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Abstract

The questions here are: can indirect methods relying on estimating Engel curves provide mathematically simpler ways of estimating price elasticities in demand systems than direct methods and, will the results be an improvement over available direct methods? In answering these questions a number of issues surface. The first has to do with specifying the size of the problem. As a practical matter, separability must be assumed. It can, however, appear in various ways. Second, if an indirect method is to be used to recover demand function parameters then we must consider the functional form of the estimating equations and the number of samples needed to obtain estimates. The samples may be from the same population at different points in time, or may be from different parts of the same population at the same time. In both cases price regimes must differ among samples.

Introduction

Suppose you have the data from an extensive survey such as the U.S. Department of Agriculture food consumption survey covering 31 food groups and approximately 15,000 households. What would be the best way to estimate the set of price elasticities contained in this demand system? If the Survey is sufficiently wide ranging there may be enough price variation in the data to use a direct method (e.g. Lau, Lin and Yotopoulos). At the other extreme if there is no price variation across households only Engel curves can be estimated. And only under the most restrictive conditions of a Cobb-Douglas utility function can price elasticities be recovered. The possibility is raised later that with some price variability an indirect method may recover price elasticities of a rather less restrictive utility function.

If there is sufficient price variation a second issue arises. The equations to be estimated are econometrically intractable and some added structure must be imposed on the system. Typically this takes the form of either an ad hoc single equation approach with explanatory variables selectively deleted or a system estimation of aggregate commodity groupings. Both of these lose information, either of a theoretical nature, or that contained within the disaggregate data. Applied formally, they constitute separability restrictions. Yet to estimate a system of this size, even with separability restrictions imposed, creates immense computational requirements and is not likely to deliver results of very high quality.

Is there an alternative approach? The present paper investigates an indirect method that consists of estimating Engel curves. It requires a number of subsamples, the number depending on the size of the system. Each subsample could consist of a separate survey in which there was no price variation across households or it could be a regional (or other) grouping from a larger survey with the same property. It is essential that there be price variation between subsamples for without this price elasticities cannot be recovered.

A desirable property of the Engel curves would be that they do not need to be parallel and linear to aggregate perfectly over consumers (i.e. to achieve the representative consumer). Deaton and Muellbauer propose a model, the Almost Ideal Demand (AID) System, which posseses the property of perfect aggregation without the restriction on Engel curve shape Its disadvantage, however, is that it is highly separability-inflexible. When separability is imposed, as it inevitably must be, the number of parameter restrictions is large. This function and this issue are examined in detail in later sections.

The organization of the paper is as follows. First, a classification of the approaches to estimating demand functions is presented, including the one described in detail later. Second, since separability of some form must be assumed, the implicatiionns for the structure of demand equations of the different types of separability are discussed. For the translog and price-independent generalized linear logarithmic (PIGLOG) utility functions these consequences are spelled out in detail. Next, having imposed separability, the number of free parameters in these two functions is calculated. Surprisingly, for a given number of commodities it turns out to be the same, indicating that under these conditions the AID system is no more flexible than a translog utility function. Calculation of the number of samples required to recover all of the parameters occupies the last part of the paper. The number depends on the function specified and the number of commodities in the system.

Methods of Estimating Systems of Demand Equations.

Estimation of a system of demand equations for disaggregated commodities is difficult. This is unfortunate. It is precisely the level that would be most useful in addressing such issues as policy analysis within a sector, forecasting, and explanation of the forces influencing the demand for a commodity. The fundamental problem is that the simultaneous estimation of all the parameters of a disaggregate demand system requires the imposition of many parameter restrictions across equations.

Researchers have tackled this problem in several ways.

(1) Ignore most of the system and estimate one or perhaps two demand equations directly. Introduce such explanatory variables (prices of other commodities in the usual form) as intuition or the findings of previous researchers suggest. Also, ignore integrability conditions; if the function is viewed as approximating some true but unknown relation it need not meet them. This method is perhaps the one most widely adopted in applied work.

(2) Proceed to estimate the system in an *ad hoc* way, equation by equation, making use of arbitrary separability conditions and imposing theoretical conditions (such as Slutsky symmetry) where needed, to obtain all cross coefficients or cross elasticities. The classic study along these lines was George and King.

