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# RISK ANALYSIS FOR AGRICULTURAL PRODUCTION FIRMS: CONCEPTS, INFORMATION REQUIREMENTS AND POLICY ISSUES

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# SOME THEORETICAL AND METHODOLOGICAL ISSUES IN ANALYZING INVESTMENT DECISIONS OF FARMERS

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Many graduate school programs in agricultural economics seem to have a void in theory and methods of intertemporal choice. Thus, few agricultural economists are well trained for research in such problems. This conclusion is supported by many references to appropriate discount rates and appropriate objectives in intertemporal analysis (e.g., Castle and Hoch; Chisholm; Perrin; Boehlje and White). For economists interested in production aspects of agriculture, graduate programs, for the most part, have emphasized intraperiod choice of production activities and choice of short-run inputs.

While choice concerning production activities in the short-run is important, the choice of investment has become increasingly important as agricultural production has become more (physical) capital intensive and that capital has become more specialized to specific production activities. Therefore, the purpose of this paper is to review some basic theory of intertemporal choice for both the certain and uncertain situations. Then, some extensions of the basic theory are developed to point out some problems in analyzing farm investment choice. These theoretical problems are related to currently used methods or techniques of analysis.

## Certainty Cases

As Hirshleifer (p. 31) points out, there are two schools of thought in the development of intertemporal choice theory. One school of thought has it that consumption is the sole end of economic activity and, therefore, investment (or savings) is a means to consuming in future periods (i.e., "invest to live"

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school). The other school of thought supports investment as an object of choice independent of consumption (i.e., "live to invest" school). The former position is the one which seems most widely accepted, philisophically at least, among agricultural economists, and the position taken for this paper.

## Complete and Perfect Markets

Suppose that an individual has an endowment of wealth and, for simplicity, the endowment is only in claims to current consumption. Furthermore, suppose that complete and perfect markets exist for exchanging claims to current consumption and claims to future consumption, and that investment opportunities exist for transforming claims to current consumption to claims to future consumption through productive means. Given these assumptions, a two-period model is used to find the utility maximizing conditions for the individual.[1] The model is specified as:

(1)(a) maximize: $U(c_0, c_1)$

(b) subject to: $Q(q_0, q_1) \geq 0$

(c) $y_0 + q_0 + \phi_1 q_1 - c_0 - \phi_1 c_1 \geq 0$

where $U(c_0, c_1)$ is the intertemporal utility function, $Q(q_0, q_1)$ is the investment transformation function, $\underline{Y}(y_0, y_1)$ is the endowment vector (for this case $y_1 = 0$), $c_t$ is the amount of consumption in period t (which may be generalized by thinking in terms of "funds" for consumption in period t), $q_t$ is the input or output values of the investment in period t, and $\phi_t$ are market prices of funds for consumption in period t in terms of current funds (i.e., $\phi_0 = 1$).

Constraint 1b is the implicit investment transformation function which ensures that the investment taken is a technically efficient one. Constraint 1c is the wealth constraint which ensures that consumption value does not exceed the value of the endowment plus productive investment gains.

The optimizing conditions[2] can be found by Lagrangian multiplier methods and, assuming that borrowing cannot exceed the value of period 1 funds, the Kuhn-Tucker conditions must be examined. The result for consumption is:

$$\frac{\partial c_1}{\partial c_0} = \frac{1}{\phi_1} .$$

This result indicates that utility is maximized when the individual has used current funds to the point that current funds will be traded for period 1 funds at the market exchange rate for trading such funds. Note that $\phi_1 = 1/(1+r)$, where r is the market interest rate. Thus, the individual's maximum utility is attained when the rate of time preference is equal to the market rate of interest.

The investment decision which maximizes utility occurs when

$$\frac{\partial q_1}{\partial q_0} = \frac{1}{\phi_1} .$$

This result indicates that investment of current funds continues until the rate of return on the marginal funds invested is equal to the market rate for trading such funds--the market interest rate.

The foregoing results indicate that the marginal rate of time preference for a consumptive optimum equals the marginal rate of return for an investment optimum, both of which are equal to the market interest rate, i.e.,

$$\frac{\partial c_1}{\partial c_0} = 1+r = \frac{\partial q_1}{\partial q_0} .$$

However, this result does not mean that only the current funds from the endowment that are not currently consumed are invested, i.e., this result does not mean $y_0 + q_0 = c_0$ ($q_0$ is negative). Since borrowing is allowed, current funds can be borrowed at the market rate up to the value of period 1 funds, i.e, all wealth could be consumed currently by borrowing. Thus, the optimal amount of current consumption and the optimal amount of current investment may occur at different points, meaning that the optimal levels of investment and consumption are independently determined. Furthermore, maximizing wealth attained through productive investment opportunities leads to the same investment choice as maximizing utility, i.e., $\partial q_1/\partial q_0 = 1+r$.[3] These results lead to the Fisher Separation Theorem which may be stated as follows:

> Given perfect and complete capital markets, the production decision is governed solely by an objective market criterion (represented by attained wealth) without regard to the individuals' subjective preference which enter into their consumption decision (Copeland and Weston, p. 10).

