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## **Per Unit and Ad Valorem Royalties in a Patent Licensing Game**

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### Per Unit and Ad Valorem Royalties in a Patent Licensing Game

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#### Summary

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**Keywords:** Licensing, Product Innovation, Bertrand, Cournot, Network Effects

**JEL Classification:** L13, L20, D43

*We wish to thank Noriaki Matsushima, Cong Pan and Yoshihiro Tomaru for their insights and suggestions, as well as the other seminar and workshop participants at Osaka University and the Nagoya University of Commerce & Business. We also thank seminar participants at the University of Pisa and the audience at Earie 2019 for useful comments. All errors are our own.*

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# Per unit and ad valorem royalties in a patent licensing game

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## ABSTRACT:

In a context of product innovation, we study two-part tariff licensing between a patentee and a potential rival which compete in a differentiated product market characterized by network externalities. The latter are shown to crucially affect the relative profitability of Cournot vs. Bertrand when a per unit royalty is applied. By contrast, we find that Cournot yields higher profits than Bertrand under ad valorem royalties, regardless of the strength of network effects.

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# 1 Introduction

Empirical evidence suggests that firms often license to direct competitors their patented innovation (Jiang and Shi, 2018). Investments in either new technologies or in new product development allow firms to advance in economic performance, gaining a competitive advantage *via* innovation. The literature on patent licensing considers contracts that assume different forms such as either a fixed-fee or a per-unit/ad valorem royalty, as well as a two-part tariff including both a fixed fee and a royalty component, generally focusing on the optimality of a license scheme over the other.<sup>1</sup>

The present paper investigates, in a framework of product innovation, the optimal patent licensing by an incumbent when consumers' preferences exhibit network effects. Such consumption externalities, which are typical of markets such as telecommunications, on-line games, digital music/movies, payment systems, software and e-commerce platforms, imply that the value of a good to a consumer increases as the number of its users grows. Network effects are argued to lie behind the success of the most dynamic and impactful companies in the world such as Microsoft, PayPal, Facebook, Uber, Twitter and Salesforce. Most recent Industrial Organization literature points out the key role of network effects in affecting *via* expectations firms' equilibrium network size and the adoption of innovations, thus achieving a critical mass (David, 1985; Farrell and Saloner, 1985; Arthur, 1989; Choi, 1994; Economides, 1996a; Cabral et al., 1999).<sup>2</sup> The intensity of network effects has been also shown to impact product pricing and the strength of firms' market power (Cabral, 2011; Katz, Shapiro, 1985 and 1986), the strategic choices of product characteristics (Lambertini and Orsini, 2001; Baake and Boom, 2001; Gabszewicz and Garcia, 2007) and the determinants of market structure through firm entry (Economides, 1996b) and vertical integration (Dogan, 2009).

Licensing of new products, brands and services has become a crucial revenue source in network industries. Recent evidence suggests that licensing is a powerful value driver for Nokia, with brand and technology licensing net sales of 1.6 billion Euros in 2017 (Nokia Corporation Financial Report, 2018), and a revenue generator for Microsoft, Ericsson, IBM, Qualcomm and Texas instruments (Ludlow, 2014). Also, earnings of on-line games' developers have massively increased over the last years (State of the Developer Nation, 2018). In a lot of cases licensing occurs between firms that are direct competitors. See Microsoft that licensed mobile operating system features to Samsung and HTC (Hoffman, 2014) or Apple that obtained from Microsoft an eight-year license for Applesoft Basic that is a dialect of Microsoft Basic, adapted to the Apple II services of personal computers. Moreover General Motors (GM) licensed its OnStar ser-

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<sup>1</sup>Literature shows that the optimality of licensing schemes depends on whether the patentee is external to the market (Kamien and Tauman, 1986; Muto, 1993; Erutku, and Richelle, 2007) or rather is a producer within the market (Wang, 1998), on product differentiation (Kabiraj and Lee, 2011; Bagchi and Mukherjee, 2014), on whether firms compete with respect to quantities or price (Muto, 1993; Bagchi and Mukherjee, 2014).

<sup>2</sup>See Gandal (2008) for empirical studies emphasizing the role of network effects in boosting firm success.

vice, a satellite-based mapping service, to other automobile manufacturers as Toyota and Honda.

Despite network effects have received wide attention in recent years both in practice and academic research, the analysis of their effects on licensing behavior in oligopolistic markets has been limited to very few studies dealing with the optimality of licensing strategies in a quantity competition framework. Wang et al. (2012) introduce network externalities in a Cournot model of process innovation, showing how they may let the patentee exploit the advantages of a larger market size achieved by favoring the competitor's production through a fixed fee, rather than charge a royalty restricting the licensees' output. The same mechanism is at work in the product innovation model of Lin and Kulatilaka (2006) who demonstrate that a pure fixed-fee license dominates a two-part tariff when the network intensity is high enough. By contrast, Zhao et al (2014) find that fixed-fee licensing never dominates royalty licensing or two-part tariff licensing when network effects interact with quality differences in a vertical product innovation model.

In the present paper we aim at investigating how the presence of network effects affects the optimal behavior of an incumbent innovator that licenses a new product technology to a potential market rival through a two-part tariff.<sup>3</sup> Market competition can occur under Cournot or under Bertrand, while either a per-unit and ad valorem royalty is included in the two-part licensing scheme.<sup>4</sup> In particular, we focus on how the strength of network externalities affects the relative profitability of Cournot vs. Bertrand, for each considered contract. The comparison on profitability between Cournot and Bertrand competition is an extensively debated issue in oligopoly theory. Following Singh and Vives (1984), much literature has found a dominance of Cournot over Bertrand with substitutes (Tanaka, 2001a; Tanaka, 2001b; Tasnádi, 2006, among others). This result, however, has been reversed in several circumstances: in mixed duopolies due to the presence of social welfare maximizing firms (Ghosh and Mitra, 2010; Matsumura and Ogawa, 2012), in vertically related industries (Correa-López and Naylor, 2004; Arya et al. 2008; Mukherjee et al., 2012; Alipranti et al., 2014), under cost and demand asymmetries (Zanchettin, 2006) and substantial quality differences (Häckner, 2000). Recently, it has been raised the question of whether the Singh and Vives (1984)'s result is robust to the presence of network effects. By dealing with quantity and price competition under network effects

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<sup>3</sup>Optimal two-part licensing contracts have been studied, among the others, by Kamien and Tauman (1984), Erutku and Richelle (2007), Faulí-Oller and Sandonís (2002), Sen and Tauman (2007), Sen and Stamatopoulos (2009) and Filippini and Vergari (2017).

<sup>4</sup>Including a per unit or an ad valorem royalty in a licensing agreement is empirically observed (Macho-Stadler et al., 1996; Bousquet et al., 1998; Lim and Veugelers, 2003; Trombini and Comacchio, 2012) and theoretically justified (San Martín and Saracho, 2010, 2015, 2016; Heywood et al., 2014; Colombo and Filippini, 2015 and 2016, Fan et al., 2018). Whether a per-unit or an ad valorem royalty must be included in a licence is found to depend on the mode of competition (Colombo and Filippini, 2015), on cost convexity (Colombo and Filippini, 2016), on demand or cost uncertainty (Bousquet et al., 1998), on product differentiation and the licensee' development cost for the new product (San Martín and Saracho, 2016), on the relative efficiency of the licensee compared to the licensor (Fan et al., 2018), on asymmetric information about the value of the patent (Heywood et al., 2014).

in a symmetric duopoly, Pal (2014) has shown that the the positive effect of on one firm’s profits through consumers’ expectations on a larger market size dominates the negative effect of more aggressive behavior as long as network effects are sufficiently intense, which provides the reversal result that Bertrand profits dominate Cournot profits.<sup>5</sup>

