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On the (Mis)Use of Wealth as a Proxy for Risk Aversion

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Introduction

- Empirical tests of contractual risk sharing often rely on wealth as a proxy for risk aversion.

- Intuition:

  1. Risk sharing is monotonic in the coefficients of absolute and relative risk aversion of the principal and the agent; and

  2. These coefficients are monotonic in the wealth levels of the principal and the agent; so

  3. There is no harm in using wealth as a proxy for risk aversion.

- Final utility, however, is defined over *wealth plus income from the contract* rather than only on income from the contract (Menezes and Hanson, 1970; Zeckhauser and Keeler, 1970; Meyer and Meyer, 2005; and Guo and Ou-Yang, 2006), i.e., wealth affects the optimal contract through more than just the principal and the agent's coefficients of absolute or relative risk aversion.

- Consequently, this paper shows that tests of contractual risk sharing are unidentified when they rely on wealth as a proxy for risk aversion, as in Laffont and Matoussi, 1995; Ackerberg and Botticini, 2000 and 2002; Dubois, 2002; and Fukunaga and Huffman, 2009.

Outline

1. Principal-Agent Model with Exogenous Wealth

2. Characterization of Result

3. Simulation-Based Example

4. Conclusion and Implications for Future Research

Principal-Agent Model with Exogenous Wealth

- Principal: utility function $V(\cdot)$, with $V' > 0$ and $V'' \leq 0$

- Agent: utility function $U(\cdot)$, with $U' > 0$ and $U'' < 0$

- The principal hires the agent to produce output $q \in Q \equiv [\underline{q}, \overline{q}]$, which is linked to agent effort $e \in E$ through the conditional pdf $f(q|e)$.

- The agent's utility from the contract is additively separable in his utility from the contract and his cost of effort $\psi(e)$, with $\psi' > 0$ and $\psi'' > 0$.

- The principal maximizes by offering a contract $w(q)$ to the agent.

- The *ex ante* wealth levels of the principal and the agent are z_p and z_a, respectively.

- Thus, the principal's utility is defined over $z_p + q - w(q)$ and the agent's utility is defined over $z_a + w(q)$.

- Finally, the agent's reservation utility is $\overline{U} = U(z_a)$.

The principal solves the following maximization problem:

$$(1) \quad \max_{\{w(q),e\}} \int_Q V[z_p + q - w(q)] f(q|e) dq \text{ subject to}$$

$$(2) \quad \int_Q U[z_a + w(q)] f(q|e) dq - \psi(e) \geq U(z_a) \text{ (IR)}$$

$$(3) \quad e \in \arg\max_{\widehat{e} \in E} \left[\int_Q U[z_a + w(q)] f(q|\widehat{e}) dq - \psi(\widehat{e}) \right] \text{ (IC).}$$

Assuming that the agent's maximization problem has a unique solution and that both the MLRP and CDFC hold, one can apply the first-order approach (Rogerson, 1985) and replace the IC constraint by its first-order condition, such that IC becomes

$$(4) \quad \int_Q U[z_a + w(q)] f_e(q|e) dq - \psi'(e) = 0 \text{ (IC')}.$$

In what follows, we assume that the effort of the agent is fixed at the optimum of the agent's maximization problem.

Setting up the Lagrangian and solving by differentiating inside the integral sign with respect to contract $w(q)$ yields

$$(5) \quad \frac{V'[z_p + q - w(q)]}{U'[z_a + w(q)]} = \lambda(z_a, z_p) + \mu(z_a, z_p)\frac{f_e(q|e)}{f(q|e)},$$

where $\lambda(z_a, z_p)$ and $\mu(z_a, z_p)$ are the multipliers associated with the IR and IC' constraints, respectively, and $f_e(q|e) = \frac{\partial f(q|e)}{\partial e}$.

Let $\widetilde{z}_p = z_p + q - w(q)$ and $\widetilde{z}_a = z_a + w(q)$. Differentiating with respect to q yields

$$(6) \quad \frac{dw(q, z_a, z_p)}{dq} = \frac{\left\{\mu(z_a, z_p)U'^2(\widetilde{z}_a)\frac{d}{dq}\left[\frac{f_e(q|e)}{f(q|e)}\right] - V''(\widetilde{z}_p)U'(\widetilde{z}_a)\right\}}{-V''(\widetilde{z}_p)U'(\widetilde{z}_a) - U''(\widetilde{z}_a)V'(\widetilde{z}_p)}.$$

Let $w_q = \frac{dw(q,z_a,z_p)}{dq}$. Then, $w_q(q, z_a, z_p)$ is the degree of risk sharing (i.e., the slope of the contract, or the incentive power of the contract). In a linear contract $w(q) = aq + b$, $w_q = a$, i.e., the share of the risk that accrues to the agent.

