[QUA]NTIFYING LONG RUN AGRICULTURAL RISKS AND EVALUATING FARMER RESPONSES TO RISK

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NEW PROCEDURES IN MODELING RISK: NIHIL NOVUM SUB SOL EST

by

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Abstract

NEW PROCEDURES IN MODELING RISK: NIHIL NOVUM SUB SOL EST

An important issue in risk management is how to model random events. This study highlights two alternatives in current literature which allows for the modeling of nonnormal correlated random deviates. First, the paper presents the transformation to normality approach focusing on its uses in pooling yield variability. Second, the study examines the possibility of using a multivariate inverse Gaussian distribution function. The two marginal distributions of corn yields for Holmes county are then used to discuss the empirical implications of each approach.

keywords: nonnormality, correlation, risk, inverse hyperbolic sine, inverse Gaussian

NEW PROCEDURES IN MODELING RISK: NIHIL NOVUM SUB SOL EST

An important research issue in risk management involves how to model random events. Traditionally, economic analysis has focused on mean-variance analysis of random events. Undoubtedly, the use of mean-variance analysis is linked to two considerations. First, the normal distribution is easily quantified and possesses a well defined multivariate form. Second, mean-variance analysis is consistent with expected utility under certain assumptions (Freund, Kroll, Levy and Markowitz, and Meyers). However, certain facets of decision making under risk may be inadequately modeled by normality, such as the probability of bankruptcy or the value of insurance. Therefore, some effort has been expended in agricultural economics in the area of modeling nonnormality.

The purpose of this paper is to discuss some new approaches to modeling random events for risk analysis. However, the primary approach to be discussed brings into focus a portion of this paper's title *Nihil Novum Sub Sol Est* that translates to "nothing is new under the sun." Specifically, this study examines a transformation of random variables approach used by Moss et al. to model correlated nonnormal yields. However, a recent literature review revealed that neither the concept of transforming random variables to normality nor the multivariate context applied by Moss et al. is unique. Specifically, using transformations to model nonnormal bivariate distributions was proposed by Johnson (1949a) who built on previous work by Edgeworth, Charlier and Rietz. In keeping with the venue of this presentation, however, this manuscript will discuss how the rediscovered concept of transformations can significantly aid in modeling nonnormal, correlated random deviates. The second "new" method presented involves the use of nonnormal multivariate distributions.

Specifically, the study will present a slight reformulation of the inverse Gaussian distribution.

Transformation to Normality

The first procedure presented for modeling nonnormality involves using a transformation function to transform nonnormal random variables to normal random variables. The general game plan is to use the transformation to model skewness or kurtosis while relying on the multivariate normal distribution to model correlation in the transformed random variables. The problem of modeling nonnormal random variables is not new to the profession. Several authors have fit univariate densities such as beta or gamma distributions to agricultural data (e.g., Nelson and Preckel). However, modeling nonnormal correlated random deviates has been more problematic.

Historically, Richardson and Condra, King, and Taylor (1990) have proposed procedures for simulating correlated nonnormal variables. Richardson and Condra suggest using the observed errors from the trend line regression in modeling the nonnormality. Specifically, a regression is estimated for each equation, a correlation matrix is computed from the estimated residuals of each equation, and the estimates are used to construct an empirical cumulative probability distribution function. King proposes a similar approach in ARMS (King et al.). ARMS allows the producer to enter yield and price distributions using various options. After entering the marginal distributions, the producer is asked to supply a correlation matrix. Nonnormal correlated random deviates are then simulated by drawing correlated standard normal variables using the user supplied correlation matrix, computing the cumulative probability for these correlated standard normal draws, and then transforming the cumulative probabilities back to the marginal distributions.

One weakness of the King and Richardson and Condra approaches is the separation of the nonnormality components from the correlation components of the distribution. Put another way, there is little interaction between the parameters that control nonnormality and the parameters that control correlation. To further examine this problem, consider the approach used in Taylor (1990). Taylor proposes fitting a sequence of conditional distributions for random variables based on the nonnormal transformation he proposed in Taylor (1984). Specifically, the distribution of the first variable estimated is identical with a marginal distribution. The second distribution is then fit conditional on the first distribution, and so on. Although this approach is fairly flexible, it may be sensitive to ordering. However, the interaction between random variables and their respective nonnormality is considered in the estimation.

