WESTERN REGIONAL RESEARCH PUBLICATION

W-133
BENEFITS AND COSTS OF RESOURCES POLICIES AFFECTING PUBLIC AND PRIVATE LAND

12TH INTERIM REPORT
JUNE 1999

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INTRODUCTION

This volume contains the proceedings of the 1999 W-133 Western Regional Project Technical Meeting on "Benefits and Costs of Resource Policies Affecting Public and Private Land." Some papers from W-133 members and friends who could not attend the meeting are also included. The meeting took place February 24th - 26th at the Starr Pass Lodge in Tucson, Arizona. Approximately 50 participants attended the 1999 meeting, are listed on the following page, and came from as far away as Oslo, Norway.

The W-133 regional research project was rechartered in October, 1997. The current project objectives encourage members to address problems associated with: 1.) Benefits and Costs of Agro-environmental Policies; 2.) Benefits Transfer for Groundwater Quality Programs; 3.) Valuing Ecosystem Managment of Forests and Watersheds; and 4.) Valuing Changes in Recreational Access.

Experiment station members at most national land-grant academic institutions constitute the official W-133 project participants. North Dakota State, North Carolina State, and the University of Kentucky proposed joining the group at this year's meeting. W-133's list of academic and other "Friends" has grown, and the Universities of New Mexico and Colorado were particularly well represented at the 1999 W-133 Technical Meeting. The meeting also benefitted from the expertise and participation of scientists from many state and federal agencies including California Fish and Game, the U.S. Department of Agriculture's Economic Research and Forest Services, the U.S. Department of Interior's Fish and Wildlife Service, and the Bureau of Reclamation. In addition, a number of representatives from the nation's top environmental and resource consulting firms attended, some presenting papers at this year's meeting.

This volume is organized around the goals and objectives of the project, but organizing the papers is difficult because of overlapping themes. The last section includes papers that are very important to the methodological work done by W-133 participants, but do not exactly fit one of the objectives. -- I apologize for the lack of consistent pagination in this volume.

On A Personal Note... Any meeting or conference is successful (and fun!) only because of its participants, so I would first like to thank all the people who came and participated in 1999 - listed below. I also want to thank Jerry Fletcher for all his help at this meeting and prior to it, and John Loomis who passed on his knowledge of how to get a meeting like this to work, and who continues to have the funniest little comments to lighten the meetings up. I especially thank Paul Jakus, who helped me to organize this conference and have a lot of fun during it and afterward. Finally, I want to thank Nicki Wieseke for all her help in preparing this volume, and Billye French for administrative support on conference matters.

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June, 1999

P.S. P.F. and J.C. - As far as I can tell, that darn scorpion is still dead!

Are Revealed Preference Measures of Quality Change Benefits Statistically Significant?

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February 1999

Prepared for the Annual Meeting of Western Regional Research Group W-133:
Benefits and Costs Transfer in Natural Resource Planning
Tucson, AZ
February 24-26, 1999

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Are Revealed Preference Measures of Quality Change Benefits Statistically Significant?

ABSTRACT

Environmental economists typically invoke weak complementarity in order to measure use values of quality changes from revealed preference information such as recreation demand functions. But they do not typically associate standard errors with the estimates. The assessment of standard errors for quality change welfare measures is more complicated than for the price change case, in part because a structural assumption must be employed to explain how the constant of integration back varies with the quality change parameter of interest. Even when the widely-invoked weak complementarity assumption is used to provide this structure, the underlying quasi-preferences are unknown for many demand functions. Numerical approximations can be used, but require changes in multiple variables to maintain the structural hypothesis, which can inflate standard errors.

We explore these and other issues involved with assessing standard errors of quality change measures. Not surprisingly, we find that when the underlying quasi-preferences are unknown, the variance of compensating variation is substantially larger than when they are known. What is surprising is how large the standard errors of quality change welfare measures are relative to their means, even when the underlying quasi-preferences are known. Demand models with statistically-significant parameters frequently yield compensating variations for quality change that include zero in their 95% confidence intervals. The covariances between demand parameters play a strong role in the magnitude of the standard error of compensating variation.

Are Revealed Preference Measures of Environmental Quality Benefits Statistically Significant?

Two main avenues are available to researchers interested in measuring the value of changes in non-market amenities such as environmental quality: the stated preference approach (exemplified by contingent valuation, contingent ranking, and other direct questioning methods) and the revealed preference approach, the best-known example of which is the travel cost method of recreation demand. Each results in a willingness-to-pay function, typically based on compensating variation or surplus, whether estimated directly (as with stated preference data) or inferred from an auxiliary relationship (such as demand functions, in the case of revealed preference).

While the topic of precision of welfare measures generally has received some attention (e.g., Adamowicz, Fletcher, and Graham-Tomasi; Kling and Sexton; Kling, 1991, 1992), much, though not all, of this work has centered on price changes and access values for recreational resources. Relatively less work has focused on the precision of quality-change welfare measures, particularly those derived from revealed preference work. An exception is Kling (1988a,b), whose focus primarily is comparing alternative estimation strategies for recreation demand based on their relative error in estimating a known true welfare change measure, but who also calculates root mean squared errors that suggest the empirical quality change measures are often statistically insignificant.

The reason why insights about quality change welfare measures might differ from those obtained from evaluating price changes is that the integrability problem generally poses an additional challenge for welfare measurement. For quality changes, it is well known that unique welfare measures cannot be obtained from revealed preference alone without imposing some additional structure on preferences (e.g., LaFrance and Hanemann). The structure typically used is *weak complementarity* between one or more market goods and the quality characteristic. First proposed by Mäler and further articulated by Bradford and Hildebrandt, Willig, and Bockstael and McConnell (1983; 1993), weak complementarity allows the researcher to recover the value of an environmental quality change by focusing on how the market demands for the weak complements to quality change as quality changes. It corresponds to the familiar graph of the area between two (Hicksian) demand curves that shift with a quality variable.[1]

The problem this poses for welfare measurement with quality changes is that if the underlying preferences for quality are not known, as is typically the case when one estimates a demand system dependent on the quality characteristic, implementation of the welfare measurement procedure under weak complementarity requires a three-step procedure, outlined in Mäler, involving changes in price as well as the change in quality. (This is described further below. Because it would typically be implemented using numerical methods such as those outlined in Vartia or Porter-Hudak and Hayes, we refer to this as the *numerical* approach.) One would expect, intuitively, that this would inflate the standard errors of the welfare measure, relative to the case where the underlying quasi-expenditure function is known and can be evaluated directly for a change in quality alone (the *analytic* approach).

