TARGET MOTAD FOR RISK LOVERS[*]

by

Francis McCamley
and
Richard K. Rudel[**]

Present at Western Agricultural Economics Association Annual Meeting
July 11-14, 1999
Fargo, ND

[*]A paper submitted to the Selected Papers Committee for the 1999 meeting of the Western Agricultural Economics Association.

[**]The authors are associate professors of Agricultural Economics at the University of Missouri-Columbia.

Abstract

of

Target MOTAD for Risk Lovers

Although risk analyses of discrete alternatives often identify at least one efficient set for persons who prefer risk, preference for risk is usually ignored when the decision variables are continuous. This paper presents a version of Target MOTAD which can be used when there is preference for risk.

Target MOTAD for Risk Lovers

Agricultural economists use various techniques to evaluate risky alternatives. When the decision variables are continuous, risk neutrality or risk aversion is almost always assumed and mathematical programming is used to find optimal solutions. By contrast, when the number of choices is small, the assumptions about risk preferences are sometimes less restrictive and stochastic dominance criteria are often applied. It is fairly common to identify efficient sets for decision makers who prefer risk as well as for decision makers who are risk averse and/or approximately risk neutral. The recent study by Larson and Mapp is one of several for which this was done.

Friedman and others have suggested that risk seeking behavior should be rare (or limited) in financial or production decision making because it is possible to buy risk at very low prices through gambling. This view is partly supported by empirical studies of risk attitudes. These studies usually find that more decision makers are risk averse than risk loving. Nonetheless, many of them also find that some persons are risk seekers (Tauer; Love and Robison; King and Oamek).

It is appropriate to give more attention to risk aversion than to risk seeking. However, it does not seem reasonable to consider both risk aversion and preference for risk when a few discrete alternatives are feasible but to almost completely ignore preference for risk when the decision variables are continuous. This paper suggests that it is often possible to consider preference for risk even when decision variables are continuous. One way of doing that is to apply a more general version of the Target MOTAD model.

Two Simple Examples

We begin by considering two simple examples which illustrate some, but not all, aspects of the problem of finding Target MOTAD solutions for decision makers who prefer risk. For the first example, there are two equally likely states of nature and two enterprises. Let x_1 and x_2 be (nonnegative) activity levels for the enterprises. The net return received if the first state of nature occurs is

(1) $y_1 = 100x_1 + 40x_2$.

If the second state of nature occurs, a net return of

(2) $y_2 = 80x_1 + 120x_2$

is received. Assume that the sum of x_1 plus x_2 can be no greater than one. These assumptions imply that the set of feasible combinations of y_1 and y_2 is the triangle 0AE in figure 1. Table 1 presents the coordinates of the points labeled in figure 1.

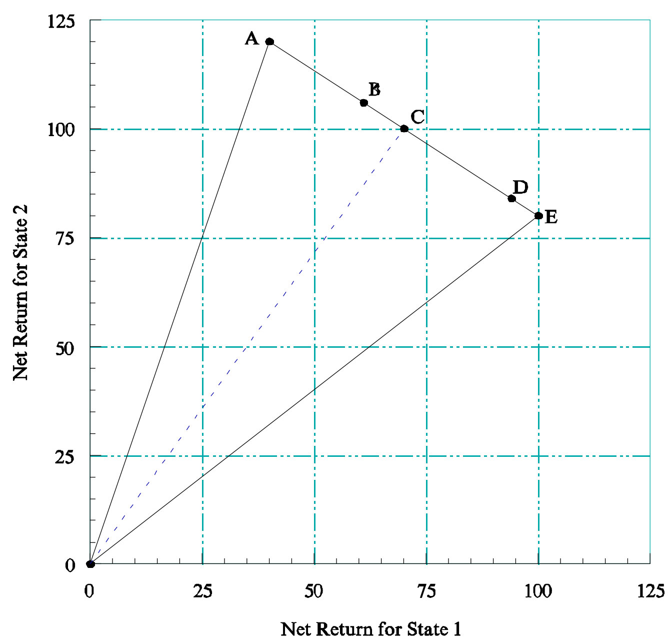

Figure 1. Feasible y Vectors for Simple Examples

Tauer's Target MOTAD model is consistent with the assumption of positive marginal utility of net return. This assumption seems reasonable for risk lovers as well. It means that only net return combinations associated with the line AE need to be considered.