Because of the similarity in names, a brief discussion of the difference between Taylor (1984)'s inverse hyperbolic tangent transformation and the inverse hyperbolic sine transformation presented in Moss et al. also the other transformations suggested by Johnson may be instructive. Taylor noted that the hyperbolic tangent resembles a cumulative probability density function with the exception that it approaches -1 as x approaches minus infinity and one as x approaches infinity as shown in Figure 1. Therefore, he defined a transformation

$$(1)\ f(x) = .5 + .5 * \tanh(x).$$

This transformation results in a valid cumulative probability density function in that it is monotonically increasing and bounded between zero and one. The transformed hyperbolic tangent function is compared with the cummulative normal density function in Figure 2. Taylor then proposes to estimate a mapping function (g) that maps yields into x based on an arbitrary function

$$(2)\quad \begin{array}{l} f(x)=.5+.5*\tanh^{-1}(x) \\ x=g(y,z) \end{array}$$

where y is a vector of observed yields and z is a vector of inputs.

Taylor's approach is different than the general approach used by Johnson (1949b). Specifically, Johnson relies on the general result that

$$(3)\ f_y(y)=\left|\frac{dg^{-1}(y)}{dy}\right| f_x(g^{-1}(y))$$

where $f_y(y)$ is the marginal distribution of y, $f_x(x)$ is the marginal distribution of x, and g(x) is a function that transforms x into y. In other words, a random variable can be transformed into another random variable with a well defined distribution. Two common examples are the chi square distribution as the square of the normal distribution and the Box-Cox transformation. However, the chi square distribution is not strictly applicable because g(x) in equation (3) must be a monotonic mapping.

The general approach of Johnson as described by Slifker and Shapiro was to define alternative functions $k(x;\lambda,\epsilon)$ that transform a nonnormal random variable, x, into a standard normal random variable. Specifically, the goal is to create a variable, z, that is distributed standard normal by

$$(4)\ z=\gamma+\eta k_i(x;\lambda,\epsilon)$$

where γ is analogous to the mean of the normal distribution and η is the standard deviation of the transformed variable. As described by Slifker and Shapiro, there are three general forms of this transformation

$$
\begin{aligned}
& k_1(x;\lambda,\epsilon) = \sinh^{-1}\left(\frac{x-\epsilon}{\lambda}\right) \\
(5)\quad & k_2(x;\lambda,\epsilon) = \ln\left(\frac{x-\epsilon}{\lambda+\epsilon-x}\right) \\
& k_3(x;\lambda,\epsilon) = \ln\left(\frac{x-\epsilon}{\lambda}\right)
\end{aligned}
$$

the k_1 distribution is typically referred to as the S_U distribution, the k_2 distribution is referred to as the S_B distribution, and the k_3 distribution is called the S_L distribution. The S_L distribution is generically known as the lognormal distribution. Thus, modeling a nonnormal random variable entails choosing γ, η, λ, and ϵ to maximize the likelihood function defined by equation (3).

The extension of this transformation to the bivariate case is given by Johnson (1949a). Specifically, Johnson proposes a bivariate normal defined as

$$
\begin{aligned}
& z_1 = \gamma_1 + \eta_1 k_i(x_1;\lambda_1,\epsilon_1) \\
& z_2 = \gamma_2 + \eta_2 k_j(x_2;\lambda_2,\epsilon_2) \\
(6)\quad & \\
& f(z_1,z_2) = \frac{1}{\sqrt{2\pi(1-\rho^2)}}\, Exp\left(-\frac{1}{2}\,\frac{z_1^2-2\rho z_1 z_2+z_2^2}{1-\rho^2}\right)
\end{aligned}
$$

where f(.) is the joint probability density function for z_1 and z_2 which determines the distribution of x_1 and x_2. In equation (6), ρ is the correlation coefficient between the two transformed random variables. Using this framework, Johnson defined ten different probability surfaces.