The debate on Cournot vs. Bertrand profitability has been recently revisited by Chang et al (2017) in a patent licensing game. In this study, a product innovator charges a fixed fee plus a per unit royalty to a potential rival. They find that the equilibrium royalty rate under Bertrand is always higher than that under Cournot and causes higher profitability in the former than in the latter, regardless of the degree of product differentiation. The optimal royalty rate, indeed, solves in Cournot the licensor’s trade-off between raising the rival’s cost, thus gaining from the direct profit channel by expanding her own output, and reducing the rival’s cost-disadvantage, thus benefiting from sufficiently large licensing revenues. In Bertrand, however, it works as a commitment to let both firms set higher market prices, thus benefiting from more relaxed market competition. By assuming a per unit royalty, our study extends Chang et al (2017) to the presence of network externalities, showing that profits may be either greater or lower under Cournot competition than under Bertrand competition, depending on the interplay between the intensity of network effects and the degree of product substitutability. In particular, increasing network effects, by raising end-users’ utility and shifting market demand through expectations, raise market profitability on both the direct sales and the licensing channel. This reduces the equilibrium royalty rate in both Cournot and Bertrand in favor of a larger fixed fee, which pushes towards higher overall profitability in Cournot vs. Bertrand as long as product substitutability is low enough. In such circumstances, indeed, the positive effect of expectations on a larger market size on Cournot profits becomes high relative to that exerted on Bertrand profits, the latter being more limited due to the greater royalty-induced downward pressure on the equilibrium prices. Moreover, we show that the type of royalty payment matters in defining the relative profitability conditions of Bertrand vs. Cournot. Indeed, the presence of network externalities is shown not to cause any reversal of market profitability when an ad valorem royalty is included in the two-part contract, which results in Cournot being more profitable than Bertrand, regardless of the strength of network effects and the degree of product substitutability. Such a result relies on the fact that, by reducing the equilibrium royalty rate, network externalities always enhance the Cournot profits to a higher extent than Bertrand profits. This is due to the positive effect of a lower royalty rate on the patentee’s output in Cournot, which lets profits increase remarkably through consumers’ expectations on a larger market size, and to the downward pressure a lower royalty rate exerts on firms’ prices in Bertrand, which refrains firms from fully exploiting higher consumers’ willingness to pay.<sup>6</sup>

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<sup>5</sup>A profit dominance of Bertrand is also found by Pal (2015) in a managerial delegation context with negative network externalities.

<sup>6</sup>As it will be discussed later in the paper, including an ad valorem royalty in the contract does not affect directly the licensee’s behavior. This leads the patentee to strategically commit

Our results contribute to the existing literature in several ways. First, they point out the role of network externalities in a licensing framework with price competition, which has been never investigated in previous literature. Second, the paper captures the implications of network effects on product innovation licensing, showing how the result of Chang et al. (2017) that Bertrand is always more profitable than Cournot under a per unit royalty does not hold under network effects when product substitutability is sufficiently low. Third, it shows that the conclusion achieved by Pal (2014) that the strength of network effects affects the relative profitability of Bertrand vs. Cournot only applies when a per unit royalty is included in a two-part licensing contract, while it does not apply under an ad valorem royalty.

The remainder of the paper is organized as follows. Section 2 develops the model, while Section 3 draws some conclusions.

## 2 The model

Firm 1 is an incumbent producer facing the following linear demand function:

$$p_1 = a + ny_1 - q_1$$

with  $a > 0$ , where  $q_1$  and  $p_1$  denote firm 1's output and price, respectively, and  $y_1$  is the consumers' expectation on firm 1's market size. Moreover,  $n$  measures the strength of the network effect. Throughout the paper, we assume  $n(\gamma) \in [0, \hat{n}(\gamma)]$  (with  $\hat{n}(\gamma) = \frac{\gamma}{1+\gamma}$ )  $\forall \gamma \in (0, 1)$ .<sup>7</sup> For the sake of simplicity, we assume that both the variable cost and the fixed costs of production are zero. Firm 1 has to decide on whether to license or not its product innovation to a potential market rival, namely firm 2. Technology licensing by firm 1 allows firm 2 to produce a differentiated network good and to compete against firm 1 either *à la* Cournot or *à la* Bertrand. Following Hoernig (2012), the inverse demand functions are:<sup>8</sup>

$$p_1 = a + n(y_1 + \gamma y_2) - q_1 - \gamma q_2 \quad (1)$$

$$p_2 = a + n(y_2 + \gamma y_1) - q_2 - \gamma q_1 \quad (2)$$

$q_2$  and  $p_2$  being respectively firm 2's output and price, whereas  $y_2$  is consumers' expectation on firm 2's sales. The parameter  $\gamma$  in the range  $(0, 1)$  measures

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to behave less aggressively following an increase of the royalty rate, thus enhancing its licensing revenues through a licensee's output expansion in Cournot and a price increase in Bertrand. A decrease of the royalty rate caused by increasing network effects, therefore, lets the patentee expand its output in Cournot and induce a price reduction in Bertrand.

<sup>7</sup>Indeed, we assume that the strength of network effects is not too high to avoid that the extreme case of a negative royalty rate chosen by the patentee to exploit higher profitability on the rival's sale channel.

<sup>8</sup>As in Chang et al. (2017) and Kitagawa et al., (2014), in our model the licensee is assumed to produce an additional differentiated product variety on the ground that the patentee is not able to produce the same variety prior to licensing because of prohibitive marketing and development costs.



the degree of substitutability between the two varieties ( $\gamma = 0$  implies that the products are unrelated, whereas  $\gamma$  approaching unity implies almost perfect product substitutability).

The direct demand functions can be written as:

$$q_1 = \frac{a(1-\gamma) + ny_1(1-\gamma^2) - p_1 + \gamma p_2}{(1-\gamma^2)} \quad (3)$$

$$q_2 = \frac{a(1-\gamma) + ny_2(1-\gamma^2) - p_2 + \gamma p_1}{(1-\gamma^2)} \quad (4)$$

The game timing is as follows. In the first stage, firm 1 chooses to license its new product technology or not through the payment of a two-part tariff, i.e. a lump sum payment plus either a per unit or an ad valorem royalty. In the second stage, if any, firm 2 accepts the contract offered by the rival and the two firms engage in either Cournot or Bertrand market competition. In Section 2.1 we derive the solution in a scenario with no licensing. The latter is then compared with the market outcome derived under per unit (ad valorem) royalty licensing in Section 2.2 (Section 2.3) where we search for the subgame perfect Nash equilibrium (SPNE) in both a Cournot and a Bertrand framework.

## 2.1 The no licensing framework

Firm 1 maximizes with respect to  $q_1$  the following profit function:

$$\pi_1 = (a + ny_1 - q_1) q_1$$

which yields firm 1's optimal quantity as a function of consumers' expectations on the market size  $y_1$ :

$$q_1 = \frac{a + ny_1}{2}$$

Following Katz and Shapiro (1985) and Hoernig (2012), we apply the fulfilled expectation condition implying that the equilibrium sales equal the expected market size, i.e.,  $y_1 = q_1$ , thus obtaining the following output:

$$q_1^{NL} = \frac{a}{2-n}$$

The equilibrium price is:

$$p_1^{NL} = \frac{a}{2-n}$$

Therefore, we obtain the following firm 1's profits:

$$\pi_1^{NL} = \frac{a^2}{(2-n)^2} \quad (5)$$

## 2.2 Two-part tariff licensing with a per unit royalty

We assume that firm 1 can license its innovation by imposing the payment of a royalty  $r \geq 0$  for each unit sold by firm 2 and a fixed amount  $F \geq 0$ . Therefore, firms' profits are:

$$\pi_1 = p_1 q_1 + r q_2 + F \quad (6)$$

$$\pi_2 = (p_2 - r) q_2 - F \quad (7)$$

### 2.2.1 Cournot competition

By using the inverse demand functions (1) and (2) and maximizing profits (9) and (10) with respect to  $q_1$  and  $q_2$ , we obtain the following reaction functions:

$$q_1 = \frac{a + n(y_1 + \gamma y_2) - \gamma q_2}{2}$$

$$q_2 = \frac{a + n(\gamma y_1 + y_2) - \gamma q_1 - r}{2}$$

Notice that each reaction function shifts outward as consumers' expectations increase, which denotes that one firm's output is positively affected by expectations on both its own sales and the rival's sales.

The solution of the system of the reaction functions, under the fulfilled expectations' conditions  $y_1 = q_1$  and  $y_2 = q_2$ , gives the optimal quantities:

$$q_1 = \frac{a(2 - (n + \gamma(1 - n))) + r\gamma(1 - n)}{4 - (n(4 - n) + \gamma^2(1 - n)^2)}$$

$$q_2 = \frac{a(2 - (n + \gamma(1 - n))) - r(2 - n)}{4 - (n(4 - n) + \gamma^2(1 - n)^2)}$$

It is immediate to verify that  $\frac{\partial q_1}{\partial r} > 0$  and  $\frac{\partial q_2}{\partial r} < 0$ , which implies that a marginal increase of  $r$  leads the patentee (licensee) firm to expand (reduce) its output.

Given the above quantities, firm 2's profits in (7) can be written as follows:

$$\pi_2(r, F) = \frac{(a(2 - (n + \gamma(1 - n))) - r(2 - n))^2}{(4 - (n(4 - n) + \gamma^2(1 - n)^2))^2} - F$$

At the royalty setting stage, maximization of firm 1's profits in (6) computed at the optimal quantities leads, under the condition  $\pi_2(r, F) \geq 0$ , to the equilibrium per unit royalty rate:

$$r^C = \frac{a(\gamma(1 - n) - n)(2 - (\gamma + n(1 - \gamma)))^2}{2(1 - n) \left( (2 - n)^2 - \gamma^2(1 - n)(3 - n) \right)}$$

Therefore, the equilibrium fixed fee solving  $\pi_2(r^C, F) = 0$  is:

$$F^{CU} = \frac{a^2 \left( (2-n)^2 - \gamma(1-n)(4-n) \right)^2}{4 \left( (1-n) \left( (2-n)^2 - \gamma^2(1-n)(3-n) \right) \right)^2}$$

Notice that both  $r^C$  and  $F^{CU}$  are positive in the assumed range of network effects (i.e.,  $n(\gamma) \in [0, \hat{n}(\gamma))$ ).

