Massachusetts
Agricultural and Resource Economics
Staff Paper

A DUAL APPROACH TO THE MEASUREMENT OF TOTAL FACTOR PRODUCTIVITY GROWTH IN THE CORN BELT REGION

by

Daniel A. Lass
and
Robert D. Weaver

Research Paper Series No. 88-3
August 1988

Department of Agricultural and Resource Economics
Draper Hall
University of Massachusetts
Amherst, MA 01003

A DUAL APPROACH TO THE MEASUREMENT OF TOTAL FACTOR PRODUCTIVITY GROWTH IN THE CORN BELT REGION

by

Daniel A. Lass
and
Robert D. Weaver

Research Paper Series No. 88-3
August 1988

Paper presented
Annual Meetings of the
American Agricultural Economics Association
Knoxville, Tennesee
August 1, 1988

The authors are Assistant Professor, Department of Agricultural and Resource Economics, University of Massachusetts and Associate Professor, Department of Agricultural Economics and Rural Sociology, The Pennsylvania State University.

A Dual Approach to the Measurement of Total Factor Productivity Growth in the Corn Belt Region

Abstract

State level estimates of total factor productivity growth were derived for the Corn Belt region. The dual profit function approach was used for estimation. The rates of growth were found to vary across states ranging from 0.0143 (Missouri) to 0.0442 (Indiana). The results also supported the contention that growth rates declined during the 1970's.

I. Introduction

Empirical measures of agricultural productivity have been derived from index number procedures and aggregate production function estimates. Previous literature has employed methodologies to characterize agricultural production and measure productivity which can be improved on at least three fronts. Initially, the methodologies used imply restrictions on the structure of production in agriculture. Both the index number (Ball) and the aggregate production function (Griliches (1964)) methodologies imply that outputs and inputs form separable quantities. The use of Cobb-Douglas and constant elasticity of substitution empirical forms for the econometric estimation of production functions limits the substitution possibilities of inputs and imposes homogeneity on the production technology. Christensen and Weaver (1980) have argued that the problems of separability, nonhomogeneity as well as the joint production of outputs should be considered in analyzing agricultural productivity.

Secondly, empirical estimates of agricultural production functions do not address the endogeneity of input choice. Recent dual approaches which employ the cost function to measure agricultural productivity (Brown) specify outputs as exogenous. Agricultural outputs are also endogenous choices and should be modeled in a manner consistent with an economic behavioral assumption.

Finally, empirical estimates of agricultural productivity have often utilized aggregate U.S. data (i.e., Ball and Brown). Such analyses imply that the production technology employed in agriculture is the same throughout the U.S. This assumption is clearly restrictive. Production and productivity should be analyzed for homogeneous regions of the U.S. to account for environmental and physical differences.

The primary objective of the research was to develop theoretical and empirical measures of total factor productivity growth ($\dot{TFP}$) from a model which characterizes agricultural production as joint and nonhomothetic. The dual profit function (Lau (1972, 1976)) provides an empirical tool which will allow the measurement of $\dot{TFP}$ consistent with our objective. The model was applied to the Corn Belt region for the period 1950-1982.

II. Theoretical Model

We assume that the firm chooses a (Mx1) vector of outputs Y and a (Nx1) vector of inputs X so as to maximize profits. The optimal quantities of outputs and inputs are chosen given expectations of the harvest time market prices (P) at the time production decisions are made and the set of input prices (R). The model implicitly assumes that farmers are risk-neutral producers.

The product transformation function is denoted by:

(1) $F\left(Y,X;\Theta,\tau\right) = 0,$

where Θ is the vector of economic and environmental constraints the firm faces and τ denotes the level of technology. The transformation function is assumed to satisfy the usual neoclassical regularity properties. Duality theory links the product transformation function and the expected profit function:

(2) $\Pi^{*} = \Pi^{*}\left(P,R;\Theta,\tau\right).$

Weaver (1980) has shown the usefulness of the profit function in measuring $\dot{TFP}$. We follow the conventional definition of $\dot{TFP}$ as the difference between the rates of growth of real output and real input (Solow, Jorgenson and Griliches). The primal measure of $\dot{TFP}$ is derived from the product transformation function. Totally differentiating (1) with respect to time and writing in growth rate form we obtain:

$$(3) \sum_i \frac{\partial F}{\partial Y_i} \cdot Y_i \cdot \dot{Y}_i + \sum_h \frac{\partial F}{\partial X_h} \cdot X_h \cdot \dot{X}_h + \sum_g \frac{\partial F}{\partial \Theta_g} \cdot \Theta_g \cdot \dot{\Theta}_g + \frac{\partial F}{\partial \tau} \cdot \tau \cdot \dot{\tau} = 0$$

where $\dot{Y}_i$ denotes the rate of growth of the i^{th} output. A measure of $\dot{TFP}$ can be derived by assuming that resources are efficiently allocated and that there are constant returns to scale:

$$(4)\ \dot{TFP}_{CRTS} = \sum_i \rho_i \dot{Y}_i - \sum_h \gamma_h \dot{X}_h = \left\{ \left(\lambda \frac{\partial F}{\partial \tau} \right) \Big/ \left(\sum_i P_i Y_i \right) \right\} \tau + \dot{\tau},$$

where:

$$\rho_i = \frac{P_i Y_i}{\left(\sum_i P_i Y_i \right)},$$ the share of total value for the i^{th} output, and

$$\gamma_h = \frac{R_h X_h}{\left(\sum_i P_i Y_i \right)},$$ the share of total value for the h^{th} input.

The measure of $\dot{TFP}$ in equation (4) is the measure employed by Jorgenson and Griliches and more recently Ball. An alternative measure is developed by allowing the existence of short-term profits and the set of constraints, Θ. The rates of growth for outputs, inputs and constraints are aggregated using profit shares as weights. The primal measure of $\dot{TFP}$ is:

$$(5)\ \dot{TFP}_{\Pi} = \sum_i \pi_i \dot{Y}_i - \sum_h \pi_h \dot{X}_h - \sum_g \pi_g \Theta_g = \pi_\tau \cdot \dot{\tau}$$

where π_i and π_h are the profit shares for the i^{th} output and h^{th} input, respectively. The terms π_g and π_τ represent implicit profit shares for the constraints and technology, respectively.

---

[1] For ease of exposition we assume fixed inputs are contained in the vector X and are paid a normal return.

The dual measure of $\dot{TFP}$ can be derived by totally differentiating the expected profit function, (2), with respect to time:

$$(6)\ \frac{d\pi^*}{dt} = \sum_i \frac{\partial\pi^* dP_i}{\partial P_i dt} + \sum_h \frac{\partial\pi^* dR_h}{\partial R_h dt} + \sum_g \frac{\partial\pi^* d\Theta_g}{\partial\Theta_g dt} + \frac{\partial\pi^* d\tau}{\partial\tau dt}$$

Applying Hotelling's Lemma and writing in growth rate form, we derive the dual measure of $\dot{TFP}$ under short-term profit maximization as:

$$(7)\ \dot{TFP}_\Pi = \dot{\Pi}^* - \sum_i \pi_i \dot{P}_i + \sum_h \pi_h \dot{R}_h - \sum_g \pi_g \dot{\theta}_g = \pi_\tau \cdot \dot{\tau}$$

The measure of $\dot{TFP}$ from the dual expected profit function is identical to the primal measure of $\dot{TFP}$ in equation (5). This result was established by Jorgenson and Griliches for the constant returns to scale case. The result here, due to Weaver (1980), is more general and does not rely on the assumption of constant returns to scale.

III. Empirical Model

Recent examples of profit function applications to U.S. agriculture include Antle, Shumway and Weaver (1983). The empirical form of the expected profit function used in this study was the normalized quadratic form. Consider the expected profit definition:

$$(8)\ \Pi = \sum_i P_i Y_i - \sum_i R_h X_h$$

Maximization of (8) is equivalent to the maximization of normalized expected profits (Lau (1978)):

$$(9)\ \tilde{\Pi} = \Pi/R_N = \sum_{i=1}^{M} \tilde{P}_i Y_i - \sum_{h=1}^{N-1} \tilde{R}_h X_h - X_N$$

where $\tilde{\Pi}$, $\tilde{P}_i$ and $\tilde{R}_h$ are the vectors of expected profits, expected output prices and input prices normalized by the N^{th} variable input price.