Burbidge et al. reformulated Johnson's univariate inverse hyperbolic sine transformation in a way that yields normality as a special case. Specifically, Burbidge et al.'s inverse hyperbolic sine transformation is

$$(7)\ z=\frac{\ln(\theta x+\sqrt{\theta^2x^2+1})}{\theta}$$

which is equivalent to Johnson's transformation such that

$$(8)\quad \begin{aligned}\gamma&=-\frac{\mu}{\sigma}\\ \eta&=\frac{1}{\sigma\theta}\end{aligned}$$

applying L'Hopital's rule to this transformation as θ approaches zero yields a straight line. Therefore, the normal distribution is a special case of inverse hyperbolic sine (see Ramirez et al. for further details). Unfortunately, Burbidge et al.'s formulation introduces a specific form of heteroscedasticity. Ramirez proposed to circumvent this problem by transforming the deviations from the regression rather than the dependent variable itself. Following this procedure, Moss et al. model nonnormal correlated trends over time using an inverse hyperbolic sine density function

$$f_{v_t} = \frac{1}{\sqrt{2\pi}} |\Omega| Exp\left(-\frac{1}{2}(z_t-\mu)\,\Omega^{-1}(z_t-\mu)'\right) \prod_{i=1}^{M} (1+v_{it}^2\theta_i^2)^{-\frac{1}{2}}$$

(9)
$$z_t = \begin{bmatrix} \ln\left(v_{1t}\theta_1 + \sqrt{1+\theta_1^2 v_{1t}^2}\right)/\theta_1 \\ \ln\left(v_{2t}\theta_2 + \sqrt{1+\theta_2^2 v_{2t}^2}\right)/\theta_2 \\ \vdots \end{bmatrix}$$

$$v_t = y_t - (\alpha + \beta t)$$

where θ_i controls the kurtosis on variable i, μ_i controls the skewness in conjunction with θ_i, Ω is the correlation matrix for the transformed random variables, z_t is a vector of transformed (normally distributed) random variables, v_t is the vector of deviations from the trend line, and α and β are parameters used to specify the trend line. Moss et al. show how this formulation can be used to represent correlated nonnormal corn, soybean, and wheat yields over time in the southeastern United States.

From an operational perspective the inverse hyperbolic sine formulation presented in equation (9) can be estimated by maximizing the natural log of the likelihood function. Moss et al. suggest using ordinary least squares estimates as initial values in an iterative maximum likelihood procedure. The formulation allows for examination of other stylized facts about risk. Specifically, we typically hypothesize that yields become more normal as they are aggregated over regions. Another way to pose this question is by looking at the pooling properties of yields within the inverse hyperbolic sine framework.

Corn yield data for Holmes, Okaloosa, and Walton counties were collected for 1961 to 1989 (1986 data were missing, these data are presented in the appendix. Following Moss et al., corn yields where detrended using ordinary least squares and a linear trend. The residuals where then tested for normality using the parametric procedures described in Spanos. These results (Table 1) indicate that corn yields in Holmes and Okaloosa counties may be nonnormal while corn-yields in Walton county cannot be distinguished from normality. Next, the multivariate distribution depicted in equation (9) was estimated allowing corn yields in Holmes and Okaloosa counties to be nonnormal while restricting yields in Walton county to be normal. These results are shown in Table 2.

The results in Table 2 indicate that the estimate of θ and μ in the Holmes equation have fairly large standard deviations. Given this, one approach is to see whether the θ and μ parameters in the Holmes and Okaloosa equations can be pooled. Pooling would allow for additional information to be focused on the nonnormality parameters. As a first step, the two nonnormality parameters where restricted to be the same. This estimation resulted in a log likelihood of 216.04 compared with a log likelihood of 215.49 in the unrestricted case. This implies a likelihood ratio statistic of 1.10 which is distributed $\chi_{(2)}^2$. At this stage, we noticed that the intercept on the time trend and the own variance were also close. These results where not obvious from Table 2 since both the mean and variance are functions of θ.[2] Therefore, a second set of restrictions where imposed to restrict the intercept and variance to be equal across equations. These results are reported in Table 3. In general, the log likelihood function for estimation after imposing all four restrictions was 216.33 yielding a likelihood ratio test of 1.68 which is distributed $\chi_{(4)}^2$. Thus, the pooling of the four parameters cannot be rejected at any conventional level of significance.