(3) Use a highly restricted utility function. The system of demand equations is therefore guaranteed to possess the theoretical conditions but should still be simple enough to estimate. Examples include the linear expenditure system (LES) and the Rotterdam system. (For a review, see Barten.)

(4) Aggregate the data to a manageable number of commodities. This allows a variety of utility functions to be specified and estimated without an excessive number of explanatory variables in any demand equation. Parameter restrictions across equations are also not too burdensome. This approach is commonly found in theoretical studies.

(5) Estimate demand systems indirectly from Engel curves, indifference curves or marginal rates of substitution. Usually this approach is forced on a researcher by lack of data. For example, a survey of consumers in a small geographic area would find little, if any, variation in prices paid for a commodity. Two cross section samples would show interperiod difference in prices for each commodity, but no intraperiod variation. If the utility function is globally additive, then, in general, two such sets of data will suffice to identify demand systems (Dybvig). The indirect approach has not been widely used. It has also been combined with aggregation. For example Pollak and Wales examined three commodity groups.

Separability.

In any econometric study of demand, some form of separability must be imposed. There are just too many commodities to be economically tractable at once. Although global definitions of separability have been proposed (e.g. Bliss) the Leontief-Sono definition, which relies on local differentiability, is still most useful. According to the latter, given a utility function, $U(X_1, \ldots, X_n)$, variables X_i and X_j are separable from variable X_k if and only if

$$\frac{\partial}{\partial X_k} \frac{\partial U/\partial X_i}{\partial U/\partial X_j} = 0$$

Separability imposes structure; however, to obtain convenient functions or to achieve dual representations of a technology, further restrictions may be required. A number of types of separability may therefore be defined. The seminal work in this area is Blackorby, Primont and Russell. The section below depends heavily on their work.

If the utility function, U, satisfies the regularity conditions of continuity, positive monotonicity and quasi concavity then it is separable ("weakly separable") if

(1) $U = \hat{U}[U^1(X^1),\ldots,U^m(X^m)]$

where $\hat{U}$, the macrofunction and each aggregator function, U^r, display continuity, positive monotonicity and quasi-concavity (BPR Theorem 4.1 p.108). Addition of the slightly stronger condition of strict quasi-concavity is sufficient for strong decentralization. That is, the consumer's problem of maximizing utility subject to the budget constraint results in systems of ordinary demand functions

(2) $X^r = \phi^r(P^r, y_r)$

where the consumer's optimal consumption vector of commodities in the r^{th} sector depends only on prices of goods in the sector, P^r, and sectoral expenditure, y_r.

The same demand systems can be derived from a conditional indirect utility function

$$H = \hat{H}[h^1(y_1,P^1),\ldots,h^m(y_m,P^m)]$$

by the use of Roy's theorem, provided the function is partitioned in the same groups as equation (1). It is referred to as conditional because both it and equation (2) depend on the sectoral expenditures, y^r. Equation (2) therefore corresponds to estimation of the second stage of Strotz's two-stage budgeting procedure. The first stage, the expenditure allocation functions,

(3) $y_r = \theta^r[\Pi^1(P^1),\ldots,\Pi^m(P^m),y] \qquad r = 1,\ldots,m,$

are derived by maximizing conditional utility subject to a budget constraint. Without them, all of the parameters of the utility function cannot be calculated. The $\Pi^r(P^r)$ are sectoral price indexes, frequently the conventional Laspeyres, Paasche or Divisia indexes (See BPR pp.285-7 and the references cited therein).

Since the arguments of the expenditure allocation functions are price indexes, a frequent strategy was to substitute equation (3) into equation (2) giving

(4) $X^r = \phi^r(P^r,P,y)$

where P is the vector of price indexes of each sector or separable group. (See, for example, George and King.) Knowledge of total income, y, is also required to estimate (4). However, if the first stage budgeting can be assumed to be optimal, then the actual sectoral expenditure, y_r, can be used instead. No other data are needed for estimation.

