Resolving Expected Utility Paradoxes

Lindon J. Robison Robert S. Shupp
Michigan State University

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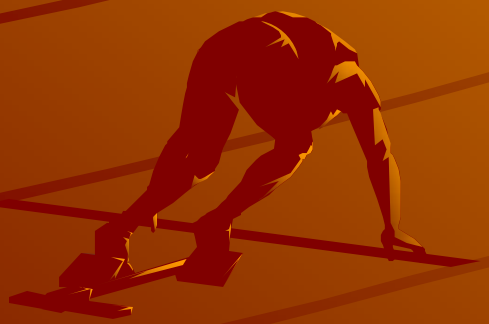
The Expected Utility Model

$$\underset{\alpha}{Max}\, EU_i[\pi(\tilde{x};\alpha)dF(\tilde{x})]$$



The problem:

- The EU model doesn't work very well.
- At the individual level EU maximization is more the exception than the rule (Schoemaker)
- Alternatives don't work well either
- Problems illustrated with equivalency frames

Can Social Capital Theory Help?

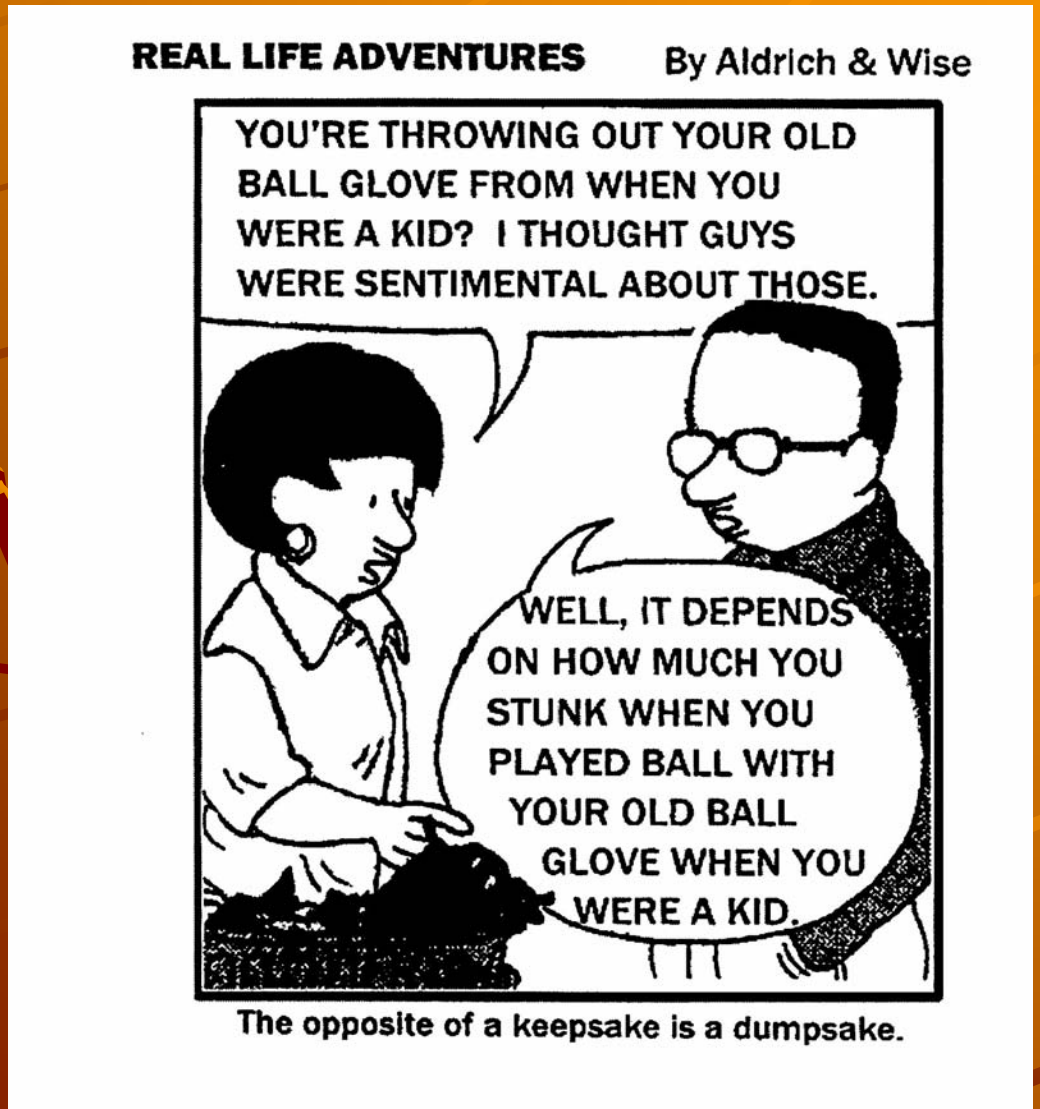
- Terms and level of trade altered by relationships
- Relationships produce intangible goods called socio-emotional goods (SEGs).
- SEGs are intangible goods that validate, express caring, or provide information that provide direction or increase self-awareness and self-regard (Robison and Flora 2003).
- SEGs have value because we all have socio-emotional needs.

