Food Demand and Consumption Behavior
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ESTIMATION OF FOOD DEMAND AND CONSUMPTION RELATIONSHIPS

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Estimation of household food demand and consumption relationships requires the integration of four basic elements: consumer behavior theory, statistical methods, data and the results/findings of relevant studies. Consumer behavior theory provides a logical framework on which to base the specification of demand-consumption models. Statistics and econometrics offer techniques for parameter estimation and hypothesis testing. Data availability affects the type of demand parameters that can be estimated while data quality directly affects the accuracy of parameter estimates. Finally, the results and findings of previous studies are useful in model specification and often provide extraneous estimates of certain model parameters.

The synergistic effects of combining the four elements of demand estimation should not be underated. Yet, it is not possible to fully examine this potential within the limits of this paper. Fortunately, other workshop participants will consider one or more of these elements. Tomek has reviewed the contributions of previous demand studies. Bayton has assessed the socio-psychological determinants of food demand and consumption behavior. In subsequent papers, Pearl and Johnson will focus on the availability and uses of various data sources. The missing element and subject matter of this paper is the theoretical and methodological aspects of food demand estimation.

OBJECTIVES

The specific objectives of this paper are:

1. To identify common problems in applying the neoclassical theory of consumer behavior to the estimation of demand relationships.

2. To review alternative methods for increasing the empirical content of neoclassical theory.

3. To assess the usefulness of these methods for achieving the estimation objectives of the regional project.

4. To document further research needs related to the development of demand models for the regional project.

CONSUMER BEHAVIOR THEORY

A distinguishing feature of available theories of consumer behavior is the emphasis placed on the major determinants of consumer

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choice. Economists typically assume that a rational consumer has a consistent preference ordering over commodity bundles. Propositions about consumer purchase behavior are then deduced from the equilibrium conditions for income constrained maximization of consumer preferences. In contrast, social psychologists and market specialists focus on the formation of consumer preferences and the dynamic interaction between preferences and choice.

Ideally, behavioral scientists should integrate their knowledge and experience with different but equally important aspects of consumer behavior into a holistic model of consumer choice. While progress in this direction has been made (e.g. Nicosia), there still remains a schism between economic and social-psychological approaches to consumer behavior. Perhaps a more serious difficulty, with respect to the objectives of this paper, is how to enhance the empirical content of consumer behavior models. The point of departure for this paper is a discussion of the empirical limitations of the neoclassical model of consumer choice. Until such limitations can be reduced, it seems inappropirate to focus on holistic models. Implicit in this judgement is the premise that holistic models do not ease the burden of empirical demand analysis although they do offer a more realistic conceptualization of consumer behavior.

Historical Perspective

Classical economists, notably Gossen, Jevons, Walras, Marshall, and Edgeworth, hypothesized that the satisfaction a consumer receives from the consumption of commodities was measurable. The unit of measurement invented for this purpose was the "util". This cardinal interpretation of utility led to the now familiar law of diminishing marginal utility. This law and the measurability of utility was later rejected by Pareto, Hicks and Allen. They showed that the inverse relationship between quantity demanded and own price implied by diminishing marginal utility could be derived from constrained maximization of an ordinal utility function. In essence, only an ordinal ranking of commodity bundles was required to demonstrate the law of demand. This less restrictive approach came to be known as the neoclassical theory of consumer choice.

In an effort to purge consumer behavior theory of the convenient but nonetheless restrictive mathematical properties assumed for an ordinal utility function, notably continuous first and second order derivatives, Samuelson (1938) developed the theory of revealed preference. This theory, which was later expanded by Houthakker (1950), stated the conditions on observable price-quantity observations required for the existence of a complete preference ordering.

The choice functions of revealed preference theory exhibited the same properties as neoclassical demand functions including the controversial integrability condition. The latter property, secured by the invoking Houthakker's strong axiom of revealed preference, insures that the choice functions could be derived by constrained maximization of a utility function. Thus, the equivalence of the choice functions of revealed preference theory and the demand functions of neoclassical theory was established.

While revealed preference theory is more platable than neoclassical theory, as judged by Occam's razor, it adds little to the empirical analysis of consumer demand. In particular, the theoretical specification of demand functions is the same using either theory. The major empirical contribution of revealed preference theory is that it provides a straightforward method for testing whether consumers have a complete or partially complete preference ordering; see, for example, Koo.

