ON THE ERGODIC PROPERTIES OF CLIMATE CHANGE WITH IMPLICATIONS FOR CLIMATE FINANCE, AGRICULTURAL RESILIENCE AND SUSTAINABILITY

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Climate Finance and Agriculture

- February 7, 2022 the USDA announced new CCC program to fund partnerships for climate-smart commodities.
- a climate-smart commodity is defined as an agricultural commodity that is produced using farming, ranching or forestry practices that reduce greenhouse gas emissions or sequester carbon .
- Initiative is to encourage farmers and landowners to
  - 1) implement climate-smart production practices, activities, and systems on working lands,
  - 2) measure/quantify, monitor and verify the carbon and greenhouse gas (GHG) benefits associated with those practices, and
  - 3) develop markets and promote the resulting climate-smart commodities

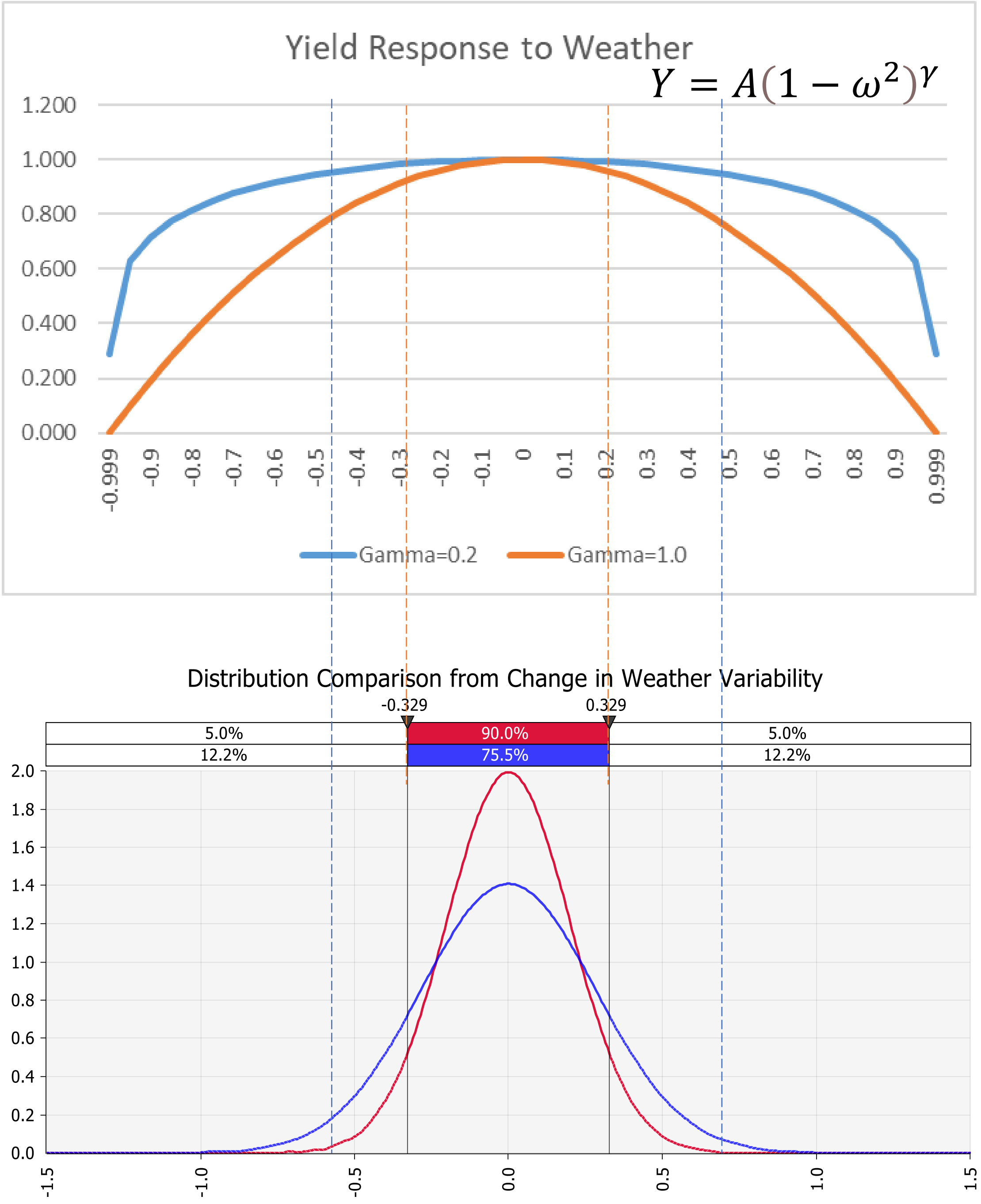
The Economic Problem: Agricultural Resilience and Climate Change

Turvey et al, 2021 (Climatic Change) argue, using a polymorphic production function, that over wide ranges of crop yields the correlation between weather and yield is weak, the response function has a plateau. Less resilient crops are mor peaked.

The climate problem is when variance increases, the probability of achieving extremes increases in time.

Research problem is to understand and measure climate variance in order to better understand climate risks, and then to better understand how climate finance will work if variance is changing.

By understanding the power laws that govern climate variability, we are better positioned to accurately price climate-smart financing and insurance products at the micro, meso and macro scales



The Problem is Larger than it Seems!!!

- Climate-smart financing appears to be a straightforward application of financial principles to new and innovative forms of credit.
- We have been working with various forms of climate finance in terms of index-based insurance and efforts at weather-linked credit.
- BUT climate change is measured globally while climate-smart financing is done locally (or at the meso level).
- Climate change can be measured by local or aggregated changes over some time scale (typically a year).
- But when climate change is considered, what about the effects of changes in climate ‘variability’?
- Answering this question has taken a year, with surprising (to us at least) results that we believe important in moving forward.
- This paper will focus more on what is the ergodic property of climate change and how this has changed the way we are thinking about climate finance in all its forms.
- But we will start with a few slides on where we are heading with Climate-smart financing

Sources of Climate-smart finance include

- Debt products
- Equity
- Hybrid mechanisms
- Environmental marketplaces
- Philanthropy
- Insurance

Cornell Atkinson
Innovation Sprint

ENVIRONMENTAL MARKETPLACES

Carbon markets

With water, carbon, and nutrient marketplaces, corporations and other entities can reduce their environmental footprint by compensating farmers for providing ecosystem services.

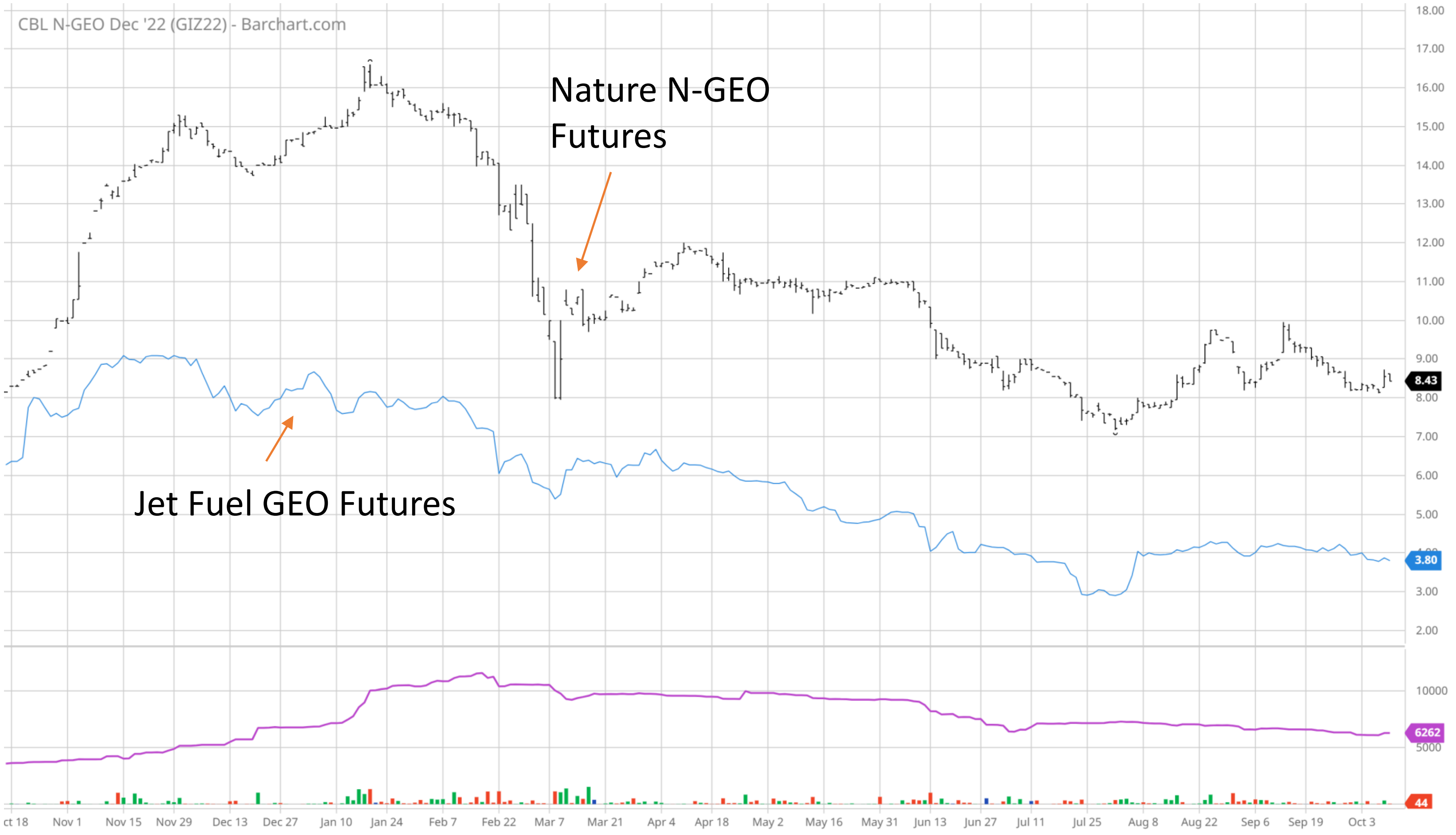
EXAMPLE:

Indigo Ag

- Farmers get support to increase soil sequestration and reduce greenhouse gases
- Farmers can then generate and sell the carbon credits
- Indigo markets the carbon credits that are generated
- Indigo also provides support to farmers through their team of agronomists and plant biologists

CME now trades GEO Carbon Futures

Climate finance in form of Green Bonds, Green Finance, Green Mortgages (and more) now has a tradeable asset that can arbitraged and used to hedge carbon risks and measure carbon risk. Will require measures of regenerative agriculture, soil conservation, technology financing to be converted to carbon equivalents.
Prices for nature GEO and GEO are priced differently.



For Climate Finance we are interested in the structure/financial engineering of operating credit, climate-linked bonds, climate-linked mortgages

- Carbon markets including Green Bonds, linked to carbon, are based on optionality and based on ex post performance relative to ex ante assessment. For a coupon bond for example, with redemption optioned to Carbon prices or futures

$$B=\sum_{t=1}^{T}\frac{F\frac{c}{n}}{\left(1+\frac{r^{*}}{n}\right)^{nT}}+\frac{F\left(1+\frac{Max(C(T)-C(0),0)}{C(0)}\right)}{\left(1+r^{*}\right)^{T}}$$

- With coupons possible be linked to carbon markets as well

$$c_{t}=cF\left(1+k\frac{Max(C(t)-C(0),0)}{C(0)}\right)$$

But how to measure climate variance?