TABLE 1. Selected Feasible Solutions for Simple Example

Label	x_1	x_2	y_1	y_2
			---$1000's--	
0	0	0	0	0
A	0	1.00	40	120
B	.35	.65	61	106
C	.50	.50	70	100
D	.90	.10	94	84
E	1.00	0	100	80

The traditional Target MOTAD model maximizes expected net return subject to an upper limit on (the absolute value of) expected negative deviations (from the Target level). This can be illustrated by assuming a target of 98 and an upper limit of 7 on expected deviations. The optimal solution is D.

It is tempting to adopt an analogous approach for risk preferrers. We might maximize expected returns subject to either a *lower* limit on expected negative deviations or a lower limit on expected positive deviations from the target level. Suppose the target level is 98 and we require expected negative deviations to be at least 18.5. B maximizes expected net return subject to the expected deviations constraint.

Although indirect utility maximization of this sort works well when risk aversion is assumed, there are at least two problems with it when preference for risk is assumed. The first problem is that although linear programming can help find optimal solutions, the approach itself is not a linear program. This difficulty may be unavoidable; it seems to "go with the territory" when preference for risk is assumed.

The second problem is that solutions obtained in this way often do not maximize expected utility for any member of the relevant class of utility functions. B may maximize expected utility for some utility function but it probably does not maximize expected utility function for any utility function consistent with preference *for* risk. It certainly does not maximize expected utility for any utility function belonging to the family of utility functions which is analogous to (and extension of) the family of utility functions consistent with conventional Target MOTAD. B is associated with a *local* maximum of expected utility for the member of the family of utility functions

(3) $U(z) = z + \alpha[\min(0, T - z)]$

for which α equals -1/3. B shares that property many of its neighbors. The "global" (constrained) maximum of expected utility for this utility function is at E. Indeed, for our simple example, only the "corners", A or E, maximize expected utility for (3) when α is smaller than or equal to zero.

A second simple example demonstrates that "corners" are not always the only optimal solutions. The second example is similar to the first. The difference is that the coefficients of x_1 are 70 and 100 rather than 100 and 80. The feasible set is now 0AC. B is now not only maximizes expected utility locally for the utility function (3) when α equals -1/3. It is also a location of the global maximum. It shares that property with all of AC. Thus, non-corners can be optimal but only if all of the corners associated with the line segment or surface are also optimal. For values of α other than -1/3, either A or C is the unique optimal y vector. Note that y_1 is smaller than the target level and y_2 is larger than the target level everywhere on AC. This absence of target "crossing" is a necessary, but not a sufficient, condition for all of an edge or surface to be optimal.

A More General Target MOTAD Model

Consider a more general Target MOTAD model:

(4) Maximize $\Sigma p_i\{y_i + \alpha[\min(0, y_i - T)]\}$

subject to

(5) $Cx - y = 0$

(6) $Ax \leq b$

(7) $x \geq 0$

In this model, summation (in (4)) is over i, p is an s-element column vector of probabilities associated with the various states of nature, y is s-element column vector of net returns associated with the states of nature, α is a risk aversion parameter, min is the minimum function, T is a target return level, C is an s by n matrix of returns associated with the enterprises for the various states of nature, x is an n-element vector of enterprise levels, A is an m by n matrix of resource or technical requirements, b is an m-element column vector of resource or technical levels, n is the number of enterprises, m is the number of resource or technical constraints and s is the number of states of nature. We assume that the set of feasible y vectors is bounded from above.