But then, multiplying the previous result by $U'(\widetilde{z}_a)V'(\widetilde{z}_p)/U'(\widetilde{z}_a)V'(\widetilde{z}_p) = 1$ and letting $\theta = \frac{d}{dq}\left[\frac{f_e(q|e)}{f(q|e)}\right]$ yields

$$(7) \quad w_q = \frac{\frac{\mu U'(\widetilde{z}_a)\theta}{V'(\widetilde{z}_p)} + A_p}{A_p + A_a},$$

where $A_p = -\frac{V''(\widetilde{z}_p)}{V'(\widetilde{z}_p)}$ and $A_a = -\frac{U''(\widetilde{z}_a)}{U'(\widetilde{z}_a)}$. In other words, one can get the degree of risk sharing expressed in terms of the (absolute) risk preferences of the principal and the agent.

Finally, multiplying the previous result by $\widetilde{z}_a\widetilde{z}_p/\widetilde{z}_a\widetilde{z}_p$ yields

$$(8) \quad w_q = \frac{\frac{\mu U'(\widetilde{z}_a)\theta}{V'(\widetilde{z}_p)}\widetilde{z}_a\widetilde{z}_p + R_p\widetilde{z}_a}{R_p\widetilde{z}_a + R_a\widetilde{z}_p}$$

where $R_p = -\frac{V''(\widetilde{z}_p)}{V'(\widetilde{z}_p)}\widetilde{z}_p$ and $R_a = -\frac{U''(\widetilde{z}_a)}{U'(\widetilde{z}_a)}\widetilde{z}_a$. In other words, one can also get the degree of risk sharing expressed in terms of the (relative) risk preferences of the principal and the agent.

Because w_q is monotonic in the coefficients of absolute and relative risk aversion of the principal and the agent, it is only natural to think that it will also be monotonic in wealth when wealth is used as a proxy for absolute or risk aversion.

Wealth, however, enters w_q through more than just the risk preferences of the principal and the agent: it also enters through the ratio of marginal utilities of the principal and the agent.

But then, monotonicity of w_q in wealth goes out the window because of the trade-off between the marginal utility and the risk preferences of the principal and the agent.

Characterization of Result

Proposition 1 *The effect on risk sharing of a change in the wealth of the agent is always unidentified, and the effect on risk sharing of a change in the wealth of the principal is identified if the principal is risk-neutral or her preferences exhibit CARA, in which case it is equal to zero. Formally, (i) with a risk-neutral principal and a risk-averse agent,* $\frac{\partial w_q}{\partial z_a}$ *is unidentified and* $\frac{\partial w_q}{\partial z_p} = 0$*; and (ii) with a risk-averse principal and a risk-averse agent,* $\frac{\partial w_q}{\partial z_a}$ *is unidentified and* $\frac{\partial w_q}{\partial z_p} = 0$ *if the preferences of the principal exhibit CARA.*

Sketch of Proof: The result for a risk-neutral principal or for a risk-neutral principal has to do with the fact that in both cases, a change in wealth only entails an affine transformation of the principal's utility function, and that such transformations do not change the utility-maximizing contract (Mas-Colell et al., 1995, p. 173).

To characterize $\frac{\partial w_q}{\partial z_a}$, assume that the principal is risk-neutral, the agent is risk-averse, and both sets of preferences exhibit CARA. Then,

$$(9) \quad \frac{\partial w_q}{\partial z_a} = \frac{\frac{\frac{\partial \mu}{\partial z_a} U'\theta}{V'} - \frac{\mu U'\theta}{V'}\left[A_a\left(1 + \frac{\partial w}{\partial z_a}\right)\right]}{A_a},$$

which is unidentified due to the indeterminacy of $\frac{\partial \mu}{\partial z_a}$. The following tables show the general case for absolute and relative risk aversion, respectively.

