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Ecosystem Services Economics

Ecosystem Services and the Macroeconomy: A Review of Linkages and Evaluation of Analytical Tools

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Preface

The development of the ecosystem services paradigm has enhanced our understanding of the fact that natural capital is an indispensable form of capital asset along with produced, human, social, and cultural capital. Sound and cost effective management of declining natural capital is a key element of sustainable economic development and poverty reduction goals. This would further require the integration of natural capital with other forms of capital in order to mainstream ecosystem services into development policy which will eventually help assessment of the sustainability and effectiveness of development options.

Most of the discussions in economics of ecosystem services however, remain in the conventional domain of microeconomic theories of correcting the externalities or market failure, designing of market based instruments and cost benefit analysis of sectoral project and policy. Economic valuation of the ecosystem services and biodiversity definitely addresses the issue of resource allocation but what it does not convey is the interlinkages and dependence of various economic sectors on services of ecosystems. The dependence of the conventional economic sectors arise not only through the use of tangible materials like water and fish but intangible services like waste minimisation, climate regulations and control of vector borne disease (MA, 2005)¹. The other relevant question which microeconomic foundation of valuation leaves unanswered is the relative scale of economic activities within the ecosystem. This scale issue is critical, if the economy has to follow the path of sustainable development. As the micro unit of the economy like individual and household functions as a part of larger macroeconomic system, the aggregate economy also operates as part of a larger system known as the ecosystem (Daly, 1991)².

The Project for Ecosystem Services (Proecoserv) - one of the flagship projects of the UNEP, aims to map and account ecosystem services, and integrate the findings of ecosystem service assessments into development planning across scales in selected regions of Chile, South Africa, Trinidad and Tobago, and Vietnam. Proecoserv considers that designing and implementing development and environmental policies require a careful alignment of environmental, social and economic goals, supported by credible scientific evidences. An economic perspective on use and conservation of ecosystem services is essential for decision-makers invariably struggling with resource constraints and conflicting choices. Furthermore, macroeconomic policy issues such as GDP, employment, fiscal policies are critical aspects of development design, however the necessary mechanics of linkages are still lacking in the conventional decision making framework. Hence, the challenge is how to incorporate ecosystem services into macroeconomic policies. In order to shed some light on this challenge, the Proecoserv catalyzed the preparation of this working paper which focuses on the linkages between macro economy and ecosystem services, and reviews the methodologies and tools used for examining the linkages.

This paper -Ecosystem Services and the Macroeconomy: A Review of Linkages and Evaluation of Analytical Tools- is published under the UNEP's Ecosystem Services Economics Working Paper Series. This working paper series is to provide a forum for disseminating works-in-progress reflecting the broad range of research activities on economics of ecosystem services. They are circulated to encourage thought and discussion and make research of Ecosystem Services Economics Unit research available to fellow economists, scientists and policy makers working in this area.

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¹ Millennium Ecosystem Assessment, (2005). *Ecosystems and Human Well-being: Synthesis*. Island Press, Washington, DC.

² Daly, H. E. (1991). *Towards an Environmental Macroeconomics*. *Land Economics*, 67(2), 255-259.

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Executive Summary

Natural capital is essential to the performance of sustainable development, supporting our way of life and achieving the Millennium Development Goals. Because of the numerous and complex links between economy and the ecosystem services, and the need to inform the public of the potential economic and social problems resulting from diminished levels of ecosystem service provision, it is important that ecosystem services are taken into account in the macroeconomic decisions.

The objectives of this report are (i) to summarize and critique the existing models used for examining macroeconomic and ecosystem linkages (meta-models); (ii) to develop an alternative model that circumvents the major criticisms of the existing meta-models; and (iii) to provide two examples of how the model has been empirically implemented.

Based on the assessment of the existing models, it can be argued that the existing macroeconomic-ecosystem models suffer from one or more of the following problems: (i) the economic side of the model does not include exchange prices that are generated within the model (hence, little confidence can be placed in how the model chooses input and output levels, and hence economic performance); the ecosystem does not feed back into the economy (implication being the economy is not affected by ecosystem or natural asset restoration or decline); (iii) the economy does not directly affect the ecosystem (implication being it ignores the impact of pollution and cannot accommodate events like fishery collapses); and (iv) one of the major determinants of per capita economic growth, savings, are exogenous to the model (major, nuanced, implication being that such models are of limited use in conducting long run analysis). Each of these shortcomings almost certainly introduces a bias in the predicted ecosystem values generated by the existing meta-models.

The model developed for this report addresses each of the above meta-model shortcomings, and provides examples of the types of insights and ecosystem value measures we can get when implementing it. The empirical findings from report-model illustrate how to use the model to measure the contribution of ecosystem service to gross domestic product (GDP), and how to begin measuring the asset value of the ecosystem. One implementation of the model looks at the economics of ecosystem service provision (of water) in the Kanto Plains region of Japan. Results suggests water use accounts for about 2.8% of Japan's GDP in 2008, and predicts water's contribution will fall to about 1.8% by 2058. The same study estimates that for 2008, the total asset value of water in the Kanto Plains of Japan, is about one quarter that of the total asset value of its man-made capital. The second implementation of the model examines the impact of electricity subsidies on water values and on agricultural and manufacturing production in The Punjab, India. The empirical results tell us a complete removal of electricity subsidies in the Punjab would lead to a 37% decrease in irrigation water pumped, a 31% drop in rice production and 11% increase in manufacturing production and a 7% drop in Punjabi agricultural and manufacturing value-added. Water's total contribution to GDP falls by 30%, but the stock of water and its value increase.

The empirical results show that the structure of the report-model seems promising as a tool for measuring the (provisioning) value of ecosystem services and for measuring part of the value of an ecosystem. The results also indicate that for successful integration of ecosystem services and macro economy, the model should be able to;

- Provide meaningful insights into how ecosystem and macroeconomic policy impact macroeconomic performance, ecosystems and ecosystem service values, and ecosystem sustainability,
- Provide information about the macroeconomic impacts of forestry, fishery, climate change, and land use policy,
- Measure both the stock and flow value of water services, forest resources, carbon sequestration, and possibly fishery resources.

Another point to consider is that the developed model has some limitations. The report-model only attempts to measure the value of provisioning services, while some meta-models attempt to measure the value of provisioning services, along with the value of regulating, cultural and supporting services. The report-model could be adapted to accommodate regulating and supporting services, but may have problems accommodating the number of ecosystem services included in some of the meta-models. Although not a limitation of the model, implementing the report-model involves a significant investment in human capital, and may serve as a deterrent for analysts with potential interest in the methodology and the problems it can tackle.

1. Introduction

As globalization proceeds at an unprecedented pace, concern with natural resource scarcity has emerged again as a concern to policymakers, and social and physical scientists. Embedded in this concern is the question of how globalization is impacting ecosystems, and the resulting ability of ecosystems to provide a level of ecosystem services concomitant with demand (see Krautkraemer, 2005). The many attempts to inform the public of the potential economic and social problems resulting from diminished levels of ecosystem service provision, the efforts of policymakers to introduce legislation to better manage ecosystem services, and the attempts by social, biological and physical scientists to develop tools to help manage ecosystems and understand the role of ecosystems in economic production is loosely referred to as efforts at “mainstreaming ecosystem services.” The increasing interest in mainstreaming ecosystem services derives from many concerns, one of the major ones being climate change.

The major objective of this report is to develop a conceptual model of macroeconomic and ecosystem linkages, that is amenable to empirical implementation. The report is divided into six sections. The introduction gives an overview of the four types of services generated by an ecosystem, the three types of capital considered in the dominant index of human wellbeing, and a short description of some of the tools behind economic/ecosystem policy analysis. The second section provides a short review of the major models used to conduct numerical ecosystem-macroeconomic analysis – meta-models – and highlights their respective shortcomings. The third section provides a simple introduction to ecosystem dynamics, and gives an overview of the setup of the formal model presented in the appendix. The fourth section presents examples of information an empirical version of the theoretical model can provide, and uses results from two studies employing variants of the model in the appendix. The fourth section also illustrates how insights into ecosystem valuation can be gleaned from the theory alone, and how policy features in such analysis. The fifth section concludes. The sixth section is an appendix that lays out the conceptual model in detail.

1.1 Ecosystem Services

The Millennium Ecosystem Assessment (2005a, 2005b) identifies four major categories of ecosystem services: provisioning, regulating, cultural and supporting. Provisioning services are benefits humans derive from nature. Examples of provisioning services are food, drinking and bathing water, timber and solar energy. The National Wildlife Federation (NWF) defines a regulating service as “the benefit provided by ecosystem processes that moderate natural phenomena.”³ Pollination, water purification and carbon sequestration are examples of regulating services. Cultural services refer to the role ecosystems have played in defining local and national cultures, and “the building of knowledge and the spreading of ideas,”⁴ and in recreational activities. Supporting services refer to natural processes like photosynthesis.

To realize the benefit of provisioning services, one usually needs to combine an ecosystem service, say water or raw plant material, with labor, and possibly capital, and produce a consumption good: water is combined with seed, fertilizer, labor and capital to produce grain; the bark of fever tree (*cinchona succiruba*) is combined with labor and capital to produce quinine. The value of a provisioning service is often embedded in measures of gross domestic product (GDP), but often ignored. For example, the water used in agricultural production contributes to economic value, but is seldom recognized as a separate component of GDP.

The fact that provisioning services often have direct, albeit often unmeasured, economic benefits provide one reason for the apparent desire of economists and policymakers to design policy that helps manage and better maintain ecosystems. Poor management almost certainly contributed to a near collapse of the Canadian Grand Banks Cod Fishery in the 1970s (Pilkey and Pilkey-Jarvis, 2007), and a complete shutdown of cod fishing in 1992, leading to one of the largest worker layoffs in Canadian history (OECD, 2011). In this case, the mismanagement of a large

³ <http://www.nwf.org/Wildlife/Wildlife-Conservation/Ecosystem-Services.aspx>

⁴ *Ibid.*

ecosystem led to an undesirable macroeconomic outcome. Such events help make sense of the current interest in ecosystem health and management, as provisioning services provide not only benefits in terms of food and clothing, but also provide jobs and investment opportunities. Hence, one objective in the efforts to mainstream ecosystem services is to develop ecosystem management tools. When an ecosystem has economic value, a desired feature of a management tool is to predict how the ecosystem and its' services evolves over time. If economic activity influences the ecosystem, then we want to know how that activity influences the ecosystem, and in turn, the level of services it provides. Linking the level of ecosystem services provided with economic activity and value, and the feedback between the two systems is a major interdisciplinary research challenge faced by economists, and physical and biological scientists.

Capital and labor are seldom required to realize the benefits of regulating services. Also, unlike provisioning services, regulating services do not often end up embedded in GDP. Exceptions, of course exist: pollination is essential in grain and fruit production, and is included in the value of grain and fruit, but is not as straightforward to measure as water's contribution to agricultural production. When an ecosystem is damaged due to natural disasters or human activities, capital and labor can often help restore a damaged ecosystem – activity that certainly contributes to GDP. Recent policy directions are attempting to provide economic incentives to encourage better management of regulating services. The carbon trading experiments implemented across several countries is one example of such policy implementation. One of the major goals of carbon trading is to decrease the levels and rates of greenhouse gas emissions generated by economic activity. In the author's opinion, it is too early to tell how successful carbon trading will be in decreasing greenhouse gas emissions. Carbon trading attempts to commoditize an ecosystem service in the hope that assigning value to an ecosystem service will lead to better management of that service – here, increasing the level of carbon storage and decreasing the amount of carbon release in the atmosphere. It remains to be seen if additional regulating services can be better managed by developing creating value to give them market value. Hence, via efforts to commoditize regulating services or efforts to restore ecosystems, regulating services can, in principle, end up having macroeconomic significance.