A more recent direction is axiomization of utility theory based on mathematical topology (Katzner). A major advancement provided by the axiomatic formulation of utility theory is that it does not require indifference curves to be continuous and convex. Unfortunately, this theoretically more appealing specification of indifference curves adds little to empirical demand analysis. A notable exception is that the axiomatic approach admits such phenomena as discontinuous, non-unique, and irreversible demand functions. Such properties would explain the phenomenon whereby quantity demanded varies only in response to threshold price changes, *ceteris paribus*.

Despite the restrictive nature of neoclassical utility theory, demand functions implied by the latter underlie much of modern demand analysis. For this reason, and in view of the limited empirical content of the revealed preference and axiomatic theories of consumer behavior, the neoclassical demand model forms the starting point for the ensuing treatment of demand estimation.

NEOCLASSICAL DEMAND THEORY

Because of the numerous expositions of neoclassical demand theory, only a brief statement is presented here. For more details see Kogiku and Samuelson (1947).

Neoclassical demand theory attempts to explain how a single individual makes consumption decisions at a given point in time. The theory asserts that a rational consumer will maximize his preferences subject to certain constraints. Under certain conditions (Debreu), preferences can be represented by a strictly quasi-concave utility function. The constraints conclude the consumer's endowment, principally income and wealth, the exogenous commodity prices and the set of commodities available to the consumer. The outcome of this decision-making process is the choice of non-negative amounts of all commodities available to the consumer.

In equilibrium, the marginal rate of substitution between every pair of commodities must equal the corresponding price ratio[1]. Utility is at a relative maximum if the utility function is strictly quasi-concave (i.e., convex indifference curves). Changes in optimum consumption with respect to prices and income can be determined from the demand functions. The latter indicate that quantity demanded of every good is a unique, homogeneous of degree zero function of all prices and money income[2]. Further manipulation of the structural and reduced form equations for the consumer optimization model yields several restrictions on the demand functions; see George and King (pp. 8-10). These restrictions will be discussed in connection with the degrees of freedom problem.

DEMAND ESTIMATION PROBLEMS

The neoclassical model of consumer behavior has numerous empirical limitations. Misahan among others, argues that the "predictive content" of neoclassical theory is very limited. The major problems encountered in the application of neoclassical theory to demand estimation are discussed below. Approaches for alleviating these problems are covered in the next section.

Degrees of Freedom Problem

The system of demand functions obtained by solving the first-order conditions for constrained utility maximization contain n^2 price slopes and n income slopes. These $n^2 + n$ demand parameters can be reduced to $\frac{1}{2}n(n+1)$ independent parameters by imposing the Engel aggregation, Cournot aggregation and Slutsky equations and the homogeneity and symmetry conditions. Even this reduced set of parameters is large relative to the typical length of time series data on quantities, prices and income. In such situations there are insufficient degrees of freedom for estimating the reduced set of demand parameters.

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1 The notable exception to these first-order conditions occurs when consumption of one of the goods in the pair is zero. This leads to an inequality between the MRS and the price ratio. Solutions to first-order conditions involving inequalities can be obtained with mathematical programming. Absence of inequalities permits use of the calculus.

2 The homogeneity property is often used to argue the use of relative prices and real income in empirical demand functions. Uniqueness follows from the assumption of a strictly quasi-concave utility function.

In cases where only a subset of demand functions or demand parameters are of interest, the demand restrictions implied by neoclassical theory cannot be imposed and the degrees of freedom problem occurs.

Other Statistical Problems

In addition to their limited length, time series data on prices and income typically exhibit high correlations. Severe multicollinearity can prevent the estimation of demand parameters or cause estimated price and income effects to be highly inefficient.

Other statistical difficulties that plague the time series estimation of demand parameters include serial correlation in the error term and non-spherical error terms (heteroskedasticity and/or non-zero covariances between error terms).

New Commodities

Optimum choice of commodities in the neoclassical model is contained for a given set of commodities. If new commodities enter the marketplace, existing commodities are modified (e.g., the addition of a vitamin supplement), or commodities are taken off the market, then a new optimum must be found. This essentially means maximizing a new utility function containing the new and/or modified set of commodities. Since food markets are continually bombarded with new or modified food items, the neoclassical model would have the consumer continually searching for a new optimum. Such behavior is terribly inefficient and unsubstantiated by casual observation.

Functional Form

Neoclassical theory specifies the demand functions in differential form only[3]. Hence, there is no theoretical basis for choosing among alternative mathematical forms for the demand functions; e.g., linear, double-log, semi-log, log-inverse, logistic, etc[4]. Yet, each of these functional forms implies somewhat different assumptions regarding satiation levels and the behavior of demand elasticities.