Note that duality between direct and (unconditional) indirect utility functions cannot be established without further restrictions. In particular, specification of separability in the indirect utility function leads to allocation functions

(5) $$X^r = \tilde{\phi}^r\left(\frac{P^r}{y}, \frac{y^r}{y}\right)$$

in which both sector and total expenditures must be known. And while functions (2) possess the conditions of homogeneity of degree zero and Slutsky symmetry, functions (3) do not.

For an indirect utility function to give rise to a demand system of equation (2) stronger restrictions are needed. Sufficient conditions can be found if duality between an indirect utility function and the separable utility function (1) can be established. If in the direct utility function (1) each aggregator function, U^r, is homothetic the utility function displays homothetic separability. Note, however, this does not imply that the utility function itself is homothetic. Dual to a homothetically separable utility function (1) is an indirect utility function

(6) $W(y,P) = \hat{W}[y,\overline{W}^1(P^1),\ldots,\overline{W}^m(P^m)]$

where $\hat{W}$ is monotonic and each $\overline{W}^r$ is positively linearly homogeneous in prices (BPR Theorem 4.4 p.123). The set of prices in aggregator function $\overline{W}^r$ are those of the set of commodities in aggregator function U^r of equation (1).

If a cost function is chosen as starting point for the imposition of separability, slightly different demand systems result. If the cost function, $C(u,P)$ satisfies the regularity conditions, then it is separable if

(7) $C = \hat{C}\ [u,C^1(u,P^1),\ldots,C^m(u,P^m)]$

where $\hat{C}$ and C^r satisfy continuity, positive monotonicity, positive linear homogeneity and concavity in prices. In addition C should also be strictly positively monotonic in u (BPR Corollary 4.1.4 p.112). The C^r are sectoral cost functions.

Differentiation of the cost function (7) results in demand functions

(8) $X^r = \zeta^r(u,P^r)y_r$

in which the consumer must know the utility of "real income," u (BPR Theorem 5.5 p.192). Flexible functional forms such as the translog have frequently been used to specify direct or indirect utility functions, but not cost functions. With a cost function specification the problem is dealing with the unobservable utility or real income. Stronger assumptions on functional form are needed.

The homothetically separable direct utility function is equivalent to a homothetically separable cost function. The latter can be written as

$$C(u,P) = \hat{C}[u,\bar{C}^1(P^1),\ldots,\bar{C}^m(P^m)]$$

where C satisfies the same regularity conditions as before, $\hat{C}$ is monotonic and each $\bar{C}^r$ is linearly homogeneous in prices (BPR Theorem 4.4 pp. 123-4). The $\bar{C}^r$ are price indexes. Finally, if the direct utility function is itself homothetic then the cost function can be written

$$C(u,P) = \Gamma(u)\hat{C}[\bar{C}^1(P^1),\ldots,\bar{C}^m(P^m)]$$

(BPR Theorem 4.6 p.125). In this case a flexible functional form cost function, for example the translog, can be estimated from share equations that are not functions of utility or "real income."

An alternative way of imposing restrictions makes use of complete separability (also referred to as strong separability). If the utility function is continuous then it is completely strictly separable if it can be written

$$U(X) = U^*[\sum_{r=1}^{m} U^r(X^r)]$$

where U* is an increasing function (BPR Theorem 4.8 p.136). The extreme case, addivity, occurs when each set X^r consists of a single commodity. Duality results can only be obtained under stronger restrictions. The utility function must be homothetic (which property is inherited by each aggregator function, (BPR Corollary 4.8.1 p.139). Under these additional restrictions there is a cost function that is separable in the same partitions and can be written as either

$$C(u,P) = \Gamma(u)[\sum_{r=1}^{m} \bar{C}^r(P^r)^\rho]^{1/\rho}, 0 \neq \rho \leq 1$$

or

$$C(u,P) = \Gamma(u)\prod_{r=1}^{m} \bar{C}^r(P^r)^{\rho^r}, \rho^r > 0, \Sigma\rho^r = 1$$

The limiting cases when each set comprises a single commodity are the CES and Cobb-Douglas functions respectively. These are clearly strong restrictions.

The Separability Inflexibility of Functional Forms.