## Imperfect Markets

Imperfect funds markets may take many forms. Perhaps the two cases discussed most often in the literature are: (1) the case of divergent lending and borrowing rates, and (2) the case of a funds acquisition limit imposed by the market. For the most part, writers on these topics have viewed the case of divergent lending and borrowing rates as the case of more practical importance, and have downplayed the practical relevance of a funds limit (e.g., Hirshleifer, p. 206; Weingartner, pp. 1404-1405).[4] For farmers, however, the case of an imposed fund limit is of practical importance. As Barry, Baker, and Sanint (p. 218) point out, lending rules by lenders in agricultural credit markets incorporate nonprice responses which produce credit limits.[5]

The theoretical solution to the case of a borrowing limited is much debated in the literature (e.g., Weingartner; Baumol and Quandt; Myers). Much of the effort for solving the problem has taken a linear programming approach. However, Weingartner graphically demonstrates that the linear programming solution makes little contribution to the general solution because of the assumption of a linear utility function. Furthermore, he graphically demonstrates the general solution to the problem of a borrowing limit. From the graphical solution conditions for maximum utility and implications for capital budgeting are not apparent. Hence, this section presents a rigorous solution to the case of a borrowing limit.

Assume the same conditions as in model (1), except that an absolute debt limit (c*) less than the current value of period 1 funds exists. Adding this constraint to model (1), the model becomes:

(2)(a) maximize: $U(c_0, c_1)$

(b) subject to: $Q(q_0, q_1) \geq 0$

(c) $y_0 + q_0 + \phi_1 q_1 - c_0 - \phi_1 c_1 \geq 0$

(d) $y_0 + q_0 + c_0^* - c_0 \geq 0.$

The optimizing condition[6] for consumption is

$$\frac{\partial c_1}{\partial c_0} = \frac{1}{\phi_1} + \frac{\lambda_3}{\lambda_2 \phi_1} .$$

Note that this solution differs from the one obtained with no debt constraint. In this situation the optimizing condition for consumption does not occur at the point at which the extra unit of current consumption funds will be traded for an extra unit of period 1 consumption funds at the market rate. Instead, the optimum occurs at a point that requires period 1 funds to be discounted at a rate higher than the market rate. This "premium" required on the market rate is the value $\lambda_3/\lambda_2\phi_1$. This value may be interpreted as the individual's subjective extra trade-off value above the market value between current and period 1 funds. This interpretation is realized by noting that $\lambda_3$ is the extra subjective value gained by obtaining more current consumption through borrowing, and $\lambda_2\phi_1$ is the extra subjective value gained from getting more wealth from period 1 funds by investing. Thus, the ratio $\lambda_3/\lambda_2\phi_1$ is the subjective extra interest rate above the market rate required at the margin to trade current for future funds.

The optimizing conditions for the investment decision is

$$\frac{\partial q_1}{\partial q_0} = \frac{1}{\phi_1} + \frac{\lambda_3}{\lambda_2 \phi_1} .$$

As in the case with no debt limit, the optimal condition for investment is the same as for consumption. However, in this case the investment and consumption decisions cannot be made independently. The trade-off for extra wealth from productive investment and extra current consumption is a subjective matter; thus, the objective market criterion of maximizing wealth cannot be used to make optimal investment decisions as in the case of no debt limit, i.e., the Fisher Separation Theorem no longer holds. Maximizing market valued wealth (before consumption), when all current funds can be invested, yields the same investment optimum as in the no debt limit case. However, maximizing wealth does not allow a consumptive optimum, if the necessary level of debt to acquire the optimum is more than the debt limit, i.e., if the debt constraint is binding.

It is worth noting, however, that, if the appropriate subjective values are known regarding consumption, maximizing the subjective value of wealth yields the appropriate investment decision. This is easily seen by substituting the subjective tradeoff values into the wealth equation, i.e., let

$$\frac{1}{\phi^*_1} = \frac{1}{\phi_1} + \frac{\lambda_3}{\lambda_2\phi_1}$$

and substitute $\phi_1^*$ into the wealth equation to get

$$W_0 = q_0 + q_1\phi_1^*.$$

Now, maximizing wealth subject to the investment possibilities will yield an optimal condition

$$\frac{\partial q_1}{\partial q_0} = \frac{1}{\phi_1^*} = \frac{1}{\phi_1} + \frac{\lambda_3}{\lambda_2\phi_1}$$

which is the same as the consumptive optimum found by maximizing utility. This demonstrates that present value methods of capital budgeting are valid with imperfect markets if the appropriate discount rate is used, but it is not the same as maximizing the market value of wealth.

## Uncertainty Cases

Intertemporal choice under uncertainty may be developed by combining the theories of choice under uncertain conditions and choice over time.[7] The von Neumann-Morganstern expected utility theory is the basis for the theory of choice under uncertainty, and the basis for the theory of choice over time is the foregoing discussion of intertemporal choice.