In addition to comparing the likelihood of the unrestricted parameters to the restricted parameters, another interesting comparison is to examine how the pooled parameters compare with average state yields. Specifically, the test for skewness and kurtosis indicate that the hypothesis that the deviations of state average corn yields from the trend are normal cannot be rejected with any degree of confidence. In addition, a region average was created by weighting the per acre yields in Holmes, Okaloosa, and Walton county by number of acres in each county and the residuals were tested for normality. Again the hypothesis could not be rejected. Hence, there is some preliminary evidence to suggest that aggregation eliminates nonnormality from the sample.

In addition to changes in the nonnormality parameters, the pooling of county level data also lets us examine changes in the variance of corn yields. The results indicate that the state level yields are normally distributed with a variance of 62.35 bushels squared. The data aggregating county yields by the number of acres indicates a slight increase in variance to 66.90 bushels squared. Table 2 indicates that the Walton county's variance on corn yields was 65.55 bushels squared before pooling. The variance declines to 64.95 bushels squared after pooling as depicted in Table 3. The variance for Holmes and Okaloosa counties decline from 136.31 and 168.48 respectively before pooling to 155.96 after pooling. These variances are computed using the variance derivation under the inverse hyperbolic sine transformation from Ramirez.

In general Johnson's formulations are useful and well behaved. However, they are not exhaustive. Specifically, the requirements for an inverse mapping as presented in equation (1) are a one to one mapping so that the inverse function is well defined. As an area for future research, we suggest a flexible third order Taylor series expansion. For example, in the simplest form a third order polynomial could be fit as a transformation function with the restriction that the first derivative is always positive. Another alternative would be to estimate a cubic root. The implications of these transformations could be defined in terms of skewness and kurtosis and may yield more flexible results than the three transformations suggested by Johnson.

Other Multivariate Distributions

Another approach for modeling agricultural risk is to use other distributions that have well defined multivariate forms. This study presents one such alternative called the inverse Gaussian distribution as described by Chhikara and Folks. Like the Burbidge et al. formulation of the inverse hyperbolic sine the inverse Gaussian distribution must be reformulated to avoid very specific heteroscedasticity implications. Similar reformulations will be required of many multivariate distributions.

The univariate inverse Gaussian probability density function can be expressed as

$$(10)\ f(x;\mu,\lambda)=\sqrt{\frac{\lambda}{2\pi}}\,x^{-\frac{3}{2}}Exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right),\ x>0$$

where x is a random variable, and λ and μ are parameters of the distribution. Unlike the inverse hyperbolic sine as reformulated by Burbidge et al., the inverse Gaussian distribution has no parameterization for the normal distribution. The μ parameter is the expected value of the distribution. In addition, the second, third, and fourth central moments of the distribution are given by Chhikara and Folks as

$$\mu_2 = \frac{\mu^3}{\lambda}$$

$$(11)\ \mu_3 = 3\frac{\mu^5}{\lambda^2}$$

$$\mu_4 = 15\frac{\mu^7}{\lambda^3} + 3\frac{\mu^6}{\lambda^2}.$$

Even though the distribution function does not have the normal distribution function as a special case, if λ/μ becomes large the inverse Gaussian distribution approaches a normal distribution function.

The next step was to model corn yields in north Florida as a function of time. This required reformulation of the inverse Gaussian function distribution function. Specifically, Chhikara and Folks present an unbiased estimator for α and β, which in this study are the constant and slope of the linear trend respectively, in which μ varies over time. Relying on the representation of the central moments presented in equation (11), this would imply that the higher moments are also a function of time. As an alternative, this study proposes a transformation such that the "residuals" from the regression are distributed inverse Gaussian. Specifically, the probability density function becomes

$$(12)\ f(y;\mu,\lambda,\alpha,\beta,t) = \sqrt{\frac{\lambda}{2\pi}}[y-(\alpha+\beta t)]^{-\frac{3}{2}} Exp\left(-\frac{\lambda([y-(\alpha+\beta t)]-\mu)^2}{2\mu^2[y-(\alpha+\beta t)]}\right).$$

Initial values are computed estimated by assuming that α and β are both zero. Table 4 presents the maximum likelihood estimates of α, β, λ, and μ for Holmes and Okaloosa counties. We attempted to estimate corn yields in Walton county using the modified inverse Gaussian, but the distribution was close enough to normality to cause difficulties.