The firms' quantities and prices at the SPNE are:

$$\begin{aligned} q_1^{CU} &= \frac{a((2-\gamma)(2-n) - \gamma^2(1-n))}{2((2-n)^2 - \gamma^2(3-n)(1-n))} \\ q_2^{CU} &= \frac{a((2-n)^2 - \gamma(1-n)(4-n))}{2 \left( (1-n) \left( (2-n)^2 - \gamma^2(1-n)(3-n) \right) \right)} \\ p_1^{CU} &= \frac{a((2-\gamma)(2-n) - \gamma^2(1-n))}{2((2-n)^2 - \gamma^2(3-n)(1-n))} \\ p_2^{CU} &= \frac{a((2-n)^2 + \gamma(1-n)((1-n)\gamma^2 - (4-n)\gamma + n))}{2((2-n)^2 - \gamma^2(1-n)(3-n))} \end{aligned}$$

Then, the equilibrium firm 1's profits are as follows

$$\pi_1^{CU} = \frac{a^2 (n^2 + (1-n)(8(1-\gamma) + \gamma(2n + \gamma(1-n))))}{4 \left( (1-n) \left( (2-n)^2 - \gamma^2(1-n)(3-n) \right) \right)} \quad (8)$$

while  $\pi_2^{CU} = 0$ . It is worth noting that all market variables increase in  $n$ : indeed,  $\frac{\partial p_1^{CU}}{\partial n} > 0$ ,  $\frac{\partial p_2^{CU}}{\partial n} > 0$ ,  $\frac{\partial q_1^{CU}}{\partial n} > 0$ ,  $\frac{\partial q_2^{CU}}{\partial n} > 0$ . Moreover, we find that the royalty rate decreases in  $n$  and the fixed fee increases in  $n$ , so that firm 1 profits' increase in  $n$ .

The following remark states the result obtained by assessing the profitability of Cournot per-unit-royalty-based licensing vs. no licensing.

**Remark 1** *In a framework of quantity competition, two-part-tariff licensing with a per unit royalty is always profitable for the patent holder as compared to no licensing.*

*Proof.* Appears in the Appendix

### 2.2.2 Bertrand competition

By engaging in Bertrand competition, firm 1 and firm 2 face the direct demand functions (3) and (4) and maximize profits in (6) and (7) with respect to  $p_1$  and  $p_2$ , respectively. We obtain:

$$\begin{aligned} p_1 &= \frac{a(1-\gamma) + ny_1(1-\gamma^2) + \gamma(p_2 + r)}{2} \\ p_2 &= \frac{a(1-\gamma) + ny_2(1-\gamma^2) + \gamma p_1 + r}{2} \end{aligned}$$

By solving the above system of the reaction functions under the fulfilled expectations' conditions  $y_1 = \frac{a(1-\gamma)-p_1+\gamma p_2}{(1-\gamma^2)(1-n)}$  and  $y_2 = \frac{a(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)(1-n)}$ , we get the following prices:

$$\begin{aligned} p_1 &= \frac{a(2 - (n(1-\gamma) + \gamma(1+\gamma))) + r\gamma(3 - 4n + n^2)}{4 - (n(4-n) + \gamma^2)} \\ p_2 &= \frac{a(2 - (n(1-\gamma) + \gamma(1+\gamma))) + r(2 - (n(3-n) - \gamma^2(1-n)))}{4 - (n(4-n) + \gamma^2)} \end{aligned}$$

Observe that  $\frac{\partial p_1}{\partial r} > 0$  and  $\frac{\partial p_2}{\partial r} > 0$ , which imply that setting a higher  $r$  leads both firms to charge higher prices.

Firm 2's profits can be written as follows:

$$\pi_2(r, F) = \frac{(a(2-(\gamma(1+\gamma)+n(1-\gamma)))-r(2-n)(1-\gamma^2))(a(2+\gamma-n)-r(2-n)(1+\gamma))}{(1+\gamma)((4-\gamma^2)-n(4-n))^2} - F$$

After computing firm 1's profits at the above optimal prices, we maximize them with respect to  $r$  under the condition  $\pi_2(r, F) \geq 0$ , getting the optimal royalty rate:

$$r^B = \frac{a(2-n+\gamma)^2(\gamma-n)}{2(1-n)\left((2-n)^2 + \gamma^2(5-2n)\right)}$$

The equilibrium fixed fee satisfies  $\pi_2(r^B, F) = 0$  and is as follows:

$$F^{BU} = \frac{a^2((2-n)(2-n-\gamma^3)-\gamma(2+n^2)(2-\gamma)+\gamma n(7-4\gamma))((2-n)(2-n+\gamma^2)+\gamma n(3-n))}{4(1+\gamma)((1-n)((2-n)^2+\gamma^2(5-2n)))^2}$$

Observe that both  $r^B$  and  $F^{BU}$  are positive in the assumed range of network effects (i.e.,  $n(\gamma) \in [0, \hat{n}(\gamma)]$ ).

The market variables at the SPNE are:

$$\begin{aligned} p_1^{BU} &= \frac{a((2-n)(2-\gamma(1+n)) + \gamma^2(7-3n))}{2\left((2-n)^2 + \gamma^2(5-2n)\right)} \\ p_2^{BU} &= \frac{a\left((2-n)^2 + \gamma^2(6-\gamma) - \gamma n(1+\gamma)\right)}{2\left((2-n)^2 + \gamma^2(5-2n)\right)} \end{aligned}$$

$$\begin{aligned}
q_1^{BU} &= \frac{a(4 + \gamma^3 + \gamma^2(5 - 3n) + (2 - n)\gamma - 2n(3 - n))}{2(1 - n)(1 + \gamma)\left((2 - n)^2 + \gamma^2(5 - 2n)\right)} \\
q_2^{BU} &= \frac{a(4 + \gamma n(3 - n) - n(4 - n) + \gamma^2(2 - n))}{2(1 - n)(1 + \gamma)\left((2 - n)^2 + \gamma^2(5 - 2n)\right)}
\end{aligned}$$

Moreover, firm 1's equilibrium profits are:

$$\pi_1^{BU} = \frac{a^2\left((1 + \gamma)n^2 + 8 + \gamma^3 + 3(3 - 2n)\gamma^2 - 2n(4 - \gamma)\right)}{4(1 - n)(1 + \gamma)\left(4 - (n(4 - n) - \gamma^2(5 - 2n))\right)} \quad (9)$$

and  $\pi_2^{BU} = 0$ . Comparative statics with respect to  $n$  reveals that  $\frac{\partial p_1^{BU}}{\partial n} > 0$  and  $\frac{\partial q_1^{BU}}{\partial n} > 0$ , as far as firm 1 is concerned, while  $\frac{\partial p_2^{BU}}{\partial n} < 0$  and  $\frac{\partial q_2^{BU}}{\partial n} > 0$  as regards firm 2. Notice that the variable royalty  $r$  and the fixed fee  $F^{BU}$  respectively decreases and increases in  $n$ , with network effects positively impacting on firm 1 profits.

The comparison between the Bertrand profits under per-unit-royalty-based licensing and the profits under no licensing allows us to introduce the following remark.

**Remark 2** *In a framework of price competition, two-part-tariff licensing with a per unit royalty is always profitable for the patent holder compared to no licensing.*

*Proof.* Appears in the Appendix

### 2.2.3 Cournot vs. Bertrand under a per unit royalty

In this section we compare the equilibrium outcomes in Cournot vs. Bertrand, focusing on the difference in the per unit royalty rates and the patentee's profits under the two competitive regimes.

**Lemma 1** *The per unit royalty rate under Bertrand is always higher than under Cournot in the feasible range of  $\gamma$  and  $n$ .*

*Proof.* Appears in the Appendix.

A comparison between the profits gained in Cournot and Bertrand yields the following proposition.

**Proposition 1** Suppose that (i)  $n(\gamma) \in [0, \hat{n}(\gamma)]$ ,  $\forall \gamma \in (0, 1)$ , where  $\hat{n}(\gamma) = \frac{\gamma}{1+\gamma}$ , and (ii) the incumbent firm licenses its innovation to the entrant by imposing two-part tariff, which involves a fixed fee and a per unit royalty. Then the following is true.