- We use the Hurst Ratio using Rescaled Range (R/S) analysis

$$\frac{R_s}{\sigma_S} = \frac{Max(v_s) - Min(v_s)}{\sigma_S}$$

$$\frac{R_s}{\sigma_S} = s^h$$

- and Scaled Variance Ratio

$$\sigma_T^2 = \sigma_1^2 T^{2H}$$

$$H = \frac{Log\left(\frac{\sigma_{t+\Delta t}^2}{\sigma_t^2}\right)}{2\log(\Delta t)}$$

$$H = \begin{cases} > 0.5 \;\; COV(t,n) > 0, \; persistent \;\; fBm \\ = 0.5 \;\; COV(t,n) = 0, \; Markov \;\; gBm \\ < 0.5 \;\; COV(t,n) < 0, \; mean\,reverting \;\; fBm \\ = 0 \quad \left|COV(t,n)\right| = \sigma_1^2, ergodic \;\; fBm \end{cases}$$

The Ornstein–Uhlenbeck (OU) process for Mean reversion

- The Vlasicek equation also proved useful and convenient

$$dx = x_t - x_{t-1} = a\left(B - x_{t-1}\right) + \varepsilon\sigma_x$$

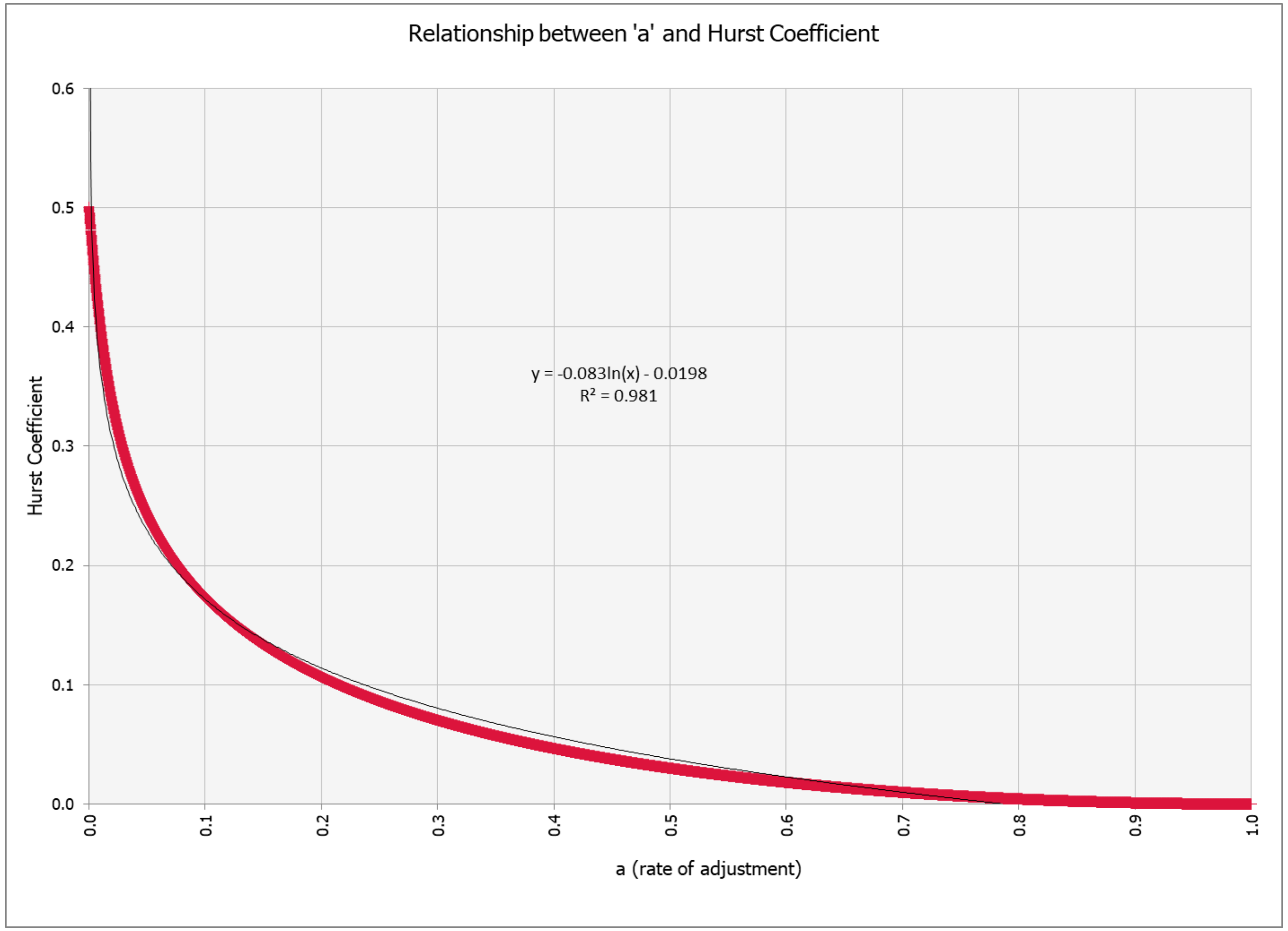
- With Variance

$$VAR\left[x_t \mid \varepsilon \sim N\left(0,1\right)\right] = \sigma_x^2\left(1 + \sum_{k=1}^{n}\left(1-a\right)^{2k}\right)$$

- And Hurst

$$H = \frac{1}{2}\frac{Log\left(1 + \sum_{k=1}^{n}\left(1-a\right)^{2k}\right)}{Log\left(n\right)}$$

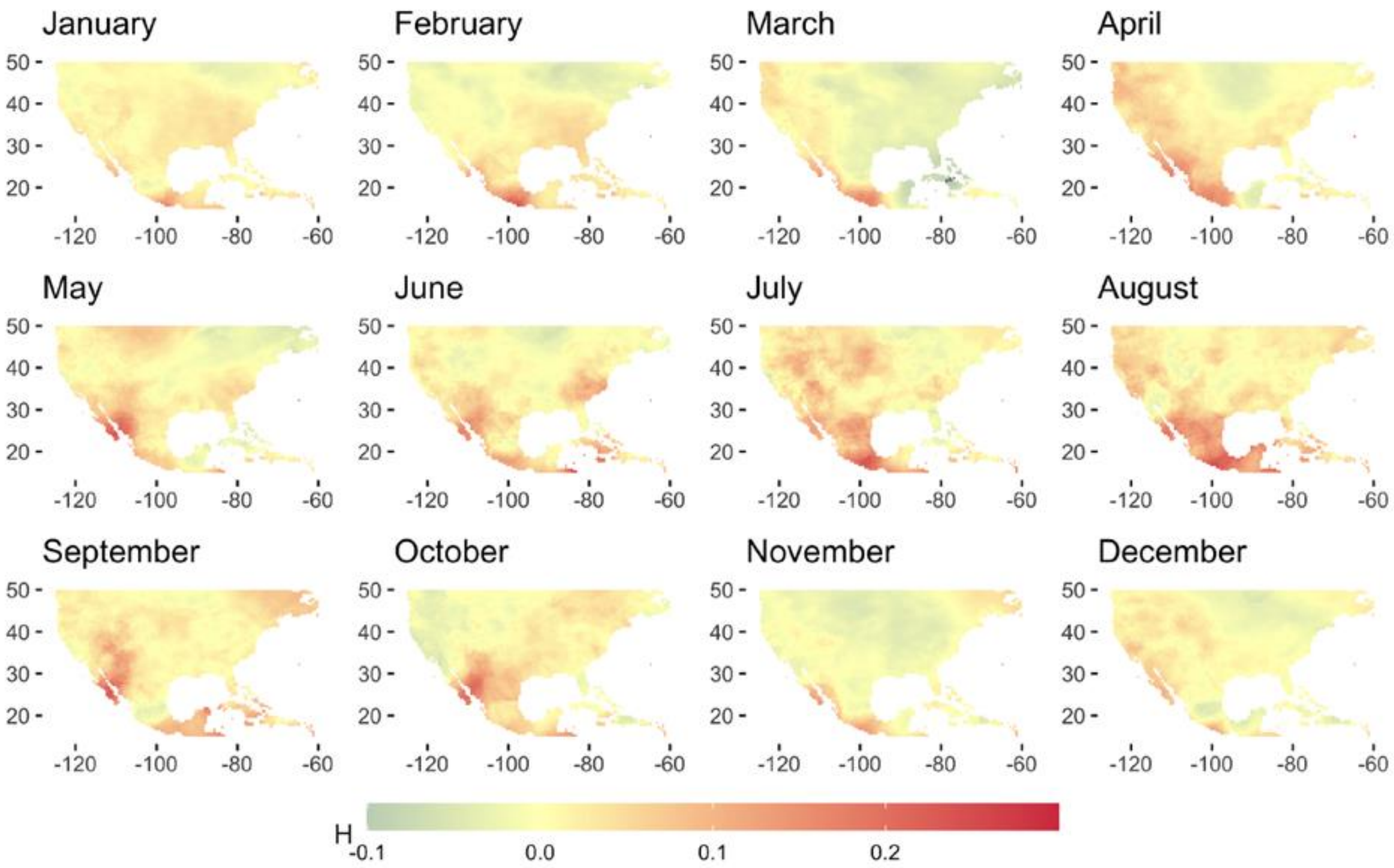
Relationship between a and Hurst



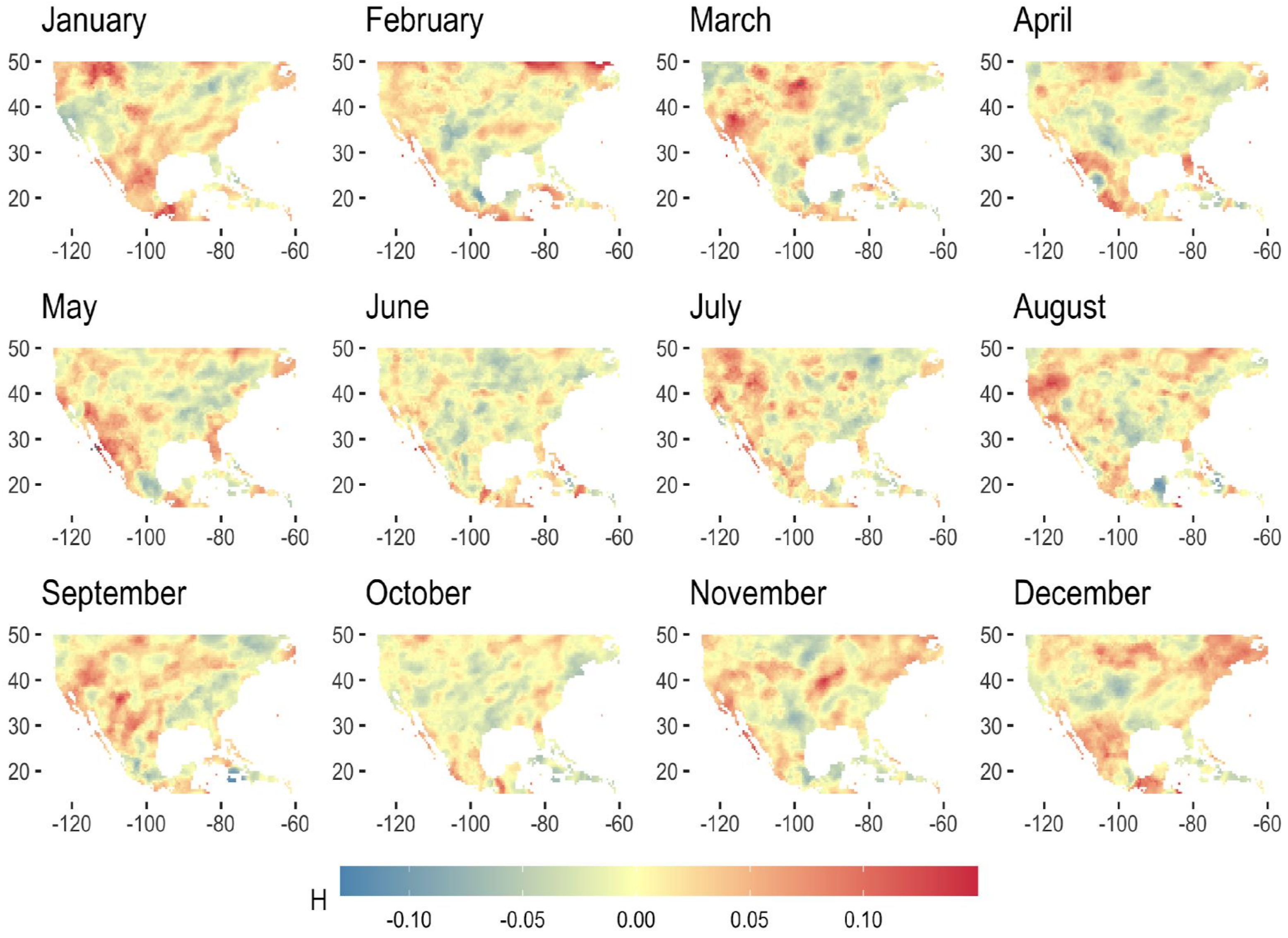
Data Source and Methods

- Monthly gridded (〖0.5〗^°) temperature and precipitation data from 1901 to 2020
- University of East Anglia Climatic Research Unit Time Series version 4.05 (CRU (n.d.),).
- The temperature is the mean daily mean temperature, and the unit of precipitation is millimeter per month.
- Programmed in R
- Will show only core results for VR Hurst measure