Table 1. Change in Risk Sharing as Wealth Levels Change: Absolute Risk Aversion

(9) $$\frac{\partial w_q}{\partial z_a} = \frac{\left\{\frac{\mu_a U'\theta}{V'} - \frac{\mu U'\theta}{V'}\left[A_a + w_a(A_p + A_a)\right] - A_p(A_p - P_P)w_a\right\}}{(A_p + A_a)} - \frac{\left\{A_a(A_a - P_a)(1 + w_a) - A_p(A_p - P_p)w_a\right\}\left(\frac{\mu U'\theta}{V'} + A_p\right)}{(A_p + A_a)^2}$$

(10) $$\frac{\partial w_q}{\partial z_p} = \frac{\left\{\frac{\mu_p U'\theta}{V'} - \frac{\mu U'\theta}{V'}\left[A_p + w_p(A_a - A_p)\right] + A_p(A_p - P_P)(1 - w_p)\right\}}{(A_p + A_a)} - \frac{\left\{A_p(A_p - P_p)(1 - w_p) + A_a(A_a - P_a)w_p\right\}\left(\frac{\mu U'\theta}{V'} + A_p\right)}{(A_p + A_a)^2}$$

Table 2. Change in Risk Sharing as Wealth Levels Change: Relative Risk Aversion

$$\frac{\partial w_q}{\partial z_a} = \frac{\left\{\frac{\mu_a U'\theta\tilde{z}_a\tilde{z}_p}{V'} - \mu\theta\tilde{z}_a\tilde{z}_p\left[A_a + w_a(A_p + A_a)\right] + \frac{\mu U'(1+w_a)\theta\tilde{z}_p}{V'} - \frac{\mu U'\theta\tilde{z}_a w_a}{V'} - A_p(A_p - P_P)w_a\tilde{z}_a\tilde{z}_p + -A_p w_a\tilde{z}_a + A_p\tilde{z}_p(1+w_a)\right\}}{(R_p\tilde{z}_a + R_a\tilde{z}_p)}$$

(11)

$$-\frac{\left\{-A_p(A_p - P_P)w_a\tilde{z}_a\tilde{z}_p + -A_p w_a\tilde{z}_a + A_p\tilde{z}_p(1+w_a) + A_a(A_a - P_a)(1+w_a)\tilde{z}_a\tilde{z}_p + A_a(1+w_a)\tilde{z}_p + A_a\tilde{z}_a w_a\right\}\left(\frac{\mu U'\theta\tilde{z}_a\tilde{z}_p}{V'} + R_p\tilde{z}_a\right)}{(R_p\tilde{z}_a + R_a\tilde{z}_p)^2}$$

$$\frac{\partial w_q}{\partial z_p} = \frac{\left\{\frac{\mu_p U'\theta\tilde{z}_a\tilde{z}_p}{V'} - \mu\theta\tilde{z}_a\tilde{z}_p\left[A_p + w_p(A_a - A_p)\right] + \frac{\mu U'\theta w_p\tilde{z}_p}{V'} + \frac{\mu U'\theta\tilde{z}_a(1+w_p)}{V'} + A_p(A_p - P_P)(1-w_p)\tilde{z}_p\tilde{z}_a + A_p(1-w_p)\tilde{z}_a + A_p\tilde{z}_p w_p\right\}}{(R_p\tilde{z}_a + R_a\tilde{z}_p)}$$

(12)

$$-\frac{\left\{A_p(A_p - P_P)(1-w_p)\tilde{z}_p\tilde{z}_a + A_p(1-w_p)\tilde{z}_a + A_p\tilde{z}_p w_p + A_a(A_a - P_a)w_p\tilde{z}_a\tilde{z}_p + A_a w_p\tilde{z}_p + A_a\tilde{z}_a(1-w_p)\right\}\left(\frac{\mu U'\theta\tilde{z}_a\tilde{z}_p}{V'} + R_p\tilde{z}_a\right)}{(R_p\tilde{z}_a + R_a\tilde{z}_p)^2}$$

Simulation-Based Example

We consider a discrete, two-outcome version of the general problem described above, so that $q \in \{\underline{q}, \overline{q}\}$ with respective probabilities $1 - P(e)$ and $P(e)$. Solving for the constrained optimal output-contingent contract levels $\underline{w}$ and $\overline{w}$ yields

$$
\begin{aligned}
(\underline{w}^*, \overline{w}^*) &= \arg\max V[z_p + \overline{q} - \overline{w}]P[e(\underline{w}, \overline{w})] \\
&\quad + V[z_p + \underline{q} - \underline{w}]\left\{1 - P[e(\underline{w}, \overline{w})]\right\}, \qquad (10)
\end{aligned}
$$

subject to the agent's IR and IC' constraints. In this case, $w_q = \frac{\Delta w}{\Delta q} = \frac{\overline{w} - \underline{w}}{\overline{q} - \underline{q}}$ measures the degree of risk sharing.