- (a) There exists a critical strength of network externalities  $\underline{n}(\gamma) \in [0, \hat{n}(\gamma)]$ ,  $\forall \gamma \in (0, 1)$ , such that the equilibrium profit of the incumbent firm under Bertrand competition is higher (lower) than that under Cournot competition, if the strength of network externalities is less (greater) than the critical level  $\underline{n}(\gamma)$  (see Figure 1).
- (b) The higher the degree of product differentiation, greater is the possibility for Bertrand profit to be higher than Cournot profit.

*Proof.* Appears in the Appendix.

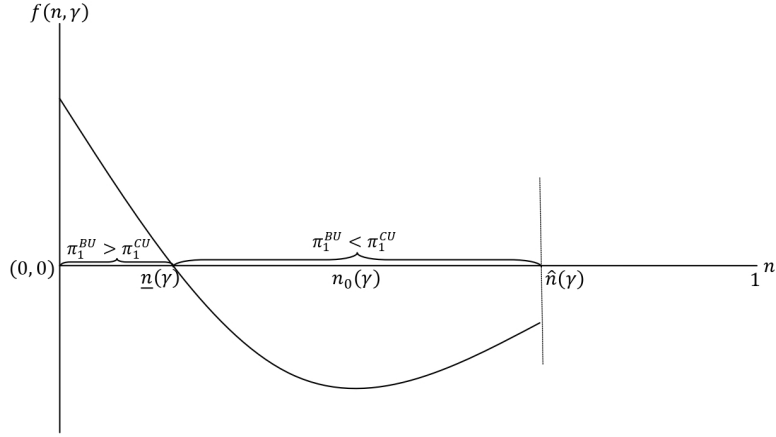


Figure 1

An explanation for the result in Proposition 1 is as follows. When  $n = 0$  (Chang et al, 2017), Bertrand competition is more profitable than Cournot due to the higher royalty rate that is optimally set in Bertrand relative to Cournot with the aim to soften downstream competition. In Bertrand we observe that more intense network effects, by enhancing downstream market profitability through expectations on a larger network size, weaken the patentee's incentive to set a relatively high royalty rate, which decreases when  $n$  rises. It turns out that the negative effect of the royalty rate's reduction on the price set by the cost-disadvantaged licensee dominates the positive effect through consumers' expectations, causing  $p_2$  to decrease in  $n$ . However, due to the cost advantage of the patentee, the downward pressure exerted by a reduced royalty on its final prices is not enough to overcome the upward pressure due to expectations,

which lets  $p_1$  increase in  $n$ . While increasing network effects limit Bertrand market profitability through the negative effect on the licensee's price, they determine an increase of Cournot market profitability by reducing the royalty rate, then positively impacting on  $q_2$ . Such an expansion, however, does not impede  $q_1$  to increase in  $n$ , which enhances both firms' ability to exploit higher consumers' willingness to pay and raises both the licensing revenues and the patentee's profits on her direct sales' channel. It follows that sufficiently low product substitutability enhances the negative impact of network effects on the licensee's price in Bertrand by reducing the patentee's incentive to relax market competition through a high royalty rate. However, it limits the negative impact of the licensee's output expansion through network effects on the patentee's output in Cournot, thus letting the equilibrium prices increase remarkably. In such circumstances, we obtain the reversal result that Cournot profits are higher than Bertrand profits.

Finally, it is worth considering that the profit-dominance of Cournot over Bertrand becomes more likely, namely occurs in a wider range of values of  $\gamma$  for any given  $n$ , as network externalities get stronger. This positive impact of more intense network effects on the Cournot higher profitability is in contrast with Pal (2014) who proves, in a standard duopoly, that higher profitability in Bertrand than in Cournot occurs under strong enough network effects.<sup>9</sup> Conversely, our model highlights the role of more intense network effects in limiting (enhancing) the firms' ability to exploit higher consumers' willingness to pay in Bertrand (Cournot), making the profit-dominance of Cournot over Bertrand more likely.

### 2.3 Two-part tariff licensing with an ad valorem royalty

In this section we keep the above assumptions on demand and firms' costs. Moreover, we assume that firm 1 uses a two-part tariff including an ad valorem royalty  $d \in (0, 1)$ , which is a fraction of rival's revenues, and a fixed amount  $F \geq 0$ . We can write firm 1's and firm 2's profits as follows:

$$\pi_1 = p_1 q_1 + d p_2 q_2 + F \quad (10)$$

$$\pi_2 = (1 - d) p_2 q_2 - F \quad (11)$$

#### 2.3.1 Cournot competition

At the market stage, firm 1 and firm 2 compete in to quantities facing the inverse demand function respectively in (1) and in (2). Maximization of firm 1's profits in (10) with respect to  $q_1$  and maximization of firm 2's profits in (11)

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<sup>9</sup>As already mentioned in the introduction, the result of Pal (2014) derives from the indirect positive effects of more aggressive conduct on profits *via* consumers' expectations dominating its direct negative effect through lower prices.

with respect to  $q_2$  lead to:

$$\begin{aligned} q_1 &= \frac{a + n(y_1 + \gamma y_2) - \gamma q_2(1 + d)}{2} \\ q_2 &= \frac{a + n(\gamma y_1 + y_2) - \gamma q_1}{2} \end{aligned}$$

Notice that the optimal licensee's behavior is not affected by the royalty rate  $d$ . By contrast, a marginal increase of  $d$  reduces the optimal patentee's output (i.e.,  $\frac{\partial q_1}{\partial d} < 0$ ).

The above reaction functions yield, under the fulfilled expectations's conditions ( $y_1 = q_1$  and  $y_2 = q_2$ ), the optimal quantities:

$$\begin{aligned} q_1 &= \frac{a(2 - n(1 - \gamma) - \gamma(1 + d))}{4 - (\gamma^2((1 - n)^2 + d(1 - n)) + n(4 - n))} \\ q_2 &= \frac{a(2 - (n + \gamma(1 - n)))}{4 - (\gamma^2((1 - n)^2 + d(1 - n)) + n(4 - n))} \end{aligned}$$

It can be verified that, following a marginal increase of  $d$ ,  $q_1$  decreases and  $q_2$  increases, namely an increasing ad valorem royalty included in a two-part contract induces the patentee to behave less aggressively in order to enhance her licensing revenues through a more aggressive reaction by the rival firm.<sup>10</sup>

After incorporating the above optimal quantities, the licensee's profits can be written as follows:

$$\pi_2(d, F) = \frac{a^2(1 - d)(2 - (n + \gamma(1 - n)))^2}{(4 - (\gamma^2((1 - n)^2 + d(1 - n)) + n(4 - n)))^2} - F$$

At the previous stage, the patentee maximizes her own profit with respect to the royalty rate  $d$  under the condition  $\pi_2(d, F) \geq 0$ , thus setting:

$$d^C = \frac{((1 - n)\gamma - n)(2 - n - (1 - n)\gamma)^2}{\gamma(1 - n)((2 - \gamma)(2 - n) - (1 - n)\gamma^2)}$$

Then, the equilibrium fixed fee satisfies  $\pi_2(d^C, F) = 0$  and is as follows:

$$F^{CV} = \frac{a^2((2 - \gamma)(2 - n) - (1 - n)\gamma^2)(n(2 - n)^2 + (1 - n)(n^2 - 4n + 2)\gamma^2 - \gamma(1 - n)(2 - n)(n(1 - \gamma^2) + \gamma^2))}{4\gamma(1 - n)((2 - n)^2 - (1 - n)(3 - n)\gamma^2)}$$

We find that  $d^C$  and  $F$  are positive in the assumed range of network effects (i.e.,  $n(\gamma) \in [0, \hat{n}(\gamma)]$ ). In this range, moreover, the condition  $d^C \leq 1$  is verified.<sup>11</sup>

<sup>10</sup>As also highlighted by Colombo and Filippini (2015, p. 9), this contrasts with the per unit royalty case in which an increase of  $r$ , by weakening the licensee, leads the patentee's to behave more aggressively.

<sup>11</sup>Indeed, we find  $d^C - 1 = -\frac{n(n - 2)^2 + (1 - n)((2 + n^2 - 4n)\gamma^2 - (2 - n)(1 - n)\gamma^3 + n(n - 2)\gamma)}{\gamma(1 - n)((2 - \gamma)(2 - n) - (1 - n)\gamma^2)} \leq 0$  for  $n(\gamma) \in [0, \hat{n}(\gamma)]$ .



