Determining Socially Optimal Nitrogen Application Rates Using a Delayed Response Model: The Case of Irrigated Corn in Western Kansas

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Abstract

A delayed response model was used to examine the optimal nitrogen application for irrigated corn in western Kansas. Results show that taking into account the effect of leached nitrate on groundwater pollution reduces the profit-based nitrogen recommendation by 12.5% with a consequent reduction of the static profit of 6.7%.

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Introduction

Groundwater contamination by nitrate is an important problem in Kansas because the state depends on groundwater more than any other state in the U.S. Roughly 51% of 2.3 million Kansans depend on groundwater for drinking, irrigation and/or industrial use (Buchanan and Buddemeier). A 1996 survey of 833 public water supply systems in Kansas showed that 4% of the systems serving 1.3% of the Kansas population were contaminated with nitrates above the maximum contaminant level (MCL) of 10 ppm (KDHE). Bukart and Kolpin's survey of wells in Midwestern and northern states showed that nitrate contamination of groundwater is greater in areas using irrigation, as is the case in Western Kansas.

Fertilizer recommendations in the Great Plains are based on yield goals and nitrate concentration in the soil profile before planting (Vanotti and Bundy, 1994a, b). However, it is not clearly shown by Vanoti and Bundy and Schlegel et al. how the soil profile nitrate concentration is used to determine an optimal N application rate using a method that is easy to apply from one site to another.

The objective of this study is to determine the socially optimal N application rates and soil profile nitrate. The study uses a delayed response model proposed by Kim et al. This research is an extension of a study by Schlegel et al., which conducted a 30 year long fertilizer experiment to determine the optimal level of nitrogen for irrigated corn in Western Kansas. The study by Schlegel et al. did not account for the social costs of N being leached into ground water.

Model

Nitrate is a stock pollutant, i.e. it accumulates and degrades over time. A number of factors influence transport of nitrates from the unsaturated zone into groundwater. Among them are precipitation and climate in general, soil type, sub-surface geology, land use and management strategies, and intensity of nitrogen use (Keeney). Because many factors impact nitrate transportation into groundwater, it is it difficult to formulate a tractable nitrate transport model. Conrad and Olson modeled mass transport of aldicarb, a stock pollutant with mass transport characteristics similar to nitrates, by making a number of simplifying assumptions. Following Conrad and Olson, the change in total contamination of groundwater by nitrate at time t is:

$$\dot{C}_t = (1-g)hN_{t-k} - dC_t, \quad \forall t \ge k \tag{1}$$

where C_t is the concentration of Nitrate in groundwater at time t; (is the rate of nitrogen loss due to volatilization and runoff; N_t is the nitrogen applied (lbs/acre) at time t; 0 is the proportion of applied nitrogen that is converted into nitrate pollutant; k is the time lag in years which measures the delay from fertilizer application to the time the leached nitrate actually reaches groundwater; and * is the degradation rate of nitrate in the groundwater due to denitrification. Equation 1 is linear and can be empirically estimated by ordinary least squares (OLS) methods if appropriate data is available.

Following Conrad and Olson, a quadratic cost function for nitrate contamination is used in this research. A quadratic function implies that the effect of the pollutant increases with an increase in its concentration. Schlegel et al. used a quadratic corn production function and that same function will be used in this research to facilitate comparison. Consider that society's goal is to manage the contamination of groundwater. It is important to consider the time lag (k) between application of nitrogenous fertilizer and entrance of leached nitrate into groundwater. Kim et al. show that if the time lag of nitrate transport is ignored, too much pollution results even at higher fertilizer tax. The time lag complicates the management of fertilizer application because concentration of nitrate in groundwater (C_t) is felt t + k years later. The time lag from when nitrogenous fertilizers are applied to the time the concentration of nitrate in groundwater responds divides the evolution of nitrate stock into 2 time periods: $0 < t_1 < k$ and $k \# t_2 < T$. Hence, the maximization problem can be solved recursively in two stages, one for each time period, t_1 and t_2. Each stage has a state equation for C_t (Kim, et al.). This model is called a delayed response model because the externality created by nitrogen application does not occur immediately.