In a timeless world of uncertainty, choice ultimately depends on the possible consumptive claims that may be forthcoming, although the actual decision may involve a choice among assets. Assets may be thought of in terms of contingent claims to consumption funds. Conceptual development in this section abstracts from choice among complex assets; instead, choices involve only simple assets that directly yield contingent quantities of consumption funds. The actual amounts of consumption funds obtained depend on the state of the world that occurs. The individual cannot choose the state of the world, but must choose, while still uncertain about which state will occur, assets (or actions) that yield consumption funds contingent on the state which occurs.

Presumably, the choice among assets considers the utility of consumption funds which occur for each possible state and the probability of occurrence of each state. This reasoning leads to the well-known expected utility rule which states that utility of an asset is the probability weighted-average of the utilities of the contingent funds. In order for a utility function of contingent funds to be represented as a weighted averge of utilities of consumption funds occurring for each possible state, these "subutility functions" must be cardinal scaling functions. In a timeless situation, four postulates are required as a basis for a cardinal scaling function and expected utility (Hirshleifer, pp. 219-220).

1. <u>Ordering and preference direction</u>: Given two amounts of consumption funds $c$ and $c'$, if $c > c'$ then $c \succ c'$.

2. Certainty equivalent: Let $X(c_a, c_b; \pi, 1-\pi)$ be a prospect for contingent consumption funds $c_a$ and $c_b$ with associated probabilities $\pi$ and $1-\pi$, respectively. There exists a certain amount of consumption funds $c \sim X(c_a, c_b; \pi, 1-\pi)$ such that the value of c is between the values $c_a$ and $c_b$.

3. Preferential independence: Let $X(c_a, c_b; \pi, 1-\pi)$ be a prospect for contingent consumption funds $c_a$ and $c_b$ with associated probabilities $\pi$ and $1-\pi$, respectively. If $X(c_a, c_b; \pi, 1-\pi) \sim X'(c_a', c_b'; \pi', 1-\pi')$, then given another prospect $X^*(c_a^*, c^*_b; \pi^*, 1-\pi^*)$, $(X, X^*; \pi, 1-\pi) \sim (X', X^*; \pi, 1-\pi)$

4. Uniqueness: Let $v_a(c_a)$ be the utility for consumption funds $c_a$ if state "a" occurs, and $v_b(c_b)$ be the utility for consumption funds $c_b$ if state "b" occurs. If $c_a = c_b$, then $v_a(c_a) = v_b(c_b)$, and further, if $c_a = c_b$ and $\pi_a = \pi_b$, then $\pi_a v_a(c_a) = \pi_b v_b(c_b)$, where $\pi_i$ is the probability of state i occurring.

Furthermore, if the subutility functions for consumption funds are concave, i.e., display diminishing marginal utility for consumption funds, then it can be shown that indifference curves between state-distributed claims to consumption funds are convex (Hirshleifer, p. 233). Therefore, a rational individual (rational in terms of displaying diminishing marginal utility for funds and the foregoing postulates) would maximize utility by holding assets that yield consumption funds, regardless of the state of the world that occurs.

To this point the discussion has been a quick review of choice in a world of uncertainty in a timeless context. Hopefully, this review sets the stage for considering intertemporal choice under uncertainty. When time is involved in the outcome of contingent consumption funds, choice can be thought of in terms of choosing among possible sequences of consumption funds. The probability of the outcome of any particular sequence is the product of the successive conditional probabilities of the occurrence of the states involved in that sequence.

In order to justify a cardinal intertemporal utility function for a sequence of dated consumption funds which can be used with the expected utility rule, some adjustments are necessary in the postulates of rational choice. Only the ordering and preference direction and certainty equivalent postulates need changing (Hirshleifer, p. 236). These two postutlates may be revised as follows:

1. <u>Intertemporal ordering and preference direction</u>: Given two sequences of consumption funds over time, $L^A(c_1^A, c_2^A, c_3^A)$ and $L^B(c_1^B, c_2^B, c_3^B)$, if $c_t^A = c_t^B$ for all t, then $L^A \sim L^B$. If $C_t^A = c_t^B$ for all t except $c_t^A > c_t^B$ when t = i, then $L^A \succ L^B$.

2. <u>Intertemporal certainty equivalent</u>: Suppose there exist two sequences of consumption funds over time, $L^A(c_1^A, c_2^A, c_3^A)$ and $L^B(c_1^B, c_2^B, c_3^B)$, where $c_t^A = c_t^B$ for all t except $c_i^A > c_i^B$ when t = i. Let $L^*(c_1^*, c^*{}_2, c_3^*)$ be another sequence identical to $L^A$ and $L^B$ except for $c_i^*$, such that $c_i^A > c^*{}_i > c_i^B$. For any prospect of sequences $(L^A, L^B; \pi, 1-\pi)$ there is a certainty equivalent $L^*$ such that $L^* \sim$

$(L^A, L^B; \pi, 1-\pi)$ and, conversely, for a certain sequence $L^*$ a prospect (lottery) of the form $(L^A, L^B, \pi, 1-\pi)$ may be found such that $(L^A, L^B; \pi, 1-\pi) \sim L^*$.

With these revised postulates, an expected utility function can be defined as a probability weighted average of cardinal intertemporal subutility functions of sequences of consumption funds. If these subutility functions are concave functions, i.e., display diminishing marginal utility for funds of any date, then indifference curves between state-distributed claims are convex and indifference curves between time-distributed claims are convex.