Normalization ensures that the homogeneity conditions will be met. The normalized expected profit function is then:

$$(10)\ \tilde{\Pi}^* = \tilde{\Pi}^*\left(\tilde{P},\tilde{R};\Theta,\tau\right) = \Pi^*\left(P,R;\Theta,\tau\right)/R_N$$

To empirically implement the model, the normalized expected profit function was approximated by the quadratic flexible form:

$$(11)\ \tilde{\Pi}^* = \alpha_0 + \sum_{i=1}^{M}\alpha_i\tilde{P}_i + \sum_{h=1}^{N-1}\alpha_h\tilde{R}_h + \sum_{g=1}^{G}\alpha_g\Theta_g + \alpha_\tau\cdot\tau$$

$$+\frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}\beta_{ij}\tilde{P}_i\tilde{P}_j + \sum_{i=1}^{M}\sum_{h=1}^{N-1}\beta_{ih}\tilde{P}_h\tilde{R}_h$$

$$+\sum_{i=1}^{M}\sum_{g=1}^{G}\beta_{ig}\tilde{P}_i\Theta_g + \sum_{i=1}^{M}\beta_{i\tau}\tilde{P}_i\tau$$

$$+\frac{1}{2}\sum_{h=1}^{N-1}\sum_{k=1}^{N-1}\beta_{hk}\tilde{R}_h\tilde{R}_k + \sum_{h=1}^{N-1}\sum_{g=1}^{G}\beta_{hg}\tilde{R}_h\Theta_g + \sum_{h=1}^{N-1}\beta_{h\tau}\tilde{R}\cdot\tau$$

$$+\frac{1}{2}\sum_{g=1}^{G}\sum_{f=1}^{G}\beta_{gf}\Theta_g\Theta_f + \sum_{g=1}^{G}\beta_{gt}\Theta_g\tau + \frac{1}{2}\beta_{\tau\tau}\tau^2$$

Hotelling's Lemma links the normalized expected profit function with the optimal supply and demand functions:

$$(12)\ Y^*_j = \alpha_j + \sum_{i=1}^{M}\beta_{ij}\tilde{P}_i + \sum_{h=1}^{N-1}\beta_{hj}\tilde{R}_h + \sum_{g=1}^{G}\beta_{gj}\Theta_g + \beta_{\tau j}$$

for all $j = 1,\ldots,M$; and

$$(13)\ -X^*_k = \alpha_k + \sum_{i=1}^{M}\beta_{ik}\tilde{P}_i + \sum_{h=1}^{N-1}\beta_{hk}\tilde{R}_h + \sum_{g=1}^{G}\beta_{gk}\Theta_g + \beta_{\tau k}$$

for $h = 1,\ldots,N-1$. We append stochastic errors to equations (11)-(13) which are assumed to be contemporaneously correlated and serially independent. A complete set of measures of the production technology can be derived from the system (11)-(13) (see, for example, Shumway or Weaver (1983)). We focus in this paper on the measure of $\dot{TFP}$.

The conceptual measure of $\dot{TFP}$ relied upon our ability to identify a single index of the level of technology. However, the process of technology change and adoption cannot be easily characterized. Factors such as research and extension efforts, operator education and managerial ability contribute to measured $\dot{TFP}$. We assume that shifts of the production surface are due to technical change (Solow). We further assume that the elements of this residual change can be decomposed into those components which are continuous over time and those which are stochastic. We assume that the continuous or systematic portion can be captured by a time trend. The empirical measure of $\dot{TFP}$ follows directly from equation (7):

$$(14)\quad \dot{TFP}_{\Pi} = \frac{\partial \Pi}{\partial \tau} \cdot \frac{\tau}{\Pi} \cdot \dot{\tau} = R_N \frac{\partial \tilde{\Pi}}{\partial t} \cdot \frac{t}{\Pi} \cdot \dot{t}.$$

The empirical measure for $\dot{TFP}$ is a combination of the shadow price for technology, $\frac{\partial \Pi}{\partial \tau}$, the exogenous variable t and the expected profit levels. The empirical measure iscalculated directly from the parametric measure of the shadow price. To the extent that the variable t characterizes systematic changes in the level technology, $\dot{TFP}$ as measured here will be free of the effects of other factors. This allows us to circumvent the Diamond-McFadden impossibility theorem (Berndt and Khaled).