Table 1. Simulation Parameterization

$V(z_p) = -\exp\{-A_p z_p\}$
$U(z_a) = -\exp\{-A_a z_a\}$
$A_p = 1$
$A_a = 1$
$\underline{q} = 0$
$\overline{q} = 1$
$P(e) = 1 - \exp\{-1.5e\}$
$\psi(e) = 0.005e^2$
$e \in [0, +\infty)$

In short, we assume that both the principal and the agent are risk-averse and that both sets of preferences exhibit CARA.

To determine how changing the wealth of the agent should affect risk sharing, we numerically solve for $e(\underline{w}, \overline{w})$ given a contract $(\underline{w}, \overline{w})$ in the IC constraint and then substitute the optimal effort into the IR constraint and the principal's objective function.

In other words, we solve by backward induction for the usual sub-game perfect Nash equilibrium in the principal-agent model.

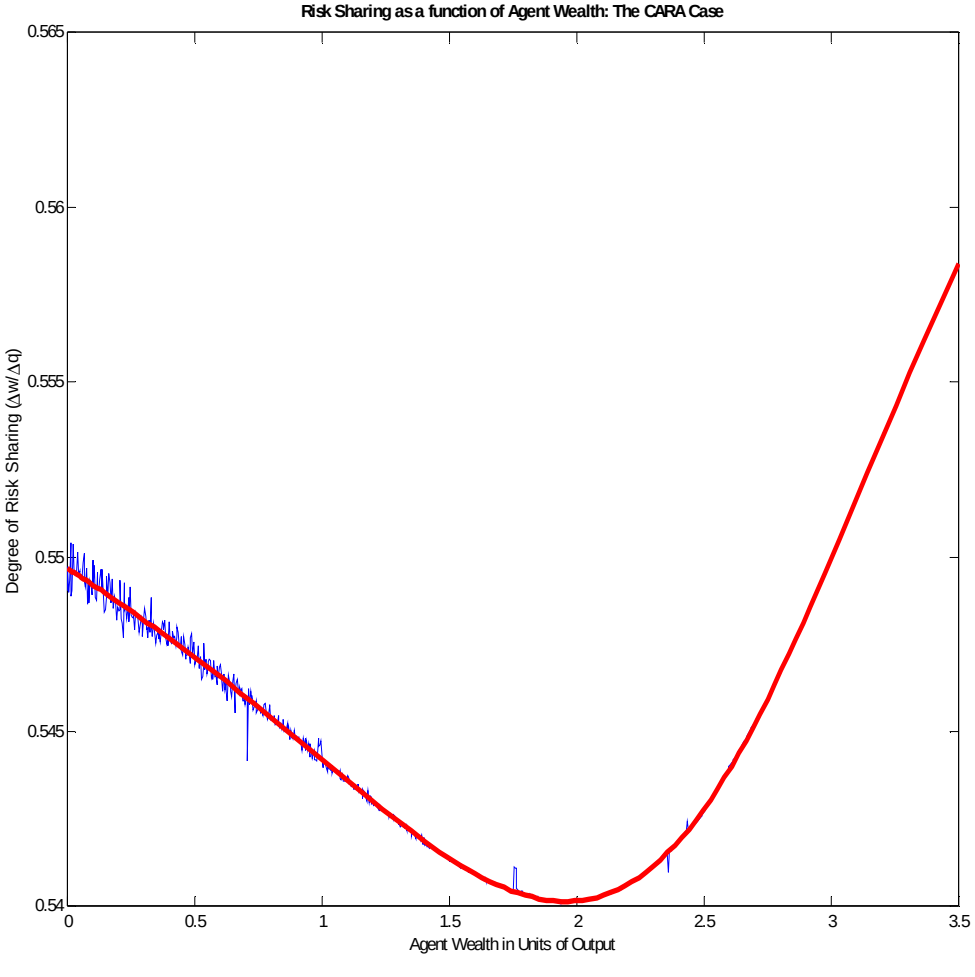

Figure 2. Risk Sharing as a Function of Agent Wealth under Constant Absolute Risk Aversion.

Conclusion

- The direction of the change in optimal risk sharing as the wealth of either party changes is rarely identified.

- Even it is identified, the statistical test has low power given that one should expect non-rejection of the null hypothesis.

- In order to properly run tests of risk sharing, then, two options are available:

  1. Use the coefficients of absolute or relative risk aversion themselves, keeping in mind the caveat in Lybbert and Just (2007); or

  2. Impose more structure on the principal-agent, which may eventually require the use of structural methods (Just 2008; Keane, 2009).