The operational question we explore is whether, in either case, the point estimates of welfare change for quality changes are sufficiently precise to be of any use in policy analysis. One dimension of the problem, clearly, is the relative increase in standard error of the welfare measure when underlying quasi-preferences are unknown, compared to when they are known. This issue is probably most relevant to revealed preference studies, where demand functions are estimated but the corresponding weakly complementary quasi-expenditure function may not be known.[2]

The other dimension we explore is the role that correlation between willingness to pay function parameters plays in determining the standard error of the resulting compensating variation. This issue is common to both revealed and stated preference methods, because each ultimately derives willingness to pay as a function of correlated random variables, namely the estimated parameters of the statistical model. This issue, somewhat to our surprise, is quite important to the precision of the resulting standard errors.

We consider these questions within the framework of a linear demand model, because this is one commonly-used functional form for which the underlying weakly-complementary quasi-expenditure function is known. This facilitates the comparison of the numerical and analytic approaches, because each can be performed on the same model and the resulting standard errors compared.

The Welfare Measurement Framework

The issue of concern is measuring the consumer's valuation of a change in exogenous quality, represented by the variable z, from an initial level z_0 to a subsequent level z_1. There are n market goods denoted by $\mathbf{x}=(x_1,...,x_n)'$ with corresponding prices $\mathbf{p}=(p_1,...,p_n)'$, and the consumer is presumed to choose market goods in a way that minimizes the cost of utility, represented by the dual problem

$$\min_{\mathbf{x}} \mathbf{p'x} \quad \text{s.t. } u^0 = u(\mathbf{x},z). \tag{1}$$

The solution to (1) is the Hicksian demands $x^h(\mathbf{p},z,u)$, which when substituted into (1) yields the minimum expenditure function $e(\mathbf{p},z,u)$. The corresponding primal problem has the form

$$\max_{\mathbf{x}} u(\mathbf{x},z) \text{ s.t. } m \geq \mathbf{p'x} \tag{2}$$

where m is the consumer's exogenous budget constraint. The solution to this problem yields Marshallian demand functions $x(\mathbf{p},z,m)$ which are estimated empirically.

In practice, we typically work with incomplete demand systems, which do not identify all the structure of $e(\mathbf{p},z,u)$. Suppose that a b-good (with b<n) incomplete demand system

$$x_i = x_i(\mathbf{p}^b,\mathbf{p}^{-b},z,m), \qquad i=1, \ldots ,b$$

is estimated, with $\mathbf{p}^b \equiv [p_1,...,p_b]'$ the vector of prices included in the estimated demand system and $\mathbf{p}^{-b} \equiv [p_{b+1},...,p_n]'$ the prices of other goods outside the empirical demand system. The empirical demand system integrates back to a quasi-expenditure function $\tilde{e}(\mathbf{p}^b,z,\theta(\mathbf{p}^{-b},z,u))$ which can be used for exact welfare measurement with respect to any of the prices in $\mathbf{p}^b$, conditional on $\mathbf{p}^{-b}$, but not for z without further structure on $\theta(\cdot)$ (LaFrance and Hanemann).

As noted above, weak complementarity of z with $\mathbf{x}^b \equiv [x_1,...,x_b]'$ is the typical assumption made about the structure of preferences that is sufficient to identify the curvature of $\theta(\cdot)$ with z and, therefore, the way that z enters the quasi-expenditure function $\tilde{e}(\mathbf{p}^b,z,\theta(\mathbf{p}^{-b},z,u))$. Mäler showed how one could use this assumption in a three-step process (raising price to the choke level given original quality level; changing quality while simultaneously adjusting choke price to keep "use" at zero; and reducing price from the new choke level to its original level) to measure the compensating variation associated with a change in quality for arbitrary demand functions.[3] Larson showed how one can analytically recover the weakly complementary quasi-expenditure function corresponding to a linear single-equation demand function.

The Demand Specification and Implied Quasi-Preferences

If we write the Marshallian demand function for a good x of interest as

$$x = \alpha + \beta p + \gamma z + \delta m, \tag{3}$$

the quasi-expenditure function obtained from integrating back from (3) is (e.g., Hausman)

$$\tilde{e}(p,q,\vartheta(z,u)) = \vartheta(z,u) \cdot e^{\delta p} - (1/\delta)[\alpha+\beta p+\gamma z+\beta/\delta], \tag{4}$$

where $\vartheta(z,u)$ is a constant of integration that may depend on all other parameters of the problem besides the variable of integration p, including the quality variable and other prices (which are not made explicit in this model).

As is well known, equation (4) does not provide a basis for unique welfare measurement for quality (z) changes without further structure being imposed. What is typically invoked is weak complementarity between the public good or quality attribute whose value is of interest, and a set of related market goods whose demand can be observed (e.g., Mäler). Weak complementary is essentially an assumption that there is no "passive-use" value. In the present model, the assumption is of weak complementarity between x and z. When this is imposed as part of the process of integrating back to recover quasi-preferences, the resulting quasi-expenditure function is

$$e(\mathrm{p,q},u) = u\mathrm{e}^{(\delta/\beta)(\gamma z+\beta p)} - (1/\delta)[\alpha+\beta \mathrm{p}+\gamma \mathrm{z}+\beta/\delta], \tag{5}$$

where $u < 0$ is the utility index, independent of both z and p (Larson). The corresponding weakly complementary indirect utility function is

$$\nu(\mathrm{p,q,m}) = [\mathrm{m}+(1/\delta)(\alpha+\beta \mathrm{p}+\gamma \mathrm{z}+\beta/\delta)]\mathrm{e}^{-(\delta/\beta)(\gamma z+\beta p)}.$$

The compensating variation for a change in quality from z_0 to z_1 is

$$\mathrm{CV} = e(\mathrm{p}_0,\mathrm{z}_0,\mathrm{u}) - e(\mathrm{p}_0,\mathrm{z}_1,\mathrm{u})$$

$$= \mathrm{M} - u\mathrm{e}^{(\delta/\beta)(\gamma z_1+\beta p)} + (1/\delta)[\alpha+\beta \mathrm{p}+\gamma \mathrm{z}_1+\beta/\delta]. \tag{6}$$

Approximating the Standard Error of CV when Quasi-Preferences are Known

In the *analytic* approach, the compensating variation is obtained directly from substituting the two levels of quality, z_0 and z_1, directly into the quasi-expenditure function, as in (6). The resulting CV is a random variable because it is a nonlinear function of a set of correlated random variables. Given the variance-covariance matrix Ω_ϕ for the vector of estimated parameters $\hat{\phi} \equiv [\hat{\alpha},\hat{\beta},\hat{\gamma},\hat{\delta}]'$ and the gradient vector $\nabla \equiv [\partial CV/\partial\alpha(\hat{\alpha}),...,\partial CV/\partial\delta(\hat{\delta})]'$, a consistent approximation to the variance of $CV(\hat{\phi})$, $V[CV(\hat{\phi})]$, is

$$V[CV(\hat{\phi})] \approx \nabla'\Omega_\phi\nabla, \tag{7}$$

provided $\hat{\phi}$ is a consistent estimate of the true underlying parameters ϕ (e.g., Greene).