Temperature VR Hurst by Month, 1901-2020



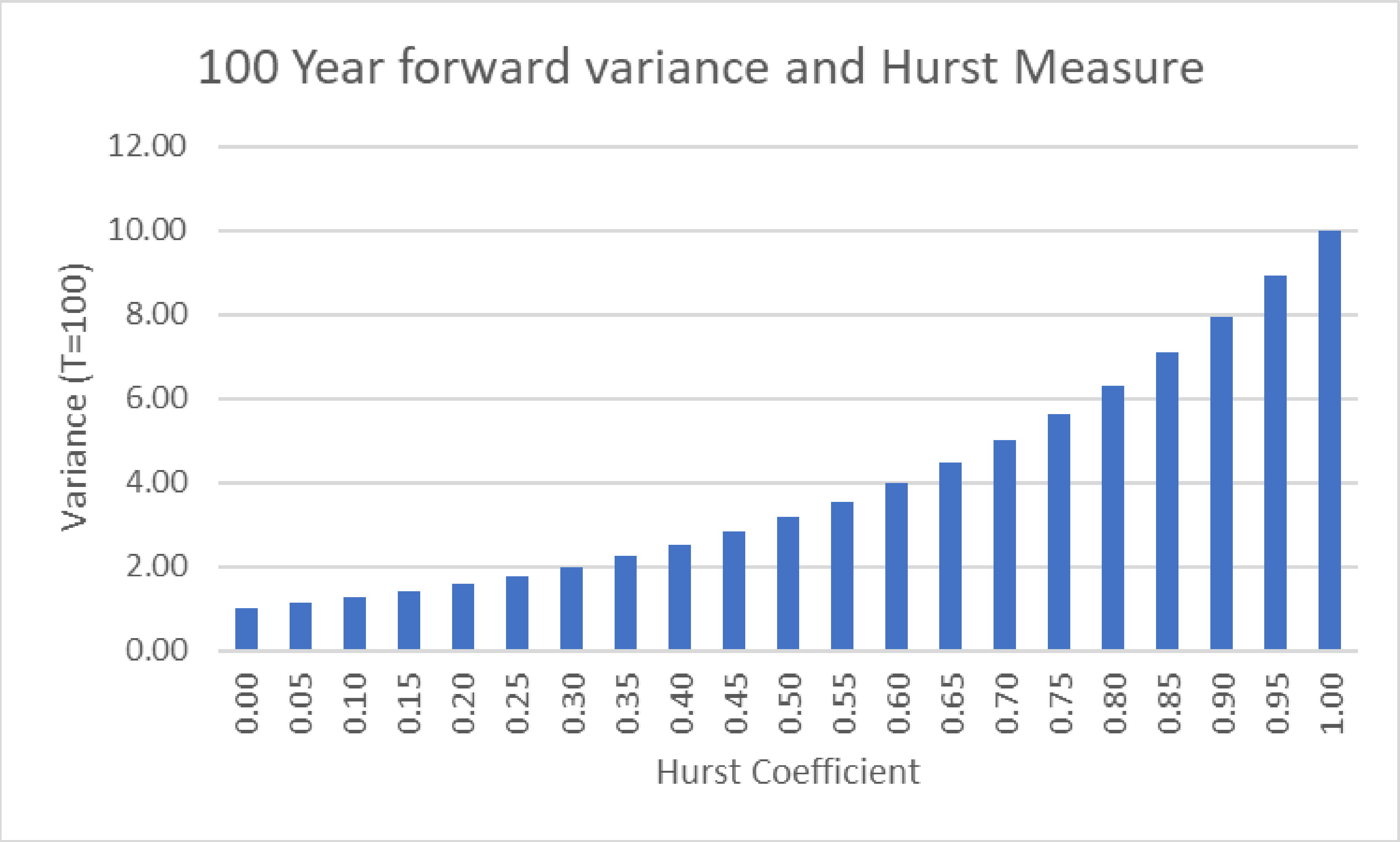
Precipitation VR Hurst by Month, 1901-2020



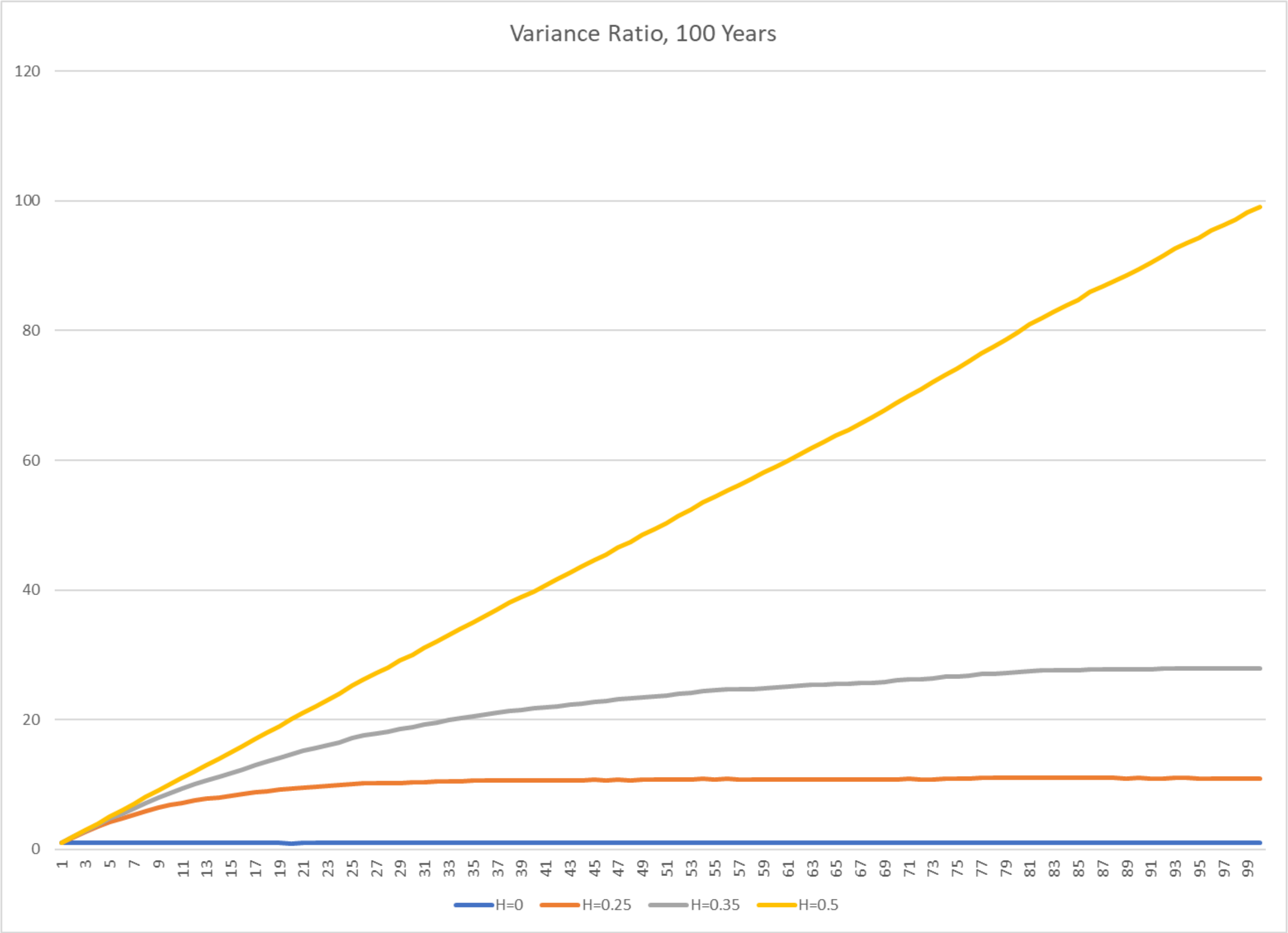
Our Surprising Find

- The literature on climate measures of Hurst generally find that $H>0.5$
- This suggests that there is long-term persistent memory in climate dynamics
- Measuring every which way and even doing hand calculations we find that $H<0.5$ in fact $H<0.35$ in most cases and $H=0$ in many cases.
- Moreover, we find a temporal dimension to changes in climate variance
- $H=0$ is important! It indicates ergodicity in climate dynamics.
- What does this ergodicity mean? It means in these regions and months variance is not changing in time!

Relationship between Hurst and Variance. If Hurst increases from 0 to 0.3, variance doubles in 100 years, or standard deviation increases by 41%

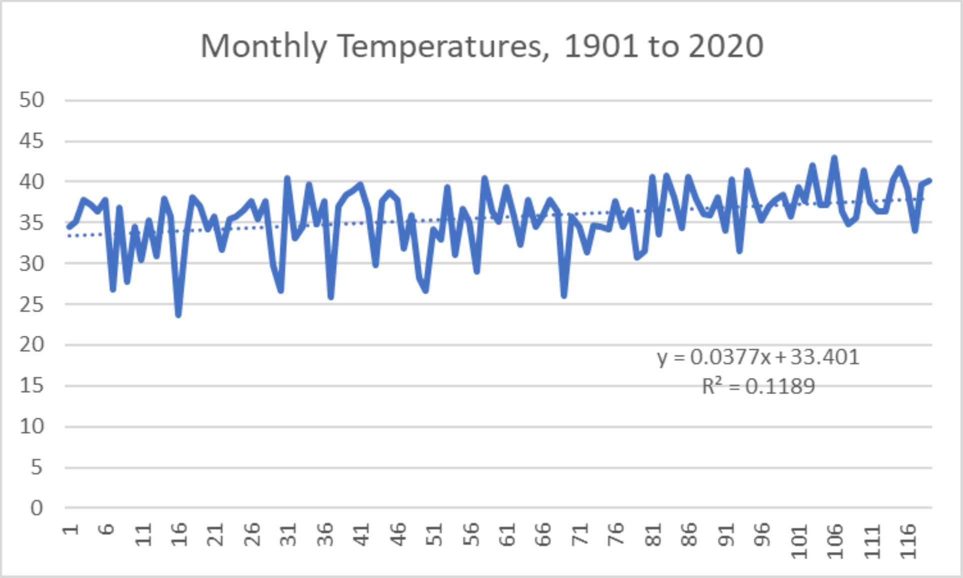


Variance Ratio

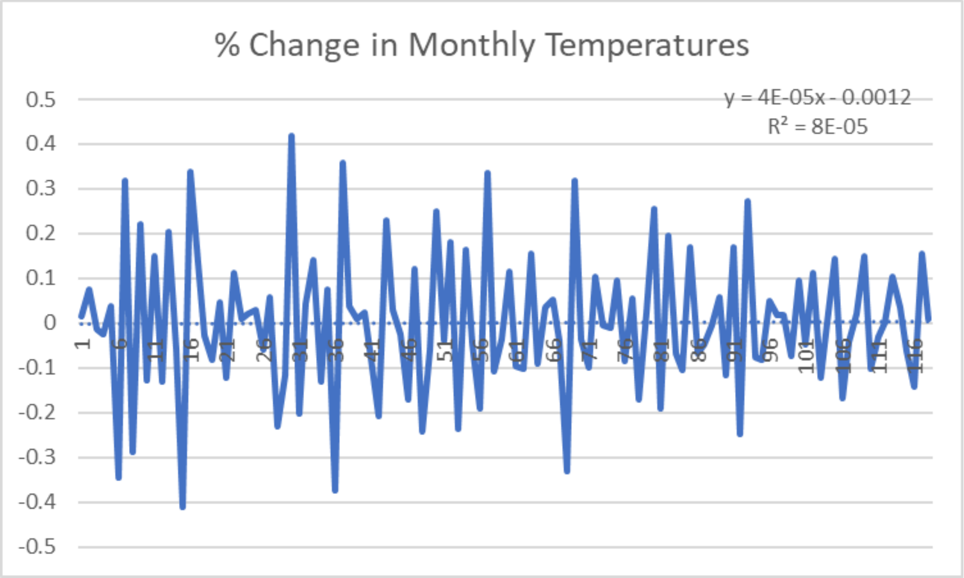


Example Patterns of Climate variability

Monthly Temperatures

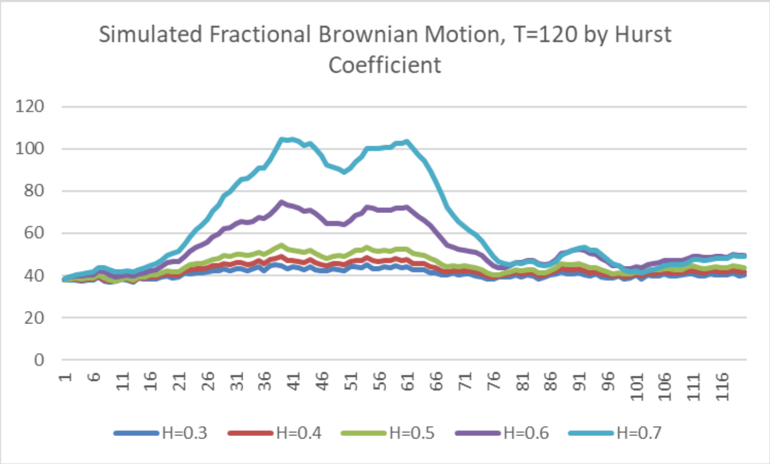


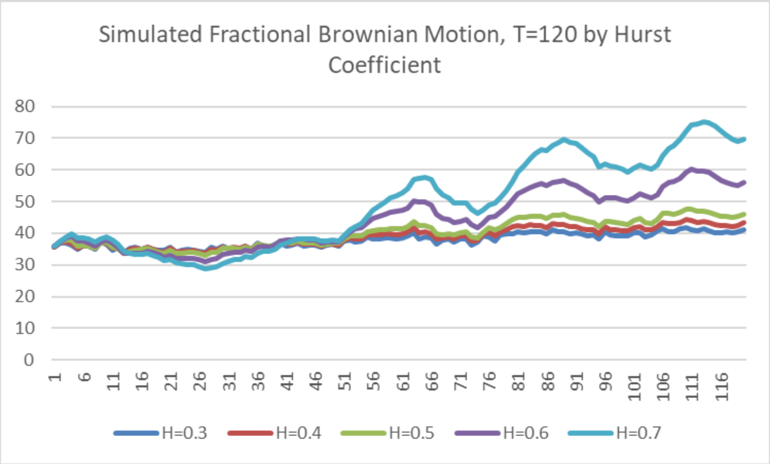
% Change in monthly temperatures

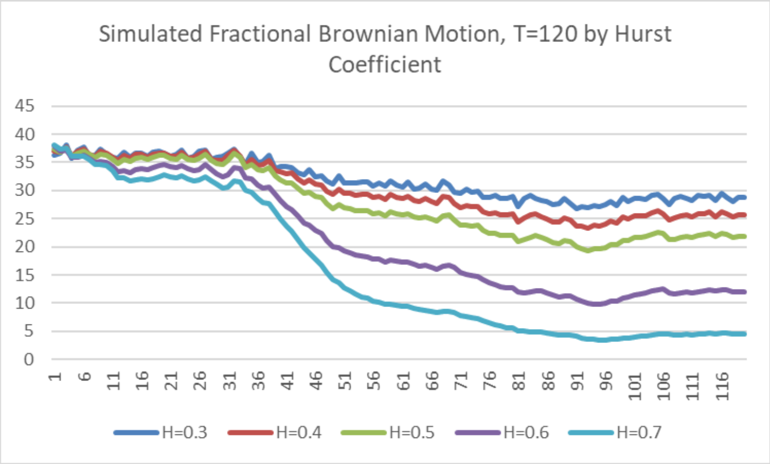


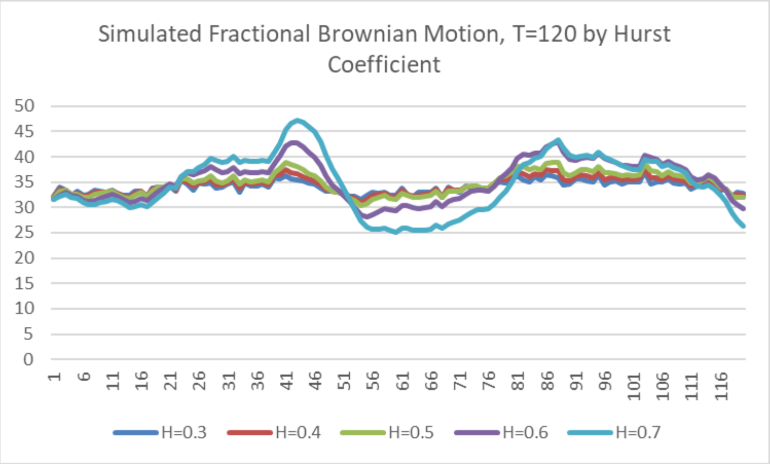
Fractional Brownian Motion by Monte Carlo Simulation

