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The Adverse Effect of Energy-Efficiency Policy

Achim Voss

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By Achim Voss, University of Hamburg, Department of Economics

Summary

I analyze energy-efficiency policy as a prescription of a minimum-efficiency standard for energy-using household goods like cars, building insulation, and home appliances. Such a policy has two effects. At the intensive margin, a household that invests will choose a more efficient device. At the extensive margin, there will be more households that choose not to invest at all. Thus, additional to and different from rebound effects, energy-efficiency policy may have unintended consequences. I analyze the equilibrium effects of a minimum-efficiency standard, taking price adjustments and household heterogeneity into account. A moderate minimum-efficiency standard increases demand for efficiency-enhancing household capital goods, and reduces energy demand. More stringent policy is shown to be less effective or even counterproductive. For the case of a fixed supply of efficiency-enhancing capital, it is shown that minimum-efficiency standards increase equilibrium energy demand. Finally, I analyze which households benefit from minimum-efficiency standards and which ones lose. A standard induces investing households to expend more for household capital and less for energy. The wedge between the induced expenditures and the private optimum is analyzed as a deadweight loss.

Keywords: Energy Efficiency, Rebound Effects, Household Heterogeneity, Extensive Margin, Gruenspecht Effect, Investment, Theory of Environmental Policy

JEL Classification: Q41, Q48, D15

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Address for correspondence:
Achim Voss
University of Hamburg
Department of Economics
Von-Melle-Park 5
20146 Hamburg
Germany
E-mail: achim.voss@uni-hamburg.de

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May 8, 2019

Abstract

I analyze energy-efficiency policy as a prescription of a minimum-efficiency standard for energy-using household goods like cars, building insulation, and home appliances. Such a policy has two effects. At the intensive margin, a household that invests will choose a more efficient device. At the extensive margin, there will be more households that choose not to invest at all. Thus, additional to and different from rebound effects, energy-efficiency policy may have unintended consequences. I analyze the equilibrium effects of a minimum-efficiency standard, taking price adjustments and household heterogeneity into account. A moderate minimum-efficiency standard increases demand for efficiency-enhancing household capital goods, and reduces energy demand. More stringent policy is shown to be less effective or even counterproductive. For the case of a fixed supply of efficiency-enhancing capital, it is shown that minimum-efficiency standards increase equilibrium energy demand. Finally, I analyze which households benefit from minimum-efficiency standards and which ones lose. A standard induces investing households to expend more for household capital and less for energy. The wedge between the induced expenditures and the private optimum is analyzed as a deadweight loss.

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*University of Hamburg, Department of Economics, Von-Melle-Park 5, 20146 Hamburg, Germany. Tel.: +49 40 42838 4529. E-mail: achim.voss@uni-hamburg.de. A previous version of this paper had the title "Energy-Efficiency Policy and its Effects at the Intensive and at the Extensive Investment Margins with Heterogenous Households".

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1 Introduction

Households make up a large share of total fuel and electricity consumption.[1] A widespread, and seemingly natural, way to reduce final energy demand is to directly target household energy efficiency via minimum-efficiency standards for durable household goods. For instance, in the European Union, light bulbs, television sets, or vaccum cleaners that use too much electricity must not be sold.[2] Similar regulations aim at cars, space-heating systems, building insulation or domestic appliances.[3]

Do these policies actually reduce aggregate energy demand? To answer this question, we have to consider the policy's target. For any given energy service (like lighting), energy efficiency usually is a fixed property of a device (like a light bulb). To increase energy efficiency for the energy service, the household usually has to scrap and replace it. At any moment, there are some households who plan such an investment because their old household devices are too inefficient. These households are the ones that are targeted by the policy, because investing is a voluntary decision, and minimum-efficiency standards usually only apply to new devices, and not to existing ones. Under these circumstances, increasing a binding standard has two countervailing effects. At the intensive margin, it will increase the energy efficiency of all households that still invest. At the extensive margin, fewer households may decide to invest at all, and instead keep their old devices. Thus, a minimum-efficiency standard increases overall energy efficiency only if the first effect dominates.

To determine under which conditions this is the case, this article formally analyzes the equilibrium effects of such a policy. I model two interdependent markets: firstly, the market for energy, which may stand for fuel or electricity, and secondly, the market for an energy-using household capital good. The household's utility depends on the energy services that the capital good and the energy input provide. If the household has a larger amount of capital, a given amount of energy translates into more energy services. Thus, energy efficiency is understood as the amount of capital the household owns for the energy service under consideration. When buying the capital good, the household can freely choose its energy efficiency – that is, buy a smaller or larger amount of capital.

Households are heterogenous with respect to their capital endowment – that is,

[1] 19% of final energy consumption in the OECD is attributed to the residential sector (IEA, 2018), and this excludes the households' part of the transport sector's 34%.

[2] See, for instance, Dehmer (2013) or Harrabin (2017) for media coverage of the topic. For analyses of the European light-bulb regulation, see Frondel and Lohmann (2011) or Perino and Pioch (2017).

[3] *The 2018 International Energy Efficiency Scorecard* (Castro-Alvarez et al., 2018) by the American Council for an Energy-Efficient Economy (ACEEE), a non-profit organization, approvingly lists several direct intervention policies and gives an overview of their implementation in 25 coutries. The U.S. car fuel-efficiency standards are more flexible than minimum-efficiency standards as they prescribe an average fuel efficiency for a car producer, but they work in a similar way (Portney et al., 2003).

with respect to their *current* energy efficiency. This implies a cutoff level of capital below which households invest; those that have more keep their endowment. Introducing a minimum-efficiency standard then increases the amount of capital an investing household chooses, but it also shifts the cutoff level downwards, reducing the share of investing households. Additionally, the cutoff level reacts to prices: If the standard increases demand for energy efficiency, its rising price will increase the cutoff level, which dampens the total effect. Cheaper energy works in the same direction.

The main insights of the current article are the following. Firstly, it is shown how introducing a minimum-efficiency standard changes the cutoff level. A standard forces households to pay too much for energy-efficiency capital relative to energy to minimize the costs of energy services, which deters them from investing. I show that the elasticity of the cutoff level with respect to the standard is proportional to the enforced difference between capital and energy expenditures. Thus, a *marginal* minimum-efficiency standard – that is, a barely binding standard – has no effect on the cutoff level. With a *stricter* standard, the cutoff level decreases: Even households with a low amount of capital forgo replacing it if the standard pushes them too far from their individually optimal investment.

Secondly, I characterize the effects of a marginal standard in the aggregate. If supply elasticities for household appliances and energy are positive but finite, then such a standard increases the equilibrium aggregate demand for energy-efficiency capital and its price, and reduces the equilibrium consumption and price of energy. By contrast, if capital is supplied inelastically, then the only effect of a marginal standard will be to *increase* the price of efficiency-enhancing capital, while the energy market will be unaffected.

Thirdly, we turn to stricter efficiency standards. A moderate standard increases aggregate energy-efficiency investment and reduces energy consumption, but less so the stricter the standard. For some critical level of the standard, energy consumption is minimized. Further increasing the standard still stimulates aggregate investment, but it is concentrated among few households, and so many other households are deterred from investing that aggregate energy demand is increased. Therefore, a positive reaction of investment to the policy is not sufficient for a reduction of energy demand.

Finally, we consider welfare and distributional issues. I illustrate how the costs and benefits of a household from minimum-efficiency standards depend on the household's current energy efficiency. A household that has high energy efficiency would not invest anyway, and therefore benefits from a minimum-efficiency standard if its introduction reduces the energy price. By contrast, a household with old, inefficient appliances that would plan to invest bears the costs of a policy that, firstly, prescribes to choose higher energy efficiency than the household would privately choose, and secondly, increases aggregate demand for such appliances, which increases their price.

A main insight here is that from the perspective of utilitarian welfare maximization, the wedge between capital expenditures and energy expenditures that the standard creates can be understood as the policy's deadweight loss. An empirical implication is that opinion polls should show support for energy-efficiency standards among households that already have efficient capital goods, for instance due to a recent investment, and opposition among the households that have inefficient capital goods and may soon plan to replace them.

My model builds on the basic insights of Gruenspecht (1982): scrappage and replacement decisions react to regulation that only applies to newly bought devices, and this reaction can be strong enough to offset the intended effects of regulation (in the short run). He analyzes the effect of pollutant emission regulation that increases the costs (and the price) of new cars. This increases the share of people keeping their old car and the price of used cars, which are substitutes to new ones. Gruenspecht thus considers standards for emissions that are an externality to car owners, and the regulation increases the costs of cars directly and, often, due to lower fuel efficiency.

The first apparent difference to the present article's focus is that the kind of policy that I consider should lead to *higher* energy efficiency. This is also the kind of policy that Jacobsen and van Benthem (2015) analyze. They apply the Gruenspecht reasoning to fuel economy standards. In their model, a representative consumer's vehicle demand is met by car suppliers whose new-car sales are subject to the standards, and by used-car supplies that are determined as last period's supply net of scrapping, where the scrap probability depends on the vehicle price. However, in their model, the elasticity of the scrap rate with respect to the vehicle price is assumed to be constant, and the consumption of energy services is exogenous.