For positive or zero values of α, the model is consistent with the traditional Target MOTAD model in the sense that any optimal solution of the traditional model is also *an* optimal solution to (4) through (7) for some value of α. Likewise, any optimal solution of (4) through (7) is also *an* optimal solution to the traditional model for some value of λ. Despite the fundamental equivalence of the two models, they appear to be different. For that and other reasons, the traditional Target MOTAD model is likely to continue to be used to examine implications of risk aversion.

This paper is more concerned with negative α values. Note that α must be greater than -1 to ensure that marginal utility is greater than zero for each state of nature. The reader will recognize that (4) through (7) generally cannot be restated as a linear program when α is negative. This fact complicates finding of optimal solutions.

Solution Procedure

Horst and Tuy suggest an iterative approach similar in spirit to, but different in detail from, approaches used to solve linear and concave programming problems which could be used to find optimal solutions. We prefer an approach which involves two or three more or less distinct phases.

The first phase identifies extreme or corner y vectors and also determines which extreme y vectors are "adjacent" to each extreme y vector. A second, optional, phase applies screening criteria. The final phase determines which extreme y vector(s) is (are) optimal for the selected combination(s) of α and T. It also determines whether non-corner solutions are also optimal. These phases are briefly discussed here. More information about them and other details of this study are available from the authors.

One method of identifying the set of extreme or corner y vectors is a multistage branching procedure. Murty (pp. 159-160) outlines a procedure of this type.

Any criterion which will not eliminate an optimal extreme y vector but which might eliminate one or more suboptimal extreme y vectors may be a suitable screening criterion. Three criteria which have these characteristics are vector efficiency, first degree stochastic dominance and (a weak version) of second stochastic inverse dominance (SSID). SSID is described by Zaraś (1987, 1989). Pairwise application of these criteria is usually less effective than applying them in a manner consistent with convex set stochastic dominance.

Another Example

Hazell's data are sometimes used to illustrate risk analysis methods. His data define a small problem which has several interesting features. We assume that each state of nature (year) is equally likely. Thus, each element of p equals one-sixth.

There are eleven extreme y vectors for the Hazell example. The screening step eliminated all but four of them. Enterprise mixtures associated with the "survivors" are presented in table 2. Adjacent extreme y vectors are also noted.

TABLE 2. Enterprise Mixtures Associated with Extreme y Vectors

	Enterprise Mixture				
Mixture Number	Carrots x_1	Celery x_2	Cucumbers x_3	Peppers x_4	Adjacent to
	---acres---				
1		27.45	100.00	72.55	2,4
2	100.00	23.53		76.47	1,7
4		100.00	100.00		1,7
7	100.00	100.00			2,4

For any combination of α and T, the optimal solution can be determined by simply computing the value of the objective function (in (4)) for each of the four surviving extreme y vectors. For example, extreme y vector/enterprise mixture 4 is optimal if α equals -.8 and T equals $81,500.

Figure 2 is included to show how the optimal solution depends on α and T. Point F is the α, T combination just mentioned. The sets of α, T combinations for which any given extreme y vector is optimal are delimited by the solid lines. The numbers in the body of figure 2 identify the optimal extreme y vector. Only α values smaller than or equal to -.6 are shown in the figure. We found that only extreme y vectors are optimal for negative α values. I.e., there are no optimal edge or surface solutions.