Denny and Fuss maintain that the translog function treated as an exact function is either a translog macrofunction of Cobb-Douglas aggregators or a Cobb-Douglas macrofunction of translog aggregators. BPR show that the situation, although restrictive, is not quite that bad. Any flexible functional form that is a Taylor's series expansion to the second order can be expressed as a mixture of the above polar cases (BPR Theorem 8.2 p.303 and for the translog, corollary 8.2.3 p.309).

Specifically, for the translog we have

$$(9)\quad \ln U(X) = \sum_{r=1}^{m} U^r(X^r) + \sum_{s=d+1}^{m}\ \sum_{t=d+1}^{m} \delta_{st} U^s(X^s) U^t(X^t)$$

Two types of aggregator function can be present

$$(10a)\quad U^r(X^r) = \sum_i \alpha_i \ln x_i + \tfrac{1}{2} \sum_{jk}\sum \beta_{jk} \ln x_j \ln x_k \qquad i,j,k \in I^r$$

$$(10b)\quad U^r(X^r) = \Sigma \gamma_i \ln x_i \qquad i \in I^s$$

The first component of (9) consists of both types of function. Those sets of commodities that are found in the second component must, however, always appear in the form of (10b). For most applications it would seem reasonable to begin with a subset of commodities specified as (10a), which would lead to demand equations like (2).

The Almost Ideal Demand System is specified through a cost function in the logarithms that also generates share equations that are price-independent generalized linear. The preferences represented are therefore members of the PIGLOG class, and the function has several desirable properties. Following Deaton and Muellbauer, it can be written.

(11a) $\ln C(u,P) = \alpha_0 + \Sigma\alpha_k \ln p_k + \frac{1}{2}\Sigma\Sigma\gamma_{kj} \ln p_k \ln p_j + u\beta_0\Pi p_k^{\beta_k}$

(11b) $\ln C(u,P) = \ln P + u\beta_0\Pi p_k^{\beta_k}$

and will be linearly homogeneous in prices if

$$\Sigma\alpha_i = 1, \sum_j\gamma_{kj} = \sum_k\gamma_{kj} = \Sigma\beta_j = 0$$

It will be noted that since the AIDS function consists of a translog cost function plus a Cobb-Douglas form, it contains more parameters than a parsimonious flexible functional form.

The AIDS function is even more separability inflexible than quadratic forms such as the translog. If a four commodity master AIDS cost function is specified, then {1,2} would be separable in lnC from {3,4} under the following conditions:

If $\beta_0 = 0$ (then the function degenerates to the translog) then with the usual translog restrictions, either

$$\alpha_1\gamma_{2k} - \alpha_2\gamma_{1k} = 0 \qquad k = 3,4$$

and

$$\gamma_{24}\gamma_{13} - \gamma_{14}\gamma_{23} = 0$$

for non-additive separability, or

$$\gamma_{13} = \gamma_{14} = \gamma_{23} = \gamma_{24} = 0$$

for additive separability.

If $\beta_0 = 0$ then

either $\beta_3 = \beta_4 = 0$

or $\beta_1 = \beta_2 = 0$

plus the restrictions above. The implication is that if a sectoral cost function takes the AIDS specification (11) then the $\beta_i (i \neq 0)$ on the excluded commodities must be zero. Therefore, the excluded sectors are either a linear combination of translog aggregators or a quadratic combination of Cobb-Douglas aggregators or both. Stated differently, one and only one of the aggregator functions of equation (9) takes the AIDS form. The desirable properties claimed for the AIDS over the translog by Deaton and Muellbauer are less impressive (With parameter restrictions on either the AIDS or the translog to ensure positive linear homogeneity in prices the additional benefits of the AIDS are Engel curves that need not be linear and parallel in different households, and the functional form is consistent with previous household budget data.).