IV. Data and Estimation

Data were collected for 19 output categories and 11 input categories for each of the five Corn Belt states. The output and input categories included in the data set are presented in Figure 1. Expected price indexes and quantity indexes were calculated for outputs. Expected revenue indexes were then calculated for each output. Divisia form indexes were calculated where adequate data existed. Outputs were then aggregated to the six groups: feed crops, soybeans, other crops, dairy, poultry and meat animals. Feed crops included hay and other crops included food grains, vegetables and cotton.

To establish the variable input accounts, state level expenditure and price data were collected for most input categories. Implicit quantity indexes were then derived using the expenditure and price indexes. Service flows were calculated for the durable inputs tractors, trucks, autos, other farm machinery and structures. The input categories were aggregated to eight final variable input accounts: purchased feeds, purchased livestock, purchased seeds, fertilizer and limestone, hired labor, pesticides, machinery operating inputs and farm operating inputs. Service flows for motor vehicles and machinery were combined with motor fuels and motor supplies to form the machinery operating category. Similarly, we aggregated service structures with farm repairs and operation into the farm operating inputs account. Input quality change was assumed to be captured by the Divisia form price indexes. Since the USDA price data for machinery has been argued to overstate quality change, the Bureau of Labor Statistics price indexes were used in calculating service flow prices.

Four factors considered exogenous were also included in the data set. Land was measured in terms of the number of acres of productive land on farms. Two categories of land were included, cropland and pastureland. Operator and family labor was measured by the USDA data for the number of operators and family workers. A binary variable was included as a measure of

Figure 1. Farm Production Accounts for the Corn Belt States.

OUTPUT ACCOUNTS

Feed Grains:
- Corn
- Oats
- Barley
- Sorghum

Food Grains:
- Wheat
- Rye

Soybeans

Hay

Vegetables:
- Processing Vegetables
- Fresh Market Vegetables

Cotton

Dairy

Meat Animals:
- Cattle and Calves
- Hogs
- Sheep and Lambs

Poultry:
- Eggs
- Chickens
- Broilers
- Turkeys

INPUT ACCOUNTS

Purchased Variable Inputs:
- Feed
- Livestock
- Seeds
- Fertilizer
- Limestone
- Hired Labor
- Pesticides
- Motor Fuels and Motor Supplies
- Farm Repairs and Operation
- Motor Vehicles and Machinery
- Service Structures

Fixed Inputs:
- Operator and Family Labor
- Land

OTHER EXOGENOUS FACTORS

Weather
Technology

weather conditions at spring planting. The binary variable was developed from USDA reports of spring weather (Krainik) and was used to indicate information available to farmers when making planting decisions. The effects of deviations from normal weather for the remainder of the season were assumed to be stochastic. The final variable included, a time trend, was assumed to measure the systematic component of technical change.

Preliminary investigations of the exogeneity of the fixed factors indicated that operator and family labor was not exogenous. If the variable was endogenous we would have anticipated a significant relationship between operator and family labor and the wage rate. This was not observed. Data limitations precluded appropriate modeling of such an economic choice. We therefore chose to proceed with caution in assuming that operator and family labor did not represent a fixed factor or constraint on production.

The final specification of the exogenous variables included relative prices, an aggregate measure of productive land, the binary weather variable and the time trend. Relative price indexes were calculated using the price of purchased feeds as the numeraire. The model defined by equations (11)-(13) was then estimated using an iterative Zellner's procedure. Symmetry restrictions were imposed to make the model consistent with the assumption of profit maximization. A complete set of price elasticities, short-run returns to size and biases of technical change, were obtained for each of the five states.[1] In general, own-price parameters were of the proper sign and were statistically significant. A measure of explanatory power for the models was provided by the approximate R^2's for expected profits. The models explained more than 80 percent of the variation for each of the five states.

---

[1] A complete set of results for each state are reported in Lass.

V. Empirical Results

The estimated total factor productivity growth rates ($T\dot{F}P$) are reported in Table 1. The estimates of $T\dot{F}P$ for each state generally show strong growth during the 1950's. Growth rates were then found to decline from the early 1970's to the end of the time period for this study. Missouri was an exception, showing stronger growth rates through the 1960's and 1970's.