Although a regulating service might not directly enter GDP, it often provides economic benefits to society. A mangrove forest provides protection to coastal regions from storm damage, and a coral reef provides shoreline protection by slowing down the rate of beach erosion. These benefits – often referred to as positive externalities – accrue to society, but not by increasing GDP. So, an accurate measure of these values, while providing a better measure of the value of an ecosystem or ecosystem service, will not provide us with a more accurate measure of GDP.

We can gather from the discussion thus far, that provisioning services typically have economic value that is directed measured, or implicitly embedded, in GDP. Regulating services can also have direct values that are directly or embedded in GDP, but will usually provide benefits that have value, but not the type that ends up in GDP. Hence, if society has an eye on the future, there are incentives to want to manage an ecosystem today so it provides for the needs of today, while leaving it with the ability to provide for future generations. Of course, there are incentives, also, for some individuals to capture as much value from the provisioning service today, with no regard for the future. Examples include: (i) overharvesting fish stocks today to maximize short-run profit, but leading to a collapse of the fishery, and (ii) high rates of groundwater extraction that drive the groundwater table low enough to be economically unavailable (i.e., too expensive to withdraw) to future generations. Profit motives can also lead to the mismanagement of non-market valued regulating services: e.g., excessive deforestation and carbon sequestration.

Cultural services provide utility to its beneficiaries, but unless the cultural services are based on land or structures earning rent, or providing some other type of consumable, the value of those services will not enter GDP. If, however, the cultural service is tied to tourism for example, then the service has measurable economic value, and is likely already measured in GDP. Cultural services might have what economists call existence value, but deriving that value is not a subject for this discussion.

We conclude this section with the following summary: (i) almost all provisioning services and some regulating services provide inputs to production processes that yield economic outputs, and hence, contribute to the creation of economic value; (ii) the health of an ecosystem can affect the level of ecosystem services available as productive

inputs, which in turn can affect the level of economic outputs – a drop in the level of ecosystem services can lead to a fall in economic production; (iii) mismanaged ecosystems can lead to precipitous drops in production and lead to nasty macroeconomic shocks (e.g., massive layoffs). On the other hand, Hence, it makes economic sense to pay close attention to the health of ecosystems, and develop tools that can assist in ecosystem management.

1.2 Natural, Physical and Human Capital in GDP and Wealth

A provisioning service is a service flow produced by the natural capital inherent in an ecosystem. An easy provisioning service to envision is that of irrigation water used in agricultural production. Irrigation water is a provisioning service produced by an ecosystem (river or groundwater), and a form of natural capital. If a farmer pays for irrigation water, the value of that water enters directly into the GDP. If the farmer does not pay for the irrigation water, and instead draws the water directly from a surface or groundwater source (paying only extraction costs), the water still contributes to agricultural value, and hence GDP. In this case, the value of water's contribution ends up embedded in GDP – hiding in the value of physical capital or in some other factor account category. An exhaustive discussion of natural capital is not a major objective of this exercise. However, the interested reader can find definitions of natural capital, and discussions on ways to measure the level of natural capital (or natural assets) in the United Nations' (UN) documentation on the System of Environmental-Economic Accounting (SEEA), and the relatively new UN effort, the SEEA Experimental Ecosystem Accounts (EEA).⁵

Physical capital refers to man-made capital: buildings, machinery, office furnishings, irrigation canals, computers and other information technology equipment, medical devices and other physical infrastructure. Physical capital also refers to assets that are not consumed in a single period. For example, in the agricultural literature fertilizer is often referred to as capital, but it is actually an intermediate input – purchased and consumed in the same period. Human capital refers to the huge variety of labor services that prevail in any given economy.

So far, our discussions of ecosystem service values focused on the contribution(s) a provisioning or regulating service made to GDP. If an ecosystem service did not provide a value that ended up in GDP, the benefit was deemed an externality and, since externalities do not enter GDP, omitted from further discussion. GDP is a concept that measures the value of the flow of services provided by “capital” and labor. In the case of labor, the flow of services is man-hours provided to firms, and the value of those services is the value of salaries, wages and benefits paid to workers over a period of time. The flow value of capital is a bit more nuanced, but examples include the amount person-A pays person-B for the right to use person-B's condo for a year, or the amount a farmer pays a landowner to use his land for a season. In the typical discussion of nation accounts, capital refers to man-made (or physical) capital.

Although GDP is a flow concept, human, physical and natural capital each have an asset (or stock) value, too, where the relationship between the flow stock value of each group is as follows:

1. The flow value of a unit of physical capital is the rent one pays for the right to use it for a period of time, e.g., the interest rate on a loan adjusted for depreciation. The stock value of a unit of physical capital is the discounted present value of current and all future rental rates.
2. The flow value of a unit of a natural asset like irrigation water stored in an aquifer, is the shadow value⁶ of water used in agricultural production during a season. The stock value of a unit of aquifer water is the shadow value of water today, plus all future shadow water values discounted appropriately.
3. The flow value of a unit of labor is that labor's wage receipt over a unit of time, while an approximate stock value of that labor is the discounted present value of the current period wage receipts plus all future wage payments adjusted for experience, additional education and remaining man-hours in the labor force.

⁵ For details of the SEEA, see http://unstats.un.org/unsd/envaccounting/White_cover.pdf, and for an overview of the SEEA-EEA, see http://unstats.un.org/unsd/envaccounting/secarev/Chapters/SEEA_EEA_v1.pdf

⁶ The change in revenue associated with a unit increase in water, or viewed another way, the price a water user would be willing to pay for the right to use/consume an additional unit of water.

As a measure of welfare, per capita GDP has many well-known shortcomings: (i) it ignores income distribution; (ii) it ignores costs of pollution and other negative externalities; (iii) it does not account for the depreciation of natural assets (e.g., land productivity and forest stocks); (iv) it ignores the value of economic production taking place in informal markets (e.g., the value of subsistence agriculture in developing countries). In addition to these well-known problems, as suggested above, GDP measurement typically confounds the relative values of the flow of services provided by physical capital, natural capital and labor.

Although several alternatives to GDP exist – e.g., the Index of Sustainable Economic Welfare, the Human Development Index, the “New” Human Development Index⁷ – the measure having the most momentum in 2013 is the inclusive wealth index (*IWI*). The *IWI* is defined as the sum of the stock values of human capital, physical capital and natural capital.⁸ Represent the price of physical capital by P_K , the unit stock value of natural and human capital by P_N and P_H respectively. If an economy is endowed with K units of physical capital, H units of natural capital and L units of human capital, then the inclusive wealth of that economy is $IWI = P_K K + P_N N + P_H H$. If the inclusive wealth index does not fall over time, then we say that economy is sustainable. While close to an ideal measure of welfare (see Dasgupta, 2009), measuring the *IWI* with the data available today is a challenging proposition.⁹

1.3 Economic Tools for Mainstreaming Ecosystem Services

As suggested above, a wide variety of activities contribute to efforts to mainstream ecosystem services. Economic analysis and policy development is a major component of mainstreaming, and is the activity to which this paper is most closely related. Implementing well thought out economic policy typically combines data with economic models to investigate the likely impact of introducing a proposed policy on an economy. When most economists talk of macroeconomic policy they have two types in mind – fiscal and monetary. Monetary policy focuses on controlling short-term interest rates to manage inflation and unemployment, and stimulate or cool down growth. Fiscal policy focuses on using taxes, quotas, tariffs, subsidies, transfers and government spending also to stimulate or cool down growth.

If industrial production generates pollution that adversely affects an ecosystem (say, through water or air pollution), then macroeconomic policy that stimulates growth in that sector can lead to a decrease in the level of that ecosystem's provisioning and regulating services. For example, increasing tariffs on industrial imports (e.g., steel) can encourage an increase in industrial production, which in turn increases the amount of effluent released into waterways and decreasing the level of fish stocks. On the other hand, failing to introduce industrial, not macroeconomic, policy to manage fish harvests can have drastic macroeconomic impacts, as witnessed by the Grand Banks Cod Fishery collapse and its subsequent high unemployment levels.

A well-designed macroeconomic/ecosystem model could, in principle at least, predict the impact of overharvesting on the economy ten to fifteen years in the future. Such a model could then be used to identify (a sequence of) harvest quota levels that could avoid a total shut down. Almost certainly, lowered catch rates would lead to lower short- and medium-run GDP for the fishing industry, but having a good idea of the magnitude of those losses could help prepare offsetting policies (e.g., fuel subsidies or relaxing net size restrictions) to lower harvest costs.

Assume one is able to estimate a production function linking physical capital, labor and natural capital levels to industrial or agricultural output. Then, natural capital accounting (i.e., a system of environmental-economic accounts), combined with the level of labor and capital employed in a sector can yield the shadow value of a natural asset or an ecosystem service (see Smith and Gemma, 2013). It follows that two types of economic data are useful in examining the links between ecosystems and macroeconomic performance: (i) a social accounting matrix, with a system of environmental-economic accounts, and (ii) production data that allows estimating how natural assets

⁷ See Bleys, 2012, for a clear summary of past candidates of wealth measures and their shortcomings.

⁸ For an exhaustive discussion of the *IWI*, see <http://www.ihdp.unu.edu/article/iwr>

⁹ Ibid.

contribute to economic output. Hence, constructing the SEEA and efforts to implement the SEEA-EEA are clearly efforts in mainstreaming ecosystem services.

In an ideal world, a modeler will also have data that explains how an ecosystem, or how the level of a natural asset evolves over time. This is discussed in more detail in section 3, but in the case of modeling a fishery, the extra data would take the form of one or more equations that explain how fish stocks evolve over time: in the case of groundwater management, the extra data would be an equation that explains how the stock of groundwater evolves as a function of groundwater extraction rate and recharge rates.

2. Overview of Ecosystem - Macroeconomic Models

For at least 30 years, scientists and economists have tried to combine forces and develop large-scale numerical models that link ecosystem dynamics with economics (meta-models). Most integrated assessment models employed by international agencies are variations of models using meta-modeling methods. The three major meta-models are the Global Unified Meta-model of Ecosystem Services (GUMBO), the Multi-scale Integrated Model of Ecosystem Services (MIMES) and Threshold 21 (T21). These models are used to assess a wide range of climate scenarios to investigate the effectiveness of different climate policies. For example, a common "business as usual" scenario predicts the future level of greenhouse-gas (GHG) emission, as well as GDP, given there are no fundamental changes in climate policies. The main advantage of employing these models is that they can capture the dynamic interactions between many economic, resource and social sectors. Furthermore, the level of subsector aggregation can be easily adjusted such that they can implement regional, national or global policy assessments.

One shortcoming of these models, however, is prices are not determined within the Model. Given that prices are the major mechanism used to allocate resources in most economies, it is difficult to place a lot of faith in the economic predictions generated by the Meta-models. The author has yet to see reported results on how the model predicts the past levels of economic (and ecosystem service) variables. The International Futures Simulator (IFS) model, another dynamic simulation model employs a similar partial equilibrium approach to determine prices.

There are some integrated assessment models that also employed the general equilibrium approach but their main objectives are for specific policy analysis which does not include feedbacks from natural assets production or ecosystem services. For instance, the Env-Linkage and the Integrated Global System (IGSM) models evaluate outputs such as economic impacts of GDP, crop productions and household consumption from determinants based on different environmental policies. On the other hand, the Global Trade Analysis Project (GTAP) analyzes only consumption and trade pattern of agricultural products with respect to different environmental or economic policies.