Socio-Emotional Goods

SEGs and Attachment Values

- When the exchange of SEGs becomes associated with or embedded in a symbol, event or object, the value and meaning of that symbol, event or object is changed.
- This changed value or meaning of the good, attributed to embedded SEGs, is the object's attachment value (Cordes, et. al., 2003).
- Goods whose value is primarily its attachment value are called high attachment value goods (AVGs).

Attachment Value Goods (AVGs)

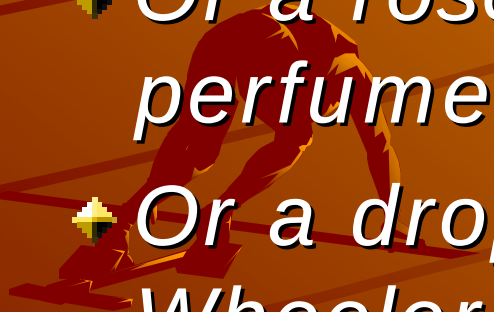


AVGs and Word Frames

- Words can become embedded with SEGs and acquire attachment value.
- Recognizing that words can become embedded with SEGs, they are carefully chosen by advertisers.
- Word frames represent a decision maker's conception of a choice.
- Frames in communication relay information.
- Word frames can become embedded with SEGs and acquire attachment value.

Attachment value of words

- *Oh, a word is a gem, or a stone, or a song,*
- *Or a flame, or a two edged sword,*
- *Or a rose in bloom, or a sweet perfume,*
- *Or a drop of gall, is a word. (Ella Wheeler Wilcox)*



Word Frames and AVGs

- We have strong evidence that alternative word frames can alter choices
- Surveys reveal the importance of word choices
- We hypothesize that the process involves embedding SEGs in words (symbols) to create attachment values
- So we intend to apply this theory to resolve important EU model paradoxes

Word Frames and Paradoxes

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed.

- If Program A is adopted, 200 people will be saved. [72%]
- If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved. [28%]

Word Frames (contd.)

Then subjects were then offered logically equivalent program C and D:

- If Program C is adopted 400 people will die. [22%]
- If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die. [78%]

The Social Expected Utility Model

$$\underset{\alpha}{Max}\, EU_i\{[\pi(\tilde{x};\alpha)dF(\tilde{x})] + s[\pi(\tilde{x},\alpha), w]\}$$

$s(\pi, w)$ refers to the SEGs experienced by the ith decision maker
triggered by the symbol, event or word

The SEU model recognizes explicitly that we process information logically and emotionally.

Paradoxes Resolved

- Allais paradox
- Insurance paradox
- Ellsberg paradox
- Cash segregation paradox
- Coalescing paradox
- Prospect theory and risk preferences for losses
- Violations of SSD

Allais Paradox (Two Stage Gamble)

$$U(\pi_{CE}) = .5[U(\pi_l) + U(\pi_h)]$$

After transformations that should leave decision makers indifferent we obtain

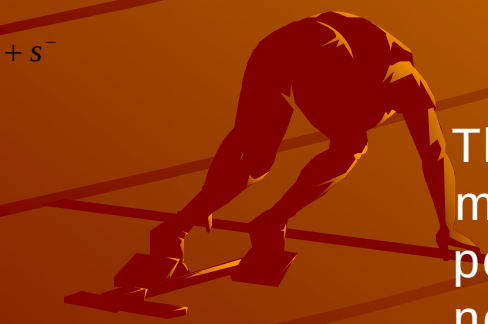
$$.5[U(\pi_{CE}) + U(\pi_h)] = .25U(\pi_l) + .75U(\pi_h)$$

Contrary to EU model predictions, decision makers are no longer indifferent—most preferring the r.h.s.

SEU model resolution

$$U(\pi_{CE}) + s^{+}(\text{sure gain}) = .5[U(\pi_l) + U(\pi_h) + s^{-}(\text{lottery})$$

$+ s^{-}$



The transformation was applied to the above model and was not the same on both sides. A positive sure gain SEG was replaced with a negative lottery SEG

Resolving the Allais Paradox

- Transformation replaces a SEG with a socio-emotional bad on the l.h.s.

$$.5[U(\pi_{CE})+U(\pi_h)]+s^{-}(\text{gamble}) \prec$$

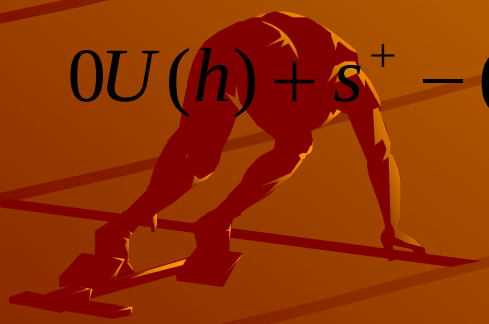
$$.25U(\pi_l)+.75U(\pi_h)+s^{-}(\text{gamble})$$