The AIDS shares the advantage of any cost function that is positively linearly homogeneous in U: it is easily inverted to the indirect utility function. Treating equation (9) as a cost function of the r^{th} sector then from (11b)

$$\ln C^r(u,P^r) = \ln y_r = \ln P^r + u\beta_0 \prod_{k\epsilon r} p_k^{\beta_k}$$

Rearranging

$$(12)\quad u\beta_0 \Pi p_k^{\beta_k} = \ln\left(\frac{y_r}{P^r}\right)$$

Differentiating (11a) and multiplying by $P_i/C^r(u,P^r)$ gives

$$(13)\quad w_i = \alpha_i + \sum_{j\epsilon r} \gamma_{ij} \ln p_j + \beta_i u\beta_0 \prod_{k\epsilon r} p_k^{\beta_k}$$

Substituting (12) into (13) gives the AIDS demand functions in budget share form

$$w_i = \alpha_i + \sum_{j\epsilon r} \gamma_{ij} \ln p_j + \beta_i \ln\left(\frac{y_r}{P^r}\right)$$

<u>Parameters in the Translog and Aids Functions</u>

If the indirect translog utility function is homothetically separable (equation (6)) it consists of a number of aggregator functions of the form

$$\ln \overline{W}^r = \alpha_0 + \Sigma\alpha_i \ln p_i + \tfrac{1}{2}\Sigma\Sigma\beta_{ij} \ln p_i \ln p_j$$

where the prices are those of the n commodities in the partition I^r. As the result of utility maximization subject to the budget constraint $p_j x_j = y_r$ the n - 1 independent share equations are

$$\frac{p_i x_i}{y_r} = \frac{\alpha_i + \Sigma_j \beta_{ij} \ln p_j}{\alpha_m + \Sigma\beta_{mj} \ln p_j} \qquad i = 1,\ldots,n - 1$$

the n^{th} share equation being derived from the budget constraint. The denominator in each equation should be identical, which adds n + 1 parameters,

$$\alpha_m = \Sigma\alpha_i$$

$$\beta_{mj} = \sum_{j=1}^{n} \beta_{ij} \quad , \qquad j = 1,\ldots,n$$

giving a total of

$$(n - 1) + n(n - 1) + n + 1 = n^2 + n$$

(Since utility cannot be cardinally measured, α_0 cannot be recovered.)

Since the share equations are homogeneous of degree zero in the parameters, one must be set arbitrarily, e.g. through the normalization

$\alpha_m = \Sigma\alpha_i = -1$ (Christensen, Jorgenson, Lau) giving

$n^2 + n - 1$ parameters.

Now there are $\frac{n(n-1)}{2}$ symmetry restrictions of the form $\beta_{ij} = \beta_{ji}$

(Note that the β_{nj} are recoverable from

$$\beta_{nj} = \beta_{mj} - \sum_{i=1}^{n-1} \beta_{ij} \quad , \qquad j = 1,\ldots,n)$$

giving finally

$$n^2 + n - 1 - \frac{n(n-1)}{2}$$

$$= \frac{n^2 + 3n - 2}{2}$$

independent parameters in the non-homothetic translog utility function. The same result prevails starting from the direct utility function.

The translog expenditure function (7) has aggregator functions that are sectoral cost functions of the form

$$\ln C^r(u,P) = \gamma_0 + \Sigma\gamma_i \ln p_i + \gamma_{00} \ln u + \tfrac{1}{2}\delta_{00}(\ln u)^2$$
$$+ \tfrac{1}{2} \Sigma\Sigma\delta_{ij} \ln p_i \ln p_j + \Sigma\phi_i \ln u \ln p_i$$

with $1 + n + 1 + 1 + n^2 + n = n^2 + 2n + 3$ parameters.

There are $n(n - 1)/2$ symmetry restrictions again. Homogeneity of degree one in prices also requires

$$\Sigma\gamma_i = 1, \underset{i}{\Sigma}\delta_{ij} = \underset{j}{\Sigma}\delta_{ij} = \Sigma\phi_i = 0$$

This adds $n + 2$ parameter restrictions. In addition, although the dependent variable, expenditure, is measureable, ln u is not, so that of γ_0, γ_{00}, δ_{00} only one constant can be recovered, losing 2 more parameters so that again $(n^2 + 3n - 2)/2$ parameters can be estimated.