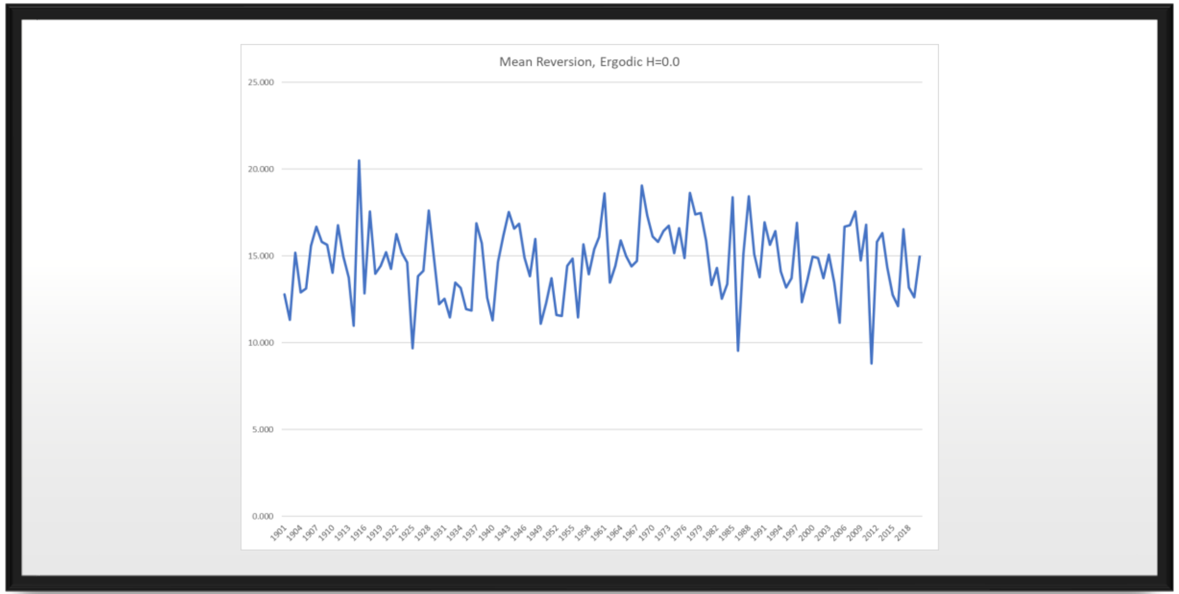
note how H>=0.5 as reported in literature have huge excursion patterns