Firms' quantities and prices at the SPNE are:

$$\begin{aligned}
q_1^{CV} &= \frac{a \left( (2-n)^2 - (1-n)(4-n)\gamma \right)}{2(1-n) \left( (2-n)^2 - (1-n)(3-n)\gamma^2 \right)} \\
q_2^{CV} &= \frac{a \left( (2-\gamma)(2-n) - \gamma^2(1-n) \right)}{2 \left( (2-n)^2 - (1-n)(3-n)\gamma^2 \right)} \\
p_1^{CV} &= \frac{a \left( (2-n)^2 + \gamma(1-n)(n - (n-1)\gamma^2 + (n-4)\gamma) \right)}{2 \left( (2-n)^2 - (3-n)(1-n)\gamma^2 \right)} \\
p_2^{CV} &= \frac{a \left( (2-\gamma)(2-n) - \gamma^2(1-n) \right)}{2(2-n)^2 - 2(3-n)(1-n)\gamma^2}
\end{aligned}$$

Finally, firm 1's equilibrium profits are:

$$\pi_1^{CV} = \frac{a^2 \left( n^2 + 8(1-n) - \gamma(1-n)(8 - 2n - \gamma(1-n)) \right)}{4(1-n) \left( (2-n)^2 - \gamma^2(1-n)(3-n) \right)} \quad (12)$$

while  $\pi_2^{CV} = 0$ .

It can be easily checked that  $d^C$  decreases in  $n$  and  $F^{CV}$  increases in  $n$ . All market variables are positive in the considered parameters' region and increase in  $n$  (i.e.,  $\frac{\partial p_1^{CV}}{\partial n} > 0$ ,  $\frac{\partial q_1^{CV}}{\partial n} > 0$ ,  $\frac{\partial p_2^{CV}}{\partial n} > 0$  and  $\frac{\partial q_2^{CV}}{\partial n} > 0$ ), as well as firm 1's profits (i.e.,  $\frac{\partial \pi_1^{CV}}{\partial n} > 0$ ).

Comparing firm 1's Cournot profits under ad valorem licensing with firm 1's profits under no licensing, we get the result highlighted in the following remark.

**Remark 3** *Under quantity competition, two-part tariff licensing with an ad valorem royalty is always profitable for the patent holder as compared to no licensing.*

*Proof.* *Appears in the Appendix*

Moreover, a comparison between the patentees' Cournot profits under the two considered licensing schemes, with either a per unit royalty or an ad valorem royalty, yields the equivalence result that is highlighted in the following proposition.

**Proposition 2** *In a Cournot setting, the patentee is indifferent between licensing through a per-unit-royalty-based contract and licensing through an ad-valorem-royalty-based contract.*

*Proof:* It directly follows from the identity between (8) and (12).

The equivalence result in Proposition 2 resembles that obtained by Niu (2013, p. 13) in a context without network effects and can be explained as follows. Let's first assume that  $n = 0$ . Due to strategic substitutability of quantities, a per unit royalty included in the contract lets the patentee commit to behave more aggressively by weakening the licensee, which reduces the licensing revenues and positively affects the patentee's direct channel profits. By contrast, an ad valorem royalty lets the patentee commit to behave less aggressively due to the positive effect of a higher royalty rate on the licensee's output. This raises the licensing revenues, while it reduces the patentee's profits on the direct channel. It turns out that, as profit maximization by the patentee under a two part tariff implies maximization of joint profits, the net effect caused by a per unit royalty is of the same magnitude but opposite sign of that caused by an ad valorem royalty, which yields the same equilibrium profits under the two contract types. Notice that the logic behind the equivalence is the same under network externalities, the presence of which affects the size but not the sign of the effects, which compensate each other as in the no-network case.

### 2.3.2 Bertrand competition

As in Section 2.2.2, in this section we assume that firm 1 and firm 2 compete with respect to prices. Moreover, we assume that in this framework the patent holder charges the licensee with a two part contract which includes an ad valorem royalty. Maximization of firm 1's and firm 2's profits in (10) and (11) with respect to  $p_1$  and  $p_2$ , respectively, yields the following reaction functions:

$$\begin{aligned} p_1 &= \frac{a(1-\gamma) + ny_1(1-\gamma^2) + \gamma p_2(1+d)}{2} \\ p_2 &= \frac{a(1-\gamma) + ny_2(1-\gamma^2) + \gamma p_1}{2} \end{aligned}$$

As in the Cournot case, we observe that the optimal licensee's behavior is not affected by the royalty rate  $d$ , a marginal increase of which raises conversely the optimal patentee's price (i.e.,  $\frac{\partial p_1}{\partial d} > 0$ ).

We solve the above system under the fulfilled expectations' conditions  $y_1 = \frac{a(1-\gamma)-p_1+\gamma p_2}{(1-\gamma^2)(1-n)}$  and  $y_2 = \frac{a(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)(1-n)}$ , thus getting the optimal prices:

$$\begin{aligned} p_1 &= \frac{a(2-\gamma^2(1+d(1-n))) - n - \gamma(1-d(1-n)-n)}{4 - (n(4-n) + \gamma^2(1+d(1-n)))} \\ p_2 &= \frac{a(2-\gamma^2 - \gamma(1-n) - n)}{4 - (n(4-n) + \gamma^2(1+d(1-n)))} \end{aligned}$$

It can be verified that, following a marginal increase of  $d$ , both  $p_1$  and  $p_2$  increase, as in the per unit royalty case.

At the previous stage, the patentee optimally chooses the two components of the licensing contract,  $d$  and  $F$ . The licensee's profit can be written as follows:

$$\pi_2(d, F) = \frac{a^2(1-d)(2-n+\gamma)(2-\gamma^2-\gamma(1-n)-n)}{(4-(n(4-n)+\gamma^2(1+d(1-n))))^2(1+\gamma)} - F$$

Subject to  $\pi_2(d, F) \geq 0$ , maximization of the patentee's profits computed at the optimal prices yields the equilibrium ad valorem royalty rate:

$$d^B = \frac{\gamma^2(4-3n+\gamma) + (2-n)(-\gamma(3n-2) - n(2-n))}{\gamma(1-n)((2+\gamma)(2-n) - \gamma^2)}$$

and thus the equilibrium fixed fee satisfying  $\pi_2(d, F) = 0$ :

$$F^{BV} = \frac{a^2(1-\gamma)((2+\gamma)(2-n)-\gamma^2)(n(2-n)^2-\gamma^2(2-n^2)+\gamma(2-n)(n-\gamma^2))}{4\gamma(1-n)(1+\gamma)((2-n)^4-\gamma^2(3-2n)(2(2-n)^2-\gamma^2(3-2n)))}$$

Notice that  $d^B \geq 0$  when  $0 < n \leq \gamma$ , that is,  $d^B$  is always positive in the considered region of the model's parameters. Moreover, we get:

$$\begin{aligned} d^B \leq 1 &\Rightarrow \frac{(2-n)(n(2-n)+\gamma(n-\gamma^2)) + (n^2-2)\gamma^2}{\gamma(1-n)((2+\gamma)(2-n) - \gamma^2)} \geq 0 \\ &\Rightarrow (n(2-n) + \gamma(n-\gamma^2)) + (n^2-2)\gamma^2 \geq 0 \end{aligned}$$

The above condition on  $d^B$  is met when  $n \geq \tilde{n}(\gamma)$ , where  $\tilde{n}(\gamma) = \frac{2+\gamma-\sqrt{4(1+\gamma-\gamma^3+2\gamma^4+\gamma^5)-7\gamma^2}}{2(1-\gamma^2)}$   $\forall \gamma \in (0, 1)$ . In the same region we find  $F^{BV} \geq 0$ .

The firms' prices and quantities at the SPNE are:

$$\begin{aligned} p_1^{BV} &= \frac{a((2-n)^2 + n\gamma(1-n) - \gamma^2(4-3n))}{2((2-n)^2 - \gamma^2(3-2n))} \\ p_2^{BV} &= \frac{a(1-\gamma)((2+\gamma)(2-n) - \gamma^2)}{2((2-n)^2 - \gamma^2(3-2n))} \\ q_1^{BV} &= \frac{a((2-n)^2 - (4-3n)\gamma^2 + \gamma(n-\gamma^2))}{2(1+\gamma)((1-n)((2-n)^2 - \gamma^2(3-2n)))} \\ q_2^{BV} &= \frac{a((2+\gamma)(2-n) - \gamma^2)}{2(1+\gamma)((2-n)^2 - \gamma^2(3-2n))} \end{aligned}$$

Moreover, firm 1's equilibrium profits are:

$$\pi_1^{BV} = \frac{a^2(8-\gamma^3-\gamma^2(7-6n) + n\gamma(2-n) - n(8-n))}{4(1+\gamma)((1-n)((2-n)^2 - \gamma^2(3-2n)))} \quad (13)$$

while  $\pi_2^{BV} = 0$ .