The approximation in (7) is based on a first-order Taylor's series approximation to the variance of a nonlinear function of random variables. We use the Taylor's series approximation approach because of our interest in comparing results when underlying quasi-preferences are known and an analytic solution can be used, to the case where underlying quasi-preferences are unknown. In this latter case, the numerical approximation methods used to assess standard errors employ a Taylor-approximation methodology (e.g., Vartia; Porter-Hudak and Hayes; Breslaw and Smith). Thus for consistency of comparison, we use the Taylor's approximation in (7) for the analytic case as well.

Approximating the Standard Error of CV when Quasi-Preferences are Unknown

In many cases, the quasi-expenditure function underlying the estimated demand function may not be known analytically, particularly as it changes with quality. The *numerical* approach is used for cases like this. Algorithms presented by Vartia, Porter-Hudak and Hayes, and

Breslaw and Smith for evaluating price changes can be adapted to the case of quality changes with weakly complementary preferences.

The principle behind these algorithms is straightforward, as they simulate the standard error of compensating variation for a series of small changes $\triangle z^i$ for i=1,...,n steps covering the interval from z_0 to z_1, following the approach of Vartia. Given the variance-covariance matrix $\Omega_\phi(\hat{\phi})$ for the estimated parameter vector $\hat{\phi} \equiv [\hat{\alpha},\hat{\beta},\hat{\gamma},\hat{\delta}]'$, for each step i, the updated estimate of the expenditure function at each step i is

$$e^i(\hat{\phi}) = e^{i-1}(\hat{\phi}) + \tfrac{1}{2}[(\partial e(\hat{\phi})/\partial z)_0 + (\partial e(\hat{\phi})/\partial z)_1] \triangle z^i \tag{8}$$

and the variance of the new estimate of the expenditure function at the i^{th} step, $e^i(\hat{\phi})$, is

$$\text{var}[e^i(\hat{\phi})] = \begin{bmatrix} 1 & \frac{\triangle z}{2} & \frac{\triangle z}{2} \end{bmatrix} \begin{bmatrix} \sigma_{ee} & \sigma_{e0} & \sigma_{e1} \\ \sigma_{e0} & \sigma_{00} & \sigma_{01} \\ \sigma_{e1} & \sigma_{01} & \sigma_{11} \end{bmatrix} \begin{bmatrix} 1 \\ \frac{\triangle z}{2} \\ \frac{\triangle z}{2} \end{bmatrix} \tag{9}$$

where the σ_{ij}, for i, j = e, 0, 1, are the variances and covariances of, and between, the previous estimate of the expenditure level, $e^{i-1}(\hat{\phi})$, and the estimates of the expenditure slope at the previous $[(\partial e(\hat{\phi})/\partial z)_0]$ and current $[(\partial e(\hat{\phi})/\partial z)_1]$ levels of quality z. Each of these is a nonlinear transformation of the regression parameters $\hat{\phi}$, with gradients denoted $\nabla_i(\hat{\phi})$, respectively, for $i = y, 0, 1$. Estimates of the asymptotic variances and covariances are then obtained from

$$\sigma_{ij} = \nabla_i(\hat{\phi})\, \Omega_\phi(\hat{\phi})\, \nabla_j(\hat{\phi})$$

following Rao and Porter-Hudak and Hayes. The compensating variation is $CV(\hat{\phi}) = M - e^n(\hat{o})$, by analogy to (6), so with initial income taken to be fixed, the variance of the compensating variation measure is

$$V[CV(\hat{\phi})] = \text{var}[e^n(\hat{\phi})],$$

from (9).

The difficulty with applying this numerical approximation technique directly when the underlying weakly complementary quasi-preferences are not known (i.e., when a different demand function is used) is that the constant of integration in (4) is not identified analytically, so the quality slopes $(\partial e/\partial z)_0$ and $(\partial e/\partial z)_1$ are unknown. The weak complementarity condition can still be imposed as part of the numerical approximation of compensating variation and its standard error, but the process is more involved than the direct simulation of a quality change in (8). The reason is that the weak complementarity condition is a statement about what happens to the expenditure function when z changes and x is *not* being consumed. That is, when the price of x is at the choke level p' and consumption of x is zero, weak complementarity of x with z means that there is no change in value and the expenditure function is stationary. Thus, weak complementarity is a condition that holds at a different set of prices (p') than those which hold at the reference point ($p < p'$, where there is positive consumption of x), so to impose it as part of the numerical approximation of CV and its standard error, prices must be changed from p to p'.

This is the three-part strategy suggested by Mäler for measuring welfare for general weakly complementary preferences (p. 173-176). To measure the welfare change associated with a change in z from z^0 to z^1, given prices p^0, one must

(a) change p from p^0 to p', given z^0; (10a)

(b) change p' to maintain consumption of x at zero as z changes from z^0 to z^1; (10b)

(c) change p from p' back to p^0 given z^1. (10c)

The compensating variation of the quality change can be measured this way because steps (a) and (c) are the area under the Hicksian demand for x given z^0 and z^1, respectively, each of which can be measured by a numerical approximation algorithm such as that given in (7), though for changes in price rather than z. The weakly complementary welfare measure is the difference between the two, since by assumption the welfare change in part (b) is zero.[4]

When underlying quasi-preferences are unknown, the welfare measures and price changes in each of these steps (a)-(c) can be measured numerically. But the result is that three numerical approximations are required, rather than just one as suggested by (7). The expectation is that having to sequentially raise and lower price in addition to changing quality to measure the welfare effects of the quality change will inflate the standard error of the resulting CV measure, resulting in more frequent occurrences of statistical insignificance for given quality changes.

Parameterizing the Simulation Model

Our starting point for developing a simulation model to compare the analytic and numerical approaches is the information which might be observed in the field about the demand for recreation by a "typical" individual, that is, the trips taken, price paid, income, and quality expected. Both the form of the demand model and its statistical significance (i.e., the variance-covariance matrix for a given parameter vector) are likely to be important to the significance of the compensating variation of a quality change, which is a nonlinear transformation of demand model parameters.

Demand Parameters

Demand model parameters are chosen to be representative of those found in recreation demand studies. Such a model might be motivated in terms of recreational fishing, where quality (e.g., expected fishing sucess) can play a prominent role. For example, consider an individual with income of $60,000 who takes 4 trips per year to a single fishing site, at a price of $55 per trip, with expected fishing success of 2.8 fish per trip. Assumptions about the demand elasticities with respect to each of these variables will yield coefficients $\phi=[\alpha,\beta,\gamma,\delta]$ for the model in equation (3).