We will maximize social welfare (SW) in stages 1 and 2 subject to the state equations in both stages, initial nitrate concentration (C_0) and the final nitrate concentration (C_T) in groundwater.

$$\text{Max SW} = \int_0^k e^{-rt}[(p(a + bN_t - dN_t^2) - fN_t - wh_t) - q\, C_t^2)]dt \; +$$

$$\int_{t>k}^{T} e^{-rt}[(p(a + bN_t - dN_t^2) - fN_t - wh_t) - q\, C_t^2]dt \qquad (2)$$

s.t.
Nitrate concentration growth in groundwater:

$$\dot{C}_t = h\,N_0(1-g) - d\,C_t \qquad \forall\, 0 \le t < k \qquad (3)$$

$$\dot{C}_t = (1-g)hN_{t\text{-}k} - d\,C_t \qquad \forall\, k \le t < T \qquad (4)$$

$$\text{Initial and Terminal Conditions}: C_0 < MCL; \qquad C_T = MCL \qquad (5)$$

where r is the discount rate; a, b, and d are constants of the production function; p is the constant price of corn; h_t is a vector of quantities of inputs other than nitrogenous fertilizers; w is a vector of constant prices for h_t; f is the constant price of nitrogen; and 2 is the marginal social cost of nitrate pollution of groundwater. Other symbols are as defined in equations and text above.

A direct estimation of 2 is difficult to obtain, because data relating the N application rate with the consequent social damage of nitrate are unavailable. However, assuming a steady state equilibrium $C_t^* = MCL$, causing 2 to equal: [1]

$$\hat{q} = \frac{h\,(r+d)(pb - f) - 2d\,dp(r+d)C_t^*}{2h^2\,C_t^*} \qquad (6)$$

where the notation is defined above. Equation 6 can be estimated empirically.

Data

Schlegel et al. using the OLS method estimated coefficients for the production function. The data for the OLS estimation was obtained from a long-term fertilizer trial at the Tribune, Kansas Experiment Station. In this study, nitrogen and phosphorus

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[1] Equation 6 is solved from the first order conditions of the current Hamiltonian of the general function:

$$\text{Max } SW = \int_0^\infty W[N(t), C(t)]dt$$

s.t.

$$\dot{N}_t = 0, \quad \dot{C}_t = hN_{t\text{-}k} - dC_t$$

Symbols are as defined above.

fertilizers were combined in a factorial experiment. Two levels of phosphorus, 0 and 40 lbs/acre were combined with 6 levels of nitrogen: 0, 40, 80, 120, 160 and 200 lbs of nitrogen per acre. Table 1 reports the OLS coefficients, prices and costs of production used in the dynamic model of this research.

Previous work shows that the time lag between application of nitrogen fertilizer to actual contamination of groundwater by the resulting nitrate is between 30 – 60 years in Southern California (Pratt) and 20 years in Buffalo Nebraska (Bentall). Most of the Midwestern states have soils formed from glacial deposits. In such soils, it is estimated that the impact of excess application of nitrogen is reflected into groundwater in 30 – 40 years (Hallberg). Therefore, in this research, the travel time of nitrate from the vadose zone to groundwater was assumed to be 35 years, which is the median of the range given by Hallberg for Midwestern States.

Table 1: Coefficients, Price and Cost Data Used in the Dynamic Model

Parameter estimates	With Phosphorus	Without Phosphorus
Constant	94.2716	70.7998
N	1.1007	0.7416
N^2	-0.0033	-0.0023
Prices and Costs		
Corn, p ($/bushel)	2.62	2.62
N Fertilizer, f ($/lb of N)	0.17	0.17
Production cost, W* ($/acre)	200.00	178.00

* Total Cost other than N fertilizer per acre (includes cost for phosphorus application).
Source: Schlegel, et al., Anon., 1997.

Data on nitrates, which actually leach into groundwater from a given farm, is difficult to measure. Yadav assumed that keeping the concentration of nitrate in the vadose zone at 10 ppm or less on water basis results in a concentration of nitrate in groundwater that does not exceed 10 ppm. The MCL of 10 ppm of nitrate on water basis is equivalent to about 50 lbs of nitrate per acre-foot of soil (soil basis).[2] The soil basis amount of nitrate in the vadose zone is the constraint that is imposed on the profit maximization problem, i.e., the concentration of nitrate in the soil vadose should not exceed 50 lbs/acre-foot of soil at any given time. The travel time from the root to the vadose zone, which represents the groundwater zone in this formulation, is expected to be within one year, implying that k = 0 for the state equation. However, the k value used in the objective function equation is the estimated travel time of nitrate from the vadose zone to groundwater, i.e. k = 35 years. This arrangement ensures that the marginal social cost of nitrate pollution of groundwater is assessed to producers according to the actual time lag of nitrate pollution.

