. If this term decreases fast enough, the response of expected output to own price can be negative. Indeed this was so for our estimates (for details, see Strauss, 1981a, Chapter 7).

[25] The dummy variable used in the equation reported here was set to one if the household lived in EA 13 and zero otherwise. Using this variable resulted in a considerably higher log-likelihood value than use of an alternative north-south regional dummy (see Strauss, 1981a, for details).

[26] The palm products produced by the sample households came almost entirely from wild oil palm trees (Spencer, Byerlee, and Franzel, 1979, p. 30).

[27] A complication arises because our study uses sales prices when estimating the production system, and a weighted average of sales and purchase prices when estimating the consumption system. Using superscripts of c for weighted consumption prices and s for sales prices, we have:

$\frac{\partial X_i^c}{\partial p_j^c} = \left.\frac{\partial X_i^c}{\partial p_j^c}\right|_{d\pi=0} + \frac{\partial X_i^c}{\partial \pi}\,\frac{\partial \pi}{p_j^s}\,\frac{\partial p_j^s}{\partial p_j^c}$. We need to make some assumption about $\frac{\partial p^s}{\partial p^c}$.

What relationship would hold over time is unclear because it depends partly on the source of the price changes, i.e., shifts in the supply schedule of marketing services versus autonomous increases in retail demand. What little evidence is in our data is inconclusive. Since an assumption must be made, it was assumed that sales prices and purchase prices are proportional. Two reasons can be offered for making this assumption. First, our entire analysis assumes fixity of firm capital and total land. This is a short- or medium-run situation. In such a short time period, it should be less likely that the marketing service's supply schedule is horizontal than for the long run. That is, one would expect some upward slope of this supply schedule for the time horizon considered here. The second reason is that the elasticity calculations which follow will be much more understandable if both weighted consumption and sales prices move by the same infinitesimal percent. This would not be true if we assumed a constant marketing margin. For instance, the mean ratio of consumption to sales price for root crops and other cereals is 3.8. The meaning of this ratio for our purposes is that if we alternatively assume a constant marketing margin, then a one percent increase in consumption price would mean that sales price increases by more than one percent. For root crops and other cereals, an increase of one percent in average consumption price for the middle expenditure group would imply a 5.5 percent increase in sales price for that group. This would result in a rather large profit effect. Worse yet, the percentage increases in sales prices would be different for different groups so that reading a table of profit effects as elasticities would be quite misleading.

[28/] The fact that we have a production block in the household-firm model makes no difference. If we estimate the latter as a system of value of input demands, output supplies, and profit equation, then summing error terms also results in a singular covariance matrix since profits equal the value of supply less the value of inputs. We then have a large system with two blocks each having one redundant equation. Barten's result on a single system applies to this situation also, so that maximum likelihood estimates are invariant to which equation is dropped in each of the subsystems.

[29/] The Jacobian of the transformation of disturbances into dependent variables is one.

[30/] Remembering that production parameters enter into the demand system through π, but not vice versa.

[31] One can use a Lagrange multiplier test which only requires restricted parameter estimates to be used.

[32] Eigenvalues of the information matrix were used to check for local optima. At a function maximum, these should all be positive.

[33] In an unrestricted maximum likelihood estimation, these values will change every iteration as parameter values, and hence, fitted values change.

[34] There were a few negative fitted values for all 138 observations. This is troublesome, but so are the solutions. We might have constrained fitted values to be positive. In our estimation, however, judging from the experience of estimating the unconstrained maximum likelihood version weighting by fitted values (actually their absolute values), we would have gotten caught on an edge of the illegal negative space. Alternatively, we might have used a Tobit procedure. However, this involves numerically evaluating multiple integrals, a very expensive procedure which would have necessitated aggregating commodities a good deal more than we did. In the raw data, there are a very few zero values for expenditures, the most being five for oils and fats, and some small negative values reflecting either errors in the data or net withdrawal from storage over the year.

[35] There is a problem in using the CET output aggregator in representing this behavior; however, the alternatives raise even more severe problems. The problem with the CET is that the marginal rate of transformation is infinite between a good not being produced and one which is (see page 12). Hence, a profit-maximizing firm would always produce an infinitesimal amount of every output. This characteristic is shared by other output aggregators which rely on few parameters; for instance, the transcendental (Mundlak, 1963). An alternative might be to use a flexible form. For instance, one might assume a translog profit function. However, then one would estimate share equations, which must add to unity. Hence, the error terms must add to zero so they cannot be assumed to be independent (see page 68). Another alternative might be a general quadratic production function, but in this case the number of parameters to be estimated is a multiple of the square of the number of specified outputs. This might result in the Tobit estimation being prohibitively expensive.

[36] Since $g_i(\beta)$, from equation 3.2, is necessarily positive, the probability that $y_i^* > 0$ is $\geq .5$. This is different from the usual Tobit model. One could get around this by letting $y_i^* = g_i(\beta) + \mu_i + \varepsilon_i$, where μ_i is a constant to be estimated. However, this would add seven parameters to be estimated and was thus not tried. One reason why excluding these parameters might not be detrimental to our results is that when evaluated at the sample average for independent variables the probability of having positive production is an estimator of the sample proportion with positive production, which is always over half.

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