The baseline model is one with unitary elasticities, with price elasticity of -1 and quality and income elasticities equal to +1. This implies the demand specification

$$x = -.07273p + 1.429z + .00006667m, \tag{11}$$

with $\alpha = 0$ by coincidence. For contrast, it is natural to consider coefficient values corresponding to both elastic and inelastic versions of the model. The inelastic version of the model has price elasticity of -.80, and income and quality elasticities of .80, yielding a demand model parameterized as

$$x = .80 - .05818p + 1.143z + .00005333m. \tag{12}$$

The elastic version of the model was also constructed for the same levels of trips, price, income, and quality, but for price elasticity of -1.2 and income and quality elasticities of 1.2. This results in a demand model of the form

$$x = -.80 - .08727p + 1.714z + .00008m. \tag{13}$$

Variance-Covariance Matrix

A key issue is how given levels of significance in the demand model translate to significance of the CV for a given quality change. We explore this by considering a variety of statistically-significant demand models, as measured by the Student's-t statistics on individual coefficients. To see how the assumptions we make about coefficient significance can be built into the model, note that the asymptotic standard error of coefficient $\widehat{\phi}_i$ can be written as

$\hat{\sigma}_{\hat{\phi}_i} = \hat{\phi}_i / t_{\hat{\phi}_i}$, where $t_{\hat{\phi}_i}$ is the Student's-t statistic for coefficient $\hat{\phi}_i$ under the common hypothesis of no association for a model with given degrees of freedom and significance level.

Using this definition of the standard error, the variance-covariance matrix for the parameter vector is

$$\Omega_o = \begin{bmatrix} \sigma^2_{\hat{\alpha}} & \cdots & \sigma_{\hat{\alpha}\hat{\delta}} \\ \vdots & \ddots & \vdots \\ \sigma_{\hat{\delta}\hat{\alpha}} & \cdots & \sigma^2_{\hat{\delta}} \end{bmatrix} = \begin{bmatrix} (\hat{\alpha}/t_{\hat{\alpha}})^2 & \cdots & \rho_{0m}(\hat{\alpha}/t_{\hat{\alpha}})(\hat{\delta}/t_{\hat{\delta}}) \\ \vdots & \ddots & \vdots \\ \rho_{0m}(\hat{\alpha}/t_{\hat{\alpha}})(\hat{\delta}/t_{\hat{\delta}}) & \cdots & (\hat{\delta}/t_{\hat{\delta}})^2 \end{bmatrix} \tag{14}$$

so the effect of varying significance level of coefficients, for a given parameter vector, on the variance-covariance matrix can be seen. The coefficients $\hat{\alpha},...,\hat{\delta}$ for the simulations are identified in equations (11)-(13). As is well known, the magnitudes of the elements of the variance-covariance matrix will vary inversely with the precision of measuring the coefficient (i.e., its Student's-t statistic) and directly with the magnitude of the absolute value of the correlation coefficients.

Equation (14) points out the potential importance of the correlation parameters ρ_{ij}, beyond the issue of how precisely they are estimated. To isolate the effects of each separately, we present results on the standard error of the quality change welfare measure for different levels of precision of estimating coefficients (given by $t_{\hat{\phi}_i} = t_{\hat{\phi}_j} = 2, 3, 5$ all i,j) and for partial correlation coefficients between price, quality, and income coefficients ranging from -1 to 1.

The first threshold of Student's-t, t=2, corresponds roughly to the asymptotic t for the 95% confidence level, as a guide to a common rule of thumb used in practice. One could, obviously, use many different thresholds for determining "significance" of the quality change welfare measure, corresponding to specific sample sizes (which imply higher threshold Student's-t statistics) and alternative significance levels. It is worth noting that our choice of a test statistic based on asymptotic distributions is conservative, in the sense that we will find more "significant" welfare measures, and hence fewer problems, than would be found in small samples.

For increments of 0.25 for the partial correlation between price and quality coefficient (ρ_{pz}) in the interval [-1,1], we consider increments of 0.1 in the partial correlations between income and quality (ρ_{mz}) and between price and income (ρ_{pm}). In addition, the constant term is uncorrelated to price, income, and quality ($\rho_{0m} = \rho_{0p} = \rho_{0z} = 0$). It is important to note that not all combinations of ρ_{pm}, ρ_{mz}, and ρ_{pz} in the unit sphere are valid representations of a partial correlation matrix, because such a matrix must be positive semidefinite; that is, it must satisfy the equation

$$\sum_i \sum_j x_i x_j \rho_{ij} \geq 0$$

for all non-zero vectors $x = [x_1,...,x_n]$.

Rousseeuw and Molenberghs analyze this feasible set and their Figure 1 (reproduced here as Figure 1) illustrates the set visually. For a three-dimensional partial correlation matrix, they show that any horizontal cross section, obtained by fixing one partial correlation, is an ellipse. If, for instance, one partial correlation, say ρ_{pm}, is held at a value within the (0,1) interval, we obtain an ellipse with a major axis in the direction of the line $\rho_{pz} = \rho_{mz}$ and a minor axis in the direction of $\rho_{pz} = -\rho_{mz}$. Conversely, if ρ_{pm} is fixed at a value between -1 and 0, the major and minor axes of the resulting ellipse are reversed. Of particular note are the extremum values, where a value of $\rho_{ij} = \pm 1$ for one partial correlation reduces the set of feasible combinations of the remaining two partial correlations, to a line of equal slope (when $\rho_{ij} = +1$) or opposite slope (when $\rho_{ij} = -1$).

When the feasible set for valid representations of a correlation matrix is imposed on the unit sphere, there are 1,893 combinations of the three pairwise correlations ρ_{pm}, ρ_{pz}, and ρ_{mz} for each parameterization of the demand model.

Comparative Statics of the Analytic Approach

When quasi-preferences are known, the asymptotic variance of the quality change welfare measure is a function of the parameters of the problem, as (7) indicates. In particular, one can examine the effects of changes in the partial correlation coefficients ρ_{ij} or the level of precision t_i with which coefficients are measured. In our simulations, the precision of each coefficient is equal, though the signs may differ, so we can simplify by writing $|t_{\phi_i}| = |t_{\phi_j}| = t$. To preserve consistency with the use of the ratios ϕ_i/t_{ϕ_i} to represent standard errors (which are non-negative) in (13), it is necessary to redefine the coefficients also in terms of absolute magnitudes, not signs; i.e., we have $|\phi_i|/|t_{\phi_i}| = \sigma_i$. Letting ∇_i denote the ith element of ∇, $\partial CV/\partial\phi_i$ (where i, j = α, β, γ, δ), and ∇_{ij} denote the second partial derivative $\partial^2 CV/\partial\phi_i\partial\phi_j$, then (7) can be written out more fully (dropping the "hats" for simplicity) as

$$V[CV(\phi)] = t^{-2}[\nabla^2_{\alpha\alpha}\alpha^2 + \nabla^2_{\beta\beta}\beta^2 + \nabla^2_{\gamma\gamma}\gamma^2 + \nabla^2_{\delta\delta}\delta^2 + 2\nabla_\beta\nabla_\gamma\rho_{pz}|\beta\gamma|$$

$$+ 2\nabla_\beta\nabla_\delta\rho_{pm}|\beta\delta| + 2\nabla_\delta\nabla_\gamma\rho_{mz}|\delta\gamma|] \quad (15)$$