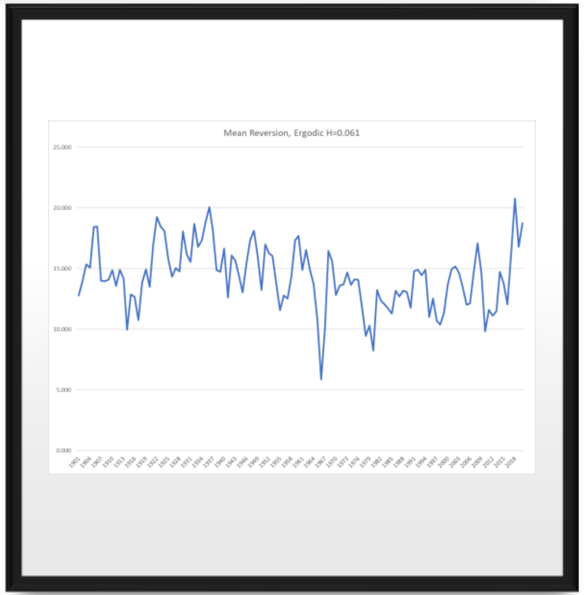


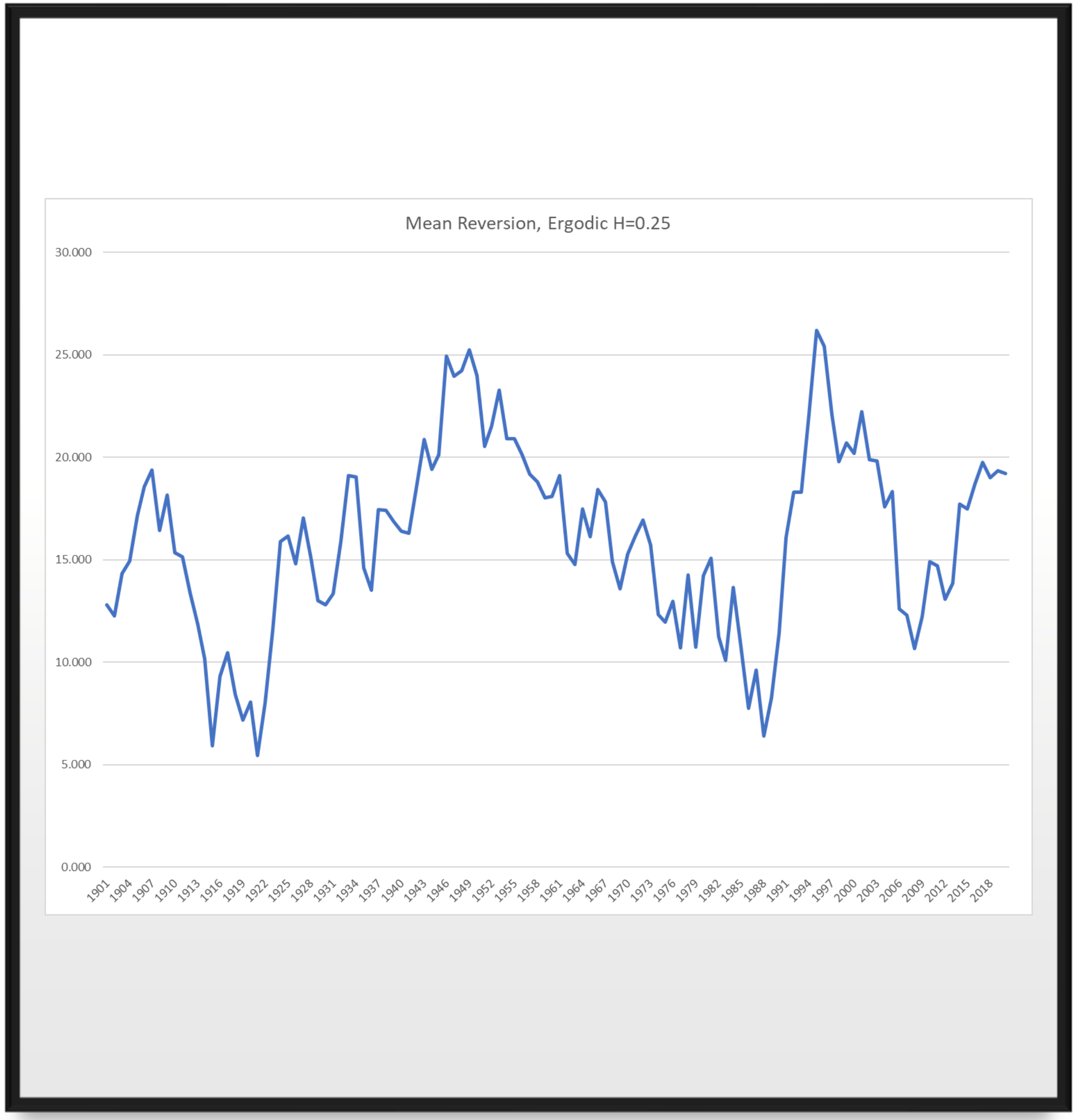


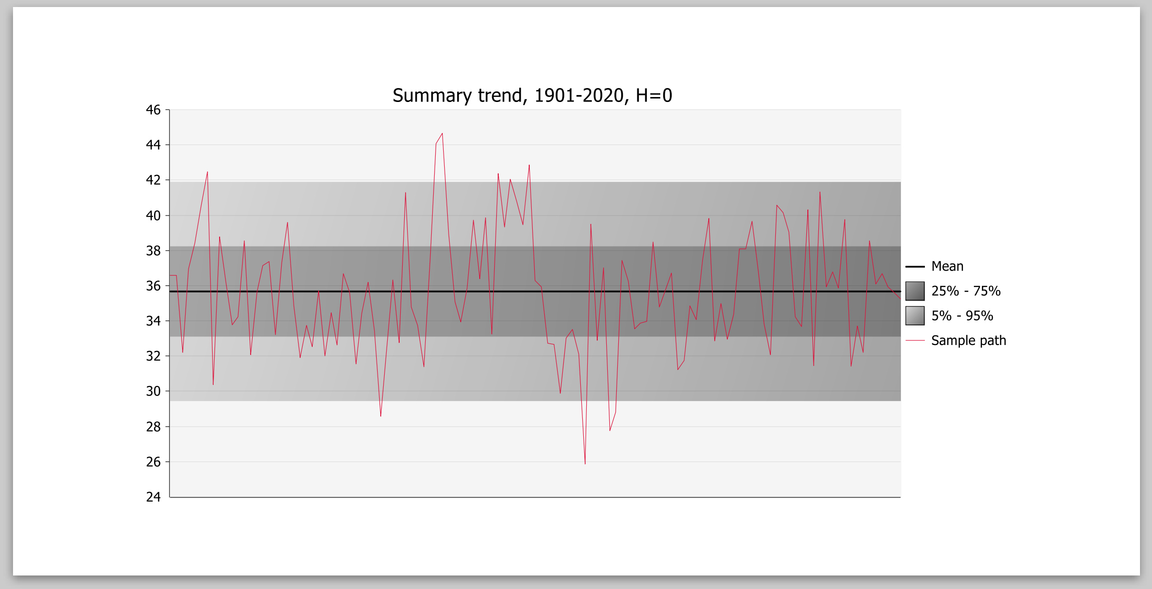


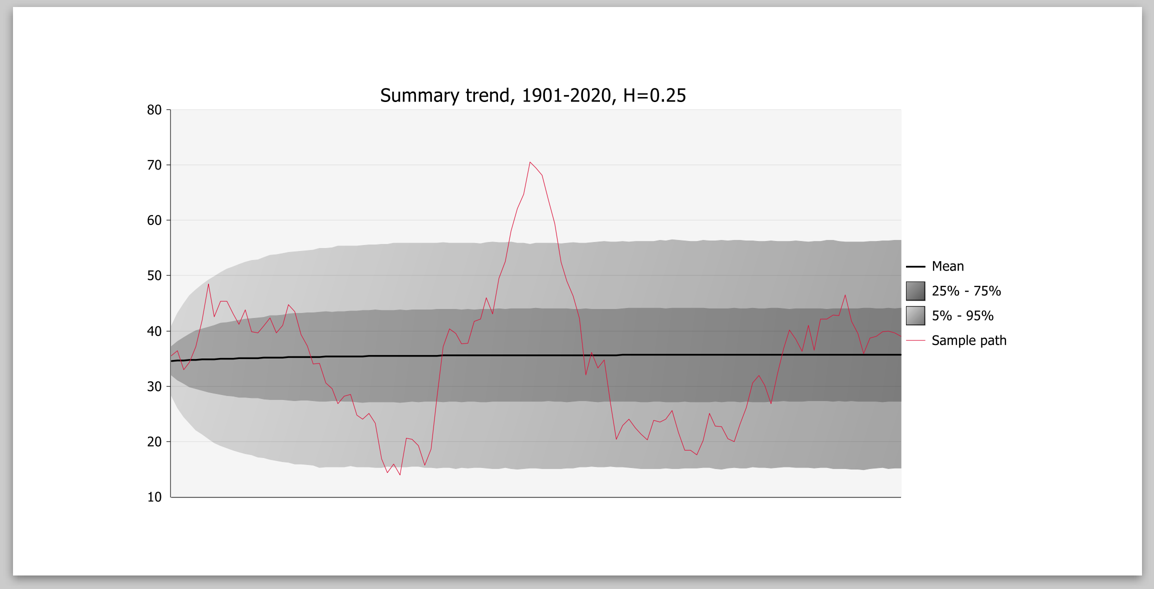

Mean Reversion, Ergodic H=0.0
25.000
20.000
15.000
10.000
5.000
0.000

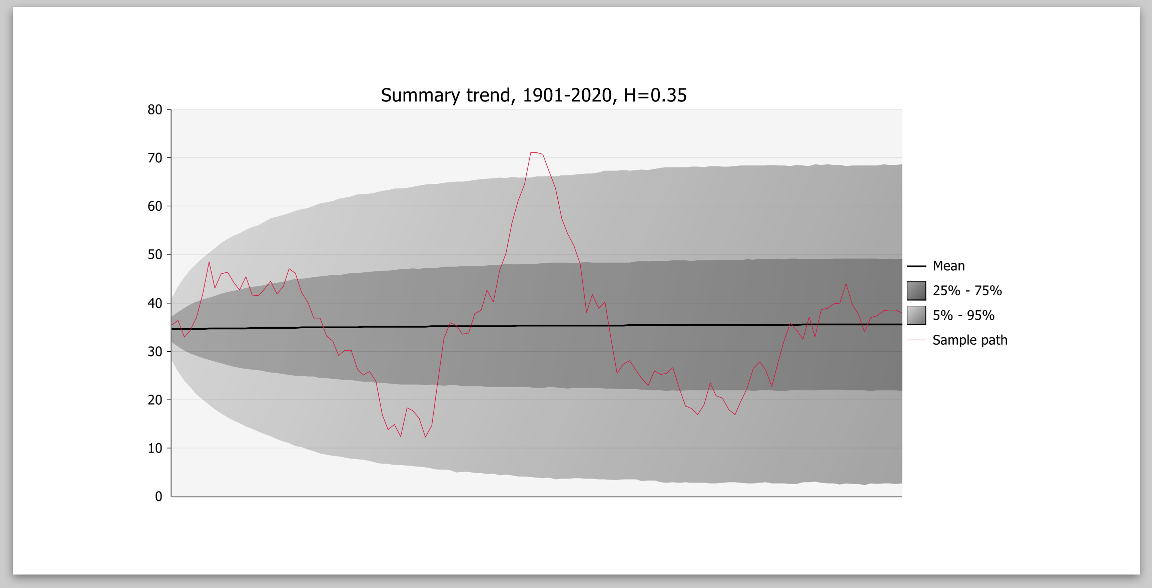

Mean Reversion, Ergodic H=0.35

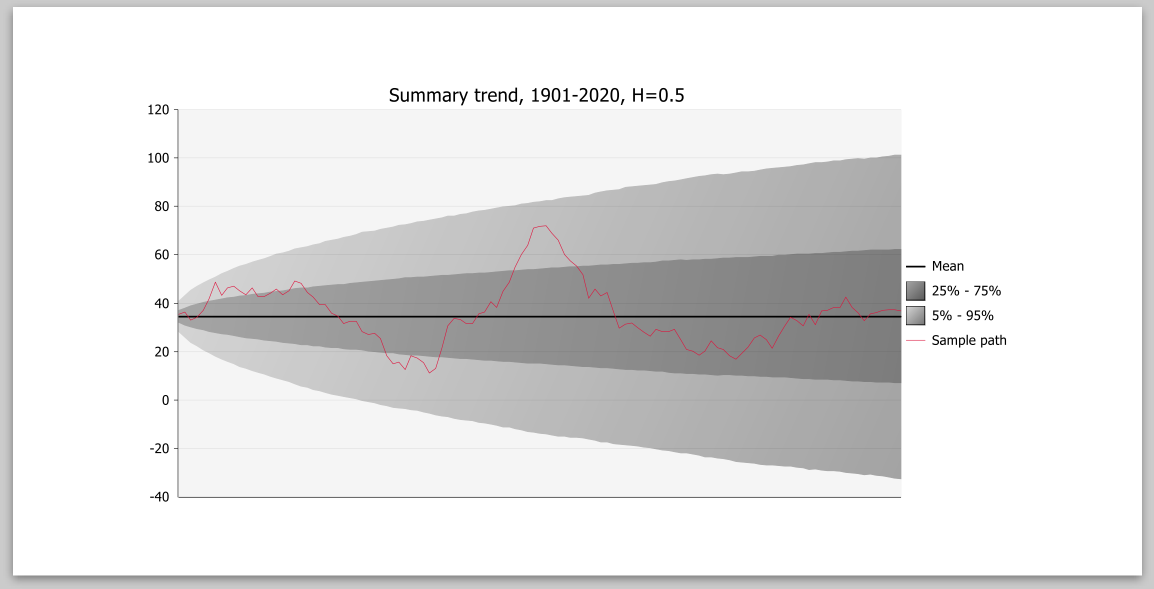

Mean Reversion, Ergodic H=0.061


Mean Reversion, Ergodic H=0.25
30.000
25.000
20.000
15.000
10.000
5.000
0.000
1901 1904 1907 1910 1913 1916 1919 1922 1925 1928 1931 1934 1937 1940 1943 1946 1949 1952 1955 1958 1961 1964 1967 1970 1973 1976 1979 1982 1985 1988 1991 1994 1997 2000 2003 2006 2009 2012 2015 2018


Summary trend, 1901-2020, H=0
46
44
42
40
38
36
34
32
30
28
26
24
Mean
25% - 75%
5% - 95%
Sample path


Summary trend, 1901-2020, H=0.25
80
70
60
50
40
30
20
10
Mean
25% - 75%
5% - 95%
Sample path


Summary trend, 1901-2020, H=0.35
80
70
60
50
40
30
20
10
0
Mean
25% - 75%
5% - 95%
Sample path


Summary trend, 1901-2020, H=0.5
120
100
80
60
40
20
0
-20
-40
Mean
25% - 75%
5% - 95%
Sample path

Background to Ergodic Theory from Comez chapter on Modern Ergodic Theory

- Originated in study of statistical mechanics by Boltzmann, Maxwell and Gibbs.
  - Boltzmann lays out the basic topology (set theory) that defines the orbital structure of state space in which all phase states are contained.
  - Boltzmann’s ergodic theory may not hold for all dynamical systems but should hold steady for our simple model
  - Boltzmann’s ergodic theory states that the
    - “*orbit of a single point in the phases space visits every point in that space*”.
  - Ehrenfest softened this to a quasi-ergodic hypothesis
    - “*the orbit of a single point comes arbitrarily close to any point in the phase space*”
  - This is probably more useful in experimental mathematics of dynamical systems like we have in the sense that the system can approximate a phase space in finite time.

Ergodic Theory

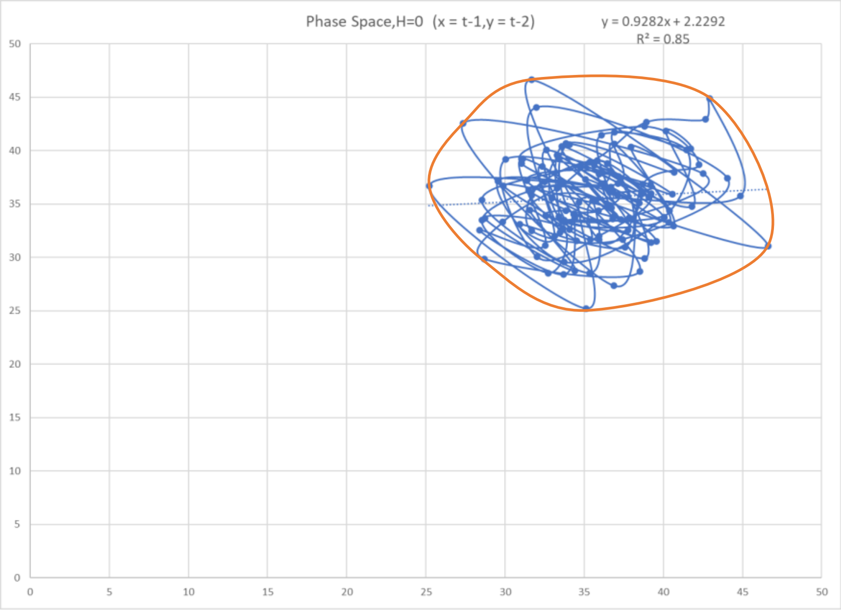
- Ergodic Theory is a field in mathematics that allows the analysis of dynamic systems using properties derived from the attractor of system behaviour.
- Allows assessment of dynamic stability of quasiperiodic system (year over year over year) (following Poincare)
- The attractor of a system is a description of the dynamics within the system and gives the trajectories of the system's development.
- It describes
    - Dynamic stability (persistence of a certain type of dynamics, e.g. mean reverting, persistent)
    - Entropy, the rate of information generation exhibited by the system
    - Complexity, which describes the system's spatial organization in phase space or the size of the attractor,
- The scaling property across time (or space) is X(t), X(t+k), X(t+2k) where, e.g. k=1 is a scale in years, k=2 is scale of 2 years, k=10 is a decadale scale and so on

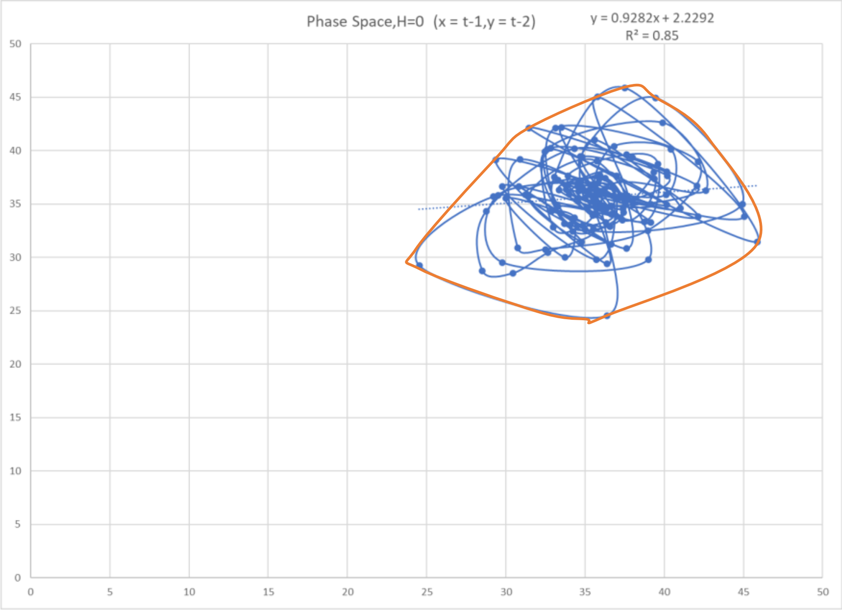
Ergodic Systems

- Ultimately there are energetic boundaries that will cause weather reversibility across time, and the deterministic nature of the multiple partial and ordinary differential equations that make up the ensemble of climate variability implies that there are limits to variance.
- Changes in climate variability come about by ‘forcings’ on the ensemble, e.g. green house gas effects
- Ergodic systems are recurrent mean-reverting dynamical systems in which the underlying dynamics constantly drive and self correct towards an ‘attractor’.
- The attractor in simple dynamical systems in the long run mean (in 2-dimensional time scale, think of an autoregressive structure, or Nerlovian dynamic that when left alone reaches equilibrium or steady state
- Regardless of initial condition (position) the dynamical system self corrects with common reversals in the direction of the attractor
- Ergodic systems appear to be more erratic but with increasingly stable variance, while less ergodic systems appear to be less erratic with a variance that increases at a rate less, than, equal to, or greater than the time scale.
- There is a need to separate the often synonym usage of erratic and variance into distinct meanings perhaps expressed by excursion theory.
- However, this distinction may be difficult when fractional scaling is considered because of the self-affine scaling properties which would reveal the same ergodic pattern when ‘the thing’ is measured in seconds or minutes or hours or days or in months.

Phase Space

- Phase space is a geometric representation of system dynamics, in our case between two points in time,
- The evolution of time in a dynamical system will thus be represented by a trajectory in phase space.
- Since system dynamics is unknowable at any moment in time and for any initial value, phase space represents an ensemble of points describing the various dynamic states compatible with the information available about the system (e.g. monthly average temperatures for 120 years)
- The space represents the maximum knowledge we have of the system
  - If ergodic then the system appears to be captured by a circle of diameter x
  - If Markov it is an ellipse
  - If persistent (H>0.5) the ellipse elongates and widens with respect to stopping time and local time.
  - There is a scaling to the circle/ellipse that should be to the power of H somewhere???
- Given what is knowable about phase space the outer points will be contained within the elliptical structure (circle included) as defined by the physical limits of what is knowable about the system. But at the level of individual pathways, the 2-dimensional geometry of the system (represented perhaps by joining the extreme points of the system) is approximately mean preserving in the neighbourhood of the attractor and variance preserving in the neighborhood of the knowable variance, but will be no greater than the variance exposed by the extreme points of the maximal phase space.