Complete and Perfect Markets

Intertemporal decisions in an uncertain environment require the choice of a preferred consumption sequence. This choice involves consideration of possible occurrences of states of the world, which affect the amounts of consusmption fund received, and dates, which affect the timing of consumption funds received. The time-state preference approach to intertemporal choice under uncertainty was chosen because it allows analysis within a framework similar to that used for intertemporal choice under certainty.

Let current consumption funds be certain claims, while future consumption funds are contingent claims. Paralleling the situation in the certainty case, suppose that an individual has an endowment of wealth and, for simplicity, the endowment is only in claims to current funds. Furthermore, suppose that complete and perfect markets exist for exchanging current funds and contingent future funds and for exchanging funds of various contingencies, and that productive investment opportunities exist for transforming current claims to future claims of various contingencies. Given these assumptions, a two-period, two-state model is used to find the utility maximizing conditions for the individual.[8] The model is specified as

(3)(a) maximize: $U(c_0, c_{1a}, c_{1b})$

(b) subject to: $Q(q_0, q_{1a}, q_{1b}) \geq 0$

$$(c)\ y_0 + q_0 + \phi_{1a}q_{1a} + \phi_{1b}q_{1b} - c_0 - \phi_{1a}c_{1a} - \phi_{1b}c_{1b} \geq 0$$

where definitions are the same as in model (1), except the double subscripted variables refer to the time and state of occurrence, e.g., $c_{ts}$ is the amount of consumption funds in period t if state s occurs.

The optimizing conditions[9] again can be found by Lagrangian multiplier methods. Assuming that debt cannot exceed the value of period 1 funds, the Kuhn-Tucker conditions must be examined for constraints involving the exchange of current and period 1 funds. However, there is no reason to assume that in trading various contingent claims to funds (i.e., the underlying security) an individual would choose not to go "short"; therefore, a strict equality would always hold. The optimizing conditions for consumption are:

$$(a)\ \frac{\partial c_{1a}}{\partial c_0} = \frac{1}{\phi_{1a}},\ (b)\ \frac{\partial c_{1b}}{\partial c_0} = \frac{1}{\phi_{1b}},\ \text{and (c)}\ \frac{\partial c_{1b}}{\partial c_{1a}} = \frac{\phi_{1a}}{\phi_{1b}}.$$

Note that three conditions must be satisfied for optimal consumption. Result (a) and (b) indicate that the individual maximizes utility when current funds are consumed to the point that current funds will be traded for period 1 contingent funds at the market price of exchange-- either $\phi_{1a}$ or $\phi_{1b}$ in this case depending on the contingency. When results (a) and (b) are determined, result (c) will also be determined, that is, choosing between current funds and funds of each contingent state automatically determines the optimum between funds of contingent states. Note that $\phi_{1s} = {}^1/(1+r_s)$ where $r_s$ is the market discount rate required for trading funds contingent on the occurrence of state s for current funds. This discount rate reflects, not only the time-value aspect of future funds, but also risk involved in accepting contingent claims to funds.

The investment decision which maximizes utility occurs when

$$(a)\ \frac{\partial q_{1a}}{\partial q_0} = \frac{1}{\phi_{1a}};\ (b)\ \frac{\partial q_{1b}}{\partial q_0} = \frac{1}{\phi_{1b}},\ \text{and (c)}\ \frac{\partial q_{1b}}{\partial q_{1a}} = \frac{\phi_{1a}}{\phi_{1b}}.$$

Results (a) and (b) indicate that optimal investment occurs when current funds continue to be invested until each contingent rate of return of the marginal funds invested is equal to the market rate for trading such funds. Furthermore, result (c) shows that the optimum requires the marginal rate of substitution between contingent funds in the productive process to equal the market rate of substitution between such contingencies in the market.

As in the certainty case with complete and perfect markets, the marginal conditions for the consumptive optimum and investment optimum are equal, i.e.,

$$\frac{\partial c_{1s}}{\partial c_0} = 1+r_s = \frac{\partial q_{1s}}{\partial q_0} \text{ and } \frac{\partial c_{1b}}{\partial c_{1a}} = \frac{\phi_{1a}}{\phi_{1b}} = \frac{\partial q_{1b}}{\partial q_{1a}} .$$

Again, these results do not mean that $y_0 + q_0 = c_0$ and, furthermore, these results do not indicate that $c_{1s} = q_{1s}$. Since borrowing from future contingent funds is allowed and a market exists for trading contingent funds, the optimal levels of consumption and investment may occur at different points by market transactions. Also, as in the certainty case with complete and perfect markets, optimal investment can be guided by the market objective of maximizing attained wealth, since the maximizing conditions are the same as in maximizing utility.[10] Therefore, this result leads to an extension of the Separation Theorem now to encompass, not only independence between the level of consumption and productive investment, but also independence between the level of consumption and the levels of productive investments yielding various contingent funds, i.e., a separation between the level of consumption and the level of investment, and a separation between the level of consumption and the level of risk in productive investment.