Another group of integrated-assessment models examine the economics of climate-change on various regions. For instance, the Asia Pacific Integrated Model (AIM) analyzes the value and cost of water supplies in the Asian Pacific region, while the Integrated Model to Assess the Global Environment (IMAGE) estimates the cost in terms of foregone Netherland GDP due to climate-change induced sea level increases. Climate scenario assessment models also include the Integrated Model to Assess the Global Environment – 2 (IMAGE-2), the International Model for Policy Analysis of Agricultural Commodities and Trade (IMPACT-WATER), the POLE STAR, the TARGETS, and the WORLD 3. Each of these models performs cost and benefit analysis for different climate policies scenarios. A characteristic each of the integrated-assessment models share is they treat most economic variables, including prices, parametrically, and inter-temporal feedback between ecosystem services and the economy are not specified or non-existent. Other biodiversity and ecosystem models, they usually ignore economic impacts and concentrate on specific scientific questions such as the dynamics of carbon sequestration, ecosystem habitats and pollution level.

The Dynamic Integrated Climate-Economy (DICE) model, of Nordhaus (1992a, 1992b) and the Regional Dynamic General Equilibrium Model of Optimal Climate-change Policy (RICE) of Nordhaus & Yang (1995) are models that endogenize factor prices and endogenize savings (i.e., are Ramsey-type models). The major criticism of the DICE model is there is no feedback between the ecosystem (climate/temperatures) and economic performance (Costanza, et al., 2007).

One set of models that appear to have most of the features one would want in a macro-ecosystem modeling effort is that by Tschirhart and Finnoff (2007), who develop dynamic models that link computable general equilibrium (CGE) economic models with general ecosystem equilibrium models (GEEM).¹⁰ This model estimates prices endogenously, but capital stock growth is exogenous. The ecosystem side of these models includes multiple species (Alaskan marine ecosystem) and ecosystem services (Western livestock systems subject to plant invasion). Similarly, Mumby and Sanchirico (2009) have developed an interesting ecological-economic model to show how mangrove-sea grass-coral reef systems enhance the productivity of marine fisheries in the Caribbean, albeit without the desired general equilibrium features of the model developed here or in Tschirhart and Finnoff (2007).

In sum, the existing set of models that combine macroeconomic dynamics with ecosystem dynamics have one or more of the following shortcomings: (i) economic prices are not endogenous; (ii) the ecosystem does not feed back into the economy; (iii) the economy does not directly affect the ecosystem; and (iv) savings (i.e., additions to the capital stock) are exogenous. In response to these omissions, a major objective of this report is to outline a conceptual model that links macroeconomic dynamics with ecosystem dynamics, and that has none of the noted four shortcomings. The underlying economic theory has its roots in the dynamic, general equilibrium models found in Roe, Smith, and Saraçoğlu (2010). Additionally, the model is empirically implementable (at least, in principle). The model can be implemented empirically by linking the multi-species, dynamic general equilibrium ecosystem model (GEEM) discussed in Tschirhart (2000) and Tschirhart and Finnoff (2007) with the dynamic, multisector macroeconomic models presented in Roe, Smith and Saraçoğlu (RSS). The resulting model – presented in detail in the appendix – is one that is based on sound conceptual underpinnings. In addition, variants of the model have been implemented and used to measure the contributions of ecosystem services to GDP, examples of which are provided in section 4.

3. The Conceptual Model

This section provides an overview of the conceptual model presented in the appendix. It begins with a brief justification for using what economists call a neo-classical model of economic growth to describe how the economy behaves. I then discuss my approach to modeling an ecosystem. I first introduce a simple model of ecosystem dynamics in the absence of economic influence. I then give an overview of the features of the economic model developed in the appendix, and show how an economist might link economic behavior with an ecosystem. This discussion includes a minimal amount of mathematics, but I attempt to explain, clearly, what the math means and provide examples of the types of ecosystems I have in mind.

The economic side of the dynamic model below assumes there are two types of agents, consumers and producers. At each point in time, a consumer makes consumption and saving decisions to maximize utility, while producers make production decisions to maximize profit or minimize the cost of producing goods and services (i.e., the economic agents are rational, optimizing agents). The model implicitly assumes consumers and producers have perfect foresight (know current and future prices). Finally, at each point in time the supply of each input (e.g., capital and labor) is equal to the demand of the inputs, and the supply of each final good (e.g., agricultural output), is equal to the demand for that good. Such models are called a neoclassical model.

Critics of neoclassical models point out that economic agents do not always optimize, are not always rational, and markets are not typically in equilibrium. Clearly, models of an entire economy are prone to specification and estimation biases. A dynamic model presuming perfect foresight and altruistic behavior of households is almost surely violated in reality, and, models with sector detail seldom account for stochastic events. One justification for employing a neoclassical analysis is provided by the Nobel Laureate Robert Aumann (1985), who advances the view that a model is a caricature, or metaphor, of an economic environment. Instead of asking whether the model is right or wrong, the more germane question is “how useful is it?”

¹⁰ For a critique of CGE models, see Scricciu, 2007.

With a dynamic model, usefulness depends, in part, on the model's capacity to replicate the history of some of an economy's key variables of interest, and to provide insights into puzzles embedded in the economic growth and development process (see Kehoe, 2003). An important aspect of an empirical model is how well it explains observed economic data of interest – not necessarily how accurately it captures all features of an economy. For example, if a model with profit maximizing producers and utility maximizing consumers (i.e., optimizing agents) does a good job of replicating actual economic data (e.g., explains GDP well, along with savings and sector output), we will use that model until another model with more complexity is developed that does a significantly better job of explaining observed data. For example, Li and Roe (2006) found that a three sector, open economy model of Taiwan with a manufactured, agricultural, and a non-traded good, performed reasonably well when compared to the predictions of a relatively sophisticated econometric model. They concluded that although the econometric model performed slightly better, the three-sector model allowed for a better interpretation of what was happening in the economy. Hence, although the conceptual framework below contains assumptions that are implicitly violated in the “real world,” it has been shown that it is capable of predicting observed economic phenomenon quite well. The model also has the advantage of providing a tool for interpreting the observed data. Finally, since it is able to replicate observed data, one can compare it to the predictive ability of competing models (e.g., predicting GDP growth rates and levels, and deforestation rates), and can serve as a benchmark for other models to match, or beat.

3.1 Ecosystem Dynamics without Economics

There has been a fair amount of research on the atmospheric properties associated with CO₂ and global warming (Nordhaus, 2009), and fishery dynamics (see Barbier, 2003 and 2009). Plant physiologists have also studied forest and agricultural systems extensively. Ecosystems, however, are typically the result of a complex mixture of plant, water, soil, and geological inputs that interact to produce provisioning and regulating services. Economic forces like trade, investment and exchange rate policies can influence the health and condition of ecosystems whether they are forest, coral, freshwater or wetlands. Conversely, the trend and changes in the resilience of ecosystems can also affect macroeconomic aggregates like GDP levels and growth.

Consider an ecosystem whose primary characteristics are captured by two natural asset stocks. For example, one natural asset could be a large forest and the other asset could be a lake whose water source derives from water captured by the forest during rainy seasons. The forest can provide provisioning services like fuel for heating and cooking, or timber for home construction or furniture, and the regulating services of carbon sequestration. The lake can provide the provisioning services of water for cooking, bathing and agricultural production, and regulating services in the form of habitat for fish.

In another example, one stock is the square kilometers of grassy plains and the other stock is a species like the American buffalo. The grass provides food for the buffalo, while the buffalo provides food for human consumption, protective clothing or pelts for nomadic homes. Yet another example is an ecosystem where one major stock is the square kilometers of mangrove forest and the other is the stock of fish as indexed by the estimated kilograms of fish biomass. Here, the mangrove provides several ecosystem services, one of the major ones being to provide habitat for the stock of fish and another providing protection against extreme weather events like tsunamis. The fish yields provisioning services in the form of fish for human consumption.¹¹

In principle, each of the examples above can be modeled using one or two mathematical expressions that describe how the ecosystems evolve over time. These expressions are often referred to as *equations of motion or differential (difference) equations*. Differential equations can be extremely simple, or very complex. A simple differential equation could describe how the change (i.e., the increase or decrease) in the stock of fish today depends on the level of today's fish stock. In a more complex differential equation the change in the stock of fish today depends on the level of today's fish stocks and the level of, say, the mangrove forest area that provides habitat for fish.

¹¹ One issue of concern to ecologists, but not entertained here, is irreversible ecosystem collapse. An irreversible ecosystem collapse can be characterized by a time t' combination of fish harvest, mangrove area, and existing fish stock, that triggers a sequence of \dot{S} values where $\dot{S} < 0$ for all $t > t'$. In such a case, the stock of fish might dwindle to a commercially meaningless level even if fish harvesting ceased completely (see Woodard, 2001). See Dasgupta and Mäler (2003) for a discussion of other issues related to irreversibility and non-convexities in ecosystem dynamics.

An even more complex differential equation would have the change in the stock of fish depend on the stock of fish, mangrove area and the amount of fish harvested in the current period. Ecosystems dynamics can also have “upstream--downstream” characteristics, as with the forest/lake example above: the volume of water today can depend on the forest area and biomass per unit area last year. Ecosystem dynamics are typically, quite complex, and often represented a system of differential equations. In practice, however, the more differential equations we try to model, the more complex the conceptual and numerical models become.

In what follows, we will view the ecosystem as a relationship between a species and its habitat. The first example will model ecosystem dynamics in the absence of human and economic activity. The basic model will be rather abstract, with examples provided when deemed useful.

Assume there exists a habitat area where a biological species lives. To express the habitat and species in mathematical terms use H to represent habitat and use B to represent the level of the biological species. For simplicity, assume $H = 1$ and assume it remains constant over time. Here, the habitat could be 10 square kilometers of forest area and the species could be deer. The habitat provides food for the biological species, with the amount of nutrients per unit of habitat area represented by ϕ . Here, “nutrients” can be another fish species, grass, or other vegetation: in the case of deer, nutrients are grass, leaves, bark and other foliage.

Assume the stock of “nutrients” evolves according to the differential equation

$$\dot{\phi}(t) = \bar{\Theta}(\phi(t), B(t), H) \quad (1)$$

In the above equation, $\dot{\phi}(t)$ is the rate of growth – which can be positive or negative – in the level of nutrients at time t . The expression $\bar{\Theta}(\phi(t), B(t), H)$ represents a function that tells us the rate of growth in nutrients depends on the level of the nutrients at time t , the level of the biological species at time t , and habitat area (which is assumed constant over time). The stock of species evolves according the differential equation

$$\dot{B}(t) = \tilde{Y}(\phi(t), B(t), H) \quad (2)$$

In equation (2), $\dot{B}(t)$ is the rate of growth in the biological species (e.g., deer). The function $\tilde{Y}(\cdot)$ represents a function that tells us the rate of growth in the biological species depends on the amount of habitat area, and the time t , stock of nutrients and the stock of species.¹²

Equations (1) and (2) describe how the ecosystem – here defined as the interactions between nutrients, a biological species and habitat – evolves over time. The dynamics of this system of equations can be quite complex, and beyond the scope of this discussion.¹³ In what follows, however, assume the ecosystem dynamics admit only stable, non-cyclical steady states. If one exists for this ecosystem, characterize it by the nutrient and species levels $(\bar{\phi}, \bar{B})$, where

$$\dot{\phi} = \bar{\Theta}(\bar{\phi}, \bar{B}, H) = \tilde{Y}(\bar{\phi}, \bar{B}, H) = \dot{B} = 0$$

In non-technical terms, the steady state is the level of nutrient and species one would eventually expect to see “after some time” if the system were unperturbed.

The next section explains some of the relevant economic dynamics of the formal model developed here, and then considers two ways in which an ecosystem and economy might interact.

¹² Here, $\tilde{Y}(\cdot)$ is an abbreviation of the function $\tilde{Y}(\phi(t), B(t), H)$. This “shorthand” will be used at the author’s discretion.