Phase Space,H=0 (x = t-1,y = t-2)
y = 0.9282x + 2.2292
R² = 0.85
0 5 10 15 20 25 30 35 40 45 50
0 5 10 15 20 25 30 35 40 45 50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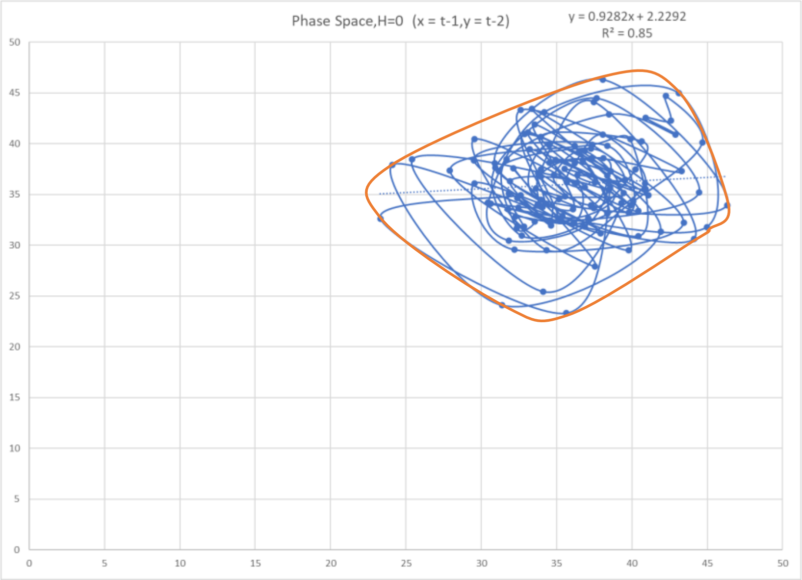
0 5 10 15 20 25 30 35 40 45 50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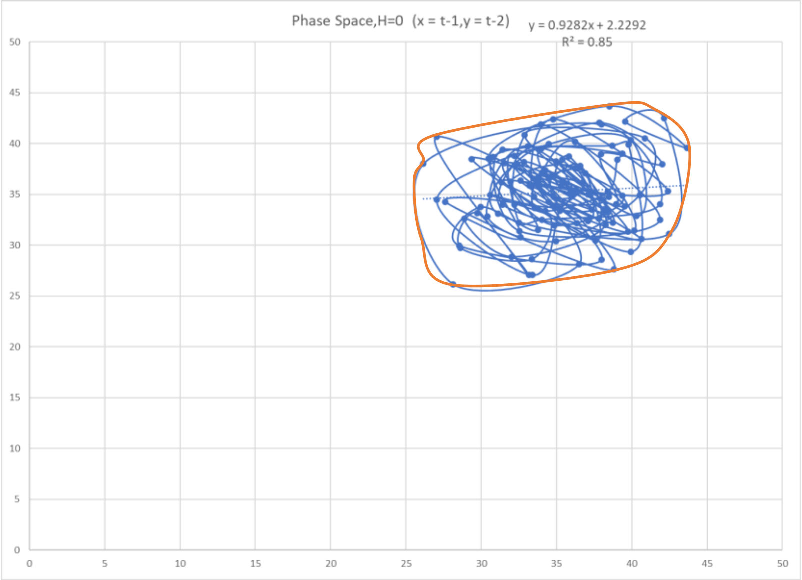
R² = 0.85
0 5 10 15 20 25 30 35 40 45 50
0 5 10 15 20 25 30 35 40 45 50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R² = 0.85
0 5 10 15 20 25 30 35 40 45 50
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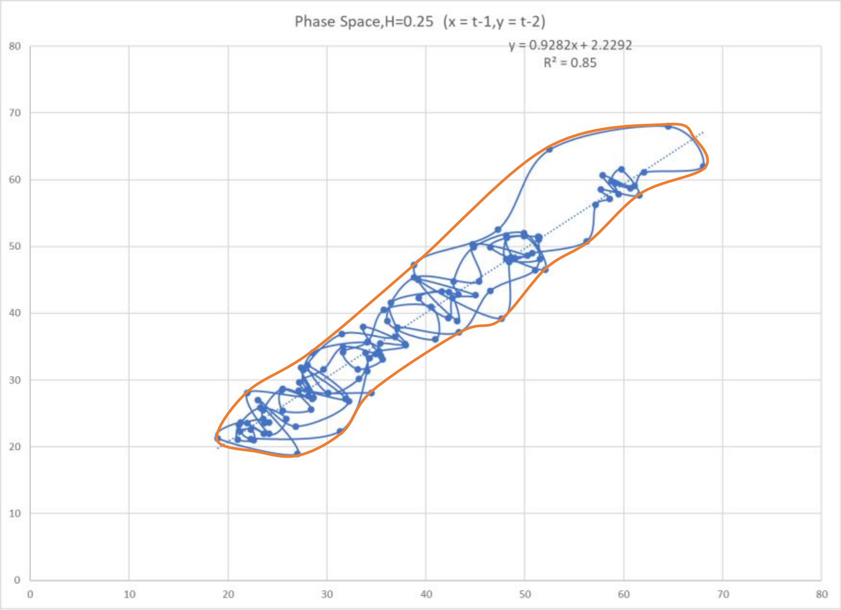
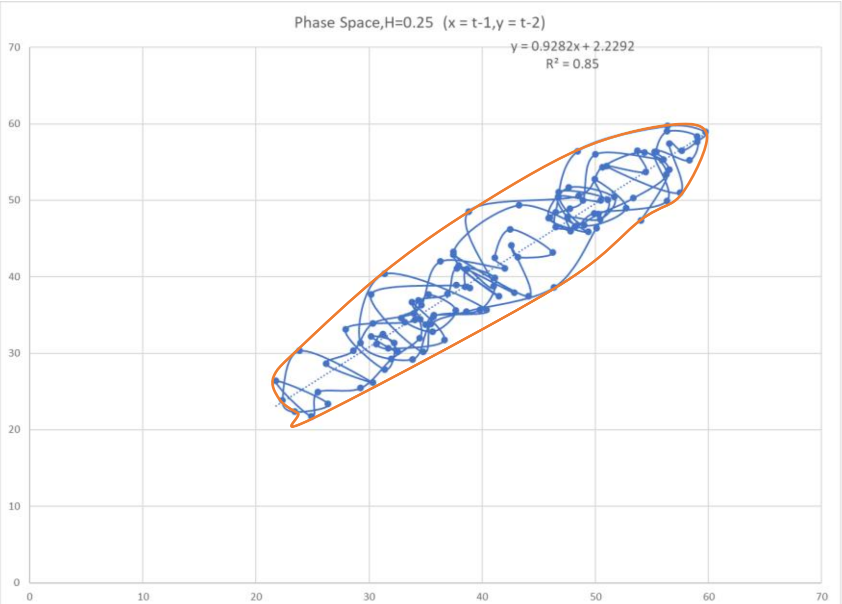
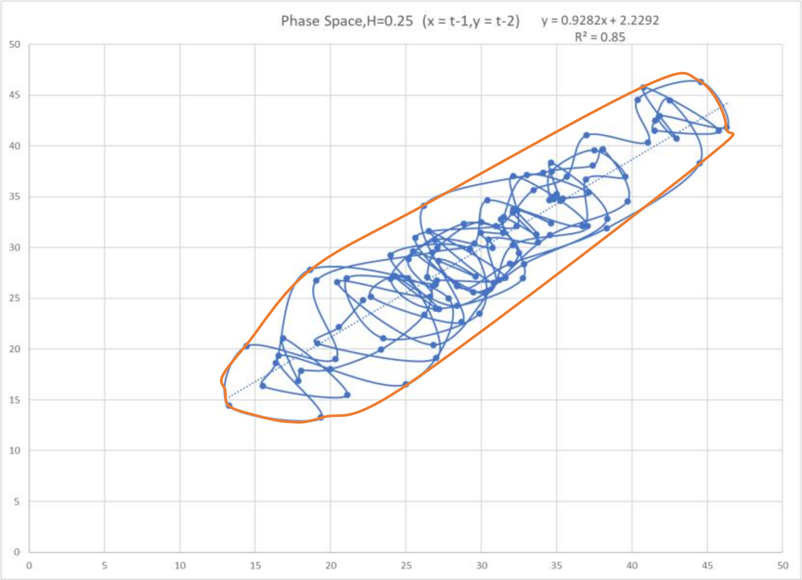
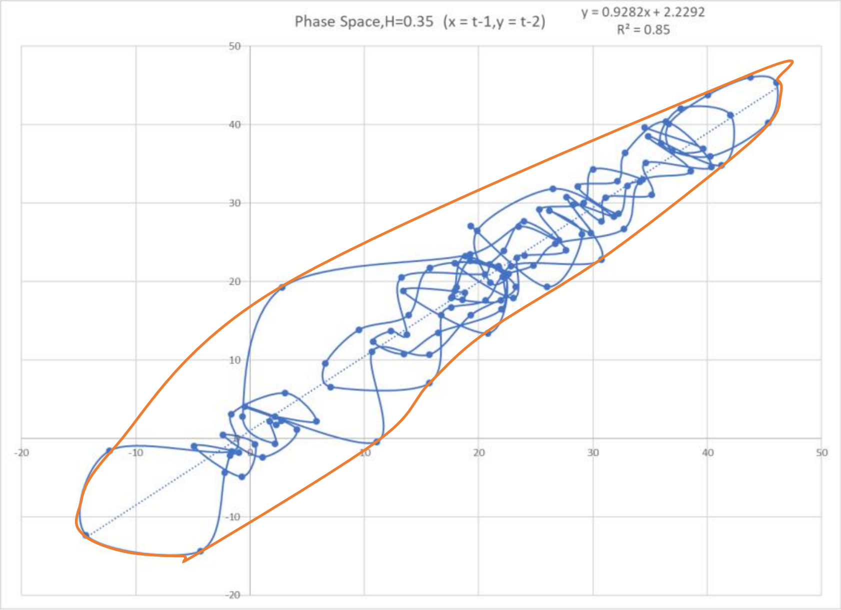
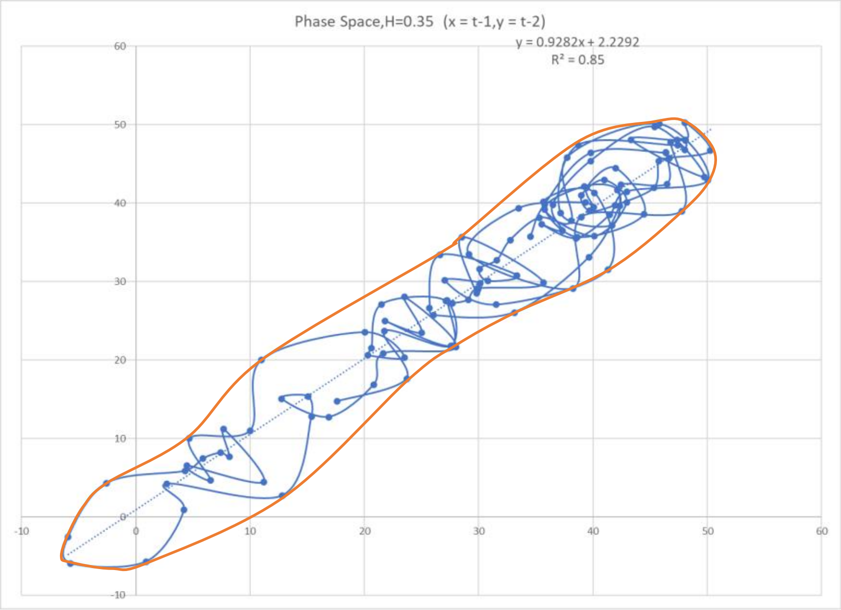

Phase Space,H=0.25 (x = t-1,y = t-2)
y = 0.9282x + 2.2292
R² = 0.85


Phase Space,H=0.25 (x = t-1,y = t-2)
y = 0.9282x + 2.2292
R² = 0.85


Phase Space,H=0.25 (x = t-1,y = t-2)
y = 0.9282x + 2.2292
R² = 0.85


Phase Space,H=0.25 (x = t-1,y = t-2)
y = 0.9282x + 2.2292
R² = 0.85


Phase Space,H=0.35 (x = t-1,y = t-2)
y = 0.9282x + 2.2292
R² = 0.85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y = 0.9282x + 2.2292
R² = 0.85