<u>Incomplete Markets</u>

In the certainty case a situation was analzyed in which the market for debt funds was imperfect in the sense of not having a unique price at which all transactions may occur. The imperfect market situation analyzed, and one of practical importance to agricultural producers, was the situation of pure capital rationing, i.e., rationing debt with an upper limit rather than a pricing mechanism. When considering risky productive investments of agricultural producers, another problem of practical significance is incomplete securities markets, i.e., incomplete in the sense that a security does not exist for every contingency.

To illustrate the problem with an incomplete securities market, assume the same conditions as in model (3), except that no $c_{1a}$ contingent funds can be obtained through existing securities being traded. Thus, all $c_{1a}$ funds must come from productive investments or the endowment (but, as assumed earlier, the endowment for this case only includes $c_0$ funds). Imposing this condition on model (3), the model becomes

(4)(a) maximize: $U(c_0, c_{1a}, c_{1b})$

(b) subject to: $Q(q_0, q_{1a}, q_{1b}) \geq 0$

(c) $y_0 + q_0 + \phi_{1b}q_{1b} - c_0 - \phi_{1b}c_{1b} \geq 0$

(d) $q_{1a} - c_{1a} = 0.$

The optimizing conditions[11] for consumption are:

(a) $\dfrac{\partial c_{1a}}{\partial c_0} = \dfrac{\lambda_2}{\lambda_3}$, (b) $\dfrac{\partial c_{1b}}{\partial c_0} = \dfrac{1}{\phi_{1b}}$ and (c) $\dfrac{\partial c_{1a}}{\partial c_{1b}} = \dfrac{\lambda_2 \phi_{1b}}{\lambda_3}$ .

Result (b) shows the individual's marginal rate of substitution of $c_{1b}$ for $c_0$ is the only personal marginal trade-off of funds made at the market rate. This, of course, is because $c_{1b}$ is the only contingent funds with underlying tradeable securities. Result (a) indicates that the individual at optimal consumption conditions will substitute $c_{1a}$ funds for $c_0$ funds at the rate $\lambda_2/\lambda_3$. That is, substitution at the margin occurs at the rate of marginal utility of current wealth from $c_0$ to marginal utility of another unit of $c_{1a}$. Result (c) is interpreted similarly.

The optimizing conditions for the investment decision are:

(a) $\dfrac{\partial q_{1a}}{\partial q_0} = \dfrac{\lambda_2}{\lambda_3}$, (b) $\dfrac{\partial q_{1b}}{\partial q_0} = \dfrac{1}{\phi_{1b}}$, and (c) $\dfrac{\partial q_{1a}}{\partial q_{1b}} = \dfrac{\lambda_2 \phi_{1b}}{\lambda_3}$ .

Result (b) shows that for optimal investment to occur current funds should be invested in productive investment yielding $c_{1b}$ funds until the rate of return on the marginal investment equals the market rate of return for securities yielding $c_{1b}$ funds. Results (a) and (c) indicate that the level of investment of current funds to yield $c_{1a}$ funds is subjectively determined. Result (a) shows that current funds should be invested in assets yielding $c_{1a}$ until the expected marginal return from such an investment equals the subjective rate of substitution between $c_0$ and $c_{1a}$ -- the ratio of the marginal utility of wealth from an increment of current funds to the marginal utility of an increment of $c_{1a}$. Similarly, result (b) shows that the amounts of assets yielding $c_{1a}$ and $c_{1b}$ should be chosen such that the marginal rate of substitution between $c_{1a}$ and $c_{1b}$ occurs at the subjective marginal rate of substitution--the ratio of the marginal utility of wealth from an increment of $c_{1b}$ to the marginal utility of wealth of an increment of $c_{1a}$.

As in the case with complete contingent funds markets, optimal conditions for investment are the same as optimal conditions for consumption. However, it should be noted that only the investment decision yielding $c_{1b}$ funds is independent of consumption decisions. Therefore, an objective market decision cannot guide all investment decisions because the desired "mix" of contingent consumption funds cannot be obtained in the securities market. Some of the desired contingent funds must come directly from the productive investment process; hence, some of the productive investments decisions specifically must be to fulfill the consumptive desire.

## Some Methodolgical Implications

The theoretical conditions examined in the previous sections have some important implications for research in farm investment and growth. First, under conditions of certainty maximizing market net worth (or net worth change) is an appropriate objective for farm investment decisions and farm firm growth strategies when markets are complete and perfect. Likewise, under conditions of uncertainty maximizing the expected utility of wealth or wealth-gain (i.e., net worth or change in net worth) are appropriate objectives when markets are perfect and complete. However, when markets are imperfect, such as the case of a debt limit, market values representing intertemporal value flows no longer are appropriate objectives for optimal investment decisions. The reason for this impropriety of the market objective stems from the fact that wealth measured at some point in time can no longer be assumed to be convertible to consumption at the desired time; thus, a maximum utility cannot be guaranteed by maximizing wealth at some date.