As in the previous settings, we find that the royalty rate  $d^B$  and the fixed fee  $F^{BV}$  respectively decreases and increases in  $n$ . Likewise, all market variables and firm 1's profits positively depend on  $n$  (i.e.,  $\frac{\partial p_1^{BV}}{\partial n} > 0$ ,  $\frac{\partial q_1^{BV}}{\partial n} > 0$ ,  $\frac{\partial p_2^{BV}}{\partial n} > 0$ ,  $\frac{\partial \pi_1^{BV}}{\partial n} > 0$ ).

Moreover, we compare firm 1's Bertrand profits under ad valorem royalty licensing in (13) with its profits under no licensing in (5), getting the result included in the following remark.

**Remark 4** *Under Bertrand competition, two-part tariff licensing with an ad valorem royalty is always profitable for the patent holder.*

*Proof:* Appears in the Appendix

Finally, we compare firm 1's Bertrand profits under ad valorem royalty licensing in (13) with Bertrand profits under per unit royalty licensing in (9), we obtain the result stated in the following proposition.

**Proposition 3** *In a Bertrand setting, a per-unit-royalty-based contract yields higher profits than an ad-valorem-royalty-based contract.*

*Proof.* Appears in the Appendix.

An intuition of the superiority of a per-unit-royalty-based contract over an ad-valorem-royalty-based contract in Bertrand follows the argument of Colombo and Filippini (2015, p. 9) who deal with the same comparison under non-drastic cost reducing innovation and no network effects. Indeed, a marginal increase of ad valorem royalty induces the patentee to behave less aggressively by setting a higher price, thus inducing a price increase from the licensee. This positively affects the patentee's profits likewise a marginal increase of a per unit royalty does by imposing a cost on the licensee and softening market competition. It turns out that, regardless of the extent of network effects, the strategic effect of a per unit royalty is higher than that induced by an ad valorem royalty, which causes higher profitability of the former contract than the latter.

### 2.3.3 Cournot vs. Bertrand under an ad valorem royalty

In this section we compare the equilibrium outcomes under Cournot vs. Bertrand, focusing on the difference in the ad valorem royalty rates and the patentee's differential profits in the two competitive regimes.

**Lemma 2** *The ad valorem royalty rate under Bertrand is always higher than under Cournot in the feasible range of  $\gamma$  and  $n$ .*

*Proof.* Appears in the Appendix.

The following proposition states the profit-dominance of Cournot over Bertrand competition.

**Proposition 4** *Assume that the incumbent firm licenses its innovation to the entrant by imposing two-part tariff, which involves a fixed fee and an ad valorem royalty. Further consider the parameter space in which the equilibrium fixed fee is positive and ad valorem royalty rate lies in the interval  $(0, 1)$ , regardless of the mode of product market competition – Cournot or Bertrand. Then, the equilibrium profits of the incumbent firm under Cournot competition are higher than that under Bertrand competition.*

Proposition 4 proves that, unlike the case of per unit royalty, market profitability is no more affected by product differentiation and the intensity of network effects. We observe that network effects reduce the optimal royalty rate in favor of the fixed component of the licensing contract, due to increased market profitability on both the direct sales channel and the licensee’s channel. The royalty rate reduction leads to lower licensee’s aggressiveness both in Cournot and in Bertrand, which positively affects the profits gained by the patentee through its output expansion in the former and negatively affects those gained in the latter due to a reduction of both prices. This results in a superiority of the Cournot profits over the Bertrand profits, regardless the strength of the network effects and product differentiation.

### 3 Concluding remarks

This paper has reconsidered the relative profitability of Cournot vs. Bertrand competition in a network market in which a patent holder licenses her product innovation to a potential rival through a two part tariff. We have found that the interplay between the intensity of network effects and the degree of product substitutability affects the relative market profitability in Cournot vs. Bertrand under a per unit royalty. The latter, indeed, has been shown to induce cost differences which refrain firms from fully exploiting a higher consumers’ willingness to pay under network effects in Bertrand, while they still enable firms to fully exploit the advantages of a larger market size in Cournot. This can determine a dominance of the Cournot profits over the Bertrand profits, which becomes more likely the lower the degree of product substitutability and the more intense the network effects. In this perspective, our findings reveal the role of network effects in causing a reversal result with respect to Chang et al. (2017) which prove that Bertrand profits are always higher than Cournot profits in a non-network market. Conversely, we demonstrate that the impact of network externalities on the strategic effect induced by an ad valorem royalty ensures higher Cournot profits than Bertrand profits, regardless of the strength of network effects and product substitutability.

Our findings can provide insights on profitability conditions in network markets which may give rise to antitrust concern. We leave to future research the analysis of social desirability of our findings, as well as their robustness to the assumption that either the licensee is also an incumbent in the market or the patentee endogenously chooses her R&D (quality improving) investment level.

## Appendix

### Proof of Remark 1

Consider the difference between firm 1's Cournot profits of licensing through a per unit royalty in (8) and the no licensing profits in (5). For all  $\gamma \in (0, 1)$  and  $n \in [0, \hat{n}(\gamma)]$ , the following holds true:

$$\pi_1^{CU} - \pi_1^{NL} = \frac{a^2((2-n)^2 - \gamma(1-n)(4-n))^2}{4(1-n)(2-n)^2((2-n)^2 - (1-n)(3-n)\gamma^2)} > 0.$$

### Proof of Remark 2

Consider the difference between firm 1's Bertrand profits of licensing through a per unit royalty in (9) and the no licensing profits in (5). For all  $\gamma \in (0, 1)$  and  $n \in [0, \hat{n}(\gamma)]$ , the following holds true:

$$\pi_1^{BU} - \pi_1^{NL} = \frac{a^2((2-n)^2((n^2 + 6n - 4)\gamma + (2-n)^2) + \gamma^2(16(1-\gamma) - 6n^3 + (25 - 7\gamma)n^2 - 8(4 - 3\gamma)n))}{4(1-n)(2-n)^2(1+\gamma)(4 - (n(4-n) - \gamma^2(5-2n)))} > 0.$$

### Proof of Lemma 1

Consider the difference between the per unit royalty rates derived in Section 2.2 in the two frameworks of Cournot and Bertrand. For all  $\gamma \in (0, 1)$  and  $n \in [0, \hat{n}(\gamma)]$ , the following holds true:

$$r^B - r^C = \frac{a(\gamma - n)((2-n)^2 - \gamma^2(1-n)(3-n))(2-n + \gamma)^2 - a(\gamma(1-n) - n)(2 - (\gamma + n(1-\gamma)))^2((2-n)^2 + \gamma^2(5-2n))}{2(1-n)((2-n)^2 + \gamma^2(5-2n))((2-n)^2 - \gamma^2(1-n)(3-n))} > 0.$$

### Proof of Proposition 1

We consider the patentee's profit difference between Cournot and Bertrand under per-unit-royalty-based licensing. From (8) and (9) we obtain the following:

$$Z = \pi_1^{BU} - \pi_1^{CU} = \frac{a^2 \gamma (1-\gamma) [(2-n)((2-n)(n^2 + \gamma^3(1-n)) - n(2-n(4-n))\gamma) - n(1-n)(8-3n)\gamma^2]}{2(1-n)(1+\gamma)[4-(n(4-n) - \gamma^2(5-2n))][(2-n)^2 - \gamma^2(1-n)(3-n)]}$$

It is easy to check that

(a)  $a^2 \gamma (1-\gamma) > 0$ , since  $a > 0$  and  $\gamma \in (0, 1)$ ; and

(b)  $2(1-n)(1+\gamma)[4-(n(4-n) - \gamma^2(5-2n))][(2-n)^2 - \gamma^2(1-n)(3-n)] > 0$ , since  $n \in [0, 1]$  and  $\gamma \in (0, 1)$ .

Therefore,  $Sign(Z) = Sign(f(n, \gamma))$ , where

$$f(n, \gamma) = (2-n)((2-n)(n^2 + \gamma^3(1-n)) - n(2-n(4-n))\gamma) - n(1-n)(8-3n)\gamma^2.$$

Now, we have the following.

(i)  $f(0, \gamma) = 4\gamma^3 > 0$ ,  $\forall \gamma \in (0, 1)$ ;

(ii)  $f(\hat{n}(\gamma), \gamma) = -\frac{\gamma^3(2-\gamma^2)}{(\gamma+1)^3} < 0$ ,  $\forall \gamma \in (0, 1)$ .

(iii)  $\frac{\partial f(n, \gamma)}{\partial n} \begin{cases} < 0, & \text{if } 0 \leq n < n_0(\gamma) \\ = 0, & \text{if } n = n_0(\gamma); \\ > 0, & \text{if } n_0(\gamma) < n \leq \hat{n}(\gamma) \end{cases}$

where  $n_0(\gamma) = \text{Root}[-4\gamma - 8\gamma^2 - 8\gamma^3 + (8 + 20\gamma + 22\gamma^2 + 10\gamma^3)\#1 + (-12 - 18\gamma - 9\gamma^2 - 3\gamma^3)\#1^2 + (4 + 4\gamma)\#1^3, 1]$ . It follows that  $0 < n_0(\gamma) < \hat{n}(\gamma)$ ,  $\forall \gamma \in (0, 1)$ .