Figure 3 depicts the estimated distribution given the inverse hyperbolic sine distribution and the inverse gaussian distribution for Holmes county. Both distributions are positively skewed. However, the inverse hyperbolic sine distribution has a positive probability for yields below 23 bushels per acre. The inverse Gaussian distribution, on the other hand, has a zero probability for yields less than 23 bushels per acre. This difference along with inability of the inverse Gaussian to model negative skewness will probably limit its applicability to agriculture. However, comparing the difference in probability between the inverse Gaussian and the inverse hyperbolic sine in Figure 3 shows that the actual probability mass between 23 bushels per acre and 0 bushels per acre is quite small. Thus, this problem is the least damning.

Finally, the multivariate form of the inverse Gaussian distribution first presented by Wasan is

$$\begin{aligned} f(y_1, y_2, \ldots y_p) &= (2\pi)^{-\frac{p}{2}} \left[\frac{\lambda_1 \lambda_2 \cdots \lambda_3}{y_1^3 (y_2 - y_1)^3 \cdots (y_p - y_{p-1})^3} \right]^{\frac{1}{2}} \\ &\cdot Exp\left[-\frac{\lambda_1 (y_1 - \mu_1)^2}{2\mu_1^2 y_1} - \frac{\lambda_2 (y_2 - y_1 - \mu_2)^2}{2\mu_2^2 y_2} \cdots - \frac{\lambda_p (y_p - y_{p-1} - \mu_p)^2}{2\mu_p^2 y_p} \right] \end{aligned} \tag{13}$$

Our suggestion is to transform this distribution function such that y is the "residual" from the time trend as presented in equation (12) in the univariate case.

Summary, Conclusions and New Directions

This paper presented two approaches to modeling random events that are new to the agricultural economics literature, although they are not new to the statistics literature. First, we presented an overview of the transformation of random variables approach presented by Moss et al. This approach allows the researcher to jointly model trends, nonnormality and correlation. Historically, Johnson proposed three transformations to normality one of which, the lognormal, is quite familiar to agricultural economists. The two other distributions, the inverse hyperbolic sine distribution and the S_U distribution have not been widely utilized by agricultural economist. Moss et al. show that the inverse hyperbolic sine has the desirable property that the normal is a special case. A similar special case may exist for the S_U distribution, but is not likely for the lognormal distribution. However, legitimate transformations are not limited to the three proposed by Johnson. Any one-to-one mapping could be used to transform random variables to normality.

To provide evidence of the potential usefulness of this transformation function approach, the study demonstrated how the inverse hyperbolic sine could be used to pool data across counties. Specifically, by restricting the nonnormality parameters for different counties to be the same it is possible to focus more information on those characteristics controlled by those parameters. Hence, it is possible to increase the precision of the estimate. In addition, our results indicate that aggregation may eliminate nonnormality from the sample.

Finally, this study examined the use of other multivariate distribution functions in modeling nonnormal correlated random deviates. Specifically, we presented a reformulation of the inverse Gaussian distribution similar to Ramirez. The reformulation transforms the distribution to a homoscedastic form as opposed to a very specific form of heteroscedasticity imposed by the original formulation. Lastly, the multivariate formulation of the inverse Gaussian is presented in an untransformed formulation.