The estimates for the five Corn Belt states are compared to the USDA estimates for the Corn Belt and the U.S. and Ball's aggregate U.S. estimates in Table 2.[2] The post-war period, 1950-1982, was broken into six time periods which Ball identified as periods corresponding to post-war business cycles. A final time period (1979-1982) not covered by Ball's study is also included. Estimates of $T\dot{F}P$ for these periods by the USDA were greater than those estimated here for all states except Indiana. The USDA estimates for the Corn Belt show an increase in $T\dot{F}P$ during the 1979-1982 period. We found that $T\dot{F}P$ declined during this period for all the Corn Belt states. The estimates of average annual $T\dot{F}P$ for this study for the entire period 1950-1982 were comparable to the estimates obtained from the USDA for the Corn Belt. Estimates reported here for several states were slightly lower while Indiana demonstrated an average annual growth rate which was nearly twice the growth rate reported by the USDA for the Corn Belt.

The state level estimates of $T\dot{F}P$ were aggregated and were compared to the USDA estimates for the Corn Belt. Figure 2 illustrates the differences between our aggregate estimates for the Corn Belt and the USDA estimates.

---

[2] The results are not directly comparable since the methodology and levels of aggregation vary substantially.

Table 1. Estimated Annual Total Factor Productivity Growth Rates for the Corn Belt States, 1950-1982.