This conclusion is an indictment for much of the farm investment and growth research. Examples of types of research incorporating this methodological problem include (a) multiperiod linear programming models with debt constraints, (b) quadratic programming models with debt constraints, (c) simulation studies incorporating debt limits and using wealth measures as indicators, and (d) stochastic dominance studies using probability distributions of wealth measures when wealth values were generated under conditions in which debt limits were effective. For research methods to appropriately consider investment decisions with debt limits, the methods must incorporate the utility of each period's cash flow along with substitution possibilities between periods, i.e., a multidimension utility approach with cash flows of each period representing the multiple attributes (Meyer as cited by Anderson, Dillon, and Hardaker). However, multiattribute utility analysis becomes very difficult when many investments are considered. Note that expected utility is a multiattribute approach for considering risky outcomes for the same period, i.e., the states of the world are the attributes. The uniqueness postulate is a crucial element in allowing the weighted average scheme involved in expected utility. Unfortunately, a uniqueness postulate in time periods is not a solution because such an assumption destroys any reason for time preference.

However, Bell (as cited by Anderson, Dillon and Hardaker) demonstrates the usefulness of preferential independence in time periods for investment decision. Furthermore, Winterfeldt and Fischer (as cited by Anderson, Dillon and Hardaker) show that, if no time period is preferentially independent, the choice among investments (cash flow sequences) becomes a matter of the decision maker's intuition. Thus, because of the difficulties and restrictions involved in methodologically correct analyses, research methods currently being used are probably the most practical. However,the limitations of such methods should be realized and more research attention should be devoted to finding out the seriousness of the shortcomings. Also, attention to development of new methods may be warranted.

Other implications of the theoretical findings deal with securities markets. If securities markets are complete and perfect, risk associated with productive investments can be offset in the market. If securities markets do not exist, the risk level must be adjusted with productive investments. The stance taken by many agricultural economists dealing with farm organization seems to be that securities markets do not exist, or exist to a very limited extent. Although a substantial body of research literature incorporates market mechanisms for offsetting contingencies (e.g., hedging), most of these studies assume a fixed production-investment decision and concentrate only on risk efficiency of marketing alternatives.

As more security assets (e.g., actuarially sound crop insurance, agricultural commodities options) become available for adjusting the risk level, it becomes increasingly important for these to be incorporated in farm production-investment decision analyses. This stems from the fact that, as farm risk can be offset in the market, productive investment decisions become less dependent on the risk inherent in the production process, because the risk can be adjusted to the utility maximizing level in the securities market. Also, with available market options for risk management and widespread participation in that market by farm producers, a risk efficient set of productive investments no longer is risk efficient in the market sense, i.e., there would exist market alternatives that would allow a higher return for the same risk level. This, of course, is the result of the second separation effect, which is the basis of the financial theoretical approach to portfolio analysis.

Furthermore, as securities markets develop, an empirical question arises as to whether or not the securities markets are complete enough to do a reasonable job in pricing agricultural production risks. Answering this question has implications for using such techniques as the capital asset pricing model (CAPM) (Sharpe) and for better practical capital budgeting by using the present value certainty equivalent method, i.e., using appropriate risky discount rates.[12] Discussion of these topics, however, extend far beyond the intent and scope of this paper.

## FOOTNOTES

1 Conditions for utility maximization are developed based on the reasoning of Hirshleifer, Chapter 3.

2 See Appendix A for derivation of these results.

3 See Appendix B for derivation of this result.

4 This point especially is shown by the title of Weingartner's article, "Capital Rationing: n Authors in Search of a Plot" which indicates much academic interest but little practical importance.

5 It may be argued that farmers may acquire funds through equity markets. However, equity markets for such funds acquisition are not easily accessible to many farmers because of high transactions costs. Direct acquisition of equity funds through negotiated partnerships also involve high costs and often are not practical. Thus, this option is ignored in this analysis.

6 See Appendix C for derivation of these results.

7 Again, this section is developed based on Hirshleifer.

8 Conditions for utility maximization are developed based on the reasoning of Hirshleifer, Chapter 9.

9 See Appendix D for derivation of these results.

10 See Appendix E for derivation of this result.

11 See Appendix F for derivation of these results.

12 Anderson, Dillon and Hardaker (p. 250) refer to using risky rates in present value analysis as naive and lacking theoretical content. However, Hirshleifer (p. 250) demonstrates that with a securities market, risky discount rates are theoretically founded, and a risky discount rate can be developed for each "risk class" of investment.

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## Appendix A

### Certainty Conditions, No Debt Constraint*

Maximize: $U(c_o, c_1)$

Subject to: $Q(q_o, q_1) \geq 0$

$y_o + q_o + \phi_1 q_1 - c_o - \phi_1 c_1 \geq 0$

$$L = U(c_o, c_1) - \lambda_1[Q(q_o, q_1)] - \lambda_2(y_o + q_o + \phi_1 q_1 - c_o - \phi_1 c_1)$$

(1) $$\frac{\partial L}{\partial c_o} = \frac{\partial U}{\partial c_o} + \lambda_2 = 0$$

(2) $$\frac{\partial L}{\partial c_1} = \frac{\partial U}{\partial c_1} + \lambda_2 \phi_1 = 0$$

(3) $$\frac{\partial L}{\partial q_o} = \lambda_1 \frac{\partial Q}{\partial q_o} - \lambda_2 = 0$$

(4) $$\frac{\partial L}{\partial q_1} = \lambda_1 \frac{\partial Q}{\partial q_1} - \lambda_2 \phi_1 = 0$$

Together, (1) and (2) imply:

$$\frac{\partial c_1}{\partial c_o} = \frac{1}{\phi_1}$$

Together, (3) and (4) imply:

$$\frac{\partial q_1}{\partial q_o} = \frac{1}{\phi_1}$$

*Only the equality conditions are shown, but all Kuhn-Tucker conditions should be examined for an optimal.