The AIDS cost function specification of equation (7) has sectoral cost functions

$$\ln C^r(u,P) = \alpha_0 + \Sigma\alpha_i \ln p_i + \tfrac{1}{2}\Sigma\Sigma\gamma_{kj} \ln p_i \ln p_j$$
$$+ u\beta_0 \Pi p_j^{\beta j}$$
$$= \ln P + u\beta_0 \Pi_{pj}^{\beta j}$$

Share equations

$$w_i = \alpha_i + \Sigma\gamma_{ij} \ln p_j + \beta_i u\beta_0 \Pi p_j^{\beta j} \quad , \qquad i = 1,\ldots,n$$
$$= \alpha_i + \Sigma\gamma_{ij} \ln p_j + \beta_i(\ln C - \ln P)$$

The cost function has $1 + n + n^2 + 1 + n = n^2 + 2n + 2$ parameters. If symmetry is imposed this gives $n(n - 1)/2$ restrictions.

Homogeneity of degree one in prices requires

$$\Sigma\alpha_i = 1, \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \beta_i = 0$$

This adds $n + 2$ restrictions. In addition, although α_0 is estimated in the process of constructing the price index, lnP, the other parameter β_0 cannot be estimated since u is unknown. This loses one more parameter, giving $n^2 + 2n + 2 - n(n - 1)/2 - (n + 2) - 1 = (2n^2 + 4n + 4 - n^2 + n - 2n - 4 - 2)/2 = (n^2 + 3n - 2)/2$ independent parameters, the same as the translog.

Recovery of parameters.

When there is no price variability in a sample, as might be anticipated from a geographically confined household budget survey, only Engel curves can be estimated within the sample. Pollak and Wales used two such cross sections to recover all the parameters of the Linear Expenditure System (LES). Dybvig shows that any globally additive function can be determined from two Engel curves, that is, from two cross sections of data.

The Cobb-Douglas utility function requires only one. Taking the self dual cost function, from Hotelling's theorem it is straightforward to establish that the budget share

$$w_i = \frac{p_i x_i}{y} = \beta_i \quad , \qquad i = 1,\ldots,n$$

where β_i is the parameter on the i^{th} commodity in the utility function $\ln u = \alpha_0 + \Sigma\beta_i \ln x_i$. The LES is an affine transformation of the C-D function. For each commodity there is an additional parameter, γ_i, sometimes referred to as the minimum required quantity.

$$\ln u = \alpha_0 + \Sigma\beta_i \ln (x_i - \gamma_i)$$

where $\Sigma\beta_i = 1, 0 < \beta_i < 1$ for all i and $x_i - \gamma_i > 0$ for all i.

Maximization of utility subject to the budget constraint gives a demand system

(14) $p_i x_i = p_i \gamma_i + \beta_i (y - \Sigma p_k \gamma_k)$, $i = 1,\ldots,n$

If sectoral expenditure is known then, because the function is separable, income can be replaced by sectoral income y_r and the summation of the minimum required quantities, $\Sigma\gamma_k$, takes place only over those commodities in the sector.

With a single cross section in which all consumers face the same prices equation (14) can be estimated as

$$p_i x_i = \theta_i + \beta_i y \quad , \qquad i = 1,\ldots,n-1$$

where $\theta_i = p_i\gamma_i - \beta_i \Sigma p_k\gamma_k$ is constant for each commodity. Even though $\Sigma p_k \gamma_k$ must be the same in each equation and β_n can be recovered from the restriction $\sum_{i=1}^{n} \beta_i = 1$, the magnitude of $\Sigma p_k\gamma_k$ is not constrained and so there is insufficient information to recover the γ_i. By introducing a second cross section with a different price vector and using maximum likelihood methods, the binary price vectors provide sufficient information to estimate the γ_i parameters directly.