Aggregation Over Consumers and Commodities

While neoclassical theory pertains to a single individual, empirical demand studies usually focus on households or a large

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3 Essentially, the demand functions are valid only for small movements around a single equilibrium.
4 Although a demand function linear in prices and income is quite popular, Quirk (pp. 80-81) shows it is inconsistent with the homogeneous of degree zero property of demand functions.

group of consumers. To the extent that preferences are identical for individuals comprising the group, consistent aggregation is possible. However, these and other conditions which permit consistent aggregation over consumers are typically not satisfied, even for individuals in the same household. When aggregation is inconsistent, the interpretation of demand parameters estimated from aggregate data is not meaningful as, for example, when income effects become confounded with price effects; see Theil (1954).

Aggregation over commodities is often done to facilitate the imposition of demand restrictions and to otherwise reduce the degrees of freedom problem. Once again, neoclassical theory offers little guidance on how commodities should be aggregated except for Hick's composite good theorem (Hicks; pp. 312-313). Improper commodity aggregation reduces the accuracy of estimated demand parameters.

Dynamic Demand Analysis

Neoclassical theory provides a static interpretation of consumer behavior. Essentially, the single time period used in neoclassical theory is assumed to be short enough so that consumer preferences are stable yet long enough so that the diversity of consumer choices can be accommodated. This single-period analysis of consumer behavior is somewhat inconsistent with reality. In general, consumers are likely to plan their purchases over a time horizon which covers several time periods. Multi-period and dynamic analysis of demand have been addressed by many authors including Henderson and Quandt, Houthakker and Taylor, Nerlove and others.

Demand models that distinguish between short- and long-run behavior have been widely applied; e.g., Nerlove's partial adjustment model. In contrast, more comprehensive models of dynamic behavior, such as the one developed by Houthakker and Taylor, have not been as widely used.

In the case of food commodities, the distinction between short- and long-run behavior can be justified in terms of food fads or food habits. However, there is little support for the hypothesis that consumers plan their food purchases over a long time horizon. For somewhat similar reasons one would not expect food purchases to be sensitive to consumer expectations concerning national economic conditions.

Consumer Choices Under Risk

In addition to its static bias, neoclassical theory assumes the consumer possesses perfect knowledge. In reality, consumers are faced with imperfect information regarding the availability, quality and prices of commodities. Decision-making models which account for risk have been developed by Friedman and Savage, von Neumann and Morgenstern, and Luce and Raiffa. While food purchase decisions are subject to some risk, the effect of risk is considered to be minor because of the recurrent nature of food purchases and the insignificance of any single food item in the consumer's budget.

For purposes of analyzing and estimating household demand for food commodities the most serious limitations of the neoclassical model appear to be the degrees of freedom problem, other statistical problems, new commodities, functional form and aggregation. This paper concentrates on this more limited set of estimation problems.

COPING WITH ESTIMATION PROBLEMS

The estimation problems discussed in this section are not equally significant in all demand analyses. Some types of demand analyses are more susceptible to these problems than others. For example, estimation of household demand relationships from cross-section data over households is much less susceptible to problems associated with insufficient degrees of freedom, multicollinearity, new commodities and aggregation over individuals than demand estimation from time series data. In a cross-section analysis, the data is typically collected over a short enough period of time so that prices are reasonably constant. This feature essentially eliminates the degrees of freedom problem and multicollinearity among prices. In addition, introduction of new commodities is insignificant over short time periods and there is no need to aggregate over households when observations on individual households are available. In contrast, estimation of demand from time series data typically involves all of the difficulties outlined in the previous section.

Why then do economists bother with time series analysis of demand? The major reason is that direct and cross price elasticities cannot be measured from cross-section data unless the utility is additive. (More on this later.) On the other hand, cross-section data provides ideal conditions for estimating income elasticities because the distribution of income is essentially constant due to the shortness of the time period. Does it then follow that price elasticities are more appropriately estimated from time series data? The answer is yes for the reason that time series price data exhibit the variation required for accurate estimation of price effects. Unfortunately, since prices tend to be highly correlated over time the precision of estimated price effects is likely to be impaired by multicollinearity.

Without belaboring the point further, it should be obvious that there is little empirical basis for preferring cross-section over time series data[5]. For this reason, this section gives a more detailed treatment of estimation difficulties inherent in cross-section and time series applications of neoclassical demand theory.

Degrees of Freedom and Other Statistical Problems

de Janvry and Bieri prescribe three approaches for dealing with the degrees of freedom and associated statistical problems. They are: a) improving statistical methods for handling multicollinearity, serial correlation, etc., b) expanding the available data base, and c) extending the neoclassical model by imposing restrictions on the utility function. A discussion of these approaches follows.