## Appendix B

## Certainty Conditions, Wealth Maximization*

Maximize: $W_o = y_o + q_o + \phi_1 q_1$

Subject to: $Q(q_o, q_1) \geq 0$

$$L = y_o + q_o + \phi_1 q_1 - \lambda[Q(q_o, q_1)]$$

(1) $$\frac{\partial L}{\partial q_o} = 1 - \lambda\frac{\partial Q}{\partial q_o} = 0$$

(2) $$\frac{\partial L}{\partial q_1} = \phi_1 - \lambda\frac{\partial Q}{\partial q_1} = 0$$

Together, (1) and (2) imply:

$$\frac{\partial q_1}{\partial q_o} = \frac{1}{\phi_1}$$

*See footnote, Appendix A.

## Appendix C

## Certainty Condition with a Debt Constraint*

Maximize: $U(c_o, c_1)$

Subject to: $Q(q_o, q_1) \geq 0$

$$y_o + q_o + \phi_1 q_1 - c_o - \phi_1 c_1 \geq 0$$

$$y_o + c_o^* - c_o - q_o \geq 0 \qquad \text{where } c_o^* < \phi_1 q_1$$

$$\mathcal{L} = U(c_o, c_1) - \lambda_1[Q(q_1, q_o)] - \lambda_2(y_o + q_o + \phi_1 q_1 - c_o - \phi_1 c_1) - \lambda_3(y_o + c_o^* - c_o - q_o)$$

(1) $$\frac{\partial \mathcal{L}}{\partial c_o} = \frac{\partial U}{\partial c_o} + \lambda_2 + \lambda_3 = 0$$

(2) $$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{\partial U}{\partial c_1} + \lambda_2 \phi_1 = 0$$

(3) $$\frac{\partial \mathcal{L}}{\partial q_o} = \lambda_1 \frac{\partial Q}{\partial q_o} - \lambda_2 - \lambda_3 = 0$$

(4) $$\frac{\partial \mathcal{L}}{\partial q_1} = \lambda_1 \frac{\partial Q}{\partial q_1} - \lambda_2 \phi_1 = 0$$

Together, (1) and (2) imply:

$$\frac{\partial c_1}{\partial c_o} = \frac{\lambda_3}{\lambda_2 \phi_1} + \frac{1}{\phi_1}$$

Together, (3) and (4) imply:

$$\frac{\partial q_1}{\partial q_o} = \frac{\lambda_3}{\lambda_2 \phi_1} + \frac{1}{\phi_1}$$

*See footnote, Appendix A.

## Appendix D

### Uncertainty Conditions, No Debt or Security Market Constraints*

Maximize: $U(c_o, c_{1a}, c_{1b})$

Subject to: $Q(q_o, q_{1a}, q_{1b}) \geq 0$

$$y_o + q_o + \phi_{1a}q_{1a} + \phi_{1b}q_{1b} - c_o - \phi_{1a}c_{1a} - \phi_{1b}c_{1b} \geq 0$$

$$L = U(c_o, c_{1a}, c_{1b}) - \lambda_1[Q(q_o, q_{1a}, q_{1b})]$$
$$- \lambda_2(y_o + q_o + \phi_{1a}q_{1a} + \phi_{1b}q_{1b} - c_o - \phi_{1a}c_{1a} - \phi_{1b}c_{1b})$$

(1) $$\frac{\partial L}{\partial c_o} = \frac{\partial U}{\partial c_o} + \lambda_2 = 0$$

(2) $$\frac{\partial L}{\partial c_{1a}} = \frac{\partial U}{\partial c_{1a}} + \lambda_2\phi_{1a} = 0$$

(3) $$\frac{\partial L}{\partial c_{1b}} = \frac{\partial U}{\partial c_{1b}} + \lambda_2\phi_{1b} = 0$$

(4) $$\frac{\partial L}{\partial q_o} = \frac{\lambda_1 \partial Q}{\partial q_o} - \lambda_2 = 0$$

(5) $$\frac{\partial L}{\partial q_{1a}} = \lambda_1\frac{\partial Q}{\partial q_{1a}} - \lambda_2\phi_{1a} = 0$$

(6) $$\frac{\partial L}{\partial q_{1b}} = \lambda_1\frac{\partial Q}{\partial q_{1b}} - \lambda_2\phi_{1b} = 0$$

Together, (1) and (2) imply:

$$\frac{\partial c_{1a}}{\partial c_o} = \frac{1}{\phi_{1a}}$$

Together, (1) and (3) imply:

$$\frac{\partial c_{1b}}{\partial c_o} = \frac{1}{\phi_{1b}}$$

Appendix D (Continued)