Compared to the Gruenspecht (1982) and Jacobsen and van Benthem (2015) contributions, my article's focus is on a theoretical model in which the investment and scrap decisions are derived from optimization behavior, such that the choice of energy efficiency and the investment decision depend on the energy price. Minimum-efficiency standards are derived and discussed for varying intensities instead of assuming constant elasticities. Additionally, I analyze equilibrium effects by includingthe influence of supply-side elasticities of energy-efficiency capital and the fuel market in the analysis. This allows to understand the effects of the regulation on aggregate fuel consumption. The equilibrium analysis in the present paper which starts from individual, heterogenous households also allows to understand welfare and distributional effects of energy-efficiency policies. In a recent contribution, Levinson (2019) analyzes how regressive efficiency standards are. I add to this another distributional dimension, namely between households that plan to invest and those that already have efficient capital goods.

The fact that the consumption of energy services and of energy are endogenous in my model connects the article to the vast literature on the aggregate effects of energy efficiency and, in particular, *rebound effects*, with early contributions by Khazzoom (1980), Brookes (1990), Saunders (1992, 2000), Wirl (1997), among others. This literature raised skepticism about energy efficiency as a policy goal, suggesting that increasing energy efficiency might raise energy demand instead of reducing it. In the many empirical studies that followed, such "backfire" behavior of energy demand was not found, and rebound effects are likely small – see Gillingham et al. (2009), Borenstein (2015), Chan and Gillingham (2015), and Gillingham et al. (2016) for an overview of the issues, for a summary of the empirics and for recent new theoretical insights. The focus of this literature is, in general, on exogenous technological increases of energy efficiency. The costs – or even the energy content – of the new efficiency-enhancing appliances are sometimes analyzed as well, for instance by Borenstein (2015); nonetheless, the decision of the households that would be affected by actual energy-efficiency policy is not analyzed. The present article aims at extending the understanding of energy-efficiency policy by modeling decisions of heterogeneous households, the effects of the policy on these decisions, and thus, the policy's equilibrium effects. Note that in my model, backfire behavior of fuel demand is assumed away, in order to focus on the interplay of intensive- and extensive-margin effects. My model emphasizes that low rebound effects are not sufficient for effective energy-efficiency policy.

Another strand of the energy-efficiency literature focuses on explanations for too low levels of energy efficiency due to either market failures or psychological explanations (see Gillingham et al., 2009 or Gerarden et al., 2017). In my model, households are perfectly rational and I do not model market failures. The model can be extended in these directions, however, as discussed in Section 5.

The paper proceeds as follows. Section 2 analyzes a household's demand for energy and for energy-efficiency capital, first without and then with a minimum-efficiency standard. Capital is chosen for a single period, anticipating how energy demand depends on the amount of capital. Section 3 then first describes aggregate supply and demand for energy and capital, and then analyzes minimum-efficiency standard, first for a marginal standard and then for stricter standards. Functional forms are introduced when necessary. Welfare and distributional effects and implications for politically determined standards are considered in Section 4. Finally, Section 5 discusses the results and possible extensions of the model.

2 Individual Demand

2.1 The Setting

Suppose that household have preferences over consumption of a numeraire good, Q, and consumption of energy services, S. Household i's utility is

$$u_i = Q_i + B(S_i), \tag{1}$$

where I assume the energy-services benefit function B to be increasing in S_i and strictly concave, with $\lim_{S\to 0} B'(S) = \infty$ and $\lim_{S\to\infty} B'(S) = 0$.[4] Energy services are obtained by combining energy E, which can represent fuel or electricity, and capital K, so that household i's energy services are $S_i \equiv K_i E_i$. K_i may represent the capital embodied in a car, a space-heating system, the house's insulation, or household appliances like fridges or washing machines. In the investment decision considered below, I assume that such household-capital goods are continuously scalable at the moment when they are bought, and that their only characteristic is the amount of energy services per unit of energy that they provide. For instance, if the household has a better space-heating system that provides a warmer house with the same amount of fuel, K_i is larger.[5] We thus equivalently say that a household has a more energy-efficient appliance (or car, space-heating system, etc) or that it has more capital.

The household's budget is $Q_i + pE_i + I_i = Y_i$, where p is the energy price, I is investment and Y is exogenous income. Thus, utility becomes

$$u_i = Y_i - pE_i - I_i + B(S_i). \tag{2}$$

In the following, I analyze the household's optimal behavior in a two-stage procedure. First, I derive energy demand given the level of capital. Afterwards, the household's capital level is endogenized: The household is endowed with some capital and decides whether to keep it or replace it.

2.2 Energy Demand

If the household has no capital ($K_i = 0 \implies S_i = 0$), consuming energy has only costs but no benefits, such that the household will choose $E_i = 0$. For $K_i > 0$, $\lim_{S\to 0} B'(S) =$

[4] However, given that u_i is a utility function, I do not restrict $B(0) = 0$ (which is in contrast to the oft-cited Inada (1963) assumptions).

[5] For cars, K_i are miles per gallon, while S_i are miles driven; $1/K_i = E_i/S_i$ would be the European version, usually stated as liters of gasoline per 100 km. The simplifying assumption in the present model is that the household can freely choose this property of its vehicle when buying it, which is a shortcut for car suppliers offering the varieties that consumers demand.

∞ implies that there is a strictly positive amount of E_i fulfillingthe first-order condition

$$B'(S_i)K_i = p. \tag{3}$$

This implicitly defines the household's energy demand $E^*(K_i, p)$ and thus energy-service demand as functions of K_i and p:

$$S_i^*(K_i, p) \equiv K_i E^*(K_i, p). \tag{4}$$

Differentiating (3) and rearranging yields the elasticities of energy demand with respect to the price of energy, $\varepsilon_{E,p}$ and with respect to the household's capital stock, $\varepsilon_{E,K}$. They directly translate to the respective elasticities of energy-services demand:

$$\varepsilon_{E,p}(K_i, p) \equiv \frac{\partial E_i^*(K_i, p)/E_i^*(K_i, p)}{\partial p/p} = \frac{1}{-\beta(S_i^*)} = \varepsilon_{S,p}(K_i, p), \tag{5a}$$

$$\varepsilon_{E,K}(K_i, p) \equiv \frac{\partial E_i^*(K_i, p)/E_i^*(K_i, p)}{\partial K_i/K_i} = \frac{1}{\beta(S_i^*)} - 1 = \varepsilon_{S,K}(K_i, p) - 1 \tag{5b}$$

where

$$\beta(S_i) \equiv S_i \frac{-B''(S_i)}{B'(S_i)} > 0 \tag{6}$$

denotes the absolute value of the elasticity of marginal benefits with respect to energy services. I assume in the following that β is bounded:

Assumption 1. $\beta(S_i) > 1$.

Then,

$$\varepsilon_{S,K}(K_i, p) = 1 + \varepsilon_{E,K}(K_i, p) = -\varepsilon_{E,p}(K_i, p) = -\varepsilon_{S,p}(K_i, p) = \frac{1}{\beta(S_i^*)} \in [0, 1). \tag{7}$$

Thus, a household with a larger amount of capital will always demand more energy services, but less energy. This assumption ensures that rebound effects are not too strong. Using the term from the literature on energy efficiency and rebound effects, I exclude a "backfire" effect of energy efficiency; backfire means that an increase in energy efficiency increases energy demand (see e.g. Voss, 2015).[6] Excluding backfire effects is in line with the empirical results of the literature (Gillingham et al., 2016). Additionally, it allows that energy-efficiency policy could at least in principle make

[6] Along the lines of Saunders (2000, 2008), the rebound would be defined as $R \equiv 1 + \varepsilon_{E,K}(K_i, p)$, which is identical to $-\varepsilon_{E,p}(K_i, p)$. In the present model, energy efficiency in $S_i \equiv K_i E_i$ is embodied in household capital, and not derived from exogenous technological progress (as it is in a large part of the rebound-effects literature). Thus, in our context, a rebound of R means that $(1 - R) \cdot 100$ % of productivity gains due to an increase in K_i are translated into actual energy conservation.

sense as a policy to reduce energy consumption, such that the present paper can focus on other aspects of such policy.

Finally, using $E^*(K_i, p)$ in u_i, the household's optimized utility is

$$U_i(K_i, p, I_i) = Y_i - pE^*(K_i, p) - I_i + B(S_i^*). \tag{8}$$

2.3 Unconstrained Investment

Suppose that the household chooses K_i in order to maximize $U_i(K_i, p, I_i)$, where now the relation between investment I_i and capital has to be taken into account. The price per unit of capital is h, such that $I_i = hK_i$:

$$U_i(K_i, p, hK_i) = Y_i - pE^*(K_i, p) - hK_i + B(S_i^*). \tag{9}$$

The household's optimal amount of capital K° is defined by

$$B'(K^\circ E^*(K^\circ, p))E^*(K^\circ, p) = h, \tag{10}$$

where we write K° instead of K_i° because nothing in the formula depends on the household's characteristics. In Appendix A.1, I show that $U_i(K_i, p, hK_i)$ is strictly concave in K_i, that K° is strictly positive, and that it is higher if the energy price is higher or the capital price is lower: $K_i^\circ = K_i^\circ(h, p)$.I suppress this functional dependence in the following in order to keep the notation clear, and proceed in the same way in similar cases below.

Note that isolating $B'(S)$ in (3) and substituting into (10) yields:

$$pE^*(K^\circ, p) = hK^\circ. \tag{11}$$

That is, the household will optimally spend as much for energy as for capital. Intuitively, to generate energy services, energy and capital are equally effective, so the household would balance their cost. The optimization yields the indirect utility function

$$U_i^\circ(h, p) = Y_i - pE^*(K^\circ, p) - hK^\circ + B(S^*(K^\circ, p)). \tag{12}$$

where $S^*(K^\circ, p) \equiv K^\circ E^*(K^\circ, p)$.