As previously mentioned, time series analysis of demand is frequently impaired by multicollinearity among commodity prices. Serial correlation in the error term is another problem encountered in time series models. While statistical methods have improved our ability to handle multicollinearity and serial correlation, they are sometimes unsatisfactory from an economic viewpoint. This dilemma is illustrated for multicollinearity.

One procedure for dealing with multicollinearity is to reduce the independent variable space to a set of orthogonal factors called principal components[6]. These principal components are then used as regressors. Unfortunately, while this method effectively removes multicollinearity and its undesirable consequences, it is difficult to relate the coefficients of the principal components to demand parameters.

The situation is somewhat different for serial correlation. Methods for handling this problem range from simple procedures like transforming data to first differences to more sophisticated techniques like the iterative method (Cochrane and Orcutt). These techniques are being used more frequently particularly with the increased appearance in computer programs of standard options for handling serial correlation.

A second approach for alleviating the degrees of freedom problem is to expand the data base. Specifically, an attempt is made to construct a consistent data base on prices, quantities and major demand determinants for a given sample of consumers. The data generated by the Household Food Consumption Surveys of 1955 and 1965-66

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5 Brown and Deaton argue that "for many practical purposes (e.g., planning and forecasting) the effects of changes in income are of greater importance than those of changes in prices. When this simplification is introduced, . . . interest centres now on the precise nature of one relationship, the Engel curve . . ." This assessment would obviously favor concentration on cross-section estimation of income effects.

6 For a more detailed treatment of multicollinearity see Prato (1976).

constitutes an internally consistent cross-section data base on household food consumption. Unfortunately, data on nonfood items must be obtained from different data sources such as the Bureau of Labor Statistics retail price series and the Survey of Consumer Expenditures. Perhaps the biggest drawback of attempting to reduce the degrees of freedom problem by expanding the data base is that it is very costly relative to other procedures.

Perhaps the most popular method for coping with insufficient degrees of freedom is to reduce the number of independent demand parameters requiring estimation by imposing further restrictions on the mathematical structure of the utility function. The most common type of restrictions are those implied by the separability hypothesis. Separable utility functions were introduced by Leontief and Sono and expanded upon by Goldman and Uzawa. Applications have been made by Brandow, de Janvry, Prato (1969), George and King and others.

Each type of separability (weak, strong, pointwise, Pearce) and additivity (pointwise and block) restricts the structure of the utility function and, hence, the form of the Slutsky substitution term for items in different groups. The more restrictive the structure of the utility function the greater the reduction in the number of independent demand parameters to be estimated.

Demand estimation under separability has proceeded along three lines (de Janvry and Bieri). Under the first approach, separability is imposed on the utility function. Demand functions derived from constrained maximization of this separable utility function are then estimated[7]. The most widely imposed types of separability are pointwise and block additivity. Use of the latter is made in studies by Brandow, and George and King.

To illustrate the strengths and weaknesses of the additivity hypothesis, consider constrained maximization of the following pointwise additive Stone-Geary utility function (Stone, Geary)[8].

$$u(q) = \prod_{i=1}^{n} (q_i - c_i)^{\beta_i}$$

where $0 \leq c_i \leq q_i$, $0 < \beta_i < 1$ and $\sum_{i=1}^{n} \beta_i = 1$.

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7 Under certain conditions, a separable utility function can be consistently maximized in two stages. In the first stage income is allocated to each group. In the second stage, group expenditure is allocated among the items in each group. This two-stage process is analogous to consumer budgeting. For more details see de Janvry and Bieri.

8 The Stone-Geary function is chosen because of its popularity in empirical demand studies.

Solving the first-order conditions for a constrained maximum yields the following linear expenditure system:

$$p_i q_i = p_i c_i + \beta_i \left(M - \sum_{j=1}^{n} p_j c_j\right) \qquad i = 1, \ldots, n$$

where the term in parentheses is residual or uncommitted income. Thus, expenditure on each item is a linear function of uncommitted income. The substitution term for different goods is

$$S_{ij} = -\left(M - \sum_{j=1}^{n} p_j c_j\right)(b_i b_j)$$

where b_i and b_j are marginal propensities to consume[9]. Since $0 < \beta_i < 1$, all marginal propensities to consumer are positive, hence $S_{ij} < 0$ for all $i \neq j$, so goods cannot be net complements. Since, c_j appears in the intercept and uncommitted income term, the parameters of the linear expenditure system must be estimated by an interative procedure. Based on the preceeding analysis, the linear expenditure system has three major limitations: a) Engel curves are linear; i.e.,

$$q_i = c_i + \beta_i \left[\left(M - \sum_{j=1}^{n} p_j c_j\right)/p_i\right],$$

b) goods cannot be complements, and c) estimation is iterative when $c_i \neq 0$. Another disadvantage of additivity which is more relevant from a theoretical than empirical viewpoint is that the equilibrium quantities are not invariant to monotonic increasing transformations of the utility function.