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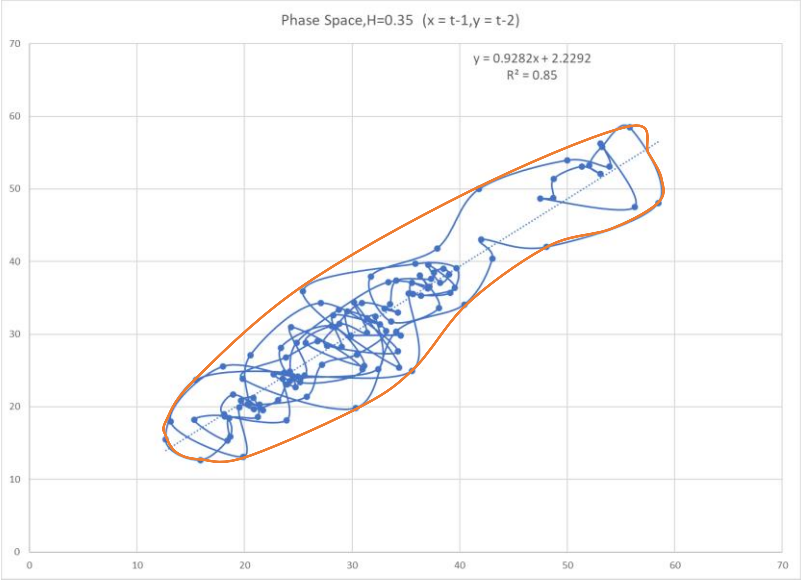
R² = 0.85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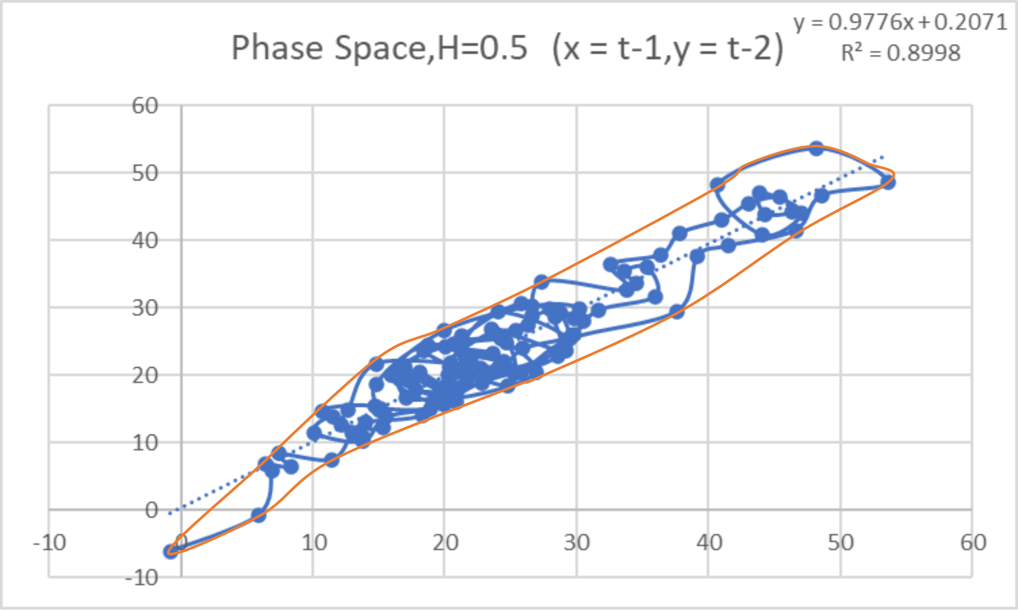
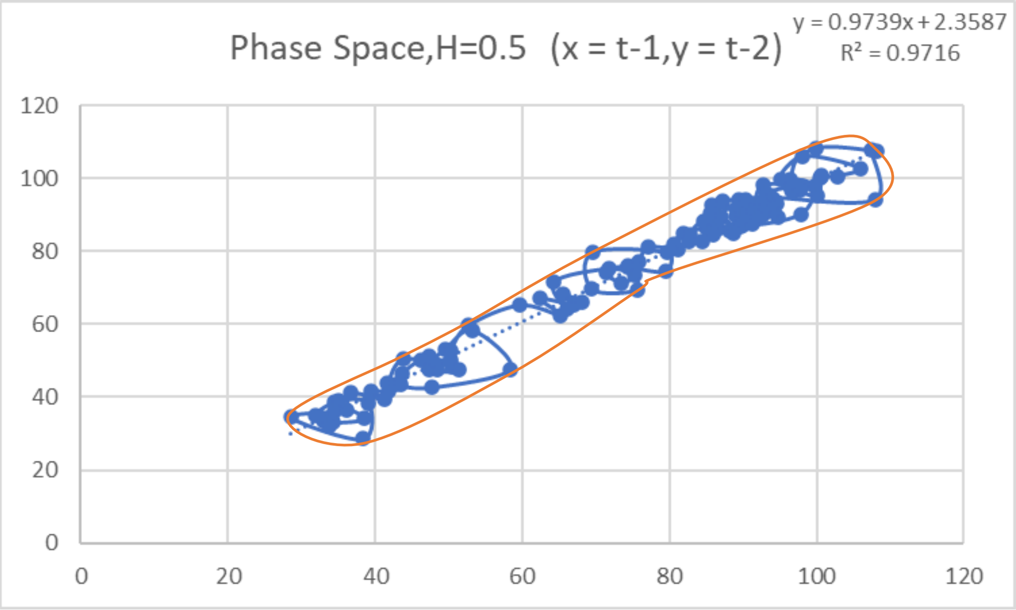
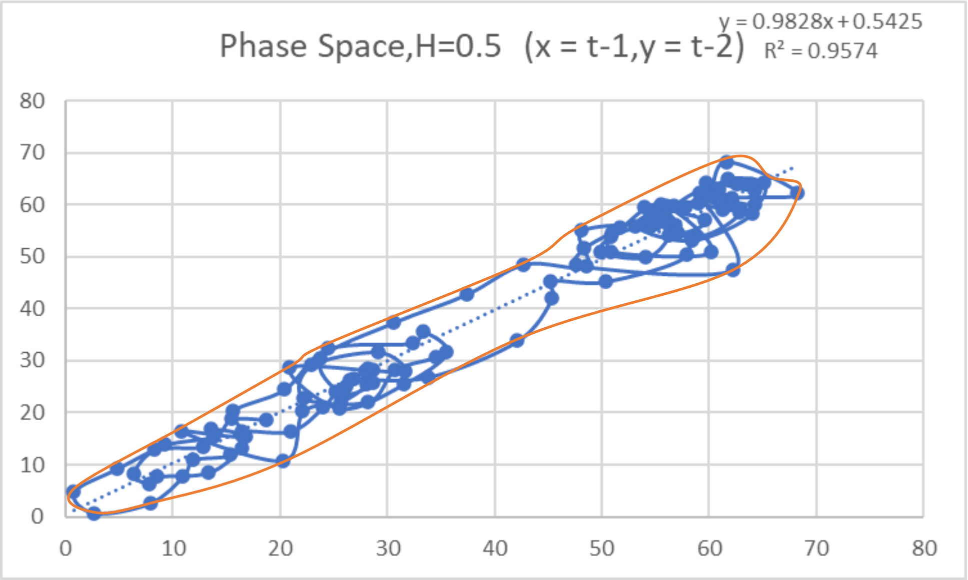
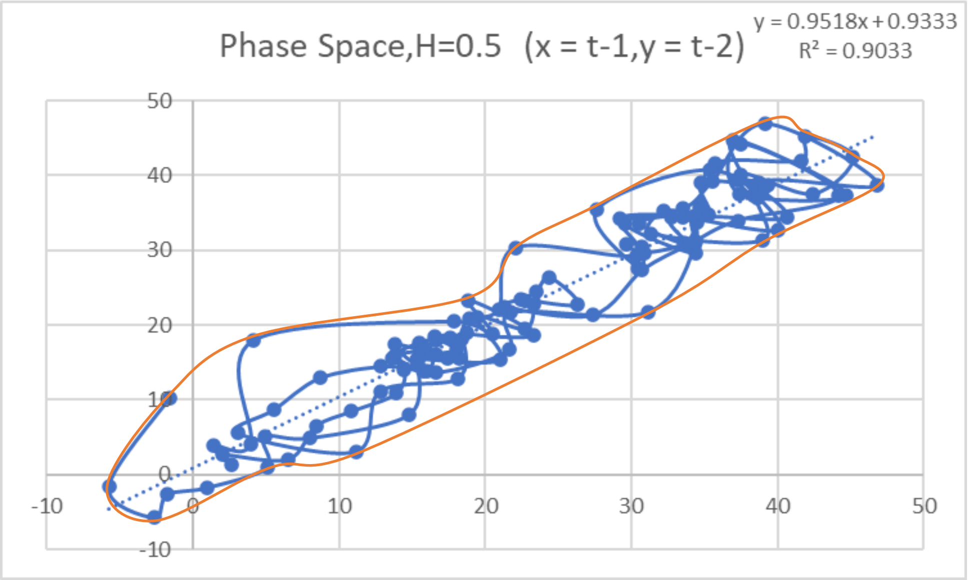
y = 0.9282x + 2.2292
R² = 0.85

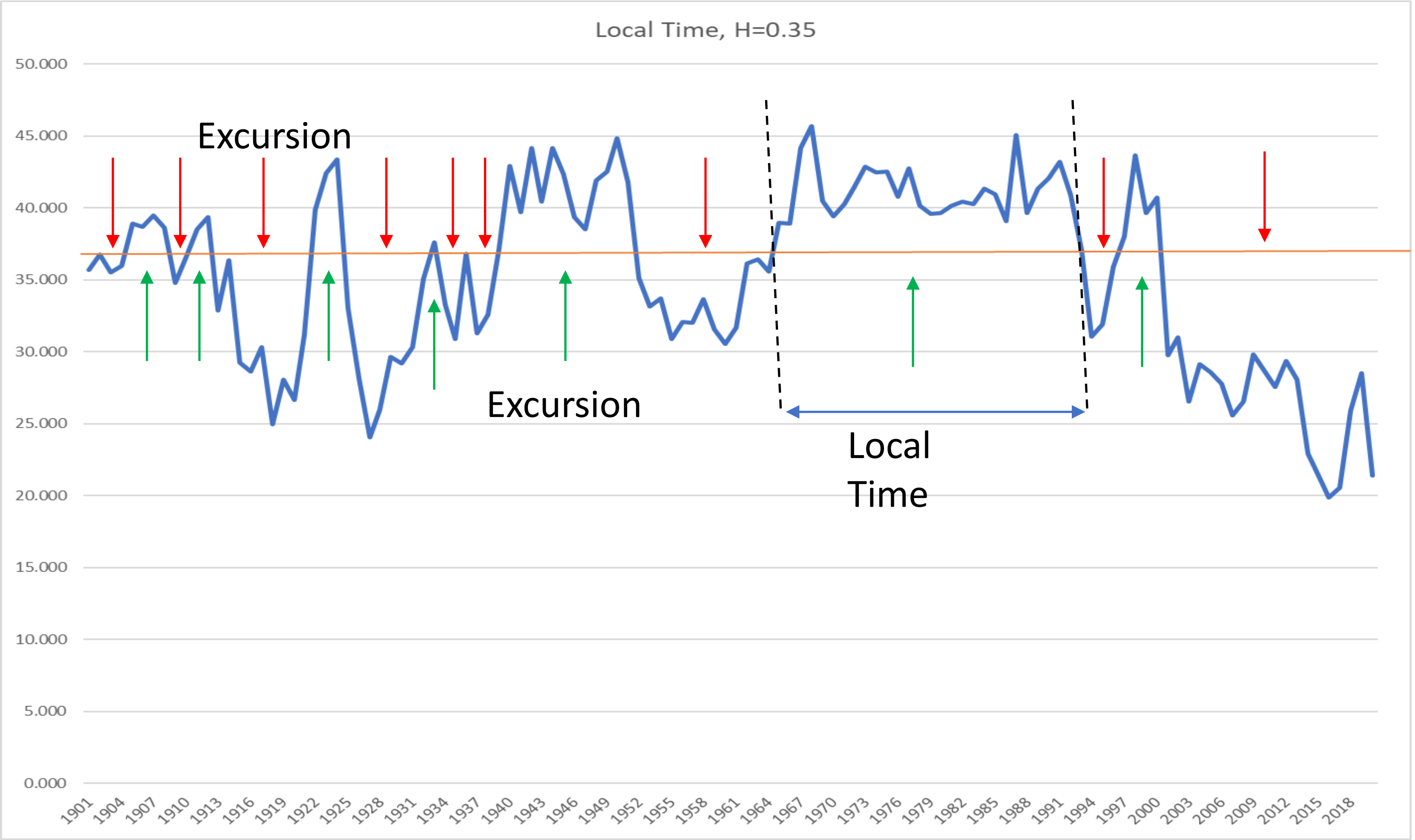

Phase Space,H=0.5 (x = t-1,y = t-2)
y = 0.9776x + 0.2071
R² = 0.8998
60
50
40
30
20
10
0
-10
-10
10
20
30
40
50
60

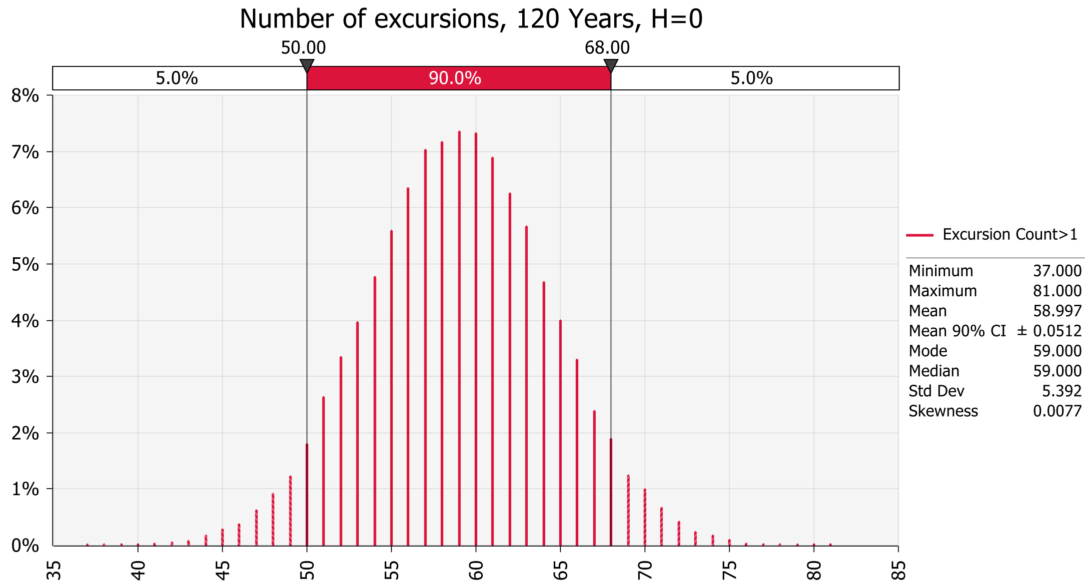

Phase Space,H=0.5 (x = t-1,y = t-2)
y = 0.9739x + 2.3587
R² = 0.9716
120
100
80
60
40
20
0
0
20
40
60
80
100
120