Now suppose that the household is endowed with an amount of capital $K_{i,0}$. It is plausible that such an equipment cannot be increased continuously. Thus, the household has two alternatives: Keep $K_{i,0}$, in which case it does not have to pay for investment, or scrap it and invest to get K°. The former option's utility is given by (8) for

$I_i = 0$ and $K_i = K_{i,0}$:

$$U_i(K_{i,0}, p, 0) = Y_i - pE^*(K_{i,0}, p) + B(K_{i,0}E^*(K_{i,0}, p)) \tag{13}$$

with

$$\frac{\partial U_i(K_{i,0}, p, 0)}{\partial K_{i,0}} = B'(K_{i,0}E^*(K_{i,0}, p))E^*(K_{i,0}, p) > 0, \tag{14}$$

which states that, naturally, the household's utility with its capital endowment is higher if this endowment is higher. Replacing the endowment is better if

$$B(S^*(K^\circ, p)) - B(K_{i,0}E^*(K_{i,0}, p)) \geq hK^\circ - p\left[E^*(K_{i,0}, p) - E^*(K^\circ, p)\right]. \tag{15}$$

The difference on the left-hand side is positive if the amount of capital the household would buy exceeds the one it has; see (7). Likewise, the energy-expenditure difference on the right-hand side is positive (with its old, small amount of capital, the household would pay more for energy). There is an endowment $K_{i,0} = \widetilde{K} > 0$ which makes the household just indifferent:[7]

$$B(S^*(K^\circ, p)) - B(\widetilde{K}E^*(\widetilde{K}, p)) - p\left[E^*(K^\circ, p) - E^*(\widetilde{K}, p)\right] - hK^\circ = 0. \tag{16}$$

This equation implicitly defines the cutoff as a function of the prices of energy and capital goods: $\widetilde{K} = \widetilde{K}(h, p)$. Differentiating and rearranging yields:

$$\frac{\partial \widetilde{K}(h, p)/\widetilde{K}}{\partial h/h} = -\frac{hK^\circ(h, p)}{pE^*(\widetilde{K}, p)} \left(= -\frac{E^*(K^\circ, p)}{E^*(\widetilde{K}, p)} \right) \in (-1, 0], \tag{17a}$$

$$\frac{\partial \widetilde{K}(h, p)/\widetilde{K}}{\partial p/p} = \frac{\Delta_E(K^\circ, \widetilde{K}, p)}{E^*(\widetilde{K}, p)} = 1 + \frac{\partial \widetilde{K}(h, p)/\widetilde{K}}{\partial h/h} \in (0, 1]. \tag{17b}$$

where we use Δ_E for the amount of energy saved if the household invests:

$$\Delta_E(K^\circ, \widetilde{K}, p) \equiv E^*(\widetilde{K}, p) - E^*(K^\circ, p). \tag{18}$$

Summarizing, the optimal amount of capital of household i is

$$K_i^*(h, p) = \begin{cases} K^\circ & \text{if } K_{i,0} \in \left[0, \widetilde{K}\right), \\ K_{i,0} & \text{if } K_{i,0} \in \left[\widetilde{K}, \infty\right). \end{cases} \tag{19a}$$

[7]Note that $K^\circ > 0$ implies that $B(S^*(K^\circ, p)) - pE^*(K^\circ, p) - hK^\circ > B(0) - pE^*(0, p)$. Thus, $\widetilde{K} > 0$.

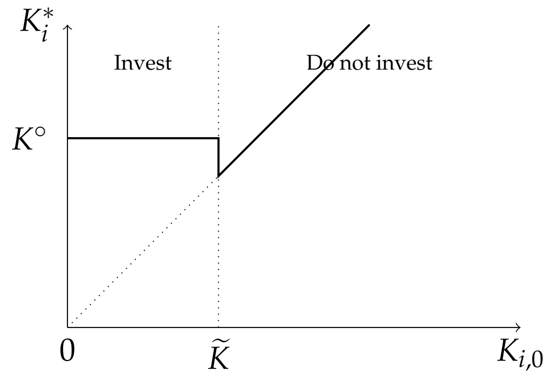

Figure 1: Household Capital with Optimal Investment

Accordingly, investment expenditure is

$$I_i^*(h,p) = \begin{cases} hK^\circ & \text{if } K_{i,0} \in \left[0, \widetilde{K}\right), \\ 0 & \text{if } K_{i,0} \in \left[\widetilde{K}, \infty\right). \end{cases} \tag{19b}$$

Note that K_i^* is discontinuous in the endowment. A household with less than $\widetilde{K}$ will invest and afterwards have K°. A household with slightly more than $\widetilde{K}$ will not accept the costs of new capital, and therefore will have less than K°. A household with much more than $\widetilde{K}$ will also keep it and therefore have a *larger* amount of capital. Figure 1 illustrates the investment decision.

The elasticity of the indifference level with respect to the capital price is clearly negative: With a higher price of capital, the household would be more willing to stick to its old space-heating system instead of replacing it. Conversely, with a higher energy price, the household is more willing to scrap its capital endowment, because energy usage with the new amount of capital would be lower.[8]

2.4 Investment with a Minimum-Efficiency Standard

We now consider a minimum-efficiency standard policy. This policy prescribes that *if* a household invests, it has to choose $K_i \geq \underline{K} \geq K^\circ$, where the last inequality has to hold strictly for the standard to have any effect, because any investing household chooses K° if there is no standard. In the following, when comparing two levels of the minimum-efficiency standard $\underline{K}' > \underline{K}$, the former standard will be called *stricter* than the latter.

Which effects will the introduction of a minimum-efficiency standard have? Firstly, there is an effect at the intensive margin: If the household still wants to invest, its investment has to be higher. This pushes the household away from privately opti-

[8]Note that (7) excludes $E^*(\widetilde{K}, p) = E^*(K^\circ, p)$.

mal cost minimization; while its energy demand is still optimal given capital ((3) still holds), the optimal-capital condition (10) does not hold. Instead,

$$\frac{\mathrm{d}U_i(\underline{K}, p, h\underline{K})}{\mathrm{d}K_i} = B'(\underline{K}E^*(\underline{K}, p))E^*(\underline{K}, p) - h < 0, \tag{20}$$

which also implies that instead of the expenditure balance (11), we have

$$h\underline{K} - pE^*(\underline{K}, p) > 0. \tag{21}$$

The minimum standard forces the household to have too much capital to minimize cost, and the investment expenditures exceed its expenditures for energy. Moreover, the stricter the standard, the larger the deviation between the expenditures:

$$\frac{\mathrm{d}\left(h\underline{K} - pE^*(\underline{K}, p)\right)}{\mathrm{d}\underline{K}} = h - p\frac{E^*(\underline{K}, p)}{\underline{K}}\varepsilon_{E,K}(\underline{K}, p) > h > 0, \tag{22}$$

where the sign stems from the fact that $\varepsilon_{E,K}(\underline{K}, p) \in (-1, 0)$ by (7).

Secondly, because the household is kept from its private optimum, the minimum standard reduces after-investment utility, and this has an effect at the extensive margin – that is, the decision whether to invest is affected. In (15), $\underline{K}$ takes the place of K°. With the minimum standard, the household will scrap the old capital and buy an amount $\underline{K}$ if

$$U_i(\underline{K}, p, h\underline{K}) \geq U_i(K_{i,0}, p, 0). \tag{23}$$

Again, the amount of capital for which this is fulfilled with equality is a cut-off level: All households whose initial amount of capital $K_{i,0}$ is lower will invest, all households with more capital will not.

We call the new cutoff $\check{K}$. It depends on prices and $\underline{K}$: $\check{K} = \check{K}(h, p, \underline{K})$. Thus, for a household who has $K_{i,0} = \check{K}$,

$$B(\underline{K}E^*(\underline{K}, p)) - B(\check{K}E^*(\check{K}, p)) + p\left[E^*(\check{K}, p) - E^*(\underline{K}, p)\right] - h\underline{K} = 0. \tag{24}$$

Thus, the optimal capital stock K_i^* and the investment expenditure of household i are

$$K_i^*(h, p, \underline{K}) = \begin{cases} \underline{K} & \text{if } K_{i,0} \in \left[0, \check{K}\right), \\ K_{i,0} & \text{if } K_{i,0} \in \left[\check{K}, \infty\right), \end{cases} \tag{25a}$$

$$I_i^*(h, p, \underline{K}) = \begin{cases} h\underline{K} & \text{if } K_{i,0} \in \left[0, \check{K}\right), \\ 0 & \text{if } K_{i,0} \in \left[\check{K}, \infty\right). \end{cases} \tag{25b}$$

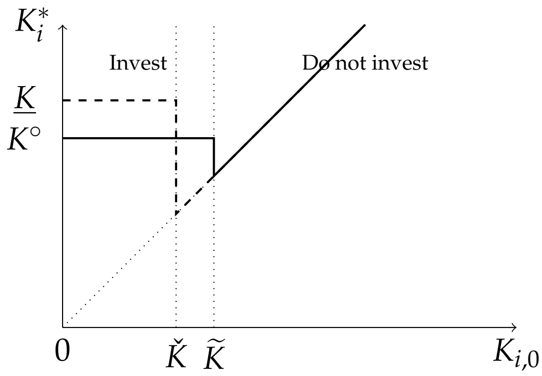

Figure 2: Household Capital with Optimal Investment, given the Minimum-Efficiency Standard

To see how the cutoff $\check{K}$ amount depends on the standard and the prices, we differentiate (24) and rearrange:

$$\varepsilon_{\check{K},\underline{K}}(h,p,\underline{K}) = \frac{\partial \check{K}/\check{K}}{\partial \underline{K}/\underline{K}} = -\frac{h\underline{K} - pE^*(\underline{K},p)}{pE^*(\check{K},p)} \leq 0, \tag{26a}$$

$$\varepsilon_{\check{K},h}(h,p,\underline{K}) = \frac{\partial \check{K}/\check{K}}{\partial h/h} = -\frac{h\underline{K}}{pE^*(\check{K},p)} \in \left(-\frac{h\underline{K}}{pE^*(\underline{K},p)}, 0\right], \tag{26b}$$

$$\begin{aligned} \varepsilon_{\check{K},p}(h,p,\underline{K}) = \frac{\partial \check{K}/\check{K}}{\partial p/p} &= \frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\check{K},p)} \\ &= 1 + \frac{pE^*(\underline{K},p)}{h\underline{K}}\varepsilon_{\check{K},h}(h,p,\underline{K}) \in (0,1]. \end{aligned} \tag{26c}$$

where $\Delta_E(\underline{K},\check{K},p)$ is defined as in (18).