Due to lack of data, we assume that (= 0 implying no loss of nitrogen by volatilization and runoff. From the above discussion and assumptions, the empirical model for estimating equation 4 is:

$$C_{t+1} = hN_t + (1 - d)\, C_t, \quad t \geq k = 0. \qquad (7)$$

[2] Assuming a weight of 1 acre-foot soil is 4 million lbs, water content of soil at field capacity is 30%, weight of cu. ft of water is 62.4 lbs and 1 acre is 43,560 sq. ft. The weight of 1 acre-foot water is 13,068*62.4 or 815,443 lbs. The volume of 1 acre-foot water at field capacity level is 0.3*43,560 or 13,068 cu. ft. Thus we can convert nitrate concentration from, water basis to, soil basis for each 1 acre-foot depth by: 4,000,000/815,443 equals 4.905. Ten ppm of Nitrate concentration in the vadose zone is equivalent to maintaining approximately 49.05 lbs of nitrate per acre-foot of soil on soil basis (Yadav, p. 117).

An OLS method was used to estimate coefficients for equation 7 using data reported by Hooker et al. and Schlegel, et al. Since the use of phosphorus alters the nitrogen use efficiency (Hooker, et al.), two sets of coefficients for equation 7 were estimated, one set with phosphorus and another without.

with phosphorus: $C_{t+1} = 0.14N_t + 0.7C_t$ (8)

and without phosphorus: $C_{t+1} = 0.16N_t + 0.6C_t$. (9)

These results imply that for the with phosphorus scenario, a unit increase in nitrogen applied increases nitrate in the vadose zone by 14%, i.e. $0 = 0.14$. For the without phosphorus scenario, $0 = 0.16$, suggesting that a higher level of nitrate is leached beyond root zone. Yadav and Viets obtained comparable figures.

The General Algebraic Modeling System (GAMS) program was used to estimate the constrained maximization problem specified in equations 2 through 5. Analysis was performed to examine the sensitivity of parameters estimated in equations 8 and 9, the price of corn (p), the price of nitrogen (f) and the time lag (k).

Results

Table 2 shows that farmers who apply phosphorus and desire to limit nitrates available for leaching would reduce nitrogenous fertilizer application by 12.5% from the 160 lbs/acre recommendation (Nelson and Dhuyvetter). This reduction in fertilizer results in a 6.7% reduction in profit from \$150 to \$140 per acre. For farmers who do not use phosphorus, accounting for the externality of nitrogenous fertilizer application leads to a 17.2% reduction in fertilizer and a 9% decline in profit.

Table 2: Private and Social Profit, Optimal N and Leached Nitrate for Irrigated Corn Production, Western Kansas

Model	Annual Profit $/acre[3]	Optimal N applied [4]
	With Phosphorus (lbs/acre)	
Private (Static)[1]	150	159
Social (Dynamic)[2]	140	138
	Without Phosphorus (lbs/acre)	
Private (Static)[1]	45	145
Social (Dynamic)[2]	41	120

1. Estimated by Schlegel, et al.
2. Estimated in this research.
3. Average of 50 year present value of annuity for the social dynamic model
4. At steady state equilibrium for the dynamic model

The dynamic and static results also show that use of phosphorus increases per acre profit by more than 100%. Hence, overwhelming incentives exist for farmers with phosphorus deficient soils to apply phosphatic fertilizers to increase corn yield and hence profit. Phosphorus also reduces nitrate leaching since it increases nitrogen use efficiency (Hooker, et al.). Table 3 shows that the level of leached nitrate in the without phosphorus plots is roughly 3 times the level found in the with phosphorus plots for farmers who apply more than 150 lbs of N/acre.[3] These findings underscore the importance of applying balanced nutrients to increase profit and environmental quality.

Farmers may use methods other than application of phosphorus to increase nitrogen use efficiency, and hence reduce nitrate leaching and probably maintain yield levels. Such methods are the split application of N, planting scavenger crops, and use of

[3] Nitrate is considered leached when it percolates beyond the root zone (0 - 5 ft for corn).

nitrate inhibitors (NI). Split application of N and use of NI are more effective in sandy soils than on finer soils (Hergert and Wiese; Maddux and Barnes). However NI and split applications of N reduces nitrate that leaches beyond the root zone. This implies, the two methods may not have a significant impact on yield on a year by year basis. However, if the benefit of reducing nitrate leaching over a long period of time is accounted for, those two methods may be beneficial.