Differentiating (15) with respect to t, the Student's t-statistic, the change in variance of the welfare measure as t increases is

$$\partial V[CV(\phi)]/\partial t = -(2/t)\cdot V[CV(\phi)] \quad \leq 0$$

since $V[CV(\phi)] \geq 0$ and $t > 0$ by construction. Not surprisingly, increasing the statistical significance of all demand parameters (by increasing the magnitude of the t-statistic) unambiguously decreases the variance of the welfare measure. Equivalently, the coefficient of variation of the welfare measure decreases as the significance of the estimated demand parameters increases.

Similarly, one can analyze how changes in the degree of correlation between variables affects the variance of the compensating variation welfare measure by differentiating (13) with respect to ρ_{pz}, ρ_{pm}, and ρ_{mz}.[5] This yields (16), (17), and (18), respectively.

$$\partial V[CV(\phi)]/\partial \rho_{pz} = 2|\beta\gamma|t^{-2}\nabla_\beta\nabla_\gamma \tag{16}$$

$$\partial V[CV(\phi)]/\partial \rho_{pm} = 2|\beta\delta|t^{-2}\nabla_\beta\nabla_\delta \tag{17}$$

$$\partial V[CV(\phi)]/\partial \rho_{mz} = 2|\delta\gamma|t^{-2}\nabla_\delta\nabla_\gamma \tag{18}$$

To determine these signs, one must know the signs of ∇_β, ∇_γ, and ∇_δ. For the linear demand model examined in this paper, one can differentiate the CV(ϕ) expression in equation (6) to determine ∇_β, ∇_γ, and ∇_δ. For this model, $\nabla_\gamma > 0$ always, and both ∇_β and $\nabla_\delta > 0$ when the combined intercept and income term is non-negative; i.e., when $\alpha + \delta m > 0$.

It is easily verified that all three versions of the demand model in (11)-(13) meet this condition. Therefore, since ∇_β, ∇_γ, and ∇_δ are all positive, V[CV(ϕ)] increases with each of the partial correlation coefficients.

Simulation Results

Analytic Approach

Table 1 presents results for the case of known quasi-preferences, where the welfare evaluation strategy in equations (6) and (7) is used. This is a tabulation of the results of simulations of the compensating variation for a 50 percent increase in the quality variable, from $z_0 = 2.8$ to $z_1 = 4.2$. These simulations cover 1,893 combinations of feasible partial correlation coefficients between the price, income, and quality coefficients, for each of the three types of demand functions (inelastic, unitary elastic, and elastic) and each of the three levels of precision in measuring coefficients (t = 2, 3, and 5). For each combination, the coefficient of variation of CV is calculated (as the standard error divided by estimated CV). When the coefficient of variation is greater than 0.5, the value of zero is within the 95% confidence bounds on CV. A conventional classical hypothesis test of difference of the CV estimate from zero would fail to reject that hypothesis.

The results are quite striking. For a model with elastic demand parameters, all statistically significant at the 95% (2-tailed) level (i.e., with t-values of 2), less than 2% of the CVs were had coefficients of variation less than 0.5. Results were comparable for the unit-elastic and inelastic cases, where slightly more than 2%, and less than 3%, of the CVs had coefficients of variation less than 0.5, respectively. When the significance of all demand coefficients increases to the .9987 level (Student's-t = 3), only 6% of the elastic demand, 7% of the unit elastic demand, and 10% of the inelastic demand simulations had coefficients of variation less than 0.5. When the t-value on all demand coefficients was increased to 5 (corresponding to a p-value of .9999997), 31% and 39% of the elastic and unit elastic demand model simulations satisfied this condition, respectively, and almost half of the inelastic demand model simulations did. The observed increase in precision of CV in all models with increases in the t-statistic is consistent with the comparative static results derived above.

Figure 2 illustrates the results from the unit-elastic case visually, with plots of the coefficient of variation of CV (COV) against the price-income [ρ(p,m)] and quality-income [ρ(q,m)] correlations, for given price-quality correlations [ρ(p,q)]. As one moves from left to right and top to bottom in each figure, the price-quality correlation relationship goes from perfect inverse correlation to perfect direct correlation. The combinations with coefficient of variation on CV less than 0.5 are illustrated with boxes, and those with higher COV are marked with pyramids. The coefficient of variation surfaces are smooth with different tilts depending on the values of the conditioning correlation, between price and quality.

These graphs emphasize the relationships found in the comparative statics results on the effects of partial correlation relationships on the precision of CV. As Figure 2 shows, the more positively correlated price and quality are (ρ_{pz} approaches 1), the less likely is there a significant coefficient of variation (i.e., COV < 0.5). Likewise, COV is higher as the correlation between price and income gets more positive (ρ_{pm} approaches 1), while COV falls as the correlation between income and quality gets more negative (ρ_{mz} approaches -1).

Other regularities are also apparent in the results. The elasticity of the demand parameters appears to play an important role in the precision of welfare measures. Table 2 highlights this effect by focusing on changes in the magnitude of all elasticities simultaneously, varying them from 0.2 to 2.0 in magnitude for a given (50%) quality change and significance level (t=2). Not surprisingly, the CV for a 50% quality change varies, from $115 to $165. As Table 2 indicates, the elasticity of the parameters clearly affects the significance of compensating variation, but it does not appear to be monotonic with the own-price elasticity. This can be seen by noting that for the linear model, changing the own-price elasticity of a demand function running through a fixed price-quantity point is equivalent to changing β; and similarly, changes in quality and income elasticities and equivalent to changes in γ and δ. But in (15) it is clear that the effect of changing the magnitude of β, γ, and δ will depend in non-trivial ways each of their magnitudes and on the pairwise correlations ρ_{pz}, ρ_{mz}, and ρ_{pm}. The non-monotonic relationship between magnitudes of elasticities and precision of the welfare measure appears to be due to the fact that all are changing simultaneously and they all affect the welfare measure precision in different ways.