¹³ For example, the Lotka-Volterra equations

$$\begin{aligned} \dot{\phi}_i &= \phi_i(a - bS_i) \\ \dot{S}_i &= S_i(c - d\phi_i) \end{aligned}$$

is one set of differential equations used in the biological sciences, where a , b , c and d are scalar parameters.

3.2 Linking Ecosystem Dynamics and Economic Activities

The model developed in the appendix has its roots in the conceptual framework found in Roe, Smith, and Saraçoğlu (2010). Roe, Smith, and Saraçoğlu (hereafter, abbreviated RSS) examine closely, the dynamics of a small open economy that uses labor, land and capital to produce three final goods – an agricultural, manufacturing and service good. The agricultural and manufacturing goods are traded, while the service good is non-traded. In such an economy, as the economy grows, the stock of capital typically grows faster than the labor force. Economists call this process capital deepening. Capital deepening has two effects: it pushes wages up over time, and pushes the unit cost of capital down over time. The service good price tends to decrease over time if the service sector relies heavily on capital (referred to as a capital intensive sector), and tends to increase over time if the service sector is labor intensive. RSS also suggest that capital deepening tends to favor sectors that rely more on capital – with the share of GDP produced by the capital intensive sector increasing over time. This occurs because capital deepening and falling capital rental rates help the capital intensive sector better compete for resources than the more labor dependent sectors.

The dynamics associated with capital deepening and capital intensity has implications for ecosystem service values. Mainly, if an ecosystem service is used by a capital intensive sector, there is a good chance the pressure for increased exploitation of that service will increase as the economy grows. This has yet to be established as a “stylized fact,” but initial empirical results suggests this to be the case (see RSS, and Smith and Gemma). On the other hand, if an ecosystem service is used by a relatively labor intensive sector there is a chance the sector will rely less on the resource as the economy grows. This means the economic value of a provisioning service would increase over time if it is used by a relatively capital intensive sector, and possibly fall if used by a relatively labor intensive sector.

Case 1 – Ecosystem Services, Economics and Health

An economy could impact an ecosystem through direct harvesting of a wildlife species having economic value, which in turn affects the viability of another wildlife species. Consider a fresh water ecosystem in Africa with two component species being fish and snails. Sources of fish nutrients include snails. Some snails, however, are intermediate hosts for schistosomiasis that infect humans (Stauffer et al., 2007). Two ecosystem services produced by this ecosystem are harvested fish for human consumption and schistosomiasis infections. In this example, let equation (1) represent a differential equation that describes the growth in nutrient (snail) levels over time: here $\dot{\phi}(t)$ is the rate of growth in snail stocks, $\phi(t)$ is the time t stock of snail harvest, $B(t)$ is the stock of fish and H is the surface area of a lake in which the fish and snail are located. In other words, equation (1) tells us that at any point in time, the rate of growth in snail stocks depends on the level of snail and fish stocks.

We now link human, i.e., economic, behavior with ecosystem dynamics. To do this, introduce fish harvesting activities into equation (2):

$$\dot{B}(t) = \tilde{Y}(\phi(t), B(t), H) - X^s(t) \quad (3)$$

where $X^s(t) \geq 0$ is the time t harvest of species – if $X^s(t) < 0$, then $X^s(t)$ represents restocking efforts. The interpretation of equation (3) is that fish harvesting offsets the rate of growth in the fish stock. Even if the value of $\tilde{Y}(\cdot)$ is greater than zero, if fish harvesting efforts are large enough, the rate of growth in fish stocks can be negative.

Typically, harvesting or restoration efforts will be a function of capital and labor, and most likely, the level of nutrient or fish stocks. The smaller the stock of fish, the more effort a fisher will likely have to make to harvest fish. On the other hand, the smaller the fish stock, the larger the number of fish a conservation agent might have to place in the lake in her restocking efforts. Note, here we implicitly allow for the possibility that ecosystem degradation is not necessarily irreversible. Furthermore, restocking could potentially emerge as a policy tool whose implementation contributes to increased investment, employment and economic growth.

The point here is that ecosystem services and economic activity are often intimately linked. This example illustrates that an activity like fish harvesting could drive down the stock of fish (an ecosystem service), which could lead

to an increase in the number of snails in the lake and hence, lead to an increase in the number of people getting schistosomiasis (another ecosystem service, albeit, an undesirable one). A policy of either restocking fish or decreasing fish harvest levels could lead to lower new infection rates, but at the cost of restocking efforts or foregone fish income. In an area with scarce employment opportunities restocking might be a welcome activity.

Case 2 - Natural Asset Consumption (or Restoration)

In case 1, the habitat level remained constant over time. This case presents a reasonably simple way to handle cases where the habitat level might change over time. Consider an ecosystem composed of forest biomass and wildlife, with the biomass serving as habitat (H) for the wildlife (B): i.e., the wildlife depends on forest biomass as its' food source. The ecosystem produces two ecosystem services: timber harvested using capital and labor, and food (turkey and deer) obtained by hunting. Harvesting timber changes the level of biomass available for the species, implying that habitat is no longer constant, but related to harvest rates.¹⁴

Since the habitat supplies wildlife food, we ignore nutrient dynamics, and link the stock of wildlife directly to habitat area:

$$\dot{B}(t) = \hat{Y}(B(t), H(t)) \quad (4)$$

Hence, one difference between the ecosystem in this case and that in case 1, is here we are ignoring the nutrient dynamics represented by $\dot{\phi}$ and ϕ – implicitly assuming nutrient availability is directly to habitat size. Equation (4) says the time t rate of growth in the stock of wildlife (say, deer) depends on the number of deer at that time and the size of the forest. For a given forest area, the stock of deer could initially grow quite fast, but as the deer herd grows, the rate of growth in the herd would decrease. Eventually, the deer herd could reach a point where additions to the herd were exactly offset by the herd's death rate.¹⁵

Represent habitat dynamics by the differential equation

$$\dot{H}(t) = -D(K_D(t), L_D(t), H(t)) + g(H(t)) \quad (5)$$

In the above equation, $D(\cdot)$ the function represents economically induced depreciation, or destruction, of the habitat (e.g., deforestation).¹⁶ The level of destruction depends on how much capital and labor is used, represented by K_D and L_D , and how much habitat is available to destroy or harvest, H . The function $g(H)$ represents the rate of habitat regeneration.

In this example, deforestation $D(\cdot)$ leads to a decrease in the level of the habitat supporting wildlife in the ecosystem.

Case 2 could also represent a coral reef ecosystem, where H represents coral reef area, $D(\cdot)$ represents economic activity that produces pollution that slowly destroys the coral reef, and B is a stock of fish to which the coral reef provides habitat. Here, we might rewrite the habitat equation as:

$$\dot{H}(t) = \Psi(D(t), H(t)) \quad (6)$$

Y_m is the amount of pollution produced by manufacturing activities, with m units of pollution produced along with each unit of manufacturing output. The interpretation of equation (6) is that the rate of change in coral reef area, depends on the amount of pollution produced by the manufacturing sector and the current size of the reef. This is a function that tells us how current levels of pollution and habitat area affect habitat growth.

¹⁴ Here, timber harvesting is an economic activity that ends up in GDP, while hunting may or may not end up in GDP. If hunters pay for the right to hunt on a nature reserve, then the hunting effort has a value that ends up in GDP. If a farmer hunts on his own land and kills a deer for food, although the deer meat has economic value, if it is not sold the value of the hunting effort will not end up in GDP.

¹⁵ Of course, it is quite possible for a herd of deer to completely collapse when it gets too large. The interested reader is invited to figure out how to modify the ecosystem equations (2) and (3) to accommodate this outcome.

¹⁶ If capital and labor are used in restoration efforts, then the $D(\cdot)$ function will be negative.

If the coral reef and fish stocks had economic value, say via tourism activities or fish harvesting, the levels of H and B would enter the economy through its tourism or fishery sectors. If the coral reef began to degrade and decrease in area because of pollution, fewer tourists might visit the region, or fewer fish would be available for harvest. In the case of deforestation, if mangrove area were being converted to agricultural uses, the effect on the fishing industry would be similar to that of pollution destroying coral reef habitat: the less mangrove forest area, the smaller the carrying capacity of the fishery (see Barbier, 2003).

The ecosystem models presented above are relatively simple, but can accommodate a wide range of ecosystem services having macroeconomic interests: e.g., forests, fisheries, surface and ground water systems, carbon storage, and coral reef and mangrove systems. The formal ecosystem-macroeconomic model presented in the appendix is based on a slight variation of the coral reef example discussed above.

The next section presents two empirical examples, each of which examines the economics of water. The first example looks at the economics of ecosystem services delivered by precipitation and a river basin in Japan, while the second looks at the ecosystem services delivered by precipitation and a statewide groundwater aquifer system in the Punjab, India.

4. Empirical Examples

As noted above, this section presents results from two empirical exercises. The first set of results is derived from a study conducted by Smith and Gemma (2013), who measure the economic value of water used in Japan. The second set of results come from an ongoing study by Nelson, Roe and Smith,¹⁷ who investigate the economics of electricity subsidies and groundwater dynamics in the Punjab, India. Both models take as their point of departure the conceptual framework found in Roe, Smith, and Saraçoğlu (2009), and both have economic dynamics that operate as described above.

The model in each study divides a country into two or more regions, and then divides each region into two or more economic subsectors. The models predict the value of production for each sector, in each region, over time, and the corresponding labor and investment demands for each sector, along with (economy-wide) wages and interest rates. In addition to the future values of these standard macroeconomic variables, the Smith and Gemma study calculates: (i) the unit flow value and the unit stock value of water (and land) used in each sector over time, and (ii) the contribution of water and land to Japan's inclusive wealth. At this point, Nelson, Roe and Smith only calculate the unit flow values of water and land, but as discussed in more detail shortly, also predict how groundwater extraction rates and groundwater depth (the distance from the surface over which the groundwater needs to be pumped) evolves over time. More on the difference between these two studies shortly.

Here, the term unit flow value refers to the price a producer would pay for the right to use an additional unit of water for one period, e.g., a year or a growing season. This price is sometimes called the unit shadow rent of water. Why measure unit shadow rent: because its value is directly linked to GDP. To illustrate, for a given year, the *unit shadow rent* of water in Tokyo manufacturing multiplied by the quantity of water used by that sector yields the total shadow rent of water used in Tokyo manufacturing for that year. This total shadow rent is water's contribution to the GDP produced by Tokyo manufacturing. When we ignore this value, we run the risk of ignoring its importance in economic production and mismanage the resource.

Another reason for wanting the unit shadow rent of water is that one cannot measure water's contribution to inclusive wealth without this information. As noted above, inclusive wealth measures the value of an economy's assets – physical, human, natural and social. Basic asset valuation techniques require a measure of current and future income streams, e.g., current and future unit shadow water rents. These unit shadow water rents are then

¹⁷ At the time this report was being prepared, there were no working papers or other published sources to reference.

“discounted” and added together to get the unit stock price. For example, assume an asset employed on January 1 (“today”) of this year is expected to last two years, and is expected to generate \$10 in net income at the end of each year. If the “discount rate” is 5%, then the \$10 earned at the end of the first year is worth $\$10/1.05 = \9.52 today. The \$10 earned two years from today is worth $\$10/1.05^2 = \9.07 today. The sum of these two discounted values is called the *discounted present value* – in this case is equal to \$18.59 – and is the value of the asset. The rules for discounting, i.e., the rules that determine the amount by which we divide future income streams, can get more complicated than choosing a single discount rate, but the idea is the same: you convert a future unit of income into a value of that income today.