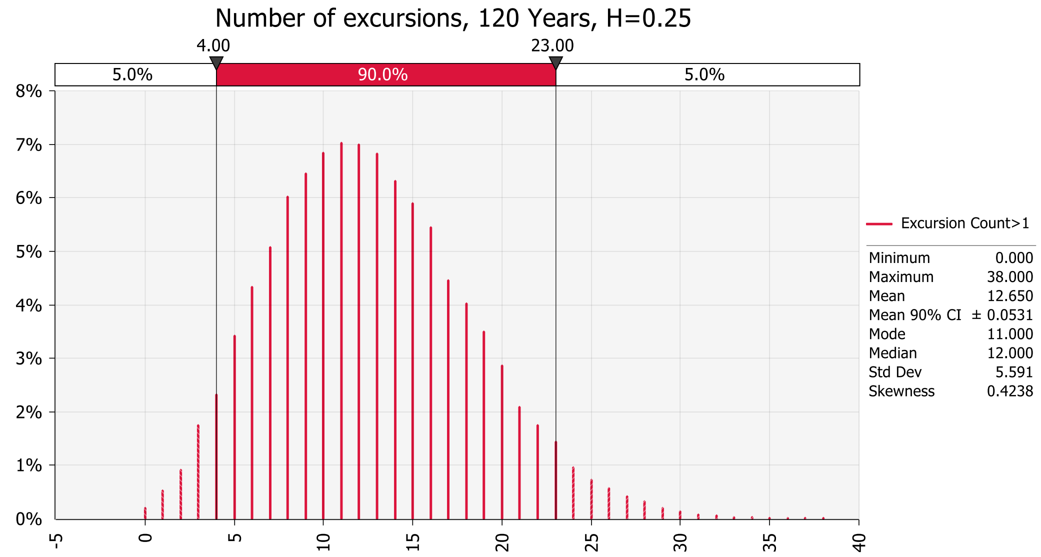

Phase Space,H=0.5 (x = t-1,y = t-2)
y = 0.9828x + 0.5425
R² = 0.9574
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80

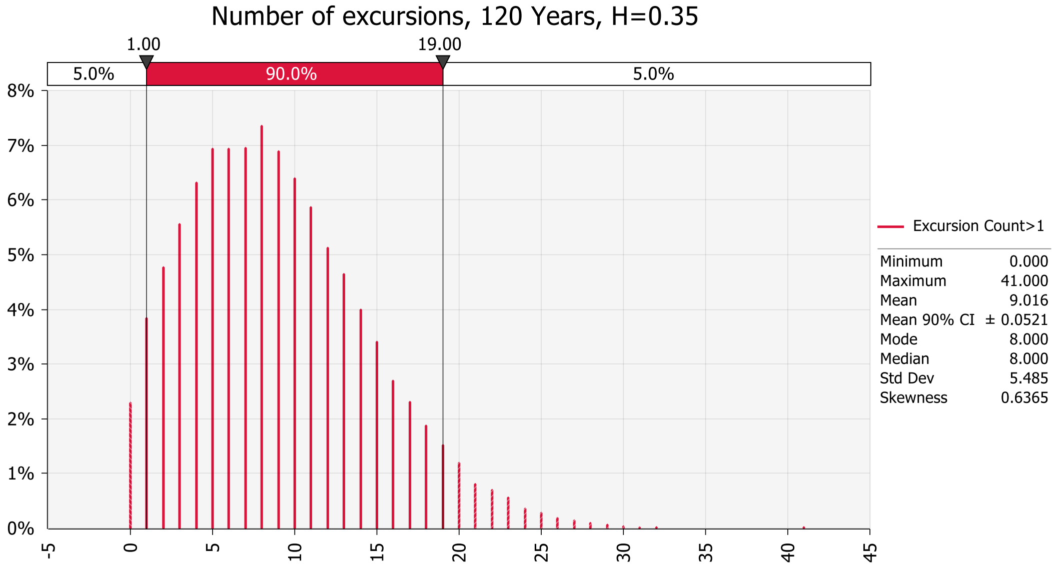

Phase Space,H=0.5 (x = t-1,y = t-2)
y = 0.9518x + 0.9333
R² = 0.9033
50
40
30
20
0
-10
-10
0
10
20
30
40
50

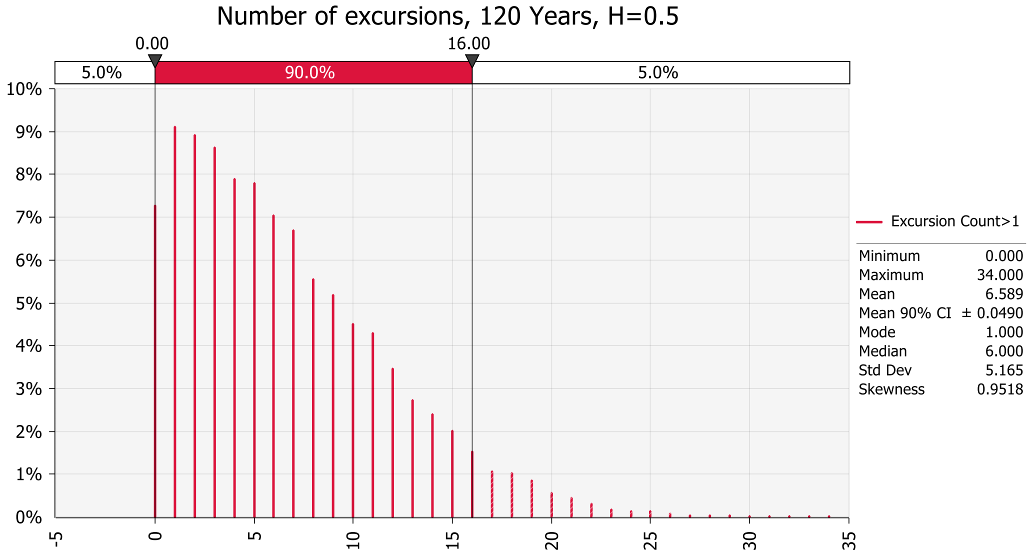
Excursion Patterns and Local Time




Number of excursions, 120 Years, H=0
50.00
68.00
5.0%
90.0%
5.0%
Excursion Count>1
Minimum 37.000
Maximum 81.000
Mean 58.997
Mean 90% CI ± 0.0512
Mode 59.000
Median 59.000
Std Dev 5.392
Skewness 0.0077


Number of excursions, 120 Years, H=0.25
4.00
23.00
5.0%
90.0%
5.0%
Excursion Count>1
Minimum 0.000
Maximum 38.000
Mean 12.650
Mean 90% CI ± 0.0531
Mode 11.000
Median 12.000
Std Dev 5.591
Skewness 0.4238

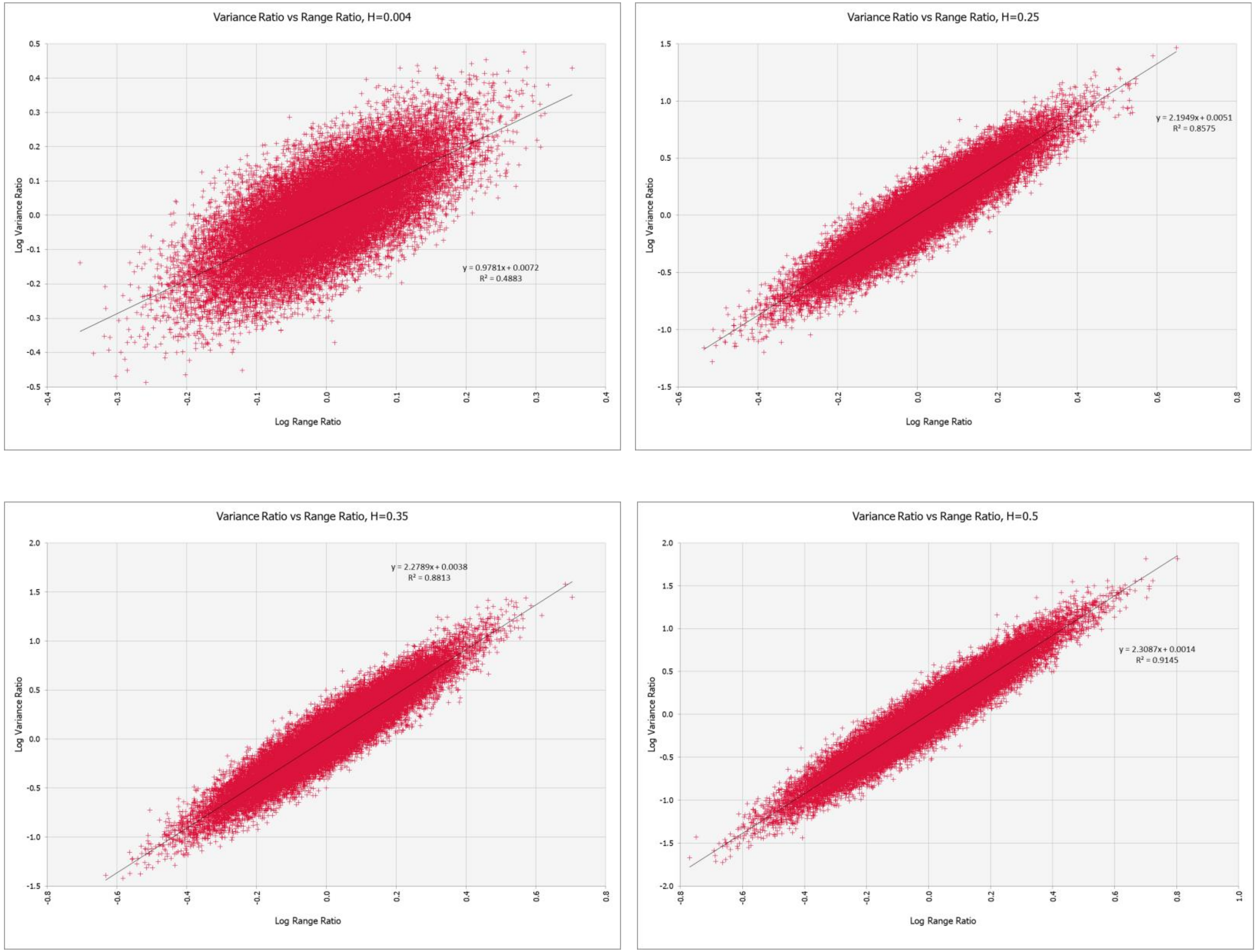

Number of excursions, 120 Years, H=0.35
1.00
19.00
5.0%
90.0%
5.0%
Excursion Count>1
Minimum 0.000
Maximum 41.000
Mean 9.016
Mean 90% CI ± 0.0521
Mode 8.000
Median 8.000
Std Dev 5.485
Skewness 0.6365


Number of excursions, 120 Years, H=0.5
0.00
16.00
5.0%
90.0%
5.0%
Excursion Count>1
Minimum 0.000
Maximum 34.000
Mean 6.589
Mean 90% CI ± 0.0490
Mode 1.000
Median 6.000
Std Dev 5.165
Skewness 0.9518

Variance and Range Ratios. Hurst scaled range by standard deviation. If scaled by average range, then VR corresponds to range analysis

$$\frac{Max(v_S)-Min(v_S)}{Max(v_1)-Min(v_1)}=\frac{R_S}{R_1}=S^h$$

$$\frac{Log\left(\frac{\sigma_S^2}{\sigma_1^2}\right)}{Log\left(\frac{R_S}{R_1}\right)}=2,\ H>0$$



So What?

- This investigation of climate variance, and the finding of ergodic, or near ergodic, dynamics in climate change is not an academic distraction. For us (at least) this work has entirely changed our thinking and approach to climate finance and understanding climate risk.
- To investigate climate risks, the conceptualized base case should be the ergodic ensemble.
- It is conceptually useful to consider the source of climate change as forcings on the boundaries of the ensemble which expand phase space, reduce ergodicity towards classical mean-reversion (H>0), and reduces the speed at which the equilibrium-attractor will be obtained.
- Climate is random and irreversible; that is the ‘Hansel and Gretel’ problem of historical markers that permit reversibility do not hold (at least under the 2^{nd} law of thermodynamics), so the reduction of climate risk should be targeted towards reducing Hurst towards H=0.
- If climate variance is increasing locally or globally then these changes have to be measured to establish the range of liability and pre-determine what Mandelbrot and Wallis refer to as Joseph and Noah effects (which we address using Ito excursion theory)
- New techniques to climate finance (most likely Monte Carlo) will have to be explored to account for changing variance (ask me in a year). The Ornstein–Uhlenbeck process is very useful so far.
- Our approach to conservation of climate-smart financing / insurance will be carbon based, but for other aspects of nature financing including promoting biodiversity the carbon linkage may be to ambiguous to measure. (ask me in a year)
- Before attempting to publish these results, we want to repeat the analysis on NOAA weather station records in USA and also examine data from China (and elsewhere if possible). (ask me in a year)