Together, (2) and (3) imply:

$$\frac{\partial c_{1b}}{\partial c_{1a}} = \frac{\phi_{1a}}{\phi_{1b}}$$

Together, (4) and (5) imply:

$$\frac{\partial q_{1a}}{\partial q_{o}} = \frac{1}{\phi_{1a}}$$

Together, (4) and (6) imply:

$$\frac{\partial q_{1b}}{\partial q_{o}} = \frac{1}{\phi_{1b}}$$

Together, (5) and (6) imply:

$$\frac{\partial q_{1b}}{\partial q_{1a}} = \frac{\phi_{1a}}{\phi_{1b}}$$

*See footnote, Appendix A.

## Appendix E

## Uncertainty Conditions, Wealth Maximization*

Maximize: $W_o = y_o + q_o + \phi_{1a}q_{1a} + \phi_{1b}q_{1b}$

Subject to: $Q(q_o, q_{1a}, q_{1b}) \geq 0$

$$L = y_o + q_o + \phi_{1a}q_{1a} + \phi_{1b}q_{1b} - \lambda[Q(q_o, q_{1a}, q_{1b})]$$

(1) $$\frac{\partial L}{\partial q_o} = 1 - \lambda\frac{\partial Q}{\partial q_o} = 0$$

(2) $$\frac{\partial L}{\partial q_{1a}} = \phi_{1a} - \lambda\frac{\partial Q}{\partial q_{1a}} = 0$$

(3) $$\frac{\partial L}{\partial q_{1b}} = \phi_{1b} - \lambda\frac{\partial Q}{\partial q_{1b}} = 0$$

Together, (1) and (2) imply:

$$\frac{\partial q_{1a}}{\partial q_o} = \frac{1}{\phi_{1a}}$$

Together, (1) and (3) imply:

$$\frac{\partial q_{1b}}{\partial q_o} = \frac{1}{\phi_{1b}}$$

Together, (2) and (3) imply:

$$\frac{\partial q_{1b}}{\partial q_{1a}} = \frac{\phi_{1a}}{\phi_{1b}}$$

*See footnote, Appendix A.

## Appendix F

## Uncertainty Conditions with No Debt Constraint but a Security Market Constraint*

Maximize: $U(c_o, c_{1a}, c_{1b})$

Subject to: $Q(q_o, q_{1a}, q_{1b}) \geq 0$

$$y_o + q_o + \phi_{1b}q_{1b} - c_o - \phi_{1b}c_{1b} \geq 0$$

$$q_{1a} - c_{1a} = 0$$

$$L = U(c_o, c_{1a}, c_{1b}) - \lambda_1[Q(q_o, q_{1a}, q_{1b})] - \lambda_2(y_o + q_o + \phi_{1b}q_{1b} - c_o - \phi_{1b}c_{1b}) - \lambda_3(q_{1a} - c_{1a})$$

(1) $$\frac{\partial L}{\partial c_o} = \frac{\partial U}{\partial c_o} + \lambda_2 = 0$$

(2) $$\frac{\partial L}{\partial c_{1a}} = \frac{\partial U}{\partial c_{1a}} + \lambda_3 = 0$$

(3) $$\frac{\partial L}{\partial c_{1b}} = \frac{\partial U}{\partial c_{1b}} + \lambda_2\phi_{1b} = 0$$

(4) $$\frac{\partial L}{\partial q_o} = \lambda_1\frac{\partial Q}{\partial q_o} - \lambda_2 = 0$$

(5) $$\frac{\partial L}{\partial q_{1a}} = \lambda_1\frac{\partial Q}{\partial q_{1a}} - \lambda_3 = 0$$

(6) $$\frac{\partial L}{\partial q_{1b}} = \lambda_1\frac{\partial Q}{\partial q_{1b}} - \lambda_2\phi_{1b} = 0$$

Together, (1) and (3) imply:

$$\frac{\partial c_{1b}}{\partial c_o} = \frac{1}{\phi_{1b}}$$

Appendix F (Continued)

Together, (4) and (6) imply:

$$\frac{\partial q_{1b}}{\partial q_o} = \frac{1}{\phi_{1b}}$$

Together, (2) and (5) imply:

$$\frac{\dfrac{\partial U}{\partial c_{1a}}}{\dfrac{\partial Q}{\partial q_{1a}}} = -\lambda_1$$

Together, (1) and (2) imply:

$$\frac{\partial c_{1a}}{\partial c_o} = \frac{\lambda_2}{\lambda_3}$$

Together, (2) and (3) imply:

$$\frac{\partial c_{1a}}{\partial c_{1b}} = \frac{\lambda_2 \phi_{1b}}{\lambda_3}$$

Together, (4) and (5) imply:

$$\frac{\partial q_{1a}}{\partial q_o} = \frac{\lambda_2}{\lambda_3}$$

Together, (5) and (6) imply:

$$\frac{\partial q_{1a}}{\partial q_{1b}} = \frac{\lambda_2 \phi_{1b}}{\lambda_3}$$

*See footnote, Appendix A.