(iv)  $f(n_0(\gamma), \gamma) < 0$ ,  $\forall \gamma \in (0, 1)$ .

From (i), (ii), (iii) and (iv) it follows that there exists an  $\underline{n}(\gamma) \in (0, n_0(\gamma))$  such that, for all  $\gamma \in (0, 1)$ ,

$f(n, \gamma) \begin{cases} < 0, & \text{if } 0 \leq n < \underline{n}(\gamma) \\ = 0, & \text{if } n = \underline{n}(\gamma) \\ > 0, & \text{if } \underline{n}(\gamma) < n \leq \hat{n}(\gamma) \end{cases}$ .

The expression for  $\underline{n}(\gamma)$  is given by

$\underline{n}(\gamma) = \text{Root}[4\gamma^3 + (-4\gamma - 8\gamma^2 - 8\gamma^3)\#1 + (4 + 10\gamma + 11\gamma^2 + 5\gamma^3)\#1^2 + (-4 - 6\gamma - 3\gamma^2 - \gamma^3)\#1^3 + (1 + \gamma)\#1^4, 1]$ . Therefore, it follows that

$\pi_1^{BU} - \pi_1^{CU} \begin{cases} < 0, & \text{if } 0 \leq n < \underline{n}(\gamma) \\ = 0, & \text{if } n = \underline{n}(\gamma) \\ > 0, & \text{if } \underline{n}(\gamma) < n \leq \hat{n}(\gamma) \end{cases}$ , for all  $\gamma \in (0, 1)$ .

It can be checked that  $\frac{\partial \underline{n}(\gamma)}{\partial \gamma} > 0$  for all  $\gamma \in (0, 1)$ .

[QED]

**Proof of Remark 3**

Consider the difference between firm 1's Cournot profits of licensing through an ad valorem royalty in (12) and the no licensing profits in (5). For all  $\gamma \in (0, 1)$  and  $n \in [0, \hat{n}(\gamma))$ , the following holds true:

$$\pi_1^{CV} - \pi_1^{NL} = \frac{a^2((2-n)^2 - \gamma(1-n)(4-n))^2}{4(1-n)(2-n)^2((2-n)^2 - \gamma^2(1-n)(3-n))} > 0.$$

**Proof of Remark 4**

Consider the difference between firm 1's Bertrand profits of licensing through an ad valorem royalty in (13) and the no licensing profits in (5). For all  $\gamma \in (0, 1)$  and  $n \in [0, \hat{n}(\gamma))$ , the following holds true:

$$\pi_1^{BV} - \pi_1^{NL} = \frac{a^2((2-n)^2((2-n)^2 + n\gamma(6-n) - 4\gamma) + 7n^2\gamma^3 - n^2\gamma^2(23-6n) - 8\gamma^2(2-\gamma)(1-2n))}{4(1-n)(1+\gamma)(2-n)^2((2-n)^2 - (3-2n)\gamma^2)} > 0.$$

**Proof of Proposition 3**

We consider the difference between (9) and (13), namely the difference between the Bertrand patentee's profits under a per unit royalty and those gained under an ad valorem royalty. For all  $\gamma \in (0, 1)$  and  $n \in [0, \hat{n}(\gamma))$ , the following holds true.

$$\pi_1^{BU} - \pi_1^{BV} = \frac{a^2\gamma(2+\gamma-n)^2(n-\gamma)^2}{2(1-n)(1+\gamma)(n^2+2\gamma^2n-4n-3\gamma^2+4)(4-4n+n^2+5\gamma^2-2\gamma^2n)} > 0.$$

**Proof of Lemma 2**

Consider the difference between the ad valorem royalty rates derived in Section 2.3 in the two frameworks of Cournot and Bertrand. For all  $\gamma \in (0, 1)$  and  $n \in [0, \hat{n}(\gamma))$ , the following holds true.

$$d^B - d^C = \frac{\gamma((2-n)(n(n-1)\gamma^3 + (2-n)(4-n(2-\gamma)(2-n))) - \gamma^2(12+18n^2-6n^3-24n+n^4))}{(1-n)((2+\gamma)(2-n) - \gamma^2)((2-\gamma)(2-n) + (n-1)\gamma^2)} > 0.$$

**Proof of Proposition 4**

First, note that, in the case of two part tariff licensing involving an ad valorem royalty and a fixed fee, the equilibrium fixed fee is positive and ad valorem royalty rate lies in the interval  $(0, 1)$  irrespective of the mode of product market competition – Cournot or Bertrand, if the following holds true.

(i)  $n \in [0, \hat{n}(\gamma)) \forall \gamma \in (0, 1)$ , where  $\hat{n}(\gamma) = \frac{\gamma}{1+\gamma}$ , and (ii)  $n \geq \tilde{n}(\gamma)$ , where  $\tilde{n}(\gamma) = \frac{2+\gamma - \sqrt{4(1+\gamma-\gamma^3+2\gamma^4+\gamma^5)-7\gamma^2}}{2(1-\gamma^2)} \forall \gamma \in (0, 1)$ .



Next, in the case of two part tariff licensing involving an ad valorem royalty and a fixed fee, the equilibrium profits of the incumbent under Cournot competition and under Bertrand competition are, respectively, given by equations (12) and (13). From (12) and (13), we get the following:

$$G = \pi_1^{CV} - \pi_1^{BV} = \frac{a^2\gamma^2[(n^2-3(n-1))\gamma^3+(2-n)(2n(1-\gamma^2)-(2-n)\gamma)]}{2(1+\gamma)[(2-n)^2-(3-2n)\gamma^2][(1-n)(3-n)\gamma^2-(2-n)^2]}$$

Now, it is easy to check that:

- (a) the denominator of  $G$  is negative, for all  $n \in [0, 1)$  and  $\gamma \in (0, 1)$ ;
- (b) the numerator of  $G$  is negative, if and only if  $0 < n \leq \bar{n}(\gamma)$  holds true, where  $\bar{n}(\gamma) = \frac{4-3\gamma^3+4\gamma(1-\gamma)-\sqrt{16(1-2\gamma^2)+\gamma^4(20-3\gamma^2)}}{2(1-\gamma^2)(2+\gamma)} \in (0, 1)$  for all  $\gamma \in (0, 1)$ .

Moreover, it can be checked that  $\bar{n}(\gamma) > \frac{\gamma}{1+\gamma} = \hat{n}(\gamma) \forall \gamma \in (0, 1)$ . Clearly,  $G > 0$  holds true for all  $\gamma \in (0, 1)$  and  $n \in [0, \hat{n}(\gamma))$ .

Next, note that both (i)  $n \in [0, \hat{n}(\gamma)) \forall \gamma \in (0, 1)$ , where  $\hat{n}(\gamma) = \frac{\gamma}{1+\gamma}$ , and (ii)  $n \geq \tilde{n}(\gamma)$ , where  $\tilde{n}(\gamma) = \frac{2+\gamma-\sqrt{4(1+\gamma-\gamma^3+2\gamma^4+\gamma^5)-7\gamma^2}}{2(1-\gamma^2)} \forall \gamma \in (0, 1)$  are satisfied, if  $\tilde{n} < \hat{n}$ . It follows that  $\tilde{n} < \hat{n} \Leftrightarrow \gamma < 0.574743$ .

[QED]

## References

- [1] Alipranti, M., Milliou, C., and Petrakis, E. 2014. "Price vs. quantity competition in a vertically related market." *Economics Letters* 124: 122–126.
- [2] Arthur, B. W. 1989. "Competing technologies, increasing returns and lock-in by historical events." *Economic Journal* 99: 116–131.
- [3] Arya, A., Mittendorf, B. and Sappington, D.E.M. 2008. "Outsourcing, vertical integration, and price vs. quantity competition." *International Journal of Industrial Organization* 26: 1–16.
- [4] Baake, P., and Boom, A. 2001. "Vertical product differentiation, network externalities, and compatibility decisions." *International Journal of Industrial Organization* 19: 267–284.
- [5] Bagchi, A, and Mukherjee, A. 2014. "Technology licensing in a differentiated oligopoly." *International Review of Economics and Finance* 29: 455–465.
- [6] Bousquet, A., Cremer, H., Ivaldi, M., and Wolkowicz, M. 1998. "Risk sharing in licensing." *International Journal of Industrial Organization* 16: 535–554.