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Endnotes

1. Originally, Burbidge et al. proposed to transform the dependent variable of the regression so that y_t was distributed inverse hyperbolic sine the mean of the transformed deviation being $x_t\beta$. Ramirez points out that this transformation implies a variance of

$$V(y_t) = (Exp(2\theta^2\sigma^2) - Exp(\theta^2\sigma^2)) \cdot (Exp(2\theta x_t\beta) + Exp(-2\theta x_t\beta)) + 2(Exp(\theta^2\sigma^2) - 1)/[4\theta^2]$$

Notice that the variance of the distribution depends on $x_t\beta$ which is by definition heteroscedastic. Under Ramirez' reformulation, only the error from the regression is distributed inverse hyperbolic sine. A constant μ becomes the mean of the transformed residuals so that homoscedasticity is imposed.

2. Given that the residuals from the trend are distributed inverse hyperbolic sine, the expected residual given θ and μ is

$$E[\epsilon_t] = \frac{\left[Exp\left(-\frac{1}{2}\theta^2\sigma^2\right)\left(Exp(\theta\mu) - Exp(-\theta\mu)\right)\right]}{2\theta}.$$

Table 1: Tests for Normality in Corn Yields in Selected Counties in North Florida.

County	Parametric Tests for Normality[a] Skewness	Kurtosis	Joint Test for Skewness and Kurtosis	Likelihood Ratio Test of IHS Transformation
Holmes	3.91	7.15	9.70	5.81
	(.94)[b]	(.93)	(.01)	(.05)
Okaloosa	5.78	2.93	8.50	5.21
	(.99)	(.72)	(.01)	(.07)
Walton	-3.78	0.07	2.45	0.08
	(.06)	(.51)	(.29)	(.96)

[a]The parametric tests given by Spanos are derived by Bera and Jaque. These tests compare the observed distribution against a general Pearson distribution. The skewness and kurtosis statistics are normal, or two tailed tests, while the joint test for skewness and kurtosis is a one tailed test.
[b]Numbers in parenthesis indicate confidence levels

Table 2: Multivariate Representation of Corn Yields in North Florida Using the Inverse Hyperbolic Sine Transformation.

	Nonnormality Parameters		Time Trend Parameters		Transformed Covariance Parameters		
County	θ	μ	α	β	ω_{Holmes}	$\omega_{Okaloosa}$	ω_{Walton}
Holmes	3.24	1.44	23.95	-0.02	0.03	0.87	0.53
	(2.81)[a]	(0.97)	(1.13)	(0.10)	(0.06)	(0.86)	(0.53)
Okaloosa	-0.09	11.18	27.42	0.67		43.71	35.97
	(0.02)	(0.97)	(1.00)	(0.12)		(1.07)	(1.05)
Walton	-	-	37.56	0.67			65.55
			(0.98)	(0.10)			(1.07)

[a]Numbers in parenthesis denote asymptotic standard errors.

Table 3: Restricted Multivariate Representation of Corn Yields in North Florida Using the Inverse Hyperbolic Sine Transformation.

County	Nonnormality Parameters		Time Trend Parameters		Transformed Covariance Parameters		
	θ	μ	α	β	ω_{Holmes}	$\omega_{Okaloosa}$	ω_{Walton}
Holmes	-0.10	11.05	26.89	0.04	34.68	14.97	30.05
	(0.04)[a]	(5.42)	(8.09)	(0.15)	(22.45)	(11.09)	(13.49)
Okaloosa	-0.10	11.05	26.89	0.67		34.68	31.24
	(0.04)	(5.42)	(8.09)	(0.16)		(22.45)	(13.23)
Walton	-	-	37.40	0.67			64.95
			(2.82)	(0.16)			(18.19)

[a]Numbers in parenthesis denote asymptotic standard errors.

Table 4: Estimated Distribution of Corn Yields Using the Modified Inverse Gaussian Distribution Function.

County	Distribution Parameters		Time Trend Parameters	
	λ	μ	α	β
Holmes	51.58	19.28	23.46	0.01
	(51.06)[a]	(4.97)	(5.07)	(0.11)
Okaloosa	456.54	39.88	1.15	0.81
	(621.55)	(16.47)	(17.29)	(0.22)

[a]Numbers in parenthesis denote asymptotic standard errors.