Table 3 follows up with a consideration of the effects of changing individual elasticities, *ceteris paribus*. The first four columns of data in Table 3 contains the results from changing the own-price coefficient, $\hat{\beta}$, in the unit-elasticity model to reflect price elasticities of $-.2$, $-.8$, -1, and -2, while keeping all other coefficient values the same. These results suggest that the more own-price elastic the demand model, the lower the precision in measuring the welfare change for a 50% increase in quality. In contrast, the last four columns in Table 2 illustrate the effect of increasing quality elasticity, *ceteris paribus*, which shows a non-monotonic relationship with the precision of the welfare measure, with the most frequent occurrences of significant results at the intermediate elasticity levels.

These results highlight the important role that parameter correlations and elasticities play in determining the magnitude of empirical standard errors of quality change measures. But the overriding point is the generally low frequency with which willingness to pay models with all significant coefficients lead to "precise" welfare estimates for quality changes (in the sense that zero is not included in the 95% confidence bounds). For a large fraction of the feasible pairwise correlations, statistically significant (at the $\alpha = .05$ level of significance) parameters of the willingness to pay function translate to imprecise quality change welfare measures.

Numeric Approach

The second issue of interest is the degree to which the standard error of compensating variation is inflated because the analytic quasi-expenditure function is unknown and the 3-step Mäler/Vartia algorithm must be used instead. Table 4 provides some perspective on this question for all the cases where the coefficient of variation was less than 0.5 in the analytic solution results, for the case where all demand parameters have t-values of 2. Of these 40 cases (resulting from 1,893 different feasible correlation combinations), only 1 (highlighted in bold) had coefficient of variation less than 0.5 under numerical approximation of the standard errror. The ratio of numerically-approximated standard error to analytic standard error showed considerable variation, depending on the particular correlations between explanatory variables. The ratio of standard errors ranged from 1.42 to 6.52, with an arithmetic mean of 3.3. This suggests that, on average, the numerical approximation routinely inflates standard errors by roughly a factor of 3.

Priors on the Correlation Combinations

One might suspect that not all correlation combinations are equally likely. In fact, one might have as priors that price and income are positively correlated (because, for example, higher income people might travel to a recreation destination in motor homes with lower gas mileage and slower travel speeds); that price and quality are positively correlated, if it is possible to price differentially for quality; and that quality and income are positively correlated, if quality is a normal good. From Table 4, one can see that none of the 40 correlation combinations that had coefficients of variation less than 0.5 were from this orthant of the correlation spheroid. Thus the problem we identify may be worse for the cases likely to occur more frequently in practice than our overall results suggest. It illustrates the decreased likelihood of "precise" welfare measures when the underlying quasi-expenditure function is unknown and numerical approximation methods must be used.

The problems we find are not due solely to the presumption of some correlation structure betweeen parameter estimates. Focusing attention on the subset of simulations where 1, 2, or 3 of the pairwise correlations are zero, none of these indicate a coefficient of variation for the welfare measure of less than 0.5 even though each demand parameter is statistically significant (with Student's-t of 2).

Conclusions

In the recreation demand literature, it is common to find benefit estimates associated with a variety of different quality characteristic changes, but much less common to find standard errors attached to these calculations. Our results suggest that were researchers to do this more frequently, they would be surprised by how routinely they are calculating statistically-insignificant welfare measures for a wide range of null hypotheses. We highlight two of the reasons why this occurs: the role of pairwise correlations between the model parameters, which is an issue with both revealed and stated preference methods of assessing willingness to pay for quality changes; and the fact that one sometimes has to numerically approximate through simultaneous price and quality changes in revealed preference studies, when the underlying quasi-expenditure function behind estimated demand is unknown.

The results suggest that strong caution is appropriate in interpreting welfare estimates of quality change for which no standard errors are provided. As a corollary, good empirical practice has to include an assessment of the precision of the welfare measure for such estimates to be taken seriously.

The results here clearly are only suggestive and not definitive. Further work is needed to assess how robust the findings from this simple, but commonly used, function really are. It is possible that the parameter values themselves, or the initial levels of consumer income, price, and quality make a difference to the magnitudes of the effects described here, though we doubt they would be reversed.

It is important to note that these results come from *correct* practice in measuring welfare change when a quality characteristic changes. This is more involved than calculating a change in consumer's surplus area from Marshallian demands, as it involves evaluating a quasi-expenditure function selected based on some prior restriction on preferences (e.g., weak complementarity). This evaluation is done either numerically or analytically depending on whether the quasi-expenditure function itself is known, or only the preference restriction used to identify the welfare measure is known. Others have noted the difficulties inherent in using consumer's surplus instead of the theoretically-correct compensating variation (or surplus) measure for quality changes(e.g., Kling 1988a,b; Bockstael and McConnell, 1993), or in incorrectly assuming weak complementarity (Bockstael and Kling; LaFrance). One would expect that the problems we identify would compound, not alleviate, these other difficulties with correct measurement of welfare changes with quality characteristics.

Footnotes

1. Furthermore, it corresponds to a story about how consumers value environmental quality that is plausible in some (though not all) contexts: if the consumer is not consuming any of the weak complements, he or she is indifferent to the quality change. This is probably reasonable for many localized resources with plentiful substitutes, that are unlikely to generate nonuse value.

2. Stated preference estimation, by contrast, typically directly estimates a compensating variation function so, in principle, the quasi-preferences are known.

3. This implicitly defines a choke price function which varies with quality and all parameters of the problem to keep quantity consumed identically at zero.

4. Graphically, the three-step procedure traces out the change in area under the Hicksian demand for the related market good. The weak complementarity assumption assures that this is the total value of the quality change. Extension to multiple goods related to quality is straightforward (Bockstael and Kling).

5. The magnitude of the variance of the welfare measure will depend on the standard errors of, and correlations among, the demand parameters, as well as the gradients of compensating variation with respect to the parameters.

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Table 1. Simulation Results for Analytic CV Estimates for 50% Increase in Quality

	Demand Elasticity								
	Inelastic[a]			Unit Elastic[b]			Elastic[c]		
Student's-t	2.00	3.00	5.00	2.00	3.00	5.00	2.00	3.00	5.00
No. "significant"	48	185	918	40	138	731	27	107	582
% "significant"	0.03	0.10	0.48	0.02	0.07	0.39	0.01	0.06	0.31
CV ($)	132	132	132	138	138	138	142	142	142

[a] Absolute value of all elasticities is 0.8. The implied demand is $x = .8 - .05818p + 1.143z + .00005333m$.
[b] Absolute value of all elasticities is 1.0. The implied demand is $x = -.07273p + 1.429z + .00006667m$.
[c] Absolute value of all elasticities is 1.2. The implied demand is $x = -.8 -.087p + 1.714z + .00008m$.

Table 2. Effect of Demand Elasticities on Precision of CV, for Student's-t = 2[a]

	Elasticity				
	0.2	0.8	1	1.2	2 .
No. "significant"	21	48	40	27	2
% "significant"	1.11	2.54	2.11	1.43	0.11
CV ($)	115	132	138	143	165

[a] All coefficients have the same elasticity.