Table 3: Level of Nitrate After 30 years (1961-1991) of Nitrogen Fertilizer Application

Fertilizer Level (lbs/acre)		Nitrate Level in Soil Profile (lbs/acre)	
Nitrogen	Phosphorus	0 - 5 ft	5 – 10 ft
0	0	7.5	7.5
40	0	7.7	9.2
80	0	10.3	14.7
120	0	45.7	35.7
160	0	71.4	81.4
200	0	94.7	87.5
0	40	7.7	7.6
40	40	7.8	6.2
80	40	6.9	8.7
120	40	10.2	12.3
160	40	17.7	29.9
200	40	29.1	32.3

Note: the nitrate levels were determined from soil samples taken in 1991.
Source: Schlegel, et al.

Table 4 shows the sensitivity of profit, level of nitrate leached and optimal nitrogen level at steady state equilibrium. All 6 parameters were changed by "30% to "67%. The nitrate contamination level constraint of 50 lbs/acre is the most limiting constraint even when the parameters are changed drastically. Consequently the maximum nitrate of 50 lbs/acre is reached over a wide range of parameter values.

However, if value of proportion of applied nitrogen that is converted into nitrate pollutant (0) falls below 0.1, the nitrate contamination constraint of 50 lbs/acre is nonbinding. In actual situations, such values are unrealistic since nitrogen utilization efficiencies of more than 90% are rare in field conditions (Viets). Corn price changes have significant impact on profit, as expected, for both the with and without phosphorus corn production scenarios. In the case of corn production using phosphorus, the per acre profit maintains a tight range of $100 - $144 for a wide range of values of almost all parameters considered. The same pattern is observed for the case of corn production without phosphorus. The optimum level of nitrogen applied in both with and without phosphorus application cases is robust across a wide range of values of marginal social cost of nitrate pollution (2), price of nitrogen (f), price of corn (p) and time lag of nitrate transport (k). This implies that the optimal nitrogen (N*) and nitrate resident in the vadose zone (C*) obtained in this research apply to a wide range of input and output prices. Schlegel et al. also observed a robust optimum nitrogen application rate over a wide range of input and output prices. The results however are sensitive to extreme values of both proportion of nitrogen that is converted to nitrate pollutant (0) and degradation rate of nitrate (*).

Table 4: Sensitivity Analysis of Profit, Optimal Nitrate Leaching and Nitrogen Application to Parameters Changes

Parameter[1]	---- Profit[2] ----		NO3 Contamination[3]		Optimal N Level[3]	
	With phosphorus	No phosphorus	With phosphorus	No phosphorus	With phosphorus	No phosphorus
0	---- Dollars/acre ----		------------------ Lbs/acre -----------------			
0.05	145	51	22	26	172	138
0.1	141	47	49	48	173	138
0.15	140	41	50	50	140	120
0.2	125	40	50	50	101	91
0.25	115	36	50	50	80	74
2						
0.003	139	45	50	50	140	120
0.006	140	41	50	50	140	120
0.009	132	40	50	50	140	120
0.012	129	38	50	50	140	120
0.015	126	35	50	50	140	120
*****						
0.22	124	33	50	50	94	83
0.27	131	40	50	50	116	101
0.32	136	41	50	50	137	120
0.37	138	42	50	50	159	138
0.42	140	44	50	50	173	138
f						
0.07	140.9	46	50	50	140	120
0.12	138.2	44	50	50	140	120
0.17	135.8	41	50	50	140	120
0.22	132.3	40	50	50	140	120
0.27	130.3	38	50	50	140	120
p						
1.62	56.5	7	50	50	140	120
2.12	94.4	24	50	50	140	120
2.62	135.8	42	50	50	140	120
3.12	175.2	60	50	50	140	120
3.62	214.2	78	50	50	140	120
k						
25	135.4	42	50	50	140	120
30	135.4	42	50	50	140	120
35	135.8	41	50	50	140	120
40	136.0	42	50	50	140	120
45	136.1	43	50	50	140	120

1 0 is the proportion of N applied that is turned into nitrate pollutant, θ is the marginal cost of nitrate contamination, δ is the proportion of nitrate that degrades in groundwater, f is the price for N, $/lb, p is the price of corn, $/bushel, and k is the lag time in years from time N is applied to time nitrate reaches groundwater.

2. Average of 50 year present value of annuity

3. Values at steady state equilibrium

Conclusions

For irrigated corn in Western Kansas, taking into account the effect of leached nitrate reduces the private profit-based N application rate recommendation by 12.5% for farmers who use phosphorus and by 17% for farmers who do not use phosphorus. Therefore, agronomists may consider reducing current nitrogen recommendations for irrigated corn for Western Kansas accordingly to limit the nitrogen available for leaching. Private profit loss resulting from reduced nitrogen application is 6.7% for farmers who use phosphorus and 9% for those who do not.