Extreme y vector 1 is optimal for all α values from -.6 to zero. In fact, extreme

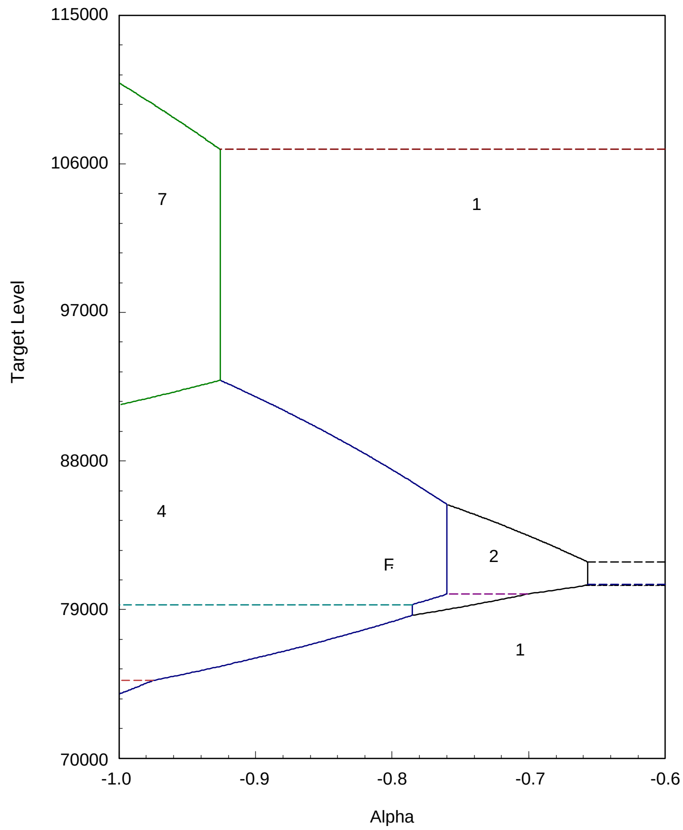

Figure 2. Alpha and T Values for Optimal Solutions

y vector 1 is also optimal for some positive values of α. The right boundary of the region for which extreme y vector 1 is optimal depends on T but at that boundary, α is larger than 1.5 for all values of T. This is consistent with our expectations. We expected that the mixture which maximizes expected returns might also be optimal for at least moderate preference for and moderate aversion to risk.

Extreme y vector 2 is not optimal for risk neutrality but is optimal both for some combinations of α and T associated with preference for risk and for some combinations associated with risk aversion. Although they use a different format than we do, McCamley and Kliebenstein's figure 1 and table 1 confirm the latter. Their line segment GN is associated with all combinations of α and T for which α is between (about) 1.74 and 2.48 and T is between $47,264.71 and $55,629.41.

Extreme y vectors 1 and 2 are unique in this respect. They are the only y vectors which are optimal for both some risk lovers and some risk averters. It easy to verify that (our) extreme y vectors 4 and 7 are not among the infinite number of y vectors included in the "complete set of Target MOTAD solutions" for risk averters.

The implications of risk programming models for marginal resource values (shadow prices) are sometimes of interest. The shadow price of any resource is a function of α and T. Within any subregion delimited by dotted and solid lines in figure 2, the shadow price vector has the form

(8) $(V_0 + \alpha V_\alpha)/(1 + \alpha P(z < T))$.

$P(z < T)$ is the probability that net return is smaller than T and is equal to the sum of the probabilities associated with the states of nature for which net return is smaller than the target level. V_0 is like an intercept vector in a system of regression or other linear equations in that it is not observed unless α can be zero. Since α can be zero for extreme y vector 1, each V_0 vector for extreme y vector 1 is equal to the (usual) shadow price vector associated with the maximization of expected net return. In the largest subregion of α and T values for which extreme vector 4 is optimal, the land component of (8) is $(363.25 + 194.67\alpha)/(1 + 2\alpha/3)$.