Table 3. The Effect of Different Price and Quality Elasticities on Precision of CV, for t = 2

	Elasticity Combinations							
Price	-0.2	-0.8	-1	-1.2	-1.0	-1.0	-1.0	-1.0
Quality	1.0	1.0	1.0	1.0	0.2	0.8	1.0	1.2
Number "significant"	79	50	40	1	3	47	40	27
Percent "significant"	4.17	2.64	2.11	0.05	0.16	2.48	2.11	1.43

Table 4. A Comparison of Standard Errors for Analytic Versus Numerical Methods Calculation of the Quality Change Welfare Measure[a]

Partial Correlations			Analytic Results		Numeric Results		Ratio of
ρ(p,q)	ρ(p,m)	ρ(q,m)	SEA(CV)[b]	COV	SEN(CV)[c]	COV	Std. Errors
-0.75	0.4	-0.9	59.66	0.43	219.40	1.58	3.68
-0.75	0.3	-0.8	60.92	0.44	213.15	1.53	3.50
-0.75	0.2	-0.7	62.15	0.45	206.70	1.49	3.33
-0.75	0.1	-0.7	49.96	0.36	170.68	1.23	3.42
-0.75	0.1	-0.6	63.36	0.46	200.05	1.44	3.16
-0.75	0	-0.6	51.45	0.37	162.56	1.17	3.16
-0.75	0	-0.5	64.55	0.47	193.17	1.39	2.99
-0.75	-0.1	-0.5	52.91	0.38	154.01	1.11	2.91
-0.75	-0.1	-0.4	65.72	0.48	186.03	1.34	2.83
-0.75	-0.2	-0.4	54.33	0.39	144.96	1.04	2.67
-0.75	-0.2	-0.3	66.86	0.48	178.61	1.28	2.67
-0.75	-0.3	-0.4	39.8	0.29	86.12	0.62	2.16
-0.75	-0.3	-0.3	55.71	0.40	135.30	0.97	2.43
-0.75	-0.3	-0.2	67.99	0.49	170.87	1.23	2.51
-0.75	-0.4	-0.3	41.66	0.30	68.64	**0.49**	1.65
-0.75	-0.4	-0.2	57.05	0.41	124.90	0.90	2.19
-0.75	-0.5	-0.1	58.37	0.42	113.55	0.82	1.95
-0.75	-0.6	0.1	59.66	0.43	100.94	0.73	1.69
-0.75	-0.7	0.1	60.92	0.44	86.50	0.62	1.42
-1	1.0	-1.0	41.42	0.30	270.00	1.94	6.52
-1	0.9	-0.9	43.21	0.31	264.94	1.91	6.13
-1	0.8	-0.8	44.93	0.33	259.79	1.87	5.78
-1	0.7	-0.7	46.59	0.34	254.52	1.83	5.46
-1	0.6	-0.6	48.2	0.35	249.15	1.79	5.17
-1	0.5	-0.5	49.75	0.36	243.66	1.75	4.90
-1	0.4	-0.4	51.25	0.37	238.04	1.71	4.64
-1	0.3	-0.3	52.71	0.38	232.29	1.67	4.41
-1	0.2	-0.2	54.13	0.39	226.39	1.63	4.18
-1	0.1	-0.1	55.52	0.40	220.33	1.59	3.97
-1	0.0	0.0	56.87	0.41	214.10	1.54	3.76
-1	-0.1	0.1	58.19	0.42	207.69	1.49	3.57
-1	-0.2	0.2	59.48	0.43	201.07	1.45	3.38
-1	-0.3	0.3	60.74	0.44	194.22	1.40	3.20
-1	-0.4	0.4	61.98	0.45	187.13	1.35	3.02
-1	-0.5	0.5	63.2	0.46	179.75	1.29	2.84
-1	-0.6	0.6	64.39	0.47	172.06	1.24	2.67
-1	-0.7	0.7	65.55	0.48	164.01	1.18	2.50
-1	-0.8	0.8	66.7	0.48	155.54	1.12	2.33
-1	-0.9	0.9	67.83	0.49	146.58	1.05	2.16
-1	-1.0	1.0	68.94	0.50	137.04	0.99	1.99

[a]This comparison is made for correlation combinations for which the coefficient of variation for the welfare measure was less than 0.5.

[b] SEA(CV) = Analytic standard error of compensating variation.

[c] SEN(CV) = Numerical approximation of standard error of compensating variation.