- [7] Cabral, L.M.B. 2011. "Dynamic price competition with network effects." *Review of Economic Studies* 78: 83–111.
- [8] Cabral, L.M.B., Salant, D. J., and Woroch, G.A. 1999. "Monopoly pricing with network externalities." *International Journal of Industrial Organization* 17: 199–214.
- [9] Chang, R.-Y., Hwang, H., and Peng, C.-H. 2017. "Competition, product innovation and licensing." *The B.E. Journal of Economic Analysis & Policy* 17: 1–14.
- [10] Choi, J.P. 1994. "Irreversible choice of uncertain technologies with network externalities." *The RAND Journal of Economics* 25: 382–401.
- [11] Colombo, S., and Filippini, L., 2015. "Patent licensing with bertrand competitors." *The Manchester School* 83: 1–16.
- [12] Colombo, S., and Filippini, L. 2016. "Revenue royalties." *Journal of Economics* 118: 47–76.
- [13] Correa-López, M., and Naylor, R.A. 2004. "The Cournot-Bertrand profit differential: a reversal result in a differentiated duopoly with wage bargaining." *European Economic Review* 48: 681–696.
- [14] David, P.A. 1985. "Clio and the economics of QWERTY." *The American Economic Review (P&P)* 75: 332–337.
- [15] Dogan, P. 2009. "Vertical networks, integration, and connectivity." *Journal of Economics and Management Strategy* 18: 347–392.
- [16] Economides, N. 1996a. "The economics of networks." *International Journal of Industrial Organization* 14: 673–699.
- [17] Economides, N. 1996b. "Network externalities, complementarities, and invitations to enter." *European Journal of Political Economy* 12: 211–233.
- [18] Erutku, C., and Richelle, Y. 2007. "Optimal licensing contract and the value of a patent, *Journal of Economics and Management Strategy* 16: 407–436.
- [19] Fan, C., Jun, B.H., and Wolfstetter, E.G. 2018. "Per unit vs. ad valorem royalty licensing." *Economics Letters* 170: 71–75.
- [20] Farrell, J., and Saloner, G. 1985. "Standardization, compatibility, and innovation." *The RAND Journal of Economics* 16, 70–83.
- [21] Faulí-Oller, R., and Sandonis, J. 2002. "Welfare reducing licensing." *Games and Economic Behavior* 41: 192–205.
- [22] Filippini, L., and Vergari, C. 2017. "Vertical Integration Smooths Innovation Diffusion". *The B.E. Journal of Economic Analysis & Policy* 17: 1-22.

- [23] Gabszewicz, J.J., and Garcia, F. 2007. "Intrinsic quality improvements and network externalities." *International Journal of Economic Theory* 3: 261–278.
- [24] Gandal, N. (2008), Network Effects - Empirical Studies, in L.E. Blume and S.N. Durlauf (eds.). The New Palgrave Dictionary of Economics, 2<sup>nd</sup> edn. Palgrave Macmillan, New York.
- [25] Ghosh, A., and Mitra, M. 2010. "Comparing Bertrand and Cournot in mixed markets." *Economics Letters* 109: 72–74.
- [26] Häckner, J. 2000. "A note on price and quantity competition in differentiated oligopolies." *Journal of Economic Theory* 93: 233–239.
- [27] Heywood J.S., Li, J., and Ye, G. 2014. "Per unit vs. ad valorem royalties under asymmetric information." *International Journal of Industrial Organization* 37: 38–46.
- [28] Hoernig, S. 2012. "Strategic delegation under price competition and network effects." *Economics Letters* 117: 487–489.
- [29] Hoffman, C. 2014. "Why Microsoft Makes \$5 to \$15 From Every Android Device Sold." (<https://www.howtogeek.com/183766/why-microsoft-makes-5-to-15-from-every-android-device-sold/>)
- [30] Jiang, B., and Shi, H. 2018. "Inter-competitor licensing and product innovation." *Journal of Marketing Research*: doi.org/10.1177/0022243718802846.
- [31] Kamien, M. I., and Tauman, Y. 1984. "The private value of a patent: A game theoretic analysis." *Journal of Economics* 4: 93–118.
- [32] Kamien, M. and Tauman, Y. 1986. "Fees versus royalties and the private value of a patent." *Quarterly Journal of Economics* 101: 471-491.
- [33] Kabiraj, T., and Lee, C.C. 2011. "Technology transfer in a duopoly with horizontal and vertical product differentiation." *Trade and Development Review* 4: 19-40.
- [34] Katz, M.L., and Shapiro, C. 1985. "Network externalities, competition, and compatibility." *The American Economic Review* 75: 424–440.
- [35] Katz, M.L., and Shapiro, C. 1986. "Technology adoption in the presence of network externalities." *Journal of Political Economy* 94: 822–841.
- [36] Lambertini, L., and Orsini, R. 2001. "Network externalities and the over-provision of quality by a monopolist." *Southern Economic Journal* 67: 969–982.
- [37] Lim, T., and Veugelers, R. 2003. "On equity as a payment device in technology licensing with moral hazard." Working Paper, Katholieke Universiteit Leuven.

- [38] Lin, L. H., and Kulatilaka, N. 2006. "Network effects and technology licensing with fixed fee, royalty, and hybrid contracts." *Journal of Management Information Systems* 23: 91–118.
- [39] Ludlow, T. 2014. "Trends In Technology IP Licensing." *IP Law Journal*, December: 1–11.
- [40] Macho-Stadler, I., Martínez-Giralt, X., and Pérez-Castrillo, J.D. 1996. "The role of information in licensing contract design." *Research Policy* 25: 43–57.
- [41] Matsumura, T., and Ogawa, A. (2012). "Price versus quantity in a mixed duopoly." *Economics Letters* 116: 174–177.
- [42] Mukherjee, A., Broll, U., and Mukherjee, D. (2012). "Bertrand versus Cournot competition in a vertical structure: a note." *Manchester School* 80: 545–559.
- [43] Muto, S. 1993. "On licensing policies in Bertrand competition." *Games and Economic Behavior* 5: 257–267.
- [44] Niu, S., 2013. "The equivalence of profit-sharing licensing and per-unit royalty licensing." *Economic Modelling* 32: 10–14.
- [45] Nokia Corporation Financial Report, 2018.
- [46] Pal R. 2014. "Price and quantity competition in network goods duopoly: a reversal result." *Economics Bulletin* 34: 1019–1027.
- [47] Pal, R. 2015. "Cournot vs. Bertrand under relative performance delegation: implications of positive and negative network externalities." *Mathematical Social Sciences* 75: 94–101.
- [48] San Martín, M., and Saracho, A.I. 2010. "Royalty licensing." *Economics Letters* 107: 284–287.
- [49] San Martín, M., and Saracho, A.I. 2015. "Optimal two-part tariff licensing mechanisms", *The Manchester School* 83: 288–306.
- [50] San Martín, M., and Saracho, A.I. 2016. "Patent strength and optimal two-part tariff licensing with a potential rival incorporating ad valorem royalties." *Economics Letters* 143: 28–31
- [51] Sen, D., and Stamatopoulos, G. 2009. "Drastic innovations and multiplicity of optimal licensing policies." *Economics Letters* 105: 7–10.
- [52] Sen, D., and Tauman, Y. 2007. "General licensing schemes for a cost-reducing innovation." *Games and Economic Behavior* 59: 163–186.
- [53] Singh, N., and Vives, X. 1984. "Price and quantity competition in a differentiated duopoly." *RAND Journal of Economics* 15: 546–554.

- [54] State of the Developer Nation 15th Edition. 2018. (<https://www.developereconomics.com/reports/state-of-the-developer-nation-15th-edition>).
- [55] Tanaka, Y. 2001a. "Profitability of price and quantity strategies in an oligopoly." *Journal of Mathematical Economics* 35: 409–418.
- [56] Tanaka, Y. 2001b. "Profitability of price and quantity strategies in a duopoly with vertical product differentiation." *Economic Theory* 17: 693–700.
- [57] Tasnádi, A. 2006. "Price vs. quantity in oligopoly games." *International Journal of Industrial Organization* 24: 541–554.
- [58] Trombini, G., and Comacchio, A. 2012. "Cooperative markets for ideas: when does technology licensing combine with R&D partnerships?," Working Papers 8, Department of Management, Università Ca' Foscari Venezia.
- [59] Wang, X.H. 1998. "Fee versus royalty licensing in a Cournot duopoly model." *Economics Letters* 60: 55-62.
- [60] Wang, K.C.A., Liang, W.J., and Lin, C.H.A. 2012. "Licensing under network externalities." *The Economic Record* 88: 585–593.
- [61] Zanchettin, P. 2006. "Differentiated duopoly with asymmetric costs." *Journal of Economics & Management Strategy* 15: 999–1015.
- [62] Zhao, D., Chen, H., Hong X., Liu J. 2014. "Technology licensing contracts with network effects". *International Journal of Production Economics* 158: 136–144.

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