For our analysis, the following wording is useful:

Definition 1. A minimum-efficiency standard that sets $\underline{K}$ only marginally above $\underline{K} = K^\circ$ is called a *marginal minimum-efficiency standard*, or marginal standard for short. Precisely, we refer to the effects of a marginal standard when evaluating the marginal effects of a minimum-efficiency standard for a value of $\underline{K} = K^\circ$.

For the application of this definition, note that $\underline{K} = K^\circ$ also implies $\check{K} = \widetilde{K}$ by the definitions of $\widetilde{K}$ in (16), and of $\check{K}$ in (24).

We now turn to the analysis of the elasticities.Firstly, consider the effect of the standard itself. A standard above K° will imply that the household is more reluctant to invest such that $\check{K} < \widetilde{K}$, but it has to choose $\underline{K} > K^\circ$ if it invests. Figure 2 illustrates these effects. For the further analysis, it is relevant how strongly $\check{K}$ reacts to an increase in $\underline{K}$, and which effect an ever-increasing efficiency standard will have. I summarize this analysis in the following proposition.

Proposition 1 (The effect of $\underline{K}$ on $\check{K}$)**.** *For a marginal standard, $\varepsilon_{\check{K},\underline{K}} = 0$. For a stricter* *standard* ($\underline{K} > K^\circ$), *we have* $\varepsilon_{\check{K},\underline{K}} < 0$. *Thus,* $\check{K}$ *is monotonically decreasing in* $\underline{K}$.*Finally,* $\lim_{\underline{K}\to\infty} \check{K}(h, p, \underline{K}) = 0$.

Proof. By (11) and (21), the numerator in (26a) is zero for a marginal standard, implying $\varepsilon_{\check{K},\underline{K}} = 0$, and positive for a stricter standard, implying $\varepsilon_{\check{K},\underline{K}} < 0$. To prove $\lim_{\underline{K}\to\infty} \check{K}(h, p, \underline{K}) = 0$, consider any endowment $K_{i,0} > 0$ and assume that the household owning it prefers to invest:

$$B(\underline{K}E^*(\underline{K}, p)) - pE^*(\underline{K}, p) - h\underline{K} > U_i(K_{i,0}, p, 0). \tag{27}$$

By (20), the left-hand side is decreasing in $\underline{K}$. Moreover, using (A.1b) in Appendix A.1, we see that $\mathrm{d}^2 U_i(\underline{K}, p, h\underline{K}) / \mathrm{d}\underline{K}^2 < 0$. Thus, for any $K_{i,0}$ there will be an increase in $\underline{K}$ such that (27) changes its sign, which proves the claim. □

Thus, a minimum-efficiency standard that is close to marginal has hardly any effect on the cutoff level $\check{K}$, but if it is increased, the capital endowment below which the household would opt for investment monotonically decreases in the standard, and if the standard is strict enough, the household will not invest.The logic of the other two elasticities follows the unconstrained case, as discussed after (17). However, the lower bound of the capital-price elasticity, $-h\underline{K}/pE^*(\underline{K}, p)$ now is smaller than -1.

2.5 Isoelastic Functions

In the following, we assume specific, isoelastic functions in order to illustrate the model and also to be able to reach concrete results later on. Thus, suppose

$$B(S_i) \equiv -\frac{1}{\beta - 1} S_i^{-(\beta-1)}. \tag{28}$$

Then (2) becomes:

$$u_i = Y_i - pE_i - I_i - \frac{1}{\beta - 1} S_i^{-(\beta-1)}. \tag{29}$$

The marginal benefit of energy services is $B'(S_i) = S_i^{-\beta} (> 0)$, and that of energy is $B'(S_i)K_i = S_i^{-\beta} K_i = E_i^{-\beta} K_i^{1-\beta}$. From (3), (4) and (8), we obtain the household's demand for energy and energy services and its optimized utility, given the household's capital:

$$E_i = p^{-\frac{1}{\beta}} K_i^{-\frac{\beta-1}{\beta}}, \tag{30a}$$

$$S_i = p^{-\frac{1}{\beta}} K_i^{\frac{1}{\beta}}, \tag{30b}$$

$$U_i(K_i, p, I_i) = Y_i - I_i - \frac{\beta}{\beta - 1} p^{\frac{\beta-1}{\beta}} K_i^{-\frac{\beta-1}{\beta}}. \tag{30c}$$

The elasticities of energy demand with respect to price and capital are thus constant: $\varepsilon_{E,p}(K_i, p) = -1/\beta < 0$ and $\varepsilon_{E,K}(K_i, p) = -(\beta - 1)/\beta$. We can see that Assumption 1, $\beta > 1$, indeed implies a negative effect of capital on energy demand. Then with $I_i = hK_i$, the utility of an investing household is $U_i(K_i, p, hK_i)$ and the optimal capital level of such a household, defined in (10), becomes

$$K^\circ = h^{-\frac{\beta}{2\beta-1}} p^{\frac{\beta-1}{2\beta-1}}, \tag{31a}$$

such that

$$E^*(K^\circ, p) = h^{\frac{\beta-1}{2\beta-1}} p^{-\frac{\beta}{2\beta-1}}, \tag{31b}$$

$$S^*(K^\circ, p) = h^{-\frac{1}{2\beta-1}} p^{-\frac{1}{2\beta-1}}. \tag{31c}$$

Using these results, we confirm the optimal expenditure balance, (11):

$$pE^*(K^\circ, p) = h^{\frac{\beta-1}{2\beta-1}} p^{\frac{\beta-1}{2\beta-1}} = hK^\circ. \tag{32}$$

Using (30c) and (31a) to derive the indirect utility function $U_i^\circ(h, p)$ from (12) and utility for the case that the household does not invest, as defined by (13), we can derive the threshold from (16):

$$\widetilde{K} = \left(\frac{2\beta - 1}{\beta}\right)^{-\frac{\beta}{\beta-1}} h^{-\frac{\beta}{2\beta-1}} p^{\frac{\beta-1}{2\beta-1}}. \tag{33}$$

We now turn to household investment given the standard $\underline{K} \geq K^\circ$. To obtain demand given the standard, (30) has to be evaluated for $K_i = \underline{K}$. For the general case, (24) defines the cutoff amount of capital, given the standard and the prices. Here it becomes

$$\check{K}(h, p, \underline{K}) = \left[\frac{\beta - 1}{\beta} \tilde{m}(h, p, \underline{K}) + 1\right]^{-\frac{\beta}{\beta-1}} \underline{K} \tag{34}$$

instead of (33), where

$$\tilde{m}(h, p, \underline{K}) = h\underline{K}^{\frac{2\beta-1}{\beta}} p^{-\frac{\beta-1}{\beta}} \begin{cases} = 1 \text{ for } \underline{K} = K^\circ, \\ > 1 \text{ for } \underline{K} > K^\circ. \end{cases} \tag{35}$$

Then,

$$\varepsilon_{\check{K},\underline{K}}(h,p,\underline{K}) = -\frac{h\underline{K} - pE^*(\underline{K},p)}{pE^*(\check{K},p)} = -\frac{\tilde{m}(h,p,\underline{K})-1}{\frac{\beta-1}{\beta}\tilde{m}(h,p,\underline{K})+1}, \tag{36a}$$

$$\varepsilon_{\check{K},h}(h,p,\underline{K}) = -\frac{h\underline{K}}{pE^*(\check{K},p)} = -\frac{\tilde{m}(h,p,\underline{K})}{\frac{\beta-1}{\beta}\tilde{m}(h,p,\underline{K})+1}, \tag{36b}$$

$$\varepsilon_{\check{K},p}(h,p,\underline{K}) = \frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\check{K},p)} = \frac{\beta-1}{\beta}\frac{\tilde{m}(h,p,\underline{K})}{\frac{\beta-1}{\beta}\tilde{m}(h,p,\underline{K})+1}. \tag{36c}$$