The major point to take from this summary of shadow rental rates and asset prices is that the two values are intimately linked, and if you want a measure of the value of a natural asset or ecosystem, the first hurdle to cross is that of getting the asset’s rental values. The major distinction between the two applications is Smith and Gemma has no ecosystem dynamics, while Nelson, Roe and Smith, do. Both studies measure water’s contribution to GDP, but at this point, only Smith and Gemma measure the asset value of the ecosystem.

4.1 Water Valuation in the Kanto Plains

Smith and Gemma divide Japan into three regions, Tokyo, the Kanto Plains without Tokyo, and the rest of Japan. They then disaggregate the Tokyo economy into four sectors – agriculture, manufacturing, services and residential water – and divide the rest of Kanto into the same subsector categories. They disaggregate the rest of Japan into three sectors: agriculture, manufacturing and services. To estimate the value of water they proceed in two steps. First, they use regression analysis to identify water’s contribution to agricultural value added. Then they employ a model whose structure is similar to that presented in the appendix.

The major difference between model of Smith and Gemma, and the one presented in the appendix, is the ecosystem in their model has a very simple structure, and can be viewed as a special version of either case 1 or case 2. From the standpoint of case 2, simply interpret H as water, not habitat, and assume they are not concerned with species. The absence of a species eliminates equation (4) from their analysis, leaving only (5) or (6). In the surface water problem they model, the amount of water available to each region remains constant over time, with no implicit change in the stock of water, H . If the amount of water taken from the river is the same each period, and there is no change in water stocks, then $\dot{H}(t) = 0$. In other words, there are no ecosystem dynamics in their model.

Still, the model yields a rich set of empirical results that provide insights into how the evolution of ecosystem service rental rates are related to the capital to labor ratios of the sector exploiting the service. In general, the higher is a sector’s capital-to-labor ratio, the faster is the rate of growth in its ecosystem service rental rates. Another insight is it is possible for the unit shadow rent of an ecosystem service to fall over time. This can happen when the level of an ecosystem service available to a sector does not vary much over time, as is possible with surface water, and the sector’s capital to labor ratio is small enough – as discussed shortly, a characteristic of Tokyo manufacturing.

Table 1 presents the predicted unit shadow water rents for agriculture, manufacturing and services for the years 2008 – 2108. Tokyo manufacturing has the lowest capital to labor ratio of all sectors, and the reader can verify that it also has the slowest rate of growth in water shadow values: in fact, for over 20 years the unit shadow rental value of water for the sector falls. This occurs primarily because Tokyo manufacturing’s low capital to labor ratio puts it at a disadvantage in competing with the other sectors for resources. This process eventually reverses itself because the cumulative effect of technological improvements enabled the sector to compete well enough for capital to actually increase output.

Table 1. Unit shadow water rental values

Year	Agriculture			Manufacturing		Services	
	Tokyo	ROK	ROJ	Tokyo	ROK	Tokyo	ROK
2008	93.88	41.02	69.52	311.14	52.44	188.76	80.06
2018	97.76	42.71	72.39	147.87	58.94	274.27	116.57
2028	111.04	48.52	82.23	125.00	68.90	353.48	150.36
2038	130.12	56.85	96.36	129.81	81.69	436.21	185.61
2048	154.37	67.45	114.32	146.35	97.40	528.98	225.12
2058	184.10	80.44	136.34	170.78	116.41	636.78	271.01
2068	220.05	96.14	162.96	202.22	139.26	764.17	325.23
2078	263.25	115.02	194.96	240.95	166.67	915.80	389.77
2088	315.07	137.67	233.33	287.88	199.51	1096.90	466.85
2098	377.16	164.79	279.31	344.35	238.85	1313.47	559.03
2108	451.52	197.28	334.38	412.10	285.95	1572.64	669.33

As noted above, a sector's unit shadow water rent multiplied by the amount of water it used, is water's contribution to the sector's GDP. Smith and Gemma note that water contributes about 30% to the Kanto region's agricultural GDP, 4% to its manufacturing GDP, and 0.7% to its service sector GDP. In 2008, Kanto GDP was equal to 144,412 billion Japanese Yen – direct calculations show water accounted for 2.8% of that GDP.¹⁸ Water's share of Kanto GDP dropped to 1.8% by 2058 – a result that occurs because over time, agriculture and manufacturing become less important to the region, while the service sector increases in importance.

Smith and Gemma also calculate the unit asset (or stock) value of water to each sector. Table 2 reports some of their results. Combining the 2008 unit shadow water prices with corresponding water stock estimates yields a lower bound measure of the Kanto region's ecosystem equal to 1,103,256 billion Yen, which they note is about one fourth the value of the region's stock of physical capital.

Table 2. Unit shadow water price values

Year	Agriculture			Manufacturing		Services	
	Tokyo	ROK	ROJ	Tokyo	ROK	Tokyo	ROK
2008	2095.19	915.46	1551.62	3017.92	1280.23	6298.74	2678.54
2018	2783.68	1216.28	2061.49	2949.86	1739.55	9156.58	3895.78
2028	3466.73	1514.73	2567.33	3352.28	2183.59	11792.33	5018.21
2038	4218.10	1843.03	3123.77	3942.29	2665.25	14545.76	6190.45
2048	5084.60	2221.63	3765.47	4686.96	3216.98	17635.18	7505.53
2058	6105.16	2667.54	4521.26	5595.13	3864.84	21227.06	9034.37

¹⁸ This is over \$1 trillion U.S., at July 2013 exchange rates.

With respect to mainstreaming ecosystem services, the Smith and Gemma study does three things: (i) it shows how to disentangle the contribution of ecosystem services embedded in GDP; (ii) it shows how to estimate the unit shadow rental value of ecosystem services across multiple sectors and over time; and (iii) it shows how to use those unit shadow rental values to estimate the asset value embedded in the ecosystem. One thing they do not do is discuss specific policy prescriptions, primarily because one of the main objectives of the chapter was to explain how to overcome potential national account data limitations and estimate ecosystem service values, and illustrate how to begin measuring the asset value of an ecosystem.

4.2 Subsidies and Groundwater Dynamics in the Punjab¹⁹

Nelson, Roe and Smith (hereafter, written NRS) divide India into two regions, the Punjab state and the rest of India. They focus on four sectors in the Punjab: manufacturing, rice, wheat and other agriculture. They then disaggregate the rest of India into five sectors: manufacturing, rice, wheat, other agriculture, and services (for all of India).

The NRS model is more complex than that of Smith and Gemma, in two ways. First, the ecosystem service from water is dynamic, and can be viewed as a variant of case 2 with no species dynamics. Their water dynamics are, essentially, a modification equation (5):

$$\dot{GWT}(t) = -D(H(t)) + g(PRECIP(t))$$

Here, $\dot{GWT}(t)$ is the change in the groundwater table, is the decrease in the groundwater table when farmers extract H cubic meters of water, and $g(PRECIP)$ is the increase in the groundwater table when the region gets $PRECIP$ centimeters of precipitation. Second, in addition to introducing groundwater dynamics, the NRS study has a distinct policy component in that it examines the impact of electricity subsidies on groundwater dynamics and Punjabi economic performance.

Table 3 presents the results of two simulations: one with farmers given a 90% subsidy on their electricity usage and the other where the subsidy is completely removed. As one would expect, removing the electricity subsidy leads to lower gross values for rice and wheat – the irrigation using sectors. With the subsidy removed, however, electricity becomes more expensive for farmers and they use less electricity. Aggregate demand for electricity falls, but with cheaper electricity, manufacturing costs fall and its output and gross value increase: in this case, the increase in manufacturing revenue offsets the losses experienced by agriculture.

On the other hand, water's contribution to GDP falls. This result highlights the fact that ecosystem service values can be influenced by policy. Here, a policy of removing electricity subsidies makes it more costly to pump water, with the result that the value of irrigation water to farmers falls. An upside of removing the subsidy is the water table is drawn down at a significantly slower, and arguably, sustainable rate. Another upside is water's asset value will increase because its unit shadow rental rate increases at each point in time, and because the water table is higher, the stock of water remaining will be larger. Unfortunately, it is too early in the study to have concrete estimates of ecosystem/water asset values.

Although beyond the scope of this report, the NRS model allows for a direct measurement of the value of precipitation embedded in GDP, and has great potential for understanding more clearly, the economic impact of climate change on an economy and its wealth.

¹⁹ The author thanks Harumi Nelson and Terry Roe for permission to report some of the preliminary results of the study. Any errors are the sole responsibility of the author.

Table 3. Simulation results from Nelson, Roe and Smith

Electricity subsidy rate	Year	Depth to water table (m)	Irrigation water demand (billion m ³)	Water's contribution to GDP	Water's Unit shadow rental rate (\$/m ²)	Irrigation energy demand (Gwh)	Unit energy price	Gross Value per Sector				Punjab Ag & Manuf value-added
								Rice	Wheat	Other Agri	Manufacture	
90% subsidy	2007	12.961	40,622	142.59	0.00351	13,472	1.850	2,685	3,526	6,023	4,381	16,921
	2012	16.253	37,785	134.66	0.00356	15,714	2.395	2,510	3,458	5,188	3,138	14,639
	2017	19.064	35,151	127.30	0.00362	17,148	2.845	2,348	3,394	4,490	2,344	13,001
	2022	21.444	32,849	120.87	0.00368	18,025	3.179	2,207	3,335	3,926	1,857	11,856
	2027	23.453	30,912	115.46	0.00373	18,551	3.402	2,088	3,283	3,476	1,566	11,059
	2032	25.154	29,310	110.98	0.00379	18,866	3.535	1,991	3,238	3,119	1,391	10,501
	2037	26.600	27,993	107.30	0.00383	19,054	3.604	1,911	3,199	2,833	1,287	10,105
No subsidy	2007	12.961	25,241	99.61	0.00395	12,564	1.626	1,688	3,397	6,025	4,884	15,713
	2012	13.738	24,396	97.25	0.00399	12,629	1.514	1,643	3,342	5,189	4,848	14,839
	2017	14.386	23,749	95.44	0.00402	12,688	1.403	1,610	3,292	4,491	4,815	14,157
	2022	14.934	23,241	94.02	0.00405	12,742	1.300	1,585	3,247	3,926	4,785	13,640
	2027	15.402	22,831	92.87	0.00407	12,791	1.205	1,566	3,207	3,477	4,758	13,256
	2032	15.805	22,495	91.93	0.00409	12,835	1.121	1,551	3,172	3,119	4,734	12,972
	2037	16.155	22,215	91.15	0.00410	12,874	1.045	1,538	3,142	2,833	4,712	12,765

4.3 More on the Model and Ecosystem Valuation

The preliminary NRS results, combined with those of Smith and Gemma suggest these models could serve as a useful tool in measuring the (provisioning) value of ecosystem services – both flow and stock values. The results also suggest these models can yield quite useful insights into how economic activities, economic structure, and economic policy can influence ecosystem valuation: meaning, a clear understanding of the model structure and its implications will tell a reasonably well trained eye a lot about how the economic value of an ecosystem service would likely evolve over time.

To illustrate, consider the coral reef, tourism economy described briefly at the end of case 2. The Roe, Smith and Saraçoğlu (2010), and Gemma and Smith models tell us capital deepening will occur, and wage rates will increase as the economy grows, while the rate of return to capital will fall – making labor more costly and capital less costly as the economy grows. Imagine, for the moment, there is no pollution, the coral ecosystem stays healthy and does not change (in size and quality), and real tourism prices remain constant over time.²⁰ If the tourism sector is the most “labor intensive” sector in the economy, then most likely, as the economy grows it will become relatively more expensive for the tourism sector to provide its services. The increase in production expenses put downward pressure on tourism profit, i.e., the shadow rent to the coral ecosystem. If tourism is the most “capital intensive” sector, the opposite would occur.²¹ In terms of stock values, we would expect the asset value of the ecosystem to be higher if tourism is capital intensive, as opposed to the case where it was labor intensive. This is because over time, the shadow rent of the coral ecosystem should be higher when the tourism sector is capital intensive. These conclusions about wage and interest rate behavior, and about how coral ecosystem shadow value dynamics are influenced by capital-to-labor ratios are reasonable guesses to make, and provide testable hypotheses on ecosystem values and value dynamics.