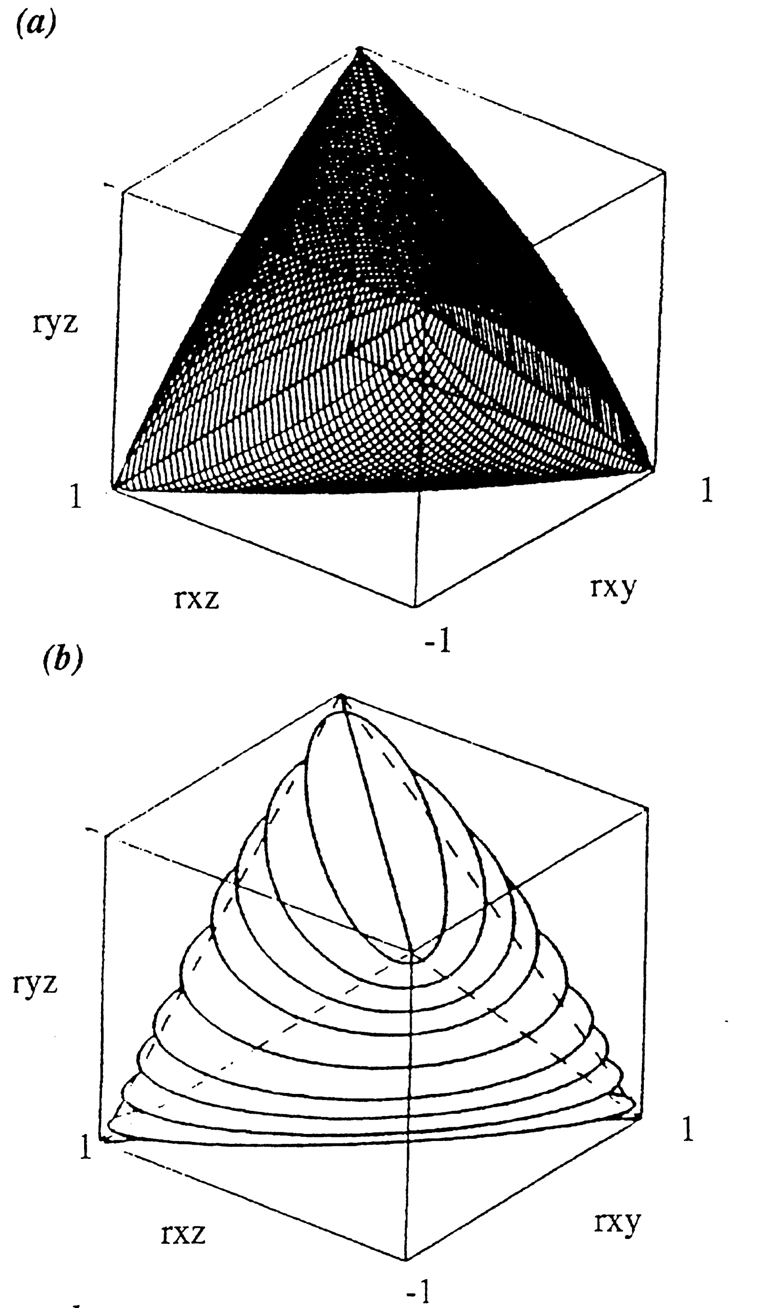
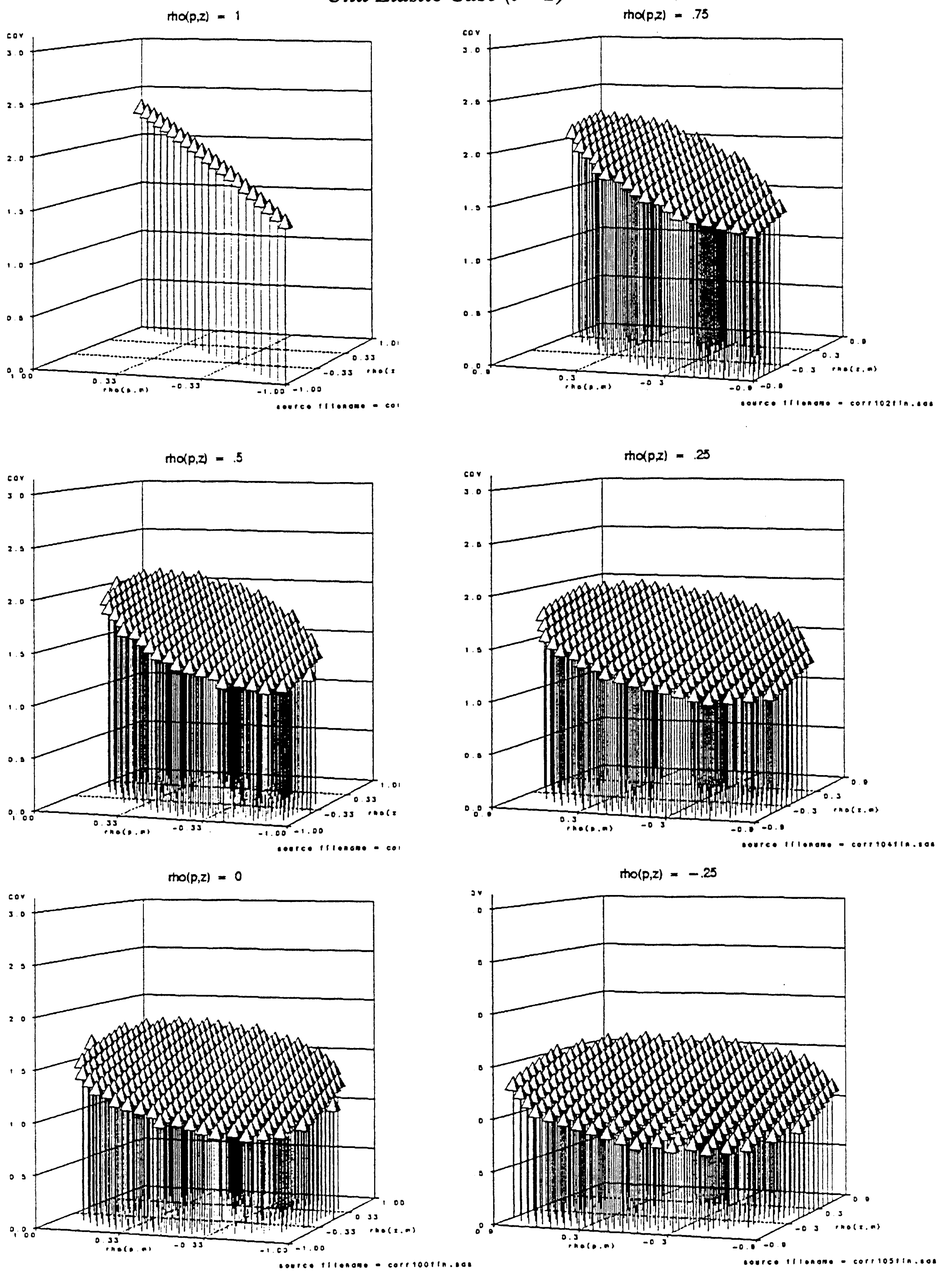


Figure 1:
(a) Set of all possible correlations between X, Y, and Z.
(b) Slicing this set at ryz yields ellipses.

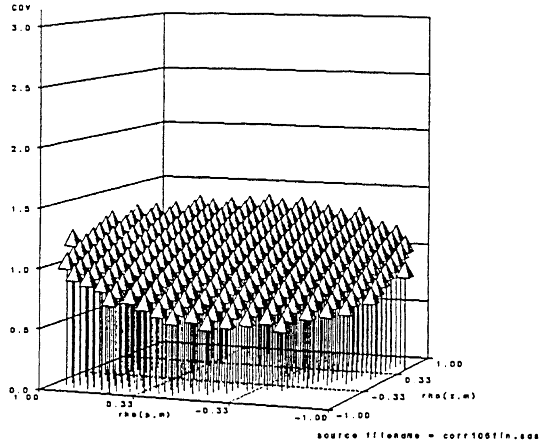
Figure 2

Scatter Plots of Coefficient of Variation for ρ_{pz}

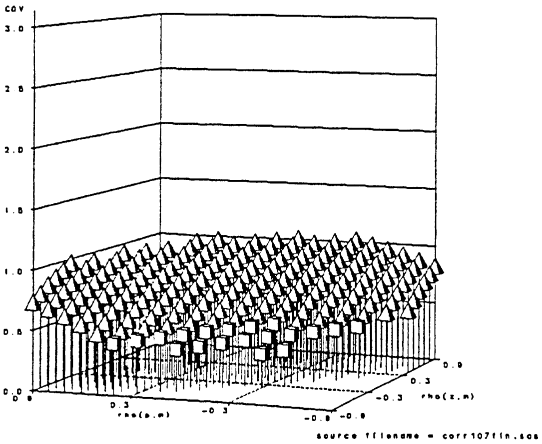
Unit Elastic Case ($t = 2$)



rho(p,z) = −.5



rho(p,z) = −.75



rho(p,z) = −1