It is now easy to confirm the results of Proposition 1. $\varepsilon_{\check{K},\underline{K}}(h,p,\underline{K})$ is indeed zero for a marginal standard, which we can confirm by substituting $\underline{K} = K^\circ$ from (31a), and negative for stricter standards; the numerator in (36a) is increasing in $\underline{K}$. Moreover, for $p = 0$, $\varepsilon_{\check{K},h}(h,p,\underline{K}) = -\frac{\beta}{\beta-1}$, and the elasticity is decreasing in the fuel price and asymptotically goes to zero. $\varepsilon_{\check{K},p}(h,p,\underline{K})$ is zero for $h = 0$ and monotonically goes to a value of 1. Moreover, the amount of energy saved if the cutoff household invests, (18), becomes

$$\Delta_E(\underline{K},\check{K},p) = \frac{\beta-1}{\beta}\frac{h}{p}\underline{K} \tag{37}$$

and, using (30), we have:

$$\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})} = \frac{\beta-1}{\beta}\tilde{m}(h,p,\underline{K}), \tag{38a}$$

$$\varepsilon_{E,K} + \frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})} = \frac{\beta-1}{\beta}\left[\tilde{m}(h,p,\underline{K})-1\right]. \tag{38b}$$

3 Aggregate Analysis

3.1 Aggregate Investment and Aggregate Energy Consumption

We turn to the analysis of aggregate energy consumption and aggregate household-capital investment, or aggregate investment for short, in equilibrium. There is a unit-size continuum of households. Suppose that the capital endowment follows a cumulative distribution function $G(K_{i,0})$ with a density $g(K_{i,0}) \equiv G'(K_{i,0})$. Without a minimum-efficiency standard, the share of households that invest is $G(\widetilde{K})$. Thus, after investment, the share $G(\widetilde{K})$ of households will have capital K°, and the share $1 - G(\widetilde{K})$ has capital distributed between $\widetilde{K}$ and ∞ (or some maximum amount of capital); this span includes K°. Aggregate investment demand thus is

$$I(h,p) = \int_0^{\widetilde{K}(h,p)} g(K_{i,0})K^\circ(h,p)\,\mathrm{d}K_{i,0} = G(\widetilde{K}(h,p))K^\circ(h,p). \tag{39}$$

Aggregate energy demand is then determined by:

$$\begin{aligned} E(h,p) &= \int_0^{\widetilde{K}(h,p)} g(K_{i,0}) E^*(K^\circ, p) \,\mathrm{d}K_{i,0} + \int_{\widetilde{K}(h,p)}^{\infty} g(K_{i,0}) E^*(K_{i,0}, p) \,\mathrm{d}K_{i,0} \\ &= G(\widetilde{K}(h,p)) E^*(K^\circ, p) + \int_{\widetilde{K}(h,p)}^{\infty} g(K_{i,0}) E^*(K_{i,0}, p) \,\mathrm{d}K_{i,0}. \end{aligned} \tag{40}$$

If we instead consider demand as influenced by the minimum-efficiency standard, we have to evaluate for $\check{K}(h, p, \underline{K})$ instead of $\widetilde{K}(h, p)$. Suppose that there are a household-capital supply function $\kappa(h)$ and an energy supply function $\Phi(p)$. In equilibrium,

$$\kappa(h^\bullet) = G(\check{K}^\bullet)\underline{K}, \tag{41a}$$

$$\Phi(p^\bullet) = G(\check{K}^\bullet) E^*(\underline{K}, p^\bullet) + \int_{\check{K}^\bullet}^{\infty} g(K_{i,0}) E^*(K_{i,0}, p^\bullet) \,\mathrm{d}K_{i,0}, \tag{41b}$$

where $\check{K}^\bullet \equiv \check{K}(h^\bullet, p^\bullet, \underline{K})$. The $^\bullet$ is for equilibrium, but we drop this in the following to keep the exposition clear.

3.2 Minimum-Efficiency Standards

We now turn to analyzing how a minimum-efficiency standard affects the equilibrium. We have already seen that a stricter standard implies that a household would need a lower capital endowment to choose investing over keeping the old capital. From the aggregate perspective, we can say that less households invest. However, we also know that any household that invests now has to choose a higher amount. If (and only if) this intensive-margin effect outweighs the extensive-margin effect, total investment is increased by the standard. Moreover, energy demand is reduced if (and only if) the increase of investment at the intensive margin also reduces energy consumption to a larger extent than the reduction of investment at the extensive margin increases it.

Define the elasticities

$$\eta(h) \equiv \frac{\partial \kappa(h)/\kappa(h)}{\partial h/h}, \tag{42a}$$

$$\rho(p) \equiv \frac{\partial \Phi(p)/\Phi(p)}{\partial p/p}, \tag{42b}$$

$$\gamma(K) \equiv Kg(K)/G(K) = \frac{\partial G(K)/G(K)}{\partial K/K}, \tag{42c}$$

where η is the elasticity of household capital goods with respect to the price, ρ is the elasticity of energy supply with respect to the price, and γ is the elasticity of the *share* of households with less than K with respect to that amount.

Then, to determine the effect of a minimum standard, we differentiate (41). After some rearrangements (see Appendix A.2.1), this yields:

$$\widehat{h} = \frac{1 + \varepsilon_{\check{K},\underline{K}}\gamma(\check{K})}{\eta - \varepsilon_{\check{K},h}\gamma(\check{K})}\widehat{\underline{K}} + \frac{\varepsilon_{\check{K},p}\gamma(\check{K})}{\eta - \varepsilon_{\check{K},h}\gamma(\check{K})}\widehat{p}, \tag{43a}$$

$$\widehat{p} = \frac{\varepsilon_{E,K}(\underline{K}) - \varepsilon_{\check{K},\underline{K}}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}}{\chi(\underline{K}) + \varepsilon_{\check{K},p}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}}\widehat{\underline{K}} - \frac{\varepsilon_{\check{K},h}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}}{\chi(\underline{K}) + \varepsilon_{\check{K},p}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}}\widehat{h}, \tag{43b}$$

where the hat notation represents relative changes (like $\widehat{h} \equiv \mathrm{d}h/h$) and where

$$\chi(\underline{K}) \equiv \rho - \varepsilon_{E,p}(\underline{K}) + \int_{\check{K}}^{\infty} \frac{g(K_{i,0})}{G(\check{K})} \left[\rho - \varepsilon_{E,p}(K_{i,0})\right] \frac{E^*(K_{i,0})}{E^*(\underline{K})} \,\mathrm{d}K_{i,0}. \tag{44}$$

For clarity of exposition, the explicit dependency of $\check{K}$, $\varepsilon_{\check{K},\underline{K}}$, $\varepsilon_{\check{K},h}$, $\varepsilon_{\check{K},p}$ on $(h, p, \underline{K})$ has been dropped and, in general, the dependency of the equilbrium quantities and elasticities on the price p and h has been dropped because these prices are the same for all agents. The denominators are positive.

The first-term fractions on the right-hand sides of (43) embody the direct effects of changing the standard, while the second terms are the indirect effects that accrue by the change of the respective other price.

First consider $\widehat{h}$. In the direct-effects numerator, the 1 represents the intensive-margin effect; the increase in the minimum standard directly increases capital demand of the households that invest. The second term represents the extensive-margin effect; the maximum level of capital at which a household chooses to invest is reduced, such that less households invest. The strength of this second effect depends on how strongly $\check{K}$ reacts to $\underline{K}$ and on how many households are affected by this because their amount of capital equals $\check{K}$. The indirect effect in the second fraction shows that an increase in the energy price will additionally increase capital demand, and vice versa.

The direct effects have counterparts in the first numerator for $\widehat{p}$. The first term shows how the intensive-margin effect operates in the energy market; the households who invest would reduce their energy demand due to their higher amount of capital ($\varepsilon_{E,K} < 0$). Likewise, the second term represents the extensive-margin effect: the households who do not invest due to a stricter standard use $\Delta_E(\underline{K}, \check{K}, p)$ more energy than they would if they invested; the relative increase of such a household's energy consumption is $\Delta_E(\underline{K}, \check{K}, p)/E^*(\underline{K})$. The indirect effect in the second fraction shows that an increase in the capital price will further decrease capital demand, leading to further energy-demand reduction reduction, and vice versa.

Solving (43) yields the elasticities of the equilibrium prices in reaction to a change in the minimum-efficiency standard. Defining the equilibrium elasticities $\Omega_{p,\underline{K}}(\underline{K}) \equiv \widehat{p}^\bullet(\underline{K})/\widehat{\underline{K}}$, $\Omega_{h,\underline{K}}(\underline{K}) \equiv \widehat{h}^\bullet(\underline{K})/\widehat{\underline{K}}$ for the prices, and $\Omega_{K,\underline{K}}(\underline{K}) \equiv \widehat{\kappa}^\bullet(\underline{K})/\widehat{\underline{K}}$ and $\Omega_{E,\underline{K}}(\underline{K}) \equiv$

$\widehat{\Phi}^\bullet/\widehat{\underline{K}}$ for the quantities, we obtain:

$$\Omega_{h,\underline{K}}(\underline{K}) = \frac{\varepsilon_{\check{K},p}\gamma(\check{K})\left[\varepsilon_{E,K}(\underline{K}) + \frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right] + \chi(\underline{K})\left[1 + \varepsilon_{\check{K},\underline{K}}\gamma(\check{K})\right]}{\chi(\underline{K})\eta - \chi(\underline{K})\varepsilon_{\check{K},h}\gamma(\check{K}) + \eta\varepsilon_{\check{K},p}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}}, \tag{45a}$$

$$\Omega_{p,\underline{K}}(\underline{K}) = \frac{-\varepsilon_{\check{K},h}\gamma(\check{K})\left[\varepsilon_{E,K}(\underline{K}) + \frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right] + \eta\left[\varepsilon_{E,K}(\underline{K}) - \varepsilon_{\check{K},\underline{K}}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right]}{\chi(\underline{K})\eta - \chi(\underline{K})\varepsilon_{\check{K},h}\gamma(\check{K}) + \eta\varepsilon_{\check{K},p}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}}, \tag{45b}$$

$$\Omega_{K,\underline{K}}(\underline{K}) = \eta\Omega_{h,\underline{K}}(\underline{K}), \tag{45c}$$

$$\Omega_{E,\underline{K}}(\underline{K}) = \rho\Omega_{p,\underline{K}}(\underline{K}) \tag{45d}$$

where we can note that the denominators are positive.