The Insurance Paradox

- The paradox consists of persons facing a "harm"
  - Compare the willingness to pay for reducing the probability of harm from 2% to 1%
  - Versus the willingness to pay for reducing the probability of harm from 1% to 0%
- Persons willing to pay much more to reduce the probability of harm to 0% even those in both cases the probability of harm is reduced by 1%

Insurance paradox resolved by comparing the SEU model for the two choices

$$.01U(h) + s^- - (.02U(h) + s^-) = -.01U(h) > 0$$



$$0U(h) + s^+ - (.01U(h) + s^-) = -.01U(h) + (s^+ - s^-) > 0$$

Ellsberg Paradox

- An urn contains a thoroughly mixed combination of 30 red balls and 60 either black or yellow balls. The proportion of black versus yellow balls is not known. In the first stage, decision makers are offered a lottery with the following outcomes.

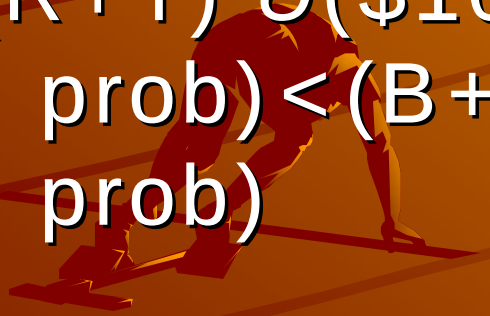
Gamble 1	Gamble 2
Draw red win $100	Draw black win $100

Gamble 3	Gamble 4
Draw red or yellow win $100	Draw black or yellow win $100

Resolving the Ellsberg Paradox

R $U(\$100) + s^{+}$ (known prob) > B $U(\$100) + s^{-}$ (unknown prob)

(R+Y) $U(\$100) + s^{-}$ (unknown prob) < (B+Y) $U(\$100) + s^{+}$ (known prob)



Symbols and Attachment Value

- Symbols as well as word can acquire attachment value.
- Think of religious symbols.
- Symbols representing dollar gains or losses can acquire attachment value.
- $100 versus -$100

Explaining Risk Preferences for Losses

$$\pi > \delta > \pi_h > \pi_m > \pi_l > 0$$

$$\pi + \pi_l = \pi_h + \pi_m$$

$$p(U(\pi) + U(\pi_l)) < p(U(\pi_h) + U(\pi_m))$$

$$U(\delta) + p(U(\pi - \delta) + U(\pi_l - \delta)) < U(\delta) + p(U(\pi_h - \delta) + U(\pi_m - \delta))$$

Explaining risk aversion concluded

$$p(U(\pi-\delta)+U(\pi_l-\delta))+s^+ + s^- >$$

$$p(U(\pi_h-\delta)+U(\pi_m-\delta))+s^- + s^-$$

Cash Segregation Paradox

Table 5. Resolving the Cash Segregation Paradox		
Probabilities	Alternative A	Alternative B
.1	$44	$98
.1	$40	$10
.8	$2	$2

Table 6. Lotteries altered by subtracting an amount from each outcome that is then awarded decision makers as a risk free endowments.

probabilities	Choice A - $25 + $25 endow-ment	Choice B - $25 + $25 endow-ment	Choice A - $50 + $50 endow-ment	Choice B - $50 + $50 endow-ment	Choice A - $100 + $100 endow-ment	Choice B - $100 + $100 endow-ment
.1	19	73	-6	48	-56	-2
.1	15	-15	-10	-40	-60	-90
.8	-23	-23	-48	-48	-98	-98
Preferred by	65% preferred A to B		76% preferred B to A		A and B were equally preferred	

Resolving the cash segregation paradox

- Negative and positive incomes are symbols embedded with SEGs and bads

Table 7. SEGs associated with lotteries described in Table 6.

Choice A - $25 + $25 endowment	Choice B - $25 + $25 endowment	Choice A - $50 + $50 endowment	Choice B - $50 + $50 endowment	Choice A - $100 + $100 endowment	Choice B - $100 + $100 endowment
s^+	s^+	s^-	s^+	s^-	s^-
s^+	s^-	s^-	s^-	s^-	s^-
s^-	s^-	s^-	s^-	s^-	s^-
65% preferred A to B		76% preferred B to A		A and B are equally preferred	

Conclusions

- The attraction of the SEU model is the wide variety of paradoxes that it resolves and the large number of experiments whose results are consistent with its prediction
- Resolving paradoxes associated with the EU model provides us increased confidence in our ability to explain and predict choices made under uncertainty

Conclusions (contd.)

- SEGs and AVGs can explain other paradoxes.
- Endowment effect experiments have already demonstrated how owning an object can alter preferences (Kahtneman, Knetsch, and Thaler, 1990).
  - We hypothesis that the endowment effect is one more example of how embedding objects with SEGs can alter preferences.
  - “Existence values may also be explained by embedding SEGs in objects.

Limitations

- One practical limitation of including SEGs in the EU model is obtaining estimates of their values.
- Theoretically it is possible to obtain estimates of SEGs embedded in word frames, but it is work in progress.
- At this point, it serves mostly to guide to applications of EUH.