Figure 1: Hyperbolic Tangent Function

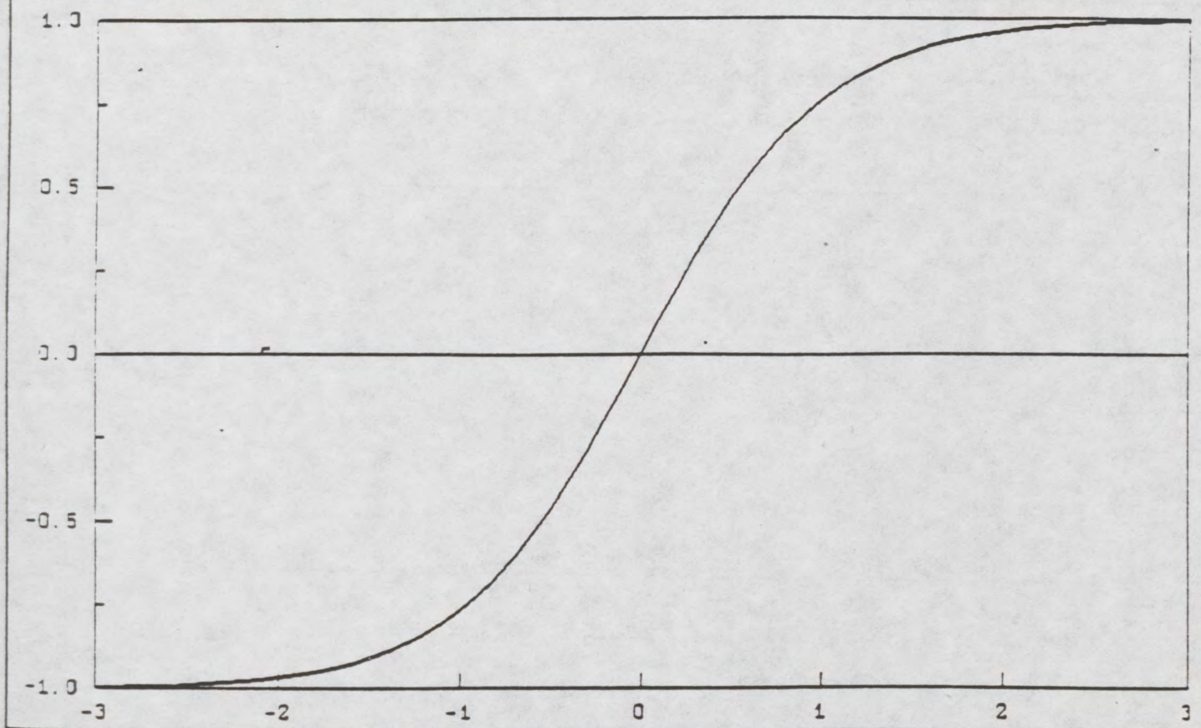


Figure 2: Transformed Hyperbolic Tangent Versus the Cummulative Normal Density Function