Consider the simplest case: infinitely elastic supply functions. The prices then are constant, but what can we say about the quantity changes? Keeping in mind that ρ is part of χ, it is easy to derive the limits

$$\Omega_{K,\underline{K}}(\underline{K})|_{\eta\to\infty,\rho\to\infty} = 1 + \varepsilon_{\check{K},\underline{K}}\gamma(\check{K}), \tag{46a}$$

$$\Omega_{E,\underline{K}}(\underline{K})|_{\eta\to\infty,\rho\to\infty} = \frac{\varepsilon_{E,K}(\underline{K}) - \varepsilon_{\check{K},\underline{K}}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}}{1 + \int_{\check{K}}^{\infty}\frac{g(K_{i,0})}{G(\check{K})}\frac{E^*(K_{i,0})}{E^*(\underline{K})}\,\mathrm{d}K_{i,0}}. \tag{46b}$$

A marginal standard ($\varepsilon_{\check{K},\underline{K}} = 0$) increases investment (with an elasticity of 1) and decreases energy consumption (with an absolute elasticity smaller than $|\varepsilon_{E,K}(\underline{K})|$). For a stricter standard ($\varepsilon_{\check{K},\underline{K}} < 0$), both effects are reduced. Whether they can change their sign depends on how many households are affected. To further explore this, I again assume functional forms.

3.3 Isoelastic Functions: A Marginal Minimum-Efficiency Standard

We continue using the isoelastic functions introduced in Section 2.5. The price elasticities of the demand side are then constant by (30a): $\varepsilon_{E,p} = -1/\beta$. Additionally, we assume that the price elasticities of the supply side are constant. We now write $m(\underline{K})$ for the equilibrium version of (35):

$$m(\underline{K}) \equiv \tilde{m}(h^\bullet, p^\bullet, \underline{K}). \tag{47}$$

$\chi(\underline{K})$ and the elasticities from (45) are explicitly stated in Appendix A.2.2.

For the further analysis, it is useful to note the following property:

Lemma 1. *m is increasing in the minimum-efficiency standard. Specifically,*

$$m'(\underline{K})\underline{K} > \frac{\beta-1}{\beta}m(\underline{K}) + 1 > 0, \tag{48a}$$

Thus,

$$m(\underline{K}) = \begin{cases} = 1 \text{ for } \underline{K} = K^\circ, \\ > 1 \text{ for } \underline{K} > K^\circ, \end{cases} \tag{48b}$$

$$\lim_{\underline{K}\to\infty} m(\underline{K}) = \infty. \tag{48c}$$

Proof. See Appendix A.2.3. □

Together with (38), this immediately allows to state a preliminary result:

Lemma 2. *For a marginal standard, it holds that*

$$\frac{\Delta_E(K^\circ, \widetilde{K}, p)}{E^*(K^\circ)} = \frac{\beta-1}{\beta} = -\varepsilon_{E,K} \qquad \Leftrightarrow \qquad \varepsilon_{E,K} + \frac{\Delta_E(K^\circ, \widetilde{K}, p)}{E^*(K^\circ)} = 0.$$

For a stricter standard ($\underline{K} > K^\circ$),

$$\frac{\Delta_E(\underline{K}, \check{K}, p)}{E^*(\underline{K})} > \frac{\beta-1}{\beta} = -\varepsilon_{E,K} \qquad \Leftrightarrow \qquad \varepsilon_{E,K} + \frac{\Delta_E(\underline{K}, \check{K}, p)}{E^*(\underline{K})} > 0.$$

As the standard is changed,

$$\frac{\mathrm{d}}{\mathrm{d}\underline{K}} \frac{\Delta_E(\underline{K}, \check{K}, p)}{E^*(\underline{K})} = \frac{\beta-1}{\beta}m'(\underline{K}) > 0.$$

The lemma states that the indifferent household's energy savings due to a marginal increase of energy efficiency and its additional energy consumption if it does not invest cancel out if the efficiency standard is marginal. In (35) and (38), we can see that the direct effect of increasing the standard is positive, and we can see that an *increase* in the capital price and a *decrease* in the energy price would work in the same direction (and vice versa). The lemma states that the gap also increases after taking equilibrium effects into account, independently of the signs of $\Omega_{h,\underline{K}}(\underline{K})$ and $\Omega_{p,\underline{K}}(\underline{K})$.

We can now, firstly, characterize the effect of a marginal standard. We talk of an increase in aggregate capital demand when either the equilibrium price or the equilibrium quantity of capital or both are increased by the policy – which of these possibilities applies depends on the supply elasticity. Likewise, an increase in aggregate energy demand means that either the equilibrium energy consumption or its equilibrium price or both increase.

Proposition 2 (The effects of a marginal minimum-efficiency standard)**.** *Assume the energy-services benefit function and the supply functions are isoelastic as stated in Section 2.5, and $\eta \in (0,\infty)$ and $\rho \in (0,\infty)$. Then a marginal standard increases aggregate capital demand and reduces aggregate energy demand relative to the situation without a standard:*

$$\Omega_{h,\underline{K}}(K^\circ) = \frac{\chi(K^\circ)}{\chi(K^\circ)\eta - \chi(K^\circ)\varepsilon_{\check{K},h}\gamma(\widetilde{K}) - \eta\varepsilon_{\check{K},p}\gamma(\widetilde{K})\varepsilon_{E,K}} > 0, \tag{49a}$$

$$\Omega_{p,\underline{K}}(K^\circ) = \frac{\eta\varepsilon_{E,K}}{\chi(K^\circ)\eta - \chi(K^\circ)\varepsilon_{\check{K},h}\gamma(\widetilde{K}) - \eta\varepsilon_{\check{K},p}\gamma(\widetilde{K})\varepsilon_{E,K}} < 0, \tag{49b}$$

$$\Omega_{K,\underline{K}}(K^\circ) = \eta\Omega_{h,\underline{K}}(K^\circ) \in (0,1), \tag{49c}$$

$$\Omega_{E,\underline{K}}(K^\circ) = \rho\Omega_{p,\underline{K}}(K^\circ) < 0. \tag{49d}$$

If $\eta \to \infty$, we have $\Omega_{h,\underline{K}}(K^\circ) = 0$ and

$$\Omega_{p,\underline{K}}(K^\circ) = \frac{\varepsilon_{E,K}}{\chi(K^\circ) - \varepsilon_{\check{K},p}\gamma(\widetilde{K})\varepsilon_{E,K}} < 0, \tag{50a}$$

$$\Omega_{K,\underline{K}}(K^\circ) = \frac{\chi(K^\circ)}{\chi(K^\circ) - \varepsilon_{\check{K},p}\gamma(\widetilde{K})\varepsilon_{E,K}} \in (0,1), \tag{50b}$$

$$\Omega_{E,\underline{K}}(K^\circ) = \frac{\rho\varepsilon_{E,K}}{\chi(K^\circ) - \varepsilon_{\check{K},p}\gamma(\widetilde{K})\varepsilon_{E,K}} < 0. \tag{50c}$$

If additionally to $\eta \to \infty$, we have $\rho \to \infty$, then $\Omega_{h,\underline{K}}(K^\circ) = \Omega_{p,\underline{K}}(K^\circ) = 0$, $\Omega_{K,\underline{K}}(K^\circ) = 1$ and

$$\Omega_{E,\underline{K}}(K^\circ) = \frac{\varepsilon_{E,K}}{1 + (K^\circ)^{-\varepsilon_{E,K}} \int_{\widetilde{K}}^{\infty} \frac{g(K_{i,0})}{G(\widetilde{K})} K_{i,0}^{\varepsilon_{E,K}} \, \mathrm{d}K_{i,0}} < 0. \tag{51}$$

If additionally to $\eta \to \infty$, we have $\rho = 0$, then $\Omega_{E,\underline{K}}(K^\circ) = 0$.

If $\eta = 0$, $\rho \in [0,\infty]$, we have:

$$\Omega_{h,\underline{K}}(K^\circ) = \frac{1}{-\varepsilon_{\check{K},h}\gamma(\widetilde{K})} > 0 = \Omega_{p,\underline{K}}(K^\circ) = \Omega_{K,\underline{K}}(K^\circ) = \Omega_{E,\underline{K}}(K^\circ). \tag{52}$$

Proof. Using Lemma 2 and $\varepsilon_{\check{K},\underline{K}}(K^\circ) = 0$, (45) reduces to (49). The cases below follow from evaluating for the respective values. □

The proposition states that with positive but finite supply elasticities, a marginal standard will do what one would expect it to do, namely to increase household capital demand, thereby driving up its price. The increase in equilibrium investment in turn reduces energy demand, and the magnitude of this effect depends on $\varepsilon_{E,K}$; this also reduces the energy price.

If both capital and energy are supplied at fixed prices, the quantity effects are strongest – in particular, the increase in $\underline{K}$ translates into a proportional increase in investment, and a strong decrease in energy consumption. By contrast, if only capital is supplied at a fixed price and energy is supplied inelastically, the increase in investment does not affect energy consumption.