Using some agronomic methods that increase nitrogen use efficiency may reduce the private profit loss. Examples of such methods are application of phosphorus, nitrate inhibitors, split application of nitrogenous fertilizers, and planting scavenger crops after harvest. For instance, applying nitrogen with phosphorus for P-deficient soils in Western Kansas significantly reduces leached nitrate and increases profit by more than 100%.

References

Anonymous, 1997. *Kansas Farm Management and Marketing Handbook*. MF 941, and MF 969. Kansas State University Department of Agricultural Economics Coop. Ext. Serv.

Bentall, R., 1975. *Physiography, Geology, Soils and Agriculture: Nebraska Mid-State Division Report*, Inst. of Agric. and Natural Resour., Univ. of Nebr., Lincoln.

Burkart, M.R. and D.W. Kolpin. 1993. "Hydrologic and Land-Use Factors Associated with Herbicides and Nitrate in Near-Surface Aquifers," *Journal of Environmental Quality*, 22(4):646-656.

Buchanan, R.; R.W. Buddemeier. 1993. Kansas Groundwater. *An Introduction to the State's Water Quantity, Quality and Management Issues,* Kansas Water Office: 2, 11-12, 22-37.

Conrad, J.M. and L.J. Olson. 1992. The Economics of a Stock Pollutant: Aldicarb on Long Island." *Environmental and Resource Economics* 2:245-58.

Hallberg, G.R. 1989. "Nitrate in Groundwater in the United States." In:Follet, R.F. (ed). Nitrogen Management and Groundwater Protection. Elsevier, New York: 58-59

Hergert, G, and R. Wiese. "Performance of Nitrate Inhibitors in the Midwest (West) States," In: Meisinger, J. and G. Randall and M.L. Vitosh. *Nitrification Inhibitors – Potentials and Limitations*, American Society of Agronomists (ASA) Special Publications No. 38, Madison WI, 1980: 96-103.

Hooker, M.L., R.E. Gwin, G.M. Herron, and P. Gallagher. 1983. "Effects of Long-Term, Annual Applications of N and P on Corn Grain Yields and Soil Chemical Properties." *Agronomy Journal* 75(February):94-8.

KDHE (Kansas Department of Health and Environment*). Annual Drinking Water Report for 1996*. Bureau of Water Supply, Topeka Kansas, 1996: 7-8.

Kim, C.S., J. Hostetler and G. Amacher. "Regulation of Groundwater Quality with Delayed Responses," *Water resources Research*, 29(5):1369-77.

Maddux, L.D. and P.L. Barnes. 1996. "Effect of Preplant and Split Nitrogen Applications on Ammonium Nutrition of Corn." In: Lamond, R.E. (ed.), 1996. *Kansas Fertilizer Research*. Agricultural Experiment Station, Kansas State University, Manhattan Kansas: 39-42.

Nelson, M.E. and K. Dhuyvetter. "Center Pivot Irrigated Corn," Kansas State University Farm management Guide MF 585, Manhattan. Kansas Cooperative Extension.

Pratt, P.F. 1984. "Nitrogen Use and Nitrate Leaching in Irrigated Agriculture," In: Hauck, R.D. *Nitrogen in Crop Production*. American Society of Agronomy, Crop Science Society of America, and Soil Science Society of America, Madison WI.

Schlegel, A.J., K.C. Dhuyvetter, and J.L. Havlin. "Economic and Environmental Impacts of Long-Term Nitrogen and Phosphorus Fertilization," *Journal of Production Agriculture*, 9 (1) 1996:114-18.

Steitchen, J., J. Koelliker, and D. Grosh. "Kansas Farmstead Well Survey for Contamination by Pesticides and Volatile Organics," In: *Proc. Agri. Impacts on Groundwater* – A Conference. National Well Association, Dublin OH, 1986:530-541.

Vanotti, M.B., L.A. Bundy. "An Alternative Rationale for Corn Nitrogen Fertilizer Recommendations," *J. Prod. Agric*. 7 (1994a):243-49.

Vanotti, M.B., L.A. Bundy. "Corn Nitrogen Fertilizer Recommendations Based on Yield Response Data," *J. Prod. Agric.* 7 (1994b):249-65.

Viets, F.G. "The Plant's Need and Use of Nitrogen," In: Bartholomew, W.V. and F.F. Clark (eds) *Soil Nitrogen*, ASA Madison WI; Agronomy 1965, 10:503-49.

Yadav, S.N. *Nitrate Contamination of Groundwater in Southern Minnesota: A Dynamic Model of Nitrogen Use*. Ph.D. Thesis, University of Minnesota (Unpublished): 1994: 91