How do these simple dynamics change, if the manufacturing sector generates a pollutant that decreases the level of the coral reef habitat? The initial effect would be to render the capital and labor employed in ecosystem service provision less productive. This, in turn, would decrease tourism demand for capital and labor. If tourism is important enough in the economy, a decrease in tourism’s labor (and capital) demand could lead to a decrease in the wage rate (and rate of return to capital). So pollution can have a negative impact on the economy by putting downward pressure on wages and rates of return to capital. Pollution would also have a negative impact on ecosystem rent: the decrease in productivity associated with a loss in ecosystem service levels makes it more difficult for the tourism sector to compete for capital and labor. With less capital and labor going to the sector, the profit/rent earned by the sector will fall. This will have two effects on ecosystem service values: (i) with tourism less competitive, ecosystem services contribution to GDP falls and (ii) the value of the ecosystem as a productive asset falls – i.e., it will contribute less to inclusive wealth.

4.4 Policy Issues

Given the myriad of policy issues, it is useful to consider two broad categories of policy. One is related to the economic forces associated with the natural evolution of an economy (e.g., capital deepening and increased labor productivity), and how these forces impact (spill over onto) the exploitation of natural assets and ecosystems. In this category, policy instruments include taxes, subsidies, quotas, licenses and property rights that are linked directly to natural assets and ecosystems. Similar instruments linked to other sectors can have indirect impacts on natural assets and ecosystems. The other set of policies are domestic macroeconomic policies that cause major fiscal and trade imbalances, which in turn distort domestic product and factor markets. Similarly, policies implemented in other countries can have effects on domestic economies and ecosystems.

²⁰ This implicitly assumes the tourism sector competes with the rest of the world for tourists, and there are many substitutes for its services.

²¹ A sector is the most labor intensive if its labor-to-capital ratio is higher than any other sector in the economy.

Noting the complexity of the sources of ecosystem degradation, policy instruments to address degradation tend to have direct and indirect impacts. For example, with the coral reef example, the expected direct effect of a tax on manufacturing output is a decrease in manufacturing output, and correspondingly, less pollution and ecosystem degradation. With a healthier ecosystem, the productivity of capital and labor should increase: the pollution tax should trigger higher tourism income, and hence, increase both the flow value and the asset value of the coral reef system. The indirect effects of the tax include wage and capital rental rate adjustments. The manner in which these adjustments take place will depend on the relative factor intensities of the three sectors. If manufacturing is capital intensive and tourism is labor intensive, when manufacturing decreases output, one could expect it to release more capital than the tourism and agricultural sector wants, and place downward pressure on capital rental rates. As capital is released to the tourism and agricultural sectors, they realize an increase in labor productivity, and being labor intensive relative to the manufacturing sector will want more labor than manufacturing releases – placing upward pressure on wage rates. In this case, tourism profit – and by extension – ecosystem service rents will likely be dampened relative to the case where wage and capital rental rates are fixed. The observation to make here is, even without an empirical model, a good understanding of the theory presented in the appendix can guide an analyst to a reasoned guess on the likely impact of some types of economic policy.

The empirical analogue to the theory below has great potential for understanding the impact of restoration policy. Consider the case where an ecosystem can be restored by investing in restoration efforts. Would simultaneously allowing pollution and subsidizing restoration efforts be a viable strategy for enhancing growth? Could such a policy restore an ecosystem to its original state without sacrificing long run economic growth? What would be the likely short-run and intermediate-run effect of taxing manufacturing output or capital use in manufacturing (and damaging an ecosystem), while subsidizing restoration? Are the growth dynamics of restoration significantly different if restoration is labor intensive or capital intensive relative to the other industries? Each of these questions can be accommodated with the empirical analog of the framework outlined in section 3.

Related to ecosystem restoration is the general notion of relying on technology to reverse damage to ecosystems and their ability to provide ecosystem services. Historically, one of the ecological costs of agricultural production has been the destruction of habitat for wildlife. Technological change in agriculture, however, has encouraged no-till or minimal-till practices, particularly in North America. As a result, habitat for various species in the upper Midwestern U.S. have expanded or improved, leading to some formerly endangered species, e.g., the wolf in North America, to be removed from the endangered species list. This is an example of how a change in technology inadvertently led to the restoration of an ecosystem, suggesting another policy tool could be to subsidize technologies that militate against ecosystem damages, giving the ecosystem a chance to restore itself.

With respect to macroeconomic performance, policymakers typically operate in economies having several policies that distort economic performance – e.g., tariffs, indirect taxes, subsidies, etc. In the coral reef example above, if manufacturing were an import competing industry protected with tariffs and quotas, more than likely decreasing the tariff or relaxing the quota would lead to lower manufacturing production, and decrease ecosystem degradation. If the economy had a subsidy on capital employed in the tourism industry, and removed the capital subsidy when it eliminated the tariff, the combined effects could lead to an outcome with more pollution than when both distortions were in place. Removing the tariff would lead to a decrease in manufacturing output, while removing the subsidy on tourism capital demand would lead to a decrease in the demand for investment capital and a fall in the unit cost of capital. The lower capital costs would provide incentives for manufacturing to increase output. The net effect of the opposing forces could yield an outcome where removal of the two distortions leads to a higher rate of pollution and ecosystem damage. The model presented in the appendix can provide a nice framework for predicting the macroeconomic impacts of a set of policy changes, and the corresponding changes in ecosystem stock and flow values.

The model presented in the appendix is also designed to address questions on the tradeoffs between ecosystem management and (i) per capita GDP (cost-benefit comparisons) and (ii) per capita GDP growth. For example, if society decides it wants to preserve an ecosystem and has implemented policy that prevents the ecosystem (say coral habitat area and fish biomass) from falling below threshold levels determined by marine scientists, a natural question to ask is: what does saving the ecosystem cost in terms of foregone per capita GDP. Furthermore, economic

impact would the policy have on the major productive sectors of the economy? What would be the net economic benefit of providing incentives to invest in ecosystem restoration efforts?

5. Conclusion

The objective of this report has been to: (i) introduce a modeling framework within which a dynamic macroeconomic system is linked with an ecosystem and preferably ecosystem service, (ii) provide a description of how the integrated economic-ecosystem would likely evolve over time, (iii) discuss how the model could be used to measure ecosystem and ecosystem service values, and provide examples of results from such an implementation, and (iv) discuss how policy could affect such a system. The conceptual framework was developed to provide a possible point-of-departure for conducting empirical analysis of two-way linkages between sectoral macroeconomic policies and ecosystem service, and policies related to ecosystem management and its impact on regional or national economies.

The fourth section of this reports shows how the modeling framework can be used to make reasons predictions on how policy would likely impact ecosystem service values and whether the policy would encourage or discourage ecosystem exploitation. It also gives the reader an idea of what types of output the empirical model can generate in terms of flow and stock values of ecosystems and the rate of ecosystem exploitation: in terms of valuation, the important outputs are the ecosystem service's contribution to GDP and their asset values. One thing not discussed above is model validation, primarily because the validation exercises of the two studies discussed above have not been completed.

Conducting, thoughtful, macro-level policy analysis on ecosystem management issues discussed above will involve several steps: (i) conceptual model development, (ii) data collection, (iii) economic and ecosystem model calibration, (iv) develop an empirical model using appropriate software, (v) validate the model and run policy scenario simulations, and (vi) interpret model results. We have discussed the first step in some detail. The remaining steps are discussed throughout the literature, and the economic side of the process summarized in RSS and empirically implemented in Smith and Gemma (2013). Macroeconomic analysis of policies targeted at management of ecosystem and ecosystem services for enhancing human well-being would provide the correct and holistic picture. The macroeconomic analysis in general equilibrium framework would also signal the imminent impact for the concerned sectors and stakeholders, something highly desirable by decision makers facing conflicting goals and choices.

The framework, while having many benefits, is not without its weaknesses. One shortcoming is it cannot accommodate problems having a transnational point of reference. For example, it is not set up to examine the economic impact of a foreign country's climate policy on a home country, or to examine the economic impact of a "large" country's fishery policy on the rest of the world. The modeling framework also does not accommodate complex intertemporal discounting schemes, like hyperbolic discounting or having a zero discount rate in the long run. Some readers may dislike the fact that technical change here is exogenous, as opposed to endogenous. This choice simply reflects the fact that the authors have more experience in constructing empirical exogenous growth models and their behaviors. Others are encouraged to develop endogenous growth model alternatives to what is presented in this report.

The conceptual framework and its empirical analog provide a platform for moving into several research directions. For example, here, we do not address transboundary problems, like the economics of managing the Great Limpopo Transfrontier Park (which spans South Africa, Zimbabwe and Mozambique) or the Mekong River Basin (which flows through China, Myanmar, Laos, Thailand, Cambodia and Vietnam). Adding a "rest of the world" or another country opens up the door to examining such issues. Understanding, better, the impact of triggers for macroeconomic imbalances on ecosystem health might be a fruitful research direction. Practical applications of the framework above include: estimating the cost to a country, say Brazil, of placing an indefinite moratorium on deforestation is possible with the framework presented; estimating the "wealth" of a country over time, and discerning whether the wealth trajectory satisfies various conditions/definitions on sustainability; calculating the (shadow) value of natural assets and ecosystems.

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Appendix: Formal Model Presentation and Derivation of Transition Dynamics

The Model

The economic environment: The economic model takes as its point of departure, the three-sector, small open economy model in chapter 4 of Roe, Smith, and Saraçoglu (2010): a model that uses land, labor and capital to produce two traded goods and a non-traded good.

The model is disaggregated into three sectors for several reasons. First, we want a theoretical model whose empirical analog can forecast growth rates and explain how the structure of an economy (sectoral composition of GDP) will likely evolve over time. Sector details are helpful in explaining how policy affects different stakeholders: e.g., an agricultural or tourism sector. Next, the model should be able to examine trade and terms of trade effects, and in other cases, international capital flows that can speed up the economic growth process. The empirical model should be able to explain how ecosystem dynamics can affect the performance and structure of an economy over time since some sectors, and some resources (such as rural unskilled labor) may be more closely linked to an ecosystem than others. In the small open economy model, the price of traded goods is exogenous, while the non-traded good price is endogenous. This dichotomy of fixed and endogenous final good prices allows the model to say something about improved (increasing non-traded good price over time) or worsening terms of trade. Finally, we want a model that can accommodate intersectoral externalities (explained shortly) in a theoretically consistent manner. The three-sector model presented below accomplishes each of these goals.

For expositional purposes, I describe the modeled economy as a tropical country/region that produces three final goods: tourism services, manufactured goods, and agricultural products (e.g., Trinidad or Thailand). Each sector uses capital and labor, while agriculture also uses land. The tourism sector is directly linked to the coral reefs and the number (biomass) of fish living among the coral. For reasons of parsimony, we assume tourism is a non-traded good, while manufacturing and agriculture are traded goods. We also ignore population growth, and set the rate of labor productivity growth equal to zero.²²

Let $j = a, m$ and s respectively index the agricultural, manufacturing, and tourism sectors. Denote the economy's initial endowment of labor, capital, and land respectively by $L, K(0)$ and Z , where the land endowment is constant over time. The economy trades the manufactured and the agricultural good internationally at fixed prices p_m and p_a , respectively. The tourism good is traded in the domestic economy at the endogenous price $p_s(t)$. Firms in each sector hire both labor and capital, while land is an input only in agricultural production. Labor earns a wage rate of $w(t)$, while capital earns a rate of return on capital denoted $r(t)$. Assume a land rental market among farmers exists, and that land is rented in or out at rate Πz per acre. Tourism revolves around a coral reef ecosystem, which we can view as interplay between coral reefs, denoted H , and exotic fish stocks denoted Φ . The stock of fish and habitat are not owned by any individual, and rental markets for these assets do not exist.