It is claimed by Deaton and Muellbauer that m cross sections provide enough information to estimate a translog system with at most $n + 1 + m(n - 1)$ parameters. This appears to be in error. For example using the indirect translog utility function

$$\frac{p_i x_i}{y_r} = \frac{\alpha_i + \Sigma\beta_{ij}(\ln p_j - \ln y)}{\alpha_m + \Sigma\beta_{mj} \ln (p_j/y)}$$

Introduce the normalization $\alpha_m = -1$. If there is no price variability within a time period t, then the estimating equation is

$$\frac{p_{it}x_{it}}{y_{rt}} = \frac{\theta_{oit} - \theta_{1i} \ln y_t}{-1 + \theta_t} \quad , \qquad i = 1,\ldots,n-1$$

where $\theta_{oit} = \alpha_i + \Sigma\beta_{ij} \ln p_{jt}$, $\quad \theta_{1i} = \beta_{i1} + \beta_{i2} + \cdots$
$+ \beta_{in}$, $\quad \theta_t = \Sigma\beta_{mj} \ln (p_{jt}/y_t)$.

There will be $m(n - 1)$ estimates $\hat{\theta}_{oit}$, $n - 1$ estimates $\hat{\theta}_{1i}$ and m estimates $\hat{\theta}_t$, a total of $m(n - 1) + n - 1 + m$ or $n(m + 1) - 1$ parameter estimates. These can be solved for no more than $n(m + 1) - 1$ structural parameters.

The AID system, on the other hand, has share equations of the form

$$w_i = \alpha_i + \Sigma\gamma_{ij} \ln p_j + \beta_i (\ln C - \ln P) \quad , \qquad i = 1,\ldots,n-1$$

(since $\Sigma w_i = 1$ is required)

or $w_i = \alpha_i + \Sigma\gamma_{ij} \ln p_j - \beta_i \ln P + \beta_i \ln C \quad i = 1, \ldots, n - 1$

If prices are fixed within a time period, $\ln P$ cannot be uniquely estimated. The estimating equations are

$$w_i = \theta_{it} + \beta_i \ln C_t$$

where $\theta_{it} = \alpha_i + \Sigma\gamma_{ij} \ln p_{jt} - \beta_i \ln P_t$. m cross sections give $m(n - 1)$ estimates of $\hat{\theta}_{it}$ and $n - 1$ estimates of β. i.e. $m(n - 1) + n - 1 = (m + 1)(n - 1)$ parameter estimates. The system can be solved for no more than $(m + 1) (n - 1)$ structural parameters.

The way in which the number of commodities in the system is related to the number of subsamples required is shown in Table 1.

The indirect approach will fail to give satisfactory estimates unless the price variability within each sample is small and the variability between samples is large. This needs to be checked prior to any estimation. The method can also be used for a single cross section if a subsampling scheme can be justified that leads to the above conditions and assumes an identical representative consumer in each subsample.

Table 1. Relation Between Number of Commodities and Number of Samples Required

Number of Commodities n	Number of Parameters (k) $\frac{n^2 + 3n-2}{2}$	Number of subsamples required (m) Translog $[n(m+1)-1]$ $m = (k-n+1)/n$	 AIDS $[(m+1)\ (n-1)]$ $m = (k-n+1)/(n-1)$
3	8	2	3
4	13	3	4
5	19	3	4
6	26	4	5
7	34	4	5
8	43	5	6
10	64	6	7
15	134	8	9
20	229	11	12
25	349	13	14
30	494	16	17

Summary

In estimating the elasticities in a demand system the question of price variability seems more important with a single cross section of data, than with time series data. A sample from a widespread survey may well contain as much price variability as a particular time series, in which case the same direct estimation methods may be employed. Where price variability is less marked, as would be expected in a small geographic region, an indirect method relying on Engel curve estimation may be preferable. This may be attainable if a number of mutually exclusive subsamples can be generated that maintain low intrasample and higher intersample price variation.

A cross section sample also poses more of a problem in creating a "representative consumer" that is consistent across subsamples. One attraction of the AID system is that this can be achieved without the requirement of parallel linear Engel curves for all consumers.

The intractability of a disaggregate system forces the investigator to adopt the assumption of separability. When this is done some of the flexibility of the AID system compared with the more familiar translog specification is lost; both end up with the same number of free parameters.

Unresolved issues are whether the gain in computational ease using the indirect method compared with the direct one is offset by a loss in accuracy of parameter estimation. This tradeoff will be influenced by changes in the relative magnitude of intersample to intrasample price variation. These issues could perhaps best be explored through a Monte Carlo type of study.

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