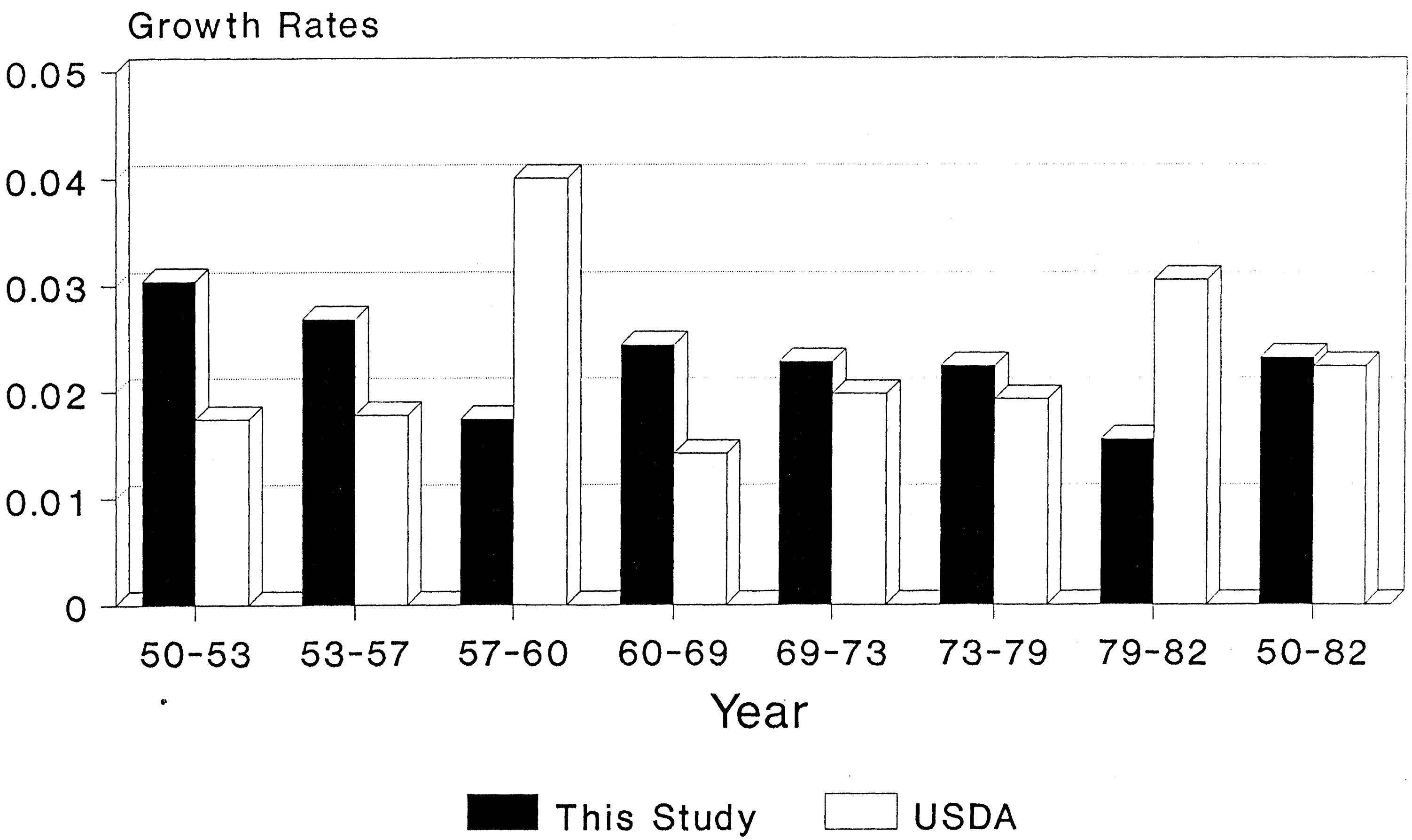
	State				
Year	Illinois	Indiana	Iowa	Missouri	Ohio
1950	0.0309	0.0621	0.0203	0.0159	0.0226
1951	0.0378	0.0700	0.0303	0.0189	0.0382
1952	0.0215	0.0567	0.0228	0.0138	0.0158
1953	0.0230	0.0549	0.0254	0.0129	0.0147
1954	0.0218	0.0540	0.0261	0.0100	0.0198
1955	0.0137	0.0464	0.0231	0.0079	0.0107
1956	0.0269	0.0578	0.0333	0.0056	0.0348
1957	0.0255	0.0638	0.0280	-0.0021	0.0291
1958	0.0131	0.0405	0.0236	0.0098	0.0173
1959	0.0053	0.0380	0.0168	0.0101	0.0041
1960	0.0059	0.0417	0.0180	0.0083	0.0075
1961	0.0208	0.0513	0.0225	0.0127	0.0296
1962	0.0160	0.0421	0.0219	0.0120	0.0235
1963	0.0152	0.0459	0.0175	0.0134	0.0282
1964	0.0137	0.0523	0.0183	0.0117	0.0276
1965	0.0217	0.0580	0.0232	0.0152	0.0441
1966	0.0210	0.0525	0.0206	0.0195	0.0370
1967	0.0163	0.0447	0.0201	0.0179	0.0144
1968	0.0113	0.0462	0.0211	0.0144	0.0054
1969	0.0103	0.0324	0.0201	0.0125	0.0021
1970	0.0044	0.0313	0.0170	0.0128	-0.0006
1971	0.0118	0.0317	0.0206	0.0150	0.0068
1972	0.0209	0.0342	0.0242	0.0211	0.0239
1973	0.0297	0.0439	0.0288	0.0295	0.0492
1974	0.0164	0.0412	0.0219	0.0211	0.0170
1975	0.0117	0.0378	0.0237	0.0134	0.0042
1976	0.0108	0.0299	0.0206	0.0174	0.0067
1977	0.0212	0.0351	0.0256	0.0206	0.0232
1978	0.0239	0.0361	0.0249	0.0226	0.0284
1979	0.0189	0.0302	0.0207	0.0187	0.0272
1980	0.0101	0.0304	0.0159	0.0173	0.0027
1981	0.0180	0.0391	0.0183	0.0142	0.0231
1982	0.0045	0.0279	0.0177	0.0115	-0.0175