If, however, capital is supplied inelastically, then the capital price has to increase to keep households from investing. Because a household that invests has to choose a higher amount of capital due to the standard, the effect of the capital price works via the extensive margin: $\check{K}$ decreases, such that less households invest. There are no effects on the energy market because (by Lemma 2) the efficiency standard's energy demand effect on the households that still invest and the effect on the households that stop investing just cancel out.

3.4 Isoelastic Functions: A Stricter Efficiency Standard

So far, we have only considered a marginal energy efficiency standard. We now consider a stricter standard. Up to now we did not need to make assumptions about the distribution of the capital endowment among the households. In the following, we will use such an assumption in order to structure the analysis. Precisely, we assume that $\gamma(K)$ is constant; the distribution function is defined as follows.

Definition 2 (Isoelastic Distribution)**.** We call the distribution of initial household capital an *isoelastic distribution* if and only if the distribution function is

$$G(K) = \begin{cases} 0 & \text{for } K < 0, \\ XK^{\gamma} & \text{for } K \in \left[0, X^{-1/\gamma}\right], \\ 1 & \text{for } K > X^{-1/\gamma}, \end{cases} \tag{53}$$

with $\gamma > 0$, $X > 0$.

This distribution function is discussed in Appendix A.3. If $\gamma < 1$, the distributional mass is concentrated at low values of $K_{i,0}$, and vice versa; a uniform distribution implies $\gamma = 1$. Figure 3 illustrates it for $\gamma = 2$, Figure 4 illustrates it for $\gamma = 1/2$. Moreover, when assuming an isolelastic distribution function, we will assume that

$$\widetilde{K} < X^{-1/\gamma}, \tag{54}$$

such that there are at least some households who would not invest even if there were no standard.

While the distribution function may not be realistic, it allows a structured discussion of the efficiency standard's effects. Given the isoelastic distribution function, we

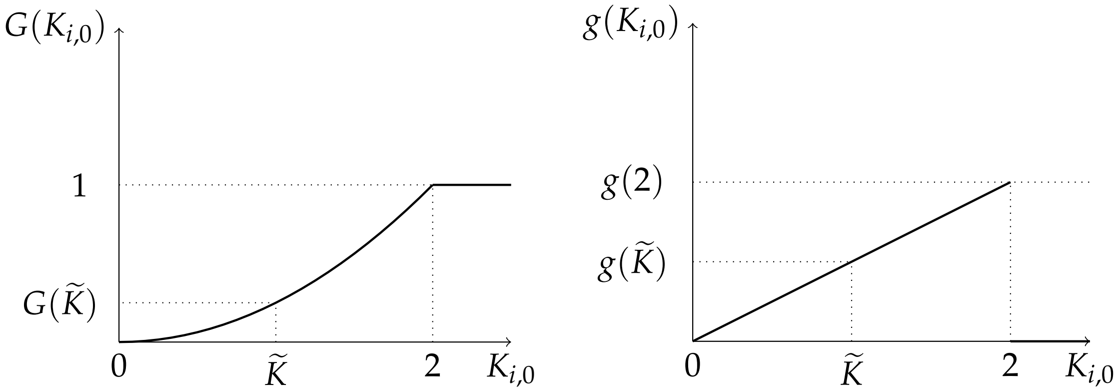

Figure 3: Isolelastic Distribution Function, example for $\gamma = 2$, $X = 1/4$.

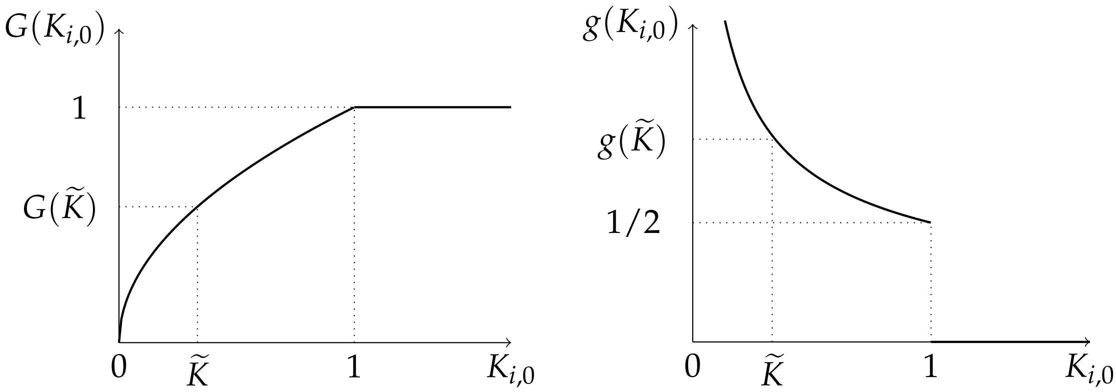

Figure 4: Isolelastic Distribution Function, example for $\gamma = 1/2$, $X = 1$.

first note that an ever-increasing efficiency standard will at some point keep all households from investing:

Lemma 3 (The equilibrium behavior of the cutoff level of capital)**.** *For isoelastic utility and supply functions, $\check{K}^\bullet(\underline{K}) \equiv \check{K}(h^\bullet, p^\bullet, \underline{K})$ is decreasing in $\underline{K}$. If additionally, the capital distribution function is isoelastic, we have $\lim_{\underline{K}\to\infty} \check{K}^\bullet(\underline{K}) = 0$.*

Proof. See Appendix A.4.2. □

We can now summarize how the elasticities from (45) change as the minimum-efficiency standard is made stricter.

Proposition 3 (The effects of a stricter minimum-efficiency standard)**.** *Assume that utility and supply functions and the capital-endowment distribution function are isoelastic. If $\gamma \leq \frac{\beta-1}{\beta}$, then increasing $\underline{K}$ always increases aggregate capital demand. By contrast, if $\gamma > \frac{\beta-1}{\beta}$, then aggregate capital demand initially increases in $\underline{K}$ but has a maximum at $\underline{K}$ which satisfies*

$$\varepsilon_{\check{K},p}\gamma \left[\varepsilon_{E,K} + \frac{\Delta_E(\underline{K}, \check{K}, p)}{E^*(\underline{K})}\right] + \chi(\underline{K}) \left[1 + \varepsilon_{\check{K},\underline{K}}\gamma\right] = 0. \tag{55}$$

Energydemand initially increases in $\underline{K}$ buthas a minimum at $\underline{K}$ which satisfies

$$-\varepsilon_{\check{K},h}\gamma\left[\varepsilon_{E,K}+\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right]+\eta\left[\varepsilon_{E,K}-\varepsilon_{\check{K},\underline{K}}\gamma\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right]=0 \tag{56}$$

This implies the following critical value:

$$\begin{aligned}
&\Omega_{p,\underline{K}}(\underline{K}),\Omega_{E,\underline{K}}(\underline{K}) \gtreqless 0\\
\Leftrightarrow & m(\underline{K}) \gtreqless \frac{1}{2}\left(1+\frac{1}{\gamma}\frac{\eta}{1+\eta}\frac{\beta-1}{\beta}\right)+\sqrt{\left[\frac{1}{2}\left(1+\frac{1}{\gamma}\frac{\eta}{1+\eta}\frac{\beta-1}{\beta}\right)\right]^2+\frac{1}{\gamma}\frac{\eta}{1+\eta}}\\
\Leftrightarrow & \underline{K} \gtreqless \underline{K}^{crit}=\left[\frac{1}{2}\left(1+\frac{1}{\gamma}\frac{\eta}{1+\eta}\frac{\beta-1}{\beta}\right)+\sqrt{\left[\frac{1}{2}\left(1+\frac{1}{\gamma}\frac{\eta}{1+\eta}\frac{\beta-1}{\beta}\right)\right]^2+\frac{1}{\gamma}\frac{\eta}{1+\eta}}\right]^{\frac{\beta}{2\beta-1}} h^{-\frac{\beta}{2\beta-1}}p^{\frac{\beta-1}{2\beta-1}}
\end{aligned} \tag{57}$$

At the value of $\underline{K}$ that minimizes aggregate energy demand, aggregate capital demand is still increasing.

Proof. See Appendix A.4.3. □

The proposition firstly states that initially, aggregate investment demand increases with a minimum-efficiency standard, but this effect gets weaker the stricter standard, and it *may* get reversed at high levels. The left-hand side of (55) is the numerator of $\Omega_{h,\underline{K}}(\underline{K})$. The first summand reflects how the change in the energy price, which is induced by the intensive and extensive margin effects, shifts $\check{K}$: A higher energy price means that more households want to invest (and vice versa), as derived in Section 2.4. The bracketed term in the second summand summarizes the direct effect on the capital market, namely, that $\underline{K}$ increases investment of the households that still invest, while this share is itself reduced by $\underline{K}$. The number of deterred households is initially zero but grows with $\underline{K}$. At some point, it dominates, such that further increases in $\underline{K}$ reduce capital demand.

This will be the case if $\gamma > \frac{\beta-1}{\beta}$, that is, if the elasticity of the capital-endowment distribution function exceeds the absolute value of the elasticity of individual energy demand with respect to capital. Intuitively, if γ is high, then many households are affected if $\check{K}$ shrinks due to an increase of $\underline{K}$. By contrast, if γ is small, then even the strongest possible reaction of $\check{K}$ to $\underline{K}$ will not affect many households.