The agricultural and tourism good are pure consumption goods, while the manufactured good is consumed or saved (augments the capital stock). Any excess supply or demand of the manufactured good trades in international markets at the price p_m . Labor services is not traded internationally and domestic residents own the entire stock of domestic assets.

²² With several model modifications not discussed here, one could also treat tourism as a traded good. It is also relatively straightforward to accommodate labor force growth and labor productivity, but comes at the cost of increased complexity in the mathematical notation.

Firm Behavior: Represent the manufacturing technology by the production function

$$Y_m(t) = F^m(L_m(t), K_m(t))$$

where Y_m is manufacturing output, and L_m and K_m represent the level of labor and capital employed by the sector. Assume the technology $F^m: \mathbb{R}_{++}^2 \rightarrow \mathbb{R}_+$ is everywhere continuous and twice differentiable, linearly homogeneous, non-decreasing and strictly concave in inputs, and satisfies the Inada conditions. The cost function corresponding to $F^m(\cdot)$ is given by

$$C^m(w(t), r^k(t))Y_m \equiv \min_{L_m, K_m} \{L_m w(t) + r^k(t)K_m : Y_m \leq F^m(L_m, K_m)\} \quad (7)$$

where $r^k = r + \delta$, with δ representing the rate of capital depreciation. Given the properties of $F^m(\cdot)$, the cost function is linearly homogeneous, concave, differentiable, and nondecreasing in input prices, and satisfies Shephard's lemma.

Agricultural production is governed by the technology

$$Y_a = F^a(L_a, K_a, Z)$$

where Y_a is agricultural output, and where $F^a(\cdot)$ satisfies the same regularity conditions as the manufacturing technology. The value-added by agriculture's sector specific resource Z is defined as

$$\pi^a(p_a, w, r^k)Z \equiv \max_{L_a, K_a} \{p_a F^a(L_a, K_a, Z) - wL_a - r^k K_a\}$$

The regularity conditions imposed on $F^a(\cdot)$ ensure the value-added function is non-decreasing in p_a , non-increasing in w and r^k , homogeneous of degree one and differentiable in input and output prices, convex in input and output prices, and satisfies Hotelling's lemma. Here, $\pi^a(p_a, w, r^k)$ is the rental rate per unit of land per worker required for the rental market among farmers to clear. Assuming differentiability, by Hotelling's lemma the gradients of $\pi^a(p_a, w, r^k)$ yield the partial equilibrium agricultural supply and derived capital and labor demand per economy-wide labor, e.g.,

$$Y^a(p_a, w, r^k)Z = \pi_{p_a}^a(p_a, w, r^k)Z$$

The tourism technology is represented by

$$Y_s(t) = F^s(K_s(t), L_s(t); \Phi(t), H(t))$$

where $Y_s(t)$ is the time t production of tourism services, and $K_s(t)$ and $L_s(t)$ are the capital and labor employed by the sector. We assume the ecosystem is not an essential input,²³ and view the coral reef area and stock of fish as an endowment. Here, we ignore the possibility of investing in coral reef restoration and restocking fish populations. Assume the function $F^s(\cdot)$ is: (i) separable in the natural assets and in K_s and L_s , i.e., the tourism technology can be represented as

$$Y_s(t) = F^s(K_s(t), L_s(t))\nu(\Phi(t), H(t))$$

(ii) twice continuously differentiable, nondecreasing and strictly concave in K_s and L_s , and (iii) K_s and L_s satisfy the Inada conditions. We assume $\nu: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ is a function that is continuously differentiable almost everywhere, and nondecreasing in Φ and in H .²⁴ The function $\nu(\cdot)$ can be thought of as a productivity index for the tourism sector.

²³ In other words, it is possible for $F^s(K_s, L_s, 0)$ to be strictly positive. For example, tourism could move away from a mix of casino and water sport activities to just casinos and gaming.

²⁴ In an empirical application, a candidate functional form for $F^s(\cdot)$ is a constant elasticity of substitution function. Here, we implicitly assume only residents of the country demand tourism services, and preclude congestion effects and the complications they can introduce into the analysis.

The value-added function corresponding to (14) is given by

$$\pi^s(p_s, w, r^k) v(\Phi, H) \equiv \max \{ p_s Y_s - r^k K_s - w L_s : Y_s(t) \leq F^s(K_s, L_s, v(\Phi, H)) \}.$$

Given the assumptions on (14), $\pi^s(\cdot)$ is linearly homogeneous and concave in w , p_s , and r^k ; nondecreasing in p_s , nondecreasing in w and r^k ; and satisfies Hotelling's lemma. In this case, the shadow value of another ecosystem unit is equal to $\pi^s(\cdot)$ while the shadow value of another unit of fish is $\pi^s(\cdot) \partial v / \partial \Phi$.

The savings and consumption behavior of households: At each instant in time, households provide labor services in exchange for a wage $w(t)$ and earn income on capital and land assets at rate $r(t)$ per unit of the asset. Households consume the agricultural, manufacturing and tourism goods, and incur expenditures $\sum_{j=a,m,s} p_j(t) Q_j(t)$. Here, $Q_j(t)$ is the level of good j consumed by households and p_j is the market price per unit of the good.

Household preferences over the consumption of goods a, m and s, are represented by

$$\int_0^\infty \frac{u(Q_a(t), Q_m(t), Q_s(t))^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \quad (10)$$

We assume the function $u(\cdot)$ is homothetic, everywhere continuous and twice continuously differentiable, nondecreasing and strictly concave in quantities. The parameter $\rho > 0$ is rate of time preference, and $\theta > 0$ is the inverse of the elasticity of inter-temporal substitution. In an empirical application, a standard representation of $u(\cdot)$ is the Cobb-Douglas function: $u(Q_a, Q_m, Q_s) = Q_a^{\lambda_a} Q_m^{\lambda_m} Q_s^{\lambda_s}$ where λ_j is the share of total expenditure spent on good- j , and $\sum_{j=a,m,s} \lambda_j = 1$.

Let Q represent an index of household consumption (i.e., utility). Then given homothetic preferences, household expenditure at an instant in time is defined as

$$E(p_a, p_s) Q \equiv \min_{Q_a, Q_m, Q_s} \left\{ \sum_{j=a,m,s} p_j Q_j : Q \leq u(Q_a, Q_m, Q_s) \right\}.$$

Here, $E(p_a, p_s(t)) Q(t)$ is the minimum cost of achieving Q . Given the properties of $u(\cdot)$ the expenditure function $E(p_a, p_s) Q$ is linearly homogeneous, concave, differentiable, and nondecreasing in prices, and satisfies Shepard's lemma.

Given the natural assets Φ and H , the total value of physical and natural asset holdings, denoted $A(t)$ is expressed as

$$A(t) = K(t) + P_z(t)Z + P_\Phi(t)\Phi(t) + P_H(t)H(t),$$

where P_z is the unit price of land, P_Φ is the unit (shadow) price of fish and P_H is the unit (shadow) price of coral reef habitat. Accounting for the two natural assets, the household's *flow budget constraint* is given by

$$\dot{K}(t) = \underbrace{w(t)L + r(t)K(t) + \Pi_z(t)Z + \Pi_\Phi(t)\Phi(t) + \Pi_H(t)H(t)}_{\text{Income}} - \underbrace{E(p_a, p_s(t))Q(t)}_{\text{Expenditure}} \quad (8)$$

where Π_Φ and Π_H are the (shadow) rental rates of fish and habitat, and \dot{K} is the time t rate of change in the capital stock. Equation (8) tells us households earn income in from wages (wL), capital rent ($r^k K$) and land rent ($\Pi_z Z$). They also realize income from the flow of services produced by the ecosystem ($\Pi_\Phi \Phi + \Pi_H H$). With this income, they purchase goods and services, with the value of those purchases equal to $E(p_a, p_s) Q$.

The household's problem is to choose the sequence of consumption levels $\{Q(t)\}_{t \in [0, \infty)}$ to maximize

$$\int_0^\infty \frac{Q(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to initial conditions, $K(0), L, Z, H(0), \Phi(0)$, the flow budget constraint (8) in each t , and the transversality condition on borrowing

$$\lim_{t \rightarrow \infty} \left\{ K(t) \cdot \exp \left[- \int_0^t r(v) dv \right] \right\} = 0.$$

Since, by assumption, Φ , and H are not assets owned by any individual agent, their evolution does not factor directly into the household's optimizing behavior. The implication here is, although the ecosystem dynamics impact the tourism sector, we do not assign a costate variable to the differential equations governing fish and habitat dynamics. The present-value Hamiltonian for this problem is

$$\Delta = \frac{Q^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \xi \left[wL + rK + \Pi_z Z + \Pi_\Phi \Phi + \Pi_H H - E(p_a, p_s) Q \right]$$

where the costate variable $\xi(t)$ is the present value shadow price of capital. The Euler condition for this problem is

$$\frac{\dot{Q}}{Q} = \frac{1}{\theta} \left[r - \rho - \lambda_s \frac{\dot{p}_s}{p_s} \right] \quad (9)$$

where λ_s , is the expenditure share on the tourism good is given by

$$\lambda_s = E_{p_s}(p_a, p_s) p_s / E(p_a, p_s).$$

Let $Q(t)$ represent an index of consumption (e.g., utility), and let $E(p_a, p_m, p_s) Q$ represent the minimum cost of achieving consumption level Q , given final good prices are p_a, p_m and p_s .

The no-arbitrage condition - the link between unit flow and stock values: The case where the capital-land markets and natural asset values are segmented is very interesting, but beyond the scope of the present discussion.²⁵ As such, assume the natural and physical asset markets are not segmented, and that arbitraging occurs for both types of assets. In such a case, RSS show the no-arbitrage condition yields the following relationship between r and land rents:

$$r = \frac{\Pi_z}{P_z} + \frac{\dot{P}_z}{P_z} \quad (9)$$

Here, r represents the return to the household from investing a unit of income in physical capital. The same unit of income can also buy $1/P_z$ units of land, generating, at time $t + dt$, a rental income of Π_z/P_z plus the rate of change in the land price. If this condition did not hold, optimizing investors could exploit the arbitrage opportunity and move investments out of land and into capital. Hence, (9) is referred to as the no-arbitrage condition.²⁶ The arbitrage condition on fish and coral habitat satisfies

$$r = \frac{\Pi_\Phi}{P_\Phi} + \frac{\dot{P}_\Phi}{P_\Phi} + \frac{\dot{\Phi}}{\Phi} = \frac{\Pi_H}{P_H} + \frac{\dot{P}_H}{P_H} + \frac{\dot{H}}{H}$$

In this case, if arbitrage conditions hold across natural and physical assets, the time t shadow price of coral habitat is given by

$$P_{\xi H}(t) = \int_t^\infty e^{-\int_t^\tau [r(v) - \frac{\dot{H}}{H}] dv} \Pi_H(\tau) d\tau$$

²⁵ Segmentation provides incentives for agents to sell an amount of one asset to acquire another, thus affecting their market values and wealth. The no-arbitrage condition presumes that asset prices adjust such that agents have no incentive to exchange one asset for another.

²⁶ In some cases, the price of capital is endogenous, and the price of land will depart from the definition given in this section.