Table 2. Estimated Average Annual Total Factor Productivity Growth Rates.

Period	Illinois	Indiana	Iowa	Missouri	Ohio	Corn Belt[a]	Aggregate U.S. USDA[a]	Aggregate U.S. Ball
1950-53	0.0274	0.0605	0.0262	0.0152	0.0228	0.0174	0.0000	0.0252
1953-57	0.0220	0.0555	0.0276	0.0053	0.0236	0.0178	0.0223	0.0272
1957-60	0.0081	0.0401	0.0195	0.0094	0.0096	0.0400	0.0406	0.0270
1960-69	0.0162	0.0472	0.0206	0.0144	0.0235	0.0142	0.0094	0.0165
1969-73	0.0166	0.0353	0.0226	0.0196	0.0196	0.0198	0.0168	0.0130
1973-79	0.0171	0.0350	0.0229	0.0190	0.0177	0.0193	0.0299	0.0202
1979-82	0.0108	0.0325	0.0173	0.0143	0.0026	0.0305	0.0180	NA
1950-82	0.0174	0.0442	0.0222	0.0144	0.0187	0.0224	0.0199	NA

[a] Calculated from the USDA indexes of Total Factor Productivity (USDA).

Figure 2. Total Factor Productivity
Average Annual Growth Rates in
the Corn Belt Region



Estimates of $\dot{TFP}$ obtained in this study were generally greater than those estimated by the USDA for the Corn Belt for five of the seven time periods. The exceptions were the 1950-1953 and 1979-1982 time periods.

The estimates of $\dot{TFP}$ reported here measure the systematic portion of the growth in outputs minus the growth in inputs and fixed factors. This has been the generally accepted measure of $\dot{TFP}$. The measure may also capture those factors which are correlated with time which were not captured by the data set. Careful attention was given to capturing changes in input quality during construction of the data set. However, the variable measuring changes in operator and family labor was dropped from the empirical models. The measures of $\dot{TFP}$ reported in this study may therefore reflect changes in the level and quality of operator and family labor.

VI. Summary

The objective of this study was to obtain estimates of total factor productivity growth for the five Corn Belt states. The dual profit function was used to measure production and total factor productivity growth. Application of the dual profit function allowed us to relax the assumptions of separability and homogeneity which were maintained in previous research.

Estimates of total factor productivity growth varied across states. Estimates of the average annual growth rates for the entire period ranged from 0.0143 (Missouri) to 0.0442 (Indiana). While our aggregate measure of $\dot{TFP}$ for the Corn Belt region was comparable for the 1950-1982 period, there were substantial differences between the two measures within this period. Our results also support the contention that $\dot{TFP}$ has declined during the 1970s and 1980s contrary to estimates by the USDA for the Corn Belt region.

References

Antle, J. M. "Incorporating Risk in Production Analysis." American Journal of Agricultural Economics 65(1983):1099-1106.

Ball, V. E. "Output, Input and Productivity Measurement in U.S. Agriculture: 1948-1979." American Journal of Agricultural Economics (forthcoming), 1985.

Berndt, E. R. and M. S. Khaled. "Parametric Productivity Measurement and Choice Among Flexible Functional Forms." Journal of Political Economy 87(1979):1220-1245.

Brown, R. S. "Productivity, Returns, and the Structure of Production in the U.S. Agriculture, 1947-1974." Ph.D. Dissertation, University of Wisconsin-Madison, 1978.

Christensen, L. R. "Concepts and Measurement of Agricultural Productivity." American Journal of Agricultural Economics 57(1975):910-915.

Griliches, Zvi. "Research Expenditures, Education and the Aggregate Agricultural Production Function." American Economic Review 54(1964):961-974.

Jorgenson, Dale W. ;and Zvi Griliches. "The Explanation of Productivity Change." Review of Economic Studies 34(1967):249-284.

Krainik, A. M. "An Analysis of the Feed Grains Programs and Their Effects on Corn Acreage in Six Midwestern States." M.S. Thesis, The Pennsylvania State University, 1979.

Lass, D. A. "Estimation of Total Factor Productivity Growth in the Corn Belt Region: A Dual Approach to Measurement." Ph.D. Dissertation, The Pennsylvania State University, 1985.

Lau, L. J. "A Characterization of the Normalized Restricted Profit Function." Journal of Economic Theory 12(1976):131-163.

Lau, L. J. "Applications of Profit Functions." In Production Economics: A Dual Approach to Theory and Application, Vol. 1, pp. 133-216. Edited by M. Fuss and D. McFadden. Amsterdam: North-Holland Publishing Company, 1978.

Lau, L. J. "Profit Functions of Technologies with Multiple Inputs and Outputs." Review of Economics and Statistics 54(1972):281-289.

Shumway, C. R. "Supply, Demand, and Technology in a Multiproduct Industry: Texas Field Crops." American Journal of Agricultural Economics 65(1983):748-760.

Solow, R. M. "Technical Change and the Aggregate Production Function." Review of Economics and Statistics 39(1957):312-320.

United States Department of Agriculture. Economic Indicators of the Farm Sector: Production and Efficiency Statistics, 1982. National Economics Division, Economic Research Service, Washington, D.C., 1984.

Weaver, R. D. "Measurement and Forecasting of Agricultural Productivity." Department of Agricultural Economics and Rural Sociology Staff Paper No. 40, The Pennsylvania State University, 1980.

Weaver, R. D. "Multiple Input, Multiple Output Production Choices and Technology in the U.S. Wheat Region." American Journal of Agricultural Economics 65(1983):45-56.