Clearly, the first case holds in particular if there are many households with high energy efficiency (cf. Figure 4). But it is also possible in the opposite case if the distribution is not *too* right-skewed. In a sensitivity analysis for rebound effects, Borenstein (2015, Table 1) considers energy-demand elasticities of -0.2, -0.4, and -0.6. In the third and fourth column of Table 1, I demonstrate the implied values of $\varepsilon_{E,K} = (\beta-1)/\beta$,

β	$\beta/(\beta-1)$	$\varepsilon_{E,p}$ $= -1/\beta$	$\varepsilon_{E,K}$ $= (\beta-1)/\beta$	$\underline{K}^{\text{crit}}(\gamma,\beta)/K^{\circ}$ for $\eta=\infty, \rho=\infty$ $\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$
5	5/4	−0.2	−0.8	1.915	1.567	1.341	1.163
5/2	5/3	−0.4	−0.6	1.942	1.581	1.347	1.165
5/3	5/2	−0.6	−0.4	1.965	1.593	1.353	1.167

Table 1: Sample Values

which then constitute the critical γ values. We can see that, for instance, the case of an energy-demand elasticity of -0.2 implies that if γ is smaller than $4/5$, investment will always increase as the standard is made stricter.

Secondly, the left-hand side of (56) is the numerator of $\Omega_{p,\underline{K}}(\underline{K})$. The first effect here is that a decrease in the capital price increases the amount of households investing (and vice versa) and thereby has the above-mentioned effects on the energy market at the intensive and extensive margins. The second effect again summarizes the (more) direct effects of $\underline{K}$. Energy consumption of the marginal (and still investing) household is reduced, and households are kept from investing such that their energy consumption is not reduced. We can see that aggregate energy demand is initially reduced by introducing a minimum-efficiency standard, but there will be a level of $\underline{K}$ minimizing it, and further increases in the standard increase energy consumption. At some point, more and more households are deterred, and the effect on these households is increasing, such that there is a value of $\underline{K}$ at which energy consumption is minimized.

The value of $m(\underline{K})$ that minimizes energy consumption equals 1 for $\eta = 0$, which corresponds to $\underline{K} = K^{\circ}$ by Lemma 1. Thus, if capital is supplied inelastically, such that the standard cannot increase its overall quantity, then it will always *increase* energy demand. By contrast, a more elastic capital supply means that it takes a stricter efficiency standard to minimize energy consumption.

From the perspective of advising second-best climate policy, we can note that the capital supply elasticity is decisive for the question whether at least moderate minimum-efficiency standards can have the effects that seem like official aims of policy. If capital supply elastically adjusts to additional demand created by a minimum-efficiency standard, then the standard reduces aggregate energy demand. By contrast, if capital is supplied inelastically, a standard cannot create additional investment. As the standard forces investing households to increase their investment, equilibrium then requires that other households are kept from investing. The net effect is increased energy demand.

Thirdly, at the standard that minimizes energy consumption, aggregate investment will still be increasing. Thus, observing additional aggregate investment is not a sufficient statistic for the influence of the policy on the energy market.

For the case in which prices are fixed due to perfectly elastic supply of both capital and energy, we can quantitatively characterize the energy-demand even without complex numerical simulations. Relative to the private optimum K°, the critical value of $\underline{K}$ from (57) then is

$$\frac{\underline{K}^{\text{crit}}(\gamma,\beta)}{K^{\circ}} = \left[\frac{1}{2}\left(1+\frac{1}{\gamma}\frac{\beta-1}{\beta}\right)+\sqrt{\left[\frac{1}{2}\left(1+\frac{1}{\gamma}\frac{\beta-1}{\beta}\right)\right]^{2}+\frac{1}{\gamma}}\right]^{\frac{\beta}{2\beta-1}}. \tag{58}$$

This is illustrated in Table 1 for the above-mentioned values of β. With increasing values of γ, $\underline{K}^{\text{crit}}$ comes close to K°.

4 Welfare, Distribution and Politics

We ultimately aim at analyzing the effect of $\underline{K}$ on utility and welfare. Up to now, we have not assumed that there are externalities or other market failures in the model. Therefore, a minimum-efficiency standard must reduce welfare in a utilitarian sense – or, as we use a partial-equilibrium model, reduce market surplus. However, its effect differs between households. We will now analyze these effects without adding market imperfections. The welfare changes can be considered costs (or benefits) to the households *before* adding potential benefits due to the correction of market failures.

For high-capital households that do not invest, utility is given by (13). The effect of the minimum-efficiency standard thus is given only by the effect of the standard on the energy price. Using (14) and the equilibrium energy-price elasticity, we have

$$\frac{\mathrm{d}U_i(K_{i,0},p,0)}{\mathrm{d}\underline{K}/\underline{K}} = -\Omega_{p,\underline{K}}(\underline{K})pE^{*}(K_{i,0}), \tag{59}$$

such that the a minimum-efficiency standard is good for such a household, up to the point where it minimizes the energy price. We can directly conclude that the household then *increases* energy consumption.[9] The effect of the price reduction on utility is stronger the higher the household's energy consumption; thus, in general households with $K_{i,0} > \check{K}$ favor a minimum standard that reduces aggregate energy demand, but the intensity of the preference (that is, the money saved due to the standard) is decreasing in the amount of capital.

To determine the effect of the standard on the low-capital households that do invest, differentiate $U_i(\underline{K},p,h\underline{K})$ and take the equilibrium-price effects into account:

$$\frac{\mathrm{d}U_i(\underline{K},p,h\underline{K})}{\mathrm{d}\underline{K}/\underline{K}} = -\left[\Omega_{p,\underline{K}}(\underline{K})pE^{*}(\underline{K})+\Omega_{h,\underline{K}}(\underline{K})h\underline{K}+h\underline{K}-pE^{*}(\underline{K})\right]. \tag{60}$$

[9] Dampening the effect of energy-consumption reduction, this is comparable to carbon leakage.

We see three effects. Firstly, like the high-capital households, the low-capital households benefit from a reduction in the energy price. The investing households benefit less from this than those non-investing households with a capital endowment between $\check{K}$ and $\underline{K}$; however, non-investors with $K_{i,0} > \underline{K}$ benefit even less. The households suffer from an increase in the capital price (which the households that do not invest do not care about). Finally, the difference $h\underline{K} - pE^*(\underline{K})$ represents that the households dislike that they are pushed towards a too high amount of capital, see (22). For a marginal standard, this effect is zero, but it gets worse the stricter the standard (if the household then still invests).

Next, the standard affects profits. Define the profit function of a representative household-capital supply firm by $\pi_K(\underline{K}) = h\kappa - C_K(\kappa)$ and that of an energy supplier by $\pi_E(\underline{K}) = p\Phi - C_E(\Phi)$, where the equilibrium quantities of κ and Φ depend on $\underline{K}$ as previously discussed, and C_K and C_E are cost functions. Differentiating π_K and π_E, we obtain

$$\frac{\partial \pi_K(\underline{K})}{\partial \underline{K}/\underline{K}} = \Omega_{h,\underline{K}}(\underline{K})h\kappa + \Omega_{K,\underline{K}}(\underline{K})\left[h - C_K'(\kappa)\right]\kappa = \Omega_{h,\underline{K}}(\underline{K})h\kappa, \tag{61a}$$

$$\frac{\partial \pi_E(\underline{K})}{\partial \underline{K}/\underline{K}} = \Omega_{p,\underline{K}}(\underline{K})p\Phi + \Omega_{E,\underline{K}}(\underline{K})\left[p - C_E'(\Phi)\right]\Phi = \Omega_{p,\underline{K}}(\underline{K})p\Phi, \tag{61b}$$

where the second step results from having assumed supply functions, and thus the price to equal marginal cost. We see that, naturally, the firms in each sector benefit from a policy that increases their respective product price, and vice versa.

To consider the total effect of the standard on welfare, we can write

$$\begin{aligned} W(\underline{K}) = &\int_0^{\check{K}} g(K_{i,0})\lambda(K_{i,0})U_i(\underline{K}, p, h\underline{K}) \\ &+ \int_{\check{K}}^{\infty} g(K_{i,0})\lambda(K_{i,0})U_i(K_{i,0}, p, 0)\,\mathrm{d}K_{i,0} \\ &+ \lambda_K\pi_K(\underline{K}) + \lambda_E\pi_E(\underline{K}) \end{aligned} \tag{62}$$

where $\lambda(K)$ is the welfare weight on a household's utility, which may depend on the capital endowment, and λ_K, λ_E are sector-specific weights on profit. These weights can be used to evaluate energy-efficiency policy for different distributional aims. Differentiating and using the previous results yields:

$$\begin{aligned} \frac{\partial W(\underline{K})}{\partial \underline{K}/\underline{K}} = &-\int_0^{\check{K}} g(K_{i,0})\lambda(K_{i,0})\left[h\underline{K} - pE^*(\underline{K})\right]\mathrm{d}K_{i,0} \\ &+ \Omega_{h,\underline{K}}(\underline{K})h\left[\lambda_K\kappa - \int_0^{\check{K}} g(K_{i,0})\lambda(K_{i,0})\underline{K}\,\mathrm{d}K_{i,0}\right] \\ &+ \Omega_{p,\underline{K}}(\underline{K})p\left[\lambda_E\Phi - \int_0^{\infty} g(K_{i,0})\lambda(K_{i,0})E^*(K_{i,0})\,\mathrm{d}K_{i,0}\right]. \end{aligned} \tag{63}$$

Firstly, suppose that all weights are identical and