Where, $P_{\xi H} = P_H / P_K$. Note, if the habitat degrades at time t' i.e., $\dot{H} / H < 0$, then the effective discount rate $r - \dot{H} / H$ increases: reflecting the loss in value associated with habitat depreciation. This effect, of course, places a downward pressure on the net rate of return to the habitat. The time t shadow price of fish is given by

$$P_{\xi\Phi}(t) = \int_t^\infty e^{-\int_t^\tau [r(v) - \frac{\dot{H}}{H}] dv} \Pi_\Phi(\tau) d\tau.$$

where, $P_{\xi\Phi} = P_\Phi / P_K$. These values, combined with predicted levels of natural assets, provide a means for estimating the trajectory of natural asset stock (shadow) values. Here, the total time t stock value of natural assets is $P_H(t)H(t) + P_\Phi(t)\Phi(t)$ (see RSS for a general discussion of asset pricing in the class of models considered in the chapter).

Definition and Characterization of Equilibrium

Given an initial home-good price, $p_f(0)$, initial resource endowments $\{K(0), L, Z, \Phi(0), H(0)\}$ and constant world market prices, p_a and p_m , a competitive equilibrium for this economy is a sequence of positive tourism good prices and capital stock levels $\{p_s(t), K(t)\}_{t \in [0, \infty)}$, household consumption plans

$$\{Q_a(t), Q_m(t), Q_s(t)\}_{t \in [0, \infty)}$$

factor rental prices for labor, capital and land

$$\{w(t), r(t), \Pi_z(t)\}_{t \in [0, \infty)}$$

shadow rental values for fish and habitat, production plans

$$\{Y_a(t), Y_m(t), Y_s(t), K_a(t), K_m(t), K_s(t), L_a(t), L_m(t), L_s(t)\}_{t \in [0, \infty)}$$

and ecosystem dynamics

$$\begin{aligned} \dot{\Phi} &= \Theta(\Phi, H, \Omega) \\ \dot{H} &= \Psi(H, \Omega) \end{aligned}$$

such that at each instant of time t

1. The representative household solves its utility maximization problem
2. Firms maximize profit subject to their technologies, yielding zero profits
3. Markets clear for the non-traded good, labor, and capital, equations, i.e.,

(a) commodities

$$Y_s - Q_s = 0$$

(b) labor

$$\sum_{j=a, m, s} L_j = L$$

(c) capital

$$\sum_{j=a, m, s} K_j = K$$

4. And the no-arbitrage condition holds between the capital, land, and natural assets

$$r = \frac{\dot{\Pi}_z}{P_z} + \frac{\dot{P}_z}{P_z} = \frac{\dot{\Pi}_\Phi}{P_\Phi} + \frac{\dot{P}_\Phi}{P_\Phi} + \frac{\dot{\Phi}}{\Phi} = \frac{\dot{\Pi}_H}{P_H} + \frac{\dot{P}_H}{P_H} + \frac{\dot{H}}{H} \quad (10)$$

Intratemporal equilibrium: Equilibrium can be characterized as follows. Given the exogenous sequence of fish and habitat levels

$$\{\Phi(t), H(t)\}_{t \in (0, \infty)} \quad (11)$$

and the endogenous sequence of capital stock and home-good price levels

$$\{K(t), p_s(t)\}_{t \in (0, \infty)} \quad (12)$$

the intratemporal equilibrium is a four-tuple sequence of positive values

$$\{w(t), r^k(t), Y_m(t), q(t)\}_{t \in (0, \infty)} \quad (13)$$

such that, for each the following four intratemporal conditions hold:

1. zero profits in production of the manufactured good

$$C^m(w, r^k) = 1 \quad (14)$$

2. labor market clearing

$$\frac{\partial}{\partial w} C^m(w, r^k) Y_m - \frac{\partial}{\partial w} \pi^s(p_s, w, r^k) v(\Phi, H) - \frac{\partial}{\partial w} \pi^a(p_s, w, r^k) Z = L \quad (15)$$

3. capital market clearing

$$\frac{\partial}{\partial r^k} C^m(w, r^k) Y_m - \frac{\partial}{\partial r^k} \pi^s(p_s, w, r^k) v(\Phi, H) - \frac{\partial}{\partial r^k} \pi^a(p_s, w, r^k) Z = K \quad (16)$$

4. and home-good market clearing

$$\frac{\partial E(p_a, p_s) q}{\partial p_s} = \frac{\partial}{\partial p_s} \pi^s(p_s, w, r^k) v(\Phi, H) \quad (17)$$

Intertemporal equilibrium: To create an empirical analog of the equilibrium characterized by equations (11) through (17), we next derive a system of four differential equations that, once solved, allow us to recover all remaining endogenous variables. The system is composed of equations of motion for the capital stock, the home-good price, the stock of fish, and the habitat area, i.e., $K, p_s, \Phi,$ and H .

The first differential equation we consider is a version of (8) that depends on p_s, K, Φ and H . Begin the derivation by using equation (14) to (implicitly) solve for the equilibrium wage rate as a function of the rate of return to capital. Denote the solution by

$$r + \delta = r^k = R(w)$$

Substitute $R(\cdot)$ into expressions (15) and (16), and solve for the equilibrium wage and manufacturing output. Denote this solution by

$$Y_m = Y^m(p_s, K, \Phi, H) \text{ and } w = W(p_s, K, \Phi, H)$$

where, here we suppress the L argument. Finally, let $\varepsilon(t) = E(p_a, p_s(t)) q(t)$ and use equation (17) to solve for $E(\cdot) q$ in terms of p_s, Φ, H and K

$$\tilde{\varepsilon}(p_s, K, \Phi, H) = \frac{p_s}{\lambda_s} Y^s(p_s, K, \Phi, H) \quad (18)$$

Where

$$Y^s(p_s, K, \Phi, H) = \frac{\partial}{\partial p_s} \pi^s(p_s, w, r^k) v(\Phi, H) \Big|_{\{w=W, r^k=R\}}$$

Let $\tilde{R}(p_s, K, \Phi, H) \equiv R(W(p_s, K, \Phi, H))$ and $\tilde{\pi}^j(p_s, K, \Phi, H) \equiv \pi^j(p_j, W(\cdot), \tilde{R}(\cdot))$, $j = a, s$. Substitute $W(\cdot)$, $\tilde{R}(\cdot)$, $\tilde{\pi}^a(\cdot)$, $\tilde{\pi}^m(\cdot)$ and $\tilde{\varepsilon}(\cdot)$ into equation (8) to get the “reduced form” differential equation

$$\begin{aligned} \tilde{K}(p_s, K, \Phi, H) = & W(p_s, K, \Phi, H)L + \tilde{R}(p_s, K, \Phi, H)K + \tilde{\pi}^a(p_s, K, \Phi, H)Z \\ & + \tilde{\pi}^s(p_s, K, \Phi, H)v(\Phi, H) - \frac{P_s}{\lambda_s} Y^s(p_s, K, \Phi, H) \end{aligned} \quad (19)$$

The arguments of \tilde{K} suggest the remaining three differential equations are required that describe the trajectory of p_s , Φ , and H .

Consider next, the derivation of the differential equation for p_s . Begin by using (18) and taking the time derivative of $E(p_s, p_s)q$ to get

$$\dot{\varepsilon} = \frac{1}{\lambda_s} \left[\dot{p}_s Y^s + Y_{p_s}^s \dot{p}_s + p_s (Y_K^s \dot{K} + Y_\Phi^s \dot{\Phi} + Y_H^s \dot{H}) \right] \quad (20)$$

Next, substitute $R(\cdot)$ into the Euler equation (9) and derive the following expression

$$\dot{\varepsilon} = \varepsilon \frac{1}{\theta} \left[\tilde{R}(p_s, K, \Phi, H) - \rho - \delta - \lambda_s (1 - \theta) \frac{\dot{p}_s}{p_s} \right] \quad (21)$$

Setting (21) equal to (22), substituting $\varepsilon = \frac{p_s}{\lambda_s} Y^s(\cdot)$ into the result, and rearranging terms yields a reduced form differential equation for p_s ,

$$\begin{aligned} \tilde{P}(p_s, K, \Phi, H) = & \frac{p_s \left[\tilde{R}(p_s, K, \Phi, H) - \rho - \delta \right] Y^s(p_s, K, \Phi, H)}{\theta \left[Y^s(p_s, K, \Phi, H) + Y_{p_s}^s(p_s, K, \Phi, H) \right] + Y^s(p_s, K, \Phi, H) \lambda_s (1 - \theta)} \\ & - \frac{p_s \left[Y_K^s(p_s, K, \Phi, H) \dot{K} + Y_\Phi^s(p_s, K, \Phi, H) \dot{\Phi} + Y_H^s(p_s, K, \Phi, H) \dot{H} \right]}{\theta \left[Y^s(p_s, K, \Phi, H) + Y_{p_s}^s(p_s, K, \Phi, H) \right] + Y^s(p_s, K, \Phi, H) \lambda_s (1 - \theta)} \end{aligned} \quad (23)$$

where denotes the reduced form of \dot{P} . The final two differential equations, then are

$$\tilde{\Phi}(\Phi, H) = \Theta(\Phi, H), \quad (24)$$

$$\tilde{H}(p_s, K, \Phi, H) = \Psi(H, \mu Y^m(p_s, K, \Phi, H)) \quad (25)$$

In principle, given a solution to the system of equations (19) and (23) - (25), one can recover the remaining endogenous variables - i.e., wage rates, rates of return to capital, sectoral output levels and input demand levels, and sectoral GDP - using the intratemporal conditions in equations (14) - (17), and exploiting Hotelling's and Shepard's lemma. Also, one can recover the shadow rental values of the environmental assets, and their underlying shadow prices.²⁷

²⁷ A system of differential equations like (19) and (23) - (25), could yield multiple solutions. This of course raises the question of which solution should society try to target and what instruments could achieve the targeted long run equilibrium? We note this issue simply to highlight an issue that could arise when trying to develop ecosystem policy in a macroeconomic setting.

The steady state: We close this section on intertemporal equilibrium with a discussion of the steady state of the system. Let r^s denote the steady state rate of return to capital and p^s denote the steady state home-good price. Also, denote the steady state level of capital, tourism price, habitat, and fish stocks by \bar{P} , \bar{K} , $\bar{\Phi}$, and \bar{H} respectively. If a steady state exists, by the Euler equation the steady state rate of return to capital is equal to

$$r^{ss} = \rho \Rightarrow r^{k,ss} = \rho + \delta$$

which implies

$$w^{ss} = R^{-1}(\rho + \delta)$$

As noted above, we only consider steady state equilibria wherein $\tilde{K} = \tilde{P} = \tilde{H} = \tilde{\Phi} = 0$. Then the steady state levels of \bar{P} , \bar{K} , $\bar{\Phi}$, and \bar{H} must satisfy:

$$\bar{K} = \frac{\frac{\bar{P}}{\lambda_s} Y^s(\bar{P}, \bar{K}, \bar{\Phi}, \bar{H}) - w^{ss} L - \tilde{\pi}^a(\bar{P}, \bar{K}, \bar{\Phi}, \bar{H}) Z - \tilde{\pi}^s(\bar{P}, \bar{K}, \bar{\Phi}, \bar{H}) v(\bar{\Phi}, \bar{H})}{\rho + \delta}$$

and

$$\begin{aligned} \tilde{R}(\bar{P}, \bar{K}, \bar{\Phi}, \bar{H}) &= \rho + \delta \\ \Theta(\bar{\Phi}, \bar{H}) &= 0 \\ \Psi(\bar{H}, \mu Y^m(\bar{P}, \bar{K}, \bar{\Phi}, \bar{H})) &= 0. \end{aligned}$$