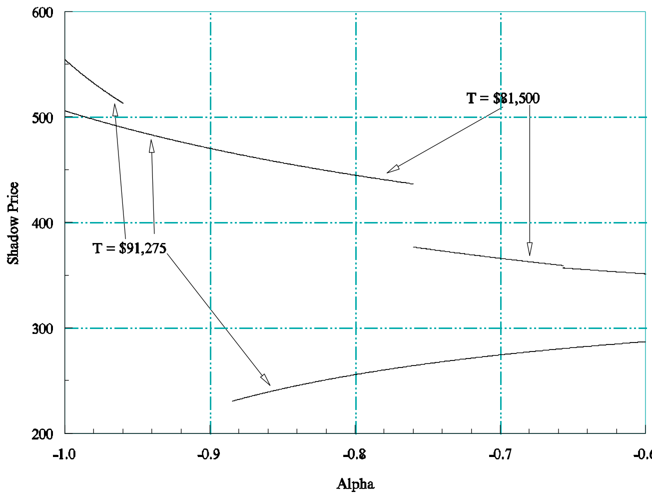

Figure 3. Land Shadow Price Functions for Two Target Levels

This expression is combined with analogous expressions for extreme y vectors 1, 2 and 7 to show how the shadow price of land varies with α for two target levels, $81,500 and $91,275. Note that in figure 3, the land shadow price function is the same for both target levels when α is between -.96 and .88. It should also be apparent that there are discontinuities in the shadow price graphs at α values for which the optimal solution shifts from one extreme y vector to another. This is one of several differences between optimal solutions of (4) through (7) when α is negative and when it is positive. Shadow prices are continuous functions of α when α is positive.

Although shadow prices are not generally not continuous functions of α (for a given T), the total (implied) value of the resources may be. For the target level, $81,500, the total value of the resource is a continuous function of α; for the target level, $91,275, it is not.

Concluding Remarks

We chose to present the optimal primal solutions and the shadow prices as functions of α and T in order to be consistent with our model. Presenting them as functions of λ (the limit on expected negative deviations) would have made them superficially more consistent with the approach used by McCamley and Kliebenstein but also more difficult to interpret than figures 2 and 3.

Many assumptions are made in this paper. Linearity of the constraints (6) is especially critical because it allows a problem with continuous decision variables to (almost) be reduced to a problem with only a finite number of discrete alternatives. The linearity assumption is not always satisfied. However, the tendency to use linear constraints (or linear approximations of nonlinear constraints) for empirical work makes the limitation of our linearity assumption more theoretical than practical.

References

Friedman, D. "Why There Are No Risk Preferrers." *Journal of Political Economy* 89(1981):600.

Hazell, P. B. R. "A Linear Alternative to Quadratic and Semivariance Programming for Farm Planning under Uncertainty." *American Journal of Agricultural Economics* 53(1971):53-62.

Horst, R., and H. Tuy. *Global Optimization: Deterministic Approaches*, New York: Springer-Verlag, 1990.

King, R. P., and G. E. Oamek. "Risk Management by Colorado Dryland Wheat Farmers and the Elimination of the Disaster Assistance Program." *American Journal of Agricultural Economics* 85(1983):247-255.

Larson, J. A., and H. P. Mapp. "Cotton Cultivar, Planting, Irrigating, and Harvesting Decisions under Risk." *Journal of Agricultural and Resource Economics* 22(1997):157-173.

Love, R. O., and L. J. Robison. "An Empirical Analysis of the Intertemporal Stability of Risk Preference." *Southern Journal of Agricultural Economics* 18(1984):159-165.

McCamley, F., and J. B. Kliebenstein. "Describing and Identifying the Complete Set of Target MOTAD Solutions." *American Journal of Agricultural Economics* 69(1987):669-676.

Murty, K. G. *Linear Programming*, New York: John Wiley and Sons, 1983.

Tauer, L. W. "Risk Preferences of Dairy Farmers." *North Central Journal of Agricultural Economics* 8(1986):7-15.

Tauer, L. W. "Target MOTAD." *American Journal of Agricultural Economics* 65(1983):606-610.

Zaraś K. "The Second Stochastic Inverse Dominance in Expected Utility Analysis." *Foundations of Control Engineering* 12(1987, no.4):195-207.

Zaraś K. "Dominances Stochastiques pour Deux Classes de Fonctions d'Utilité: Concaves et Convexes." *Recherche Opérationnelle* 23(1989, no. 1):57-65.