What then are the advantages of the linear expenditure system? Its main attraction is that it permits all price elasticities to be measured from estimates of β_i, c_i and price-quantity data. Since β_i and c_i can be estimated from an equation containing only one independent variable, the degrees of freedom problem is virtually eliminated. While the ability to measure all price elasticities from estimates of β_i and c_i appears to be advantageous, there is no way of assessing the reliability of the imputed price elasticities. Moreover, the simplified structure of the price elasticities is obtained independently of the structure of actual price elasticities as noted by Brown and Deaton. This same weakness occurs under block-additivity. In addition, the additivity assumption has been rejected for United Kingdom data (Deaton) and U.S. data (Theil, 1971). Based on these arguments, the present author does not favor use of the additivity hypothesis.

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$$b_i = \frac{\beta i}{p_i} = \frac{\partial q_i}{\partial (M - \Sigma p_j c_j)}$$

Under the second approach to demand estimation under separability, the demand function is directly specified. Restrictions implied by separability are then imposed on the demand function. This approach has been used by Boutwell and Simmons who work with a constant elasticity demand model. Assuming strong separability and using the Slutsky, Cournot and Engel aggregation equations, they reduce the number of parameters in the demand function from $n + 1$ to $n_R + 1$ ($n_R < n$). Unfortunately, the parameters of each demand equation must be estimated by an iteration process which involves the complete system of n demand equations.

The third estimation approach imposes the separability restrictions on the total differentials of the demand functions. This so-called Rotterdam demand model was developed by Theil (1975), Barten (1964, 1968, 1969), Barten and Turnovsky, and Parks. An advantage of this approach is that it does not require specification of the mathematical form of the demand function since use of differentials restricts model relevance to movements around a single equilibrium point. Secondly, the differential equation is linear in price and income slopes making it easy to impose separability restrictions. Assuming block-additivity of the utility function and approximating differentials by first differences gives,

$$\begin{aligned} w_r \Delta \log q_r = & \sum_{r' \neq r} w_r e^*_{rr'} (\Delta \log p_{r'} - \Delta \log p_r) \\ & - \frac{1}{\overset{v}{w}} w_r \eta_r \sum_{K \neq R} \sum_{K} [w_k \eta_k (\Delta \log p_k - \Delta \log p_r)] \\ & + w_r \eta_r (\Delta \log M - w' \Delta \log p) \qquad r, r' \varepsilon R \end{aligned}$$

where $w_r = p_r q_r / M$, $e^*_{rr'}$, are price elasticities (compensated to keep the marginal utility of money constant), η_r is an income elasticity and $\overset{v}{w}$ is money flexibility. This equation contains $n_R + 1$ parameters. Estimation is iterative unless prior estimates of income elasticities (η_r's) are available.

The Rotterdam model has been criticized for a) being a poor approximation of movements around a single equilibrium when the range of data is large and true demand is not approximately linear, b) the first differences approximation leads to a specification error as true demands become more non-linear, c) the demand functions are generally not integrable, and d) iterative estimation method has no known convergence properties[10].

To apply any of the three estimation approaches under separability (except for point-wise additivity), the partition of the commodity

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10 Draws heavily from de Janvry and Bieri (pp. 18-20).

vector for which separability is valid must be defined. Since no study has been able to estimate all $n^2 + n$ price and income slopes without imposing some restrictions on the demand parameters, it is not possible to directly determine the partition. Other methods have been devised to test for separability. The method used by de Janvry is to perform a cluster analysis on household budget data. Results based on German data suggest that food and non-food items are separable.

Testing for strong separability can be done for pre-defined partitions. Let Q_K and Q_R be group quantity indices, P_K and P_R group price indices and Q_{KM} and Q_{RM} income elasticity estimates for groups K and R, respectively. Given two observations on price and quantity for each group, and assuming the groups are strongly separable[11],

$$(dQ_K/dQ_{KM}) - (dQ_R/dQ_{RM}) = \theta(dP_R/P_R) - (dP_K/P_K)$$

From this equation, θ can be computed for each pair of groups considered to be strongly separable. Similarity of the θ's for all pairs of groups would indicate that the partition selected is strongly separable. This approach can be applied to situations where the group quantity and price indices and the group income elasticities have been estimated from two cross-sections at different points in time; e.g., from the 1955 and 1965-66 household food consumption surveys. An undesirable feature of this test for separability is that it may be necessary to repeat the process for a very large number of partitions. Prior information on partitions which are likely to be strongly separable would speed up the test procedure.