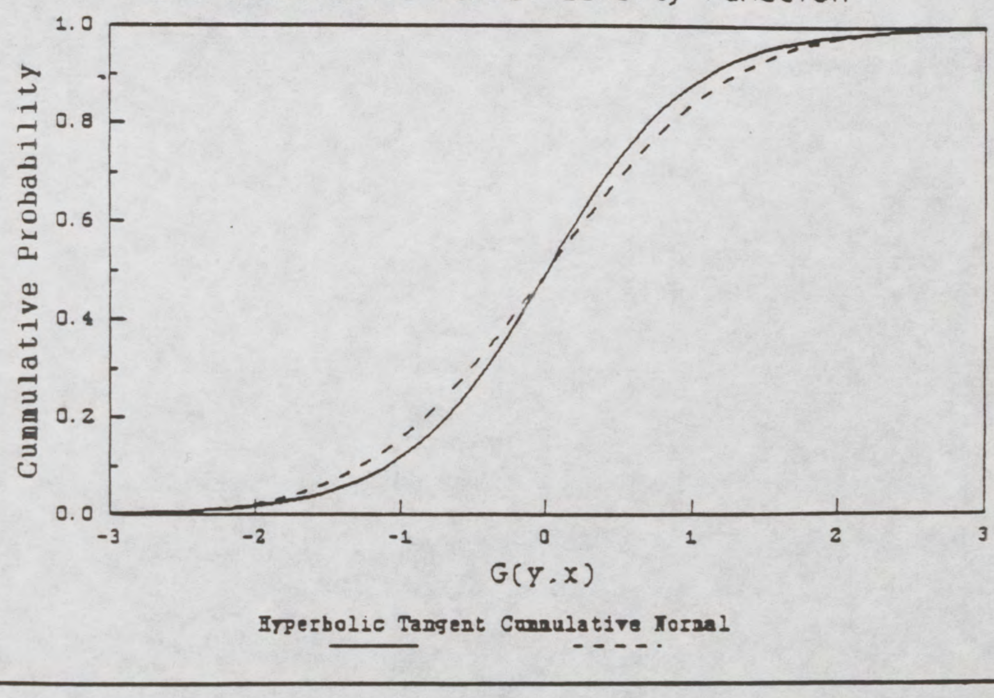
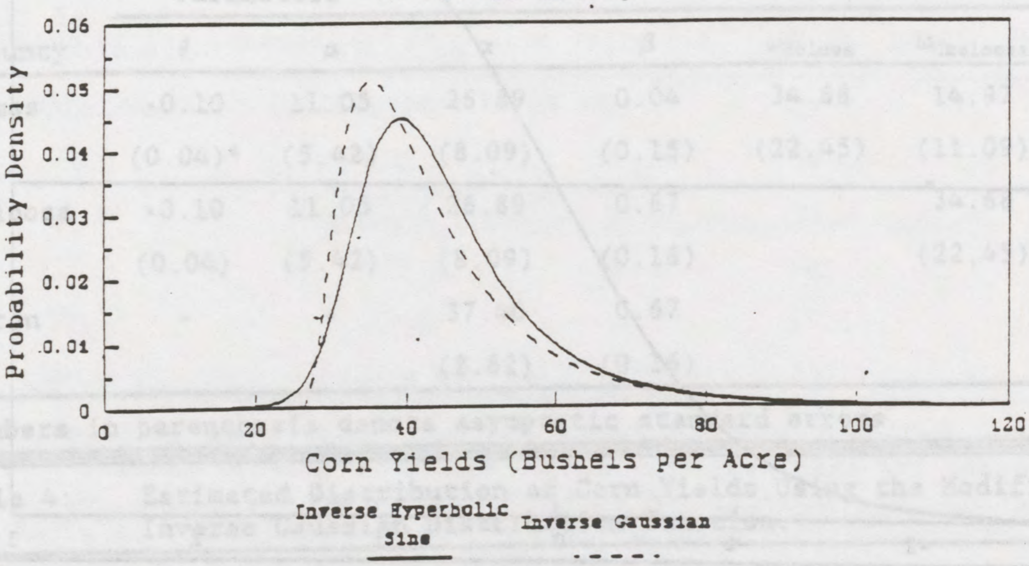


Figure 3: Comparison Between Inverse Gaussian and the Inverse Hyperbolic Sine for Corn Yields in Holmes County



Appendix: Corn Yields for North Florida

YEARS	HOLMES	OKALOOSA	WALTON	STATE	WEIGHTED YIELD
61	34	34	35	35	34.27
62	36	35	36	37	35.81
63	39	38	38	40	38.58
64	29	35	33	29	30.83
65	45	49	41	44	44.59
66	44	48	40	43	43.60
67	46	54	52	50	48.66
68	36	45	41	57	39.25
69	32	37	37	39	34.90
70	33	33	27	25	29.45
71	42	75	54	49	52.82
72	50	50	56	46	52.90
73	45	50	48	43	46.98
74	40	57	51	48	47.22
75	36	55	40	45	41.48
76	50	90	55	60	63.75
77	32	34	36	35	33.76
78	41	56	40	52	46.40
79	50	60	46	53	52.49
80	35	59	55	47	47.66
81	55	80	60	55	64.17
82	55	55	60	66	55.95
83	55	50	60	67	55.00
84	44	60	50	65	47.67
86	30	70	65	62	51.30
87	50	65	65	69	59.22
88	30	50	40	58	40.00
89	85	55	65	74	68.57