New Commodities

Changes in the number of commodities available to the consumer, say through the introduction of new products, is poorly accommodated by the neoclassical model. To overcome these and other limitations Lancaster (1966, 1971) has developed a new approach to consumer theory. Lancaster assumes that it is the characteristics of goods rather than goods themselves that are desired by consumers. Goods possess a particular combination of the specified characteristics. So, for example, food nutrients could be considered the desired characteristics of food products. Foods are related to nutrients through the consumption technology matrix which defines the vector of nutrients associated with each food item.

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11 Under strong separability local price and quantity indices exist as shown by Gorman.

The consumer optimization problem under this new approach is:

$$\begin{aligned} \underset{z}{\text{maximize}} \quad & u(z) \\ \text{subject to} \quad & z = B_q \\ & p'q \leq M \\ & q \geq 0 \end{aligned}$$

where z is a vector of characteristics, B is the consumption technology matrix, q and p are quantity and price vectors of commodities, respectively, and M is personal disposable income. Note that the utility function is defined over characteristics space, the budget set over goods space and the relationship between goods and characteristics by B. For simplicity B is assumed to be a linear transformation. Traditional theory is a special case where there is a one-to-one correspondence between goods and characteristics.

Within this framework dimensional changes in the goods space simply alter the number of columns in B. The number of characteristics and, hence, the arguments of the utility function remain the same. Hence, the new approach can accommodate new commodities without requiring the consumer to remaximize utility.

The above optimization problem cannot be solved by linear programming techniques because the objective function and budget constraint are defined over different spaces. Of course, if B were square (number of characteristics = number of goods) and non-singular, the constraint set could be rewritten as $p'B^{-1}z \leq M$ and the problem could be solved by Langrangean techniques. Furthermore, demand functions for characteristics could be derived. Measurement of these demand equations is typically not feasible because characteristics and their prices are not generally observable. In the case of food products, data on consumption of nutrients is available and their prices could be derived from knowledge of p and B. Unfortunately, since the number of nutrients is less than the number of food items B is not square and, hence, the above procedure breaks down.

An alternative tact is to redefine the consumer optimization problem. Let E be the efficiency frontier of the characteristics space. This frontier is determined from the combination of characteristics given by the income-constrained maximum consumption of each good taken separately. The goods efficiency set is defined as the set of all combinations of goods whose images lie on the efficiency frontier; i.e., $E_G = \{q|Bq \varepsilon E, p'q \leq M, q \geq 0\}$. Lancaster shows that every $q^* \varepsilon E_G$ is the solution to the following canonical linear program:

$$\underset{q}{\text{minimize}} \quad p'q$$

$$\text{subject to} \quad Bq = z^*$$

$$q \geq 0.$$

The solution to this program gives an efficient bundle of goods for the prespecified collection of characteristics. This formulation of Lancaster's characteristics-goods model has been used by Prato and Bagali to isolate the nutrition component of demand for food products. An obvious disadvantage of this model is that new or modified commodities alter the objective function and constraints. Hence, a new optimization is required.

The above model can be used to investigate the response of food consumption to price and food expenditure changes for various nutrient intake levels. To illustrate, suppose z^* is the nutrient intake desired by a particular household. At price vector $p^{(1)}$ suppose $q^{(1)}$ minimizes expenditure subject to attainment of z^*. Next, change prices to $p^{(2)}$ and find the corresponding $q^{(2)}$ which minimizes expenditure. Comparison of $q^{(1)}$ and $q^{(2)}$ to $p^{(1)}$ and $p^{(2)}$ would provide estimates of quantity response to price changes. These estimates are not based on observed behavior but rather hypothetical choice situations. In applying this estimation method the food expenditure given by the optimal solution should not exceed or fall short of budgeted food expenditure. If the original minimum expenditure given by the LP solution is below or above budgeted food expenditure for the household, the original optimum q can be adjusted upward or downward, respectively, to obtain a new optimum which exactly exhausts budgeted food expenditure[12].

In summary, Lancaster's theoretical model does provide a suitable means of handling new or modified commodities. However, in its original form, it is difficult to apply to the estimation of demand parameters. Reformulation of the model as an expenditure minimizing linear program provides some potential for estimating quantity response to price and food expenditure changes. Despite the theoretical potential of Lancaster's new approach to demand theory, it requires further refinement for empirical applications.

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12 For example, suppose the original LP solution q^* costs $k^* = p'q^*$. If budgeted food expenditure is λk^*, the corresponding optimal quantity and nutrient vector would be λq^* and λz^*. This method holds for any linear program.

Functional Form

The two most popular functional forms used in the specification of demand functions are the linear in actual variate and linear in logarithms. The former implies variable and the latter constant price and income elasticities. In specifying Engel functions, three algebraic properties seem desirable. First, there may be an "initial income" below which a commodity is not purchased. Second, consumption of some commodities may reach a "satiation level" beyond some level of income. Third, the sum of all expenditures must equal income; i.e., the "adding-up" property. The semi-log function ($y = a + b \log x$) and the reciprocal function [$Y = a + b(1/x)$] have an initial income but only the latter has as asymptote. The log-inverse function [$\log y = a + b(1/x)$] has an asymptote and a zero initial income. The double-log form which is quite popular in budget studies lacks an asymptote and is not defined for zero initial income. To satisfy the adding-up property, not all Engel curves can have asymptotes. For food products, the existence of asymptotes or satiation levels seems reasonable yet there is no reason to expect all food items to have identical Engel function.

In addition to the three properties discussed above, each of the functional forms slightly different behavior for the elasticities. Elasticity behavior for the linear and log-linear forms has already been mentioned. For the log-inverse function, the elasticity is inversely proportional to the independent variable. In the case of the semi-log function, the elasticity is inversely proportional to the dependent variable. The elasticity decreases with increases in the product of the dependent and independent variables in the case of the reciprocal function. Prais and Houthakker show that decreasing elasticity fits data better than constant elasticity Engel functions.

In empirical applications, it would be wise to test a variety of functional forms using criteria such as goodness-of-fit (R^2) and the significance of coefficients. In studies of food products, forms containing satiation levels should do quite well provided the range in income is sufficiently large. It should be noted, however, that satiation levels make more sense when applied to a household than to a large aggregate of consumers.

Aggregation

The aggregation issues judged to be most pertinent to the regional project are aggregation over individual members of a household and aggregation over non-food products. Aggregation over consumers beyond the household level is not necessary when household data is available. The extent to which non-food items can be aggregated depends on the assumptions made concerning separability between food and non-food items. This section focuses on aggregation over household members.

Variations in household consumption invariably reflect differences in household size and composition. The simplest hypothesis for accommodating differences in household size is to assume no economies of size in household consumption[13]. This implies that per capita consumption of an item is a function of per capita income. In household food expenditure analysis, the analogous statement is that per capita expenditure on a food item is a function of per capita food expenditure. Such formulations are quite restrictive because a) they do not permit economies of size in household consumption and b) differences in household composition are not accounted for.

Techniques for incorporating household size and composition effects into an Engel function, which is typically the focus of household consumption analysis, have been developed by Prais and Houthakker. Their approach essentially involves measuring household size on a specific and income scale. The specific scale is pertinent to each commodity and is used to determine the number of unit consumers of that item in the household. Household consumption or expenditure is then expressed in terms of unit consumers by dividing the former by the latter. The unit consumer scale essentially measures the relative requirements of different types of persons for various commodities. With this approach, household income or expenditure is measured on an income scale. The latter is a weighted average of the specific scales with weights approximately proportional to relative expenditures on the commodities consumed by the household.

The problem with using the specific and income scales is that they enter the Engel function in a non-linear manner. Prais and Houthakker propose an iterative method to estimate the scales. The cumbersome estimation procedures required with this approach is perhaps the biggest factor limiting more widespread use of the method. Fortunately, there are other more straightforward approaches to incorporating size and composition effects into Engel functions. The most popular method is based on the following formulation:

$$q_i = f_i\,(n_1, \ldots, n_t, m)$$

where q_i is household consumption of good i, n_j (j=1, . . ., t) is the number of household members of type j in the household and m is

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13 Economies of size in consumption implies that for households with the same income per capita, larger families would enjoy a higher standard of living in terms of, say, a larger per capita expenditure on luxuries.

household income. In this formulation the effects of different types of household members on household consumption are measured separately. This method uses up slightly more degrees of freedom than the method employed by Prais and Houthakker, but estimation is straightforward. The approach is also useful for forecasting consumption for various household types. Applications of this approach have been made by Prato (1971) and Raunikar et al.

Implications for Regional Research Effort

This section evaluates the usefulness of existing demand models for the estimation objectives of the proposed regional project and points out specific needs related to data and model development.

The first phase of objective "a" of the regional project is to "define and estimate parameters describing demand and consumption decisions with particular emphasis on food demand behavior and nutritional intake." Discussion of procedures for accomplishing this phase indicates that conceptual demand models developed for this purpose should incorporate effects due to price, income, urbanization, region, race, religion, household size and composition, etc.

It is the author's contention that while such a model can be specified, its parameters could not be estimated from the data sets identified in the proposal. In particular, the 1960-61 BLS Consumer Expenditure survey and the 1965-66 USDA Household Food Consumption survey were conducted over short enough intervals of time to justify treating the data as cross-sectional. A recognized limitation of cross section data is that it cannot be used to estimate price effects unless the utility function is assumed to possess restrictive additivity properties. However, these data are well suited to the measurement of income and expenditure elasticities. For example, Engel functions for food items could be measured from USDA household data. Because of the richness of these data, the Engel functions estimated could account for slope and/or intercept differences associated with season, region, urbanization, household size and composition and other social and ethnic characteristics of the household.

If the utility function is assumed to be strictly additive, some or all of the price effects could be measured from cross-section data. Because of the artificially induced structure of price effects under additivity, it would be preferable to impose less restrictive assumptions on the structure of the utility function such as strong separability.

Proceeding with the strong separability hypothesis, suppose time series and cross-section data were available for two strongly separable groups say food (F) and clothing (C)[14]. Income and expenditure elasticities for food and clothing items could be estimated from the cross-section data. Using these estimates along with price-quantity data, θ could be measured from:

$$\hat{\theta} = (d\tilde{Q}_F - d\tilde{Q}_C)/(d\tilde{P}_C - d\tilde{P}_F)$$

where: $d\tilde{Q}_F = \sum_f P_f dP_f / b_f$ and

$$d\tilde{P}_F = \sum_f b_{f/F} dP_f; \; b_f = \partial a_f/\partial M, \; b_{f/F} = \partial q_f/\partial M_F,$$

M_F = food expenditure. Second-stage price slopes could then be measured from time series data using the differential of the second-stage demand functions:

$$d_{qf} = \sum_{f'\varepsilon F} \left(\frac{\partial q_f}{\partial P_{f'}}\right)_{M_F} dP_{f'} + \frac{\partial q_f}{\partial M_f} dM_F \qquad f \; \varepsilon \; F$$

This function includes prices of all food items and food expenditure. Since the food expenditure slope has already been estimated from the cross-section data, it can be treated as an extraneous estimate. This would eliminate one source of multicollinearity. Finally, the two-stage price slopes for food items can be estimated from:

$$\partial q_f/\partial P_{f'} = (\partial q_f/\partial P_{f'})_{M_F} - b_{f/F} \, (1 + b_F)(q_{f'} + \theta b_{f'})$$

where f, f' ε F.

The above procedure may seem very involved. Nevertheless, a much greater effort is required to estimate price effects than income effects. The procedure is also somewhat more involved than the one used by George and King (1971) and de Janvry and Bieri. In these studies food and non-food items were assumed to be block additive which simplifies the measurement of price slopes.

There are many other types of separability that could be imposed on the utility function. In each instance, somewhat different estimation procedures and data are required. For specific treatment of these approaches see de Janvry. If one is not prepared to make any separability assumptions the estimation of price effects becomes an almost

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14 Price data are periodically published by the Bureau of Labor Statistics.

hopeless task because of insufficient degrees of freedom and other statistical problems outlined in this paper.

Research needs with respect to the estimation objectives of the regional project do not appear unwieldy. Initially, a consensus must be reached regarding the assumptions to be made concerning the structure of the utility function. Ideally, this decision should follow a careful assessment of available data sources and the results of previous studies. While it may be tempting to use some of the George and King demand estimates in the estimation models agreed upon for the regional project, the assumptions implicit in the former should not be inconsistent with those adopted for the latter. Failure to recognize this limitation of model hybridization can lead to misleading demand parameter estimates.

CONCLUDING REMARKS

It is hoped that the issues raised in this paper will provide some guidance in the specification and estimation of food demand relationships. While the literature on demand estimation problems is quite extensive, it is primarily directed at ways of bridging the gap between theory and empirical applications. The models developed for this purpose employ a wide variety of assumptions concerning the structure of the utility function and, hence, have different data requirements. Before a demand estimation model can be chosen, several issues must be settled including: the choice of demand parameters to be estimated, the data to be used in estimation, the assumptions to be made concerning separability (if any), the intended uses of the results, etc. Settlement of these issues will narrow the choice of demand models. This process will involve compromises and trade-offs since conditions for estimation are never ideal, particularly in the social sciences. On the other hand, it is the absence of ideal conditions that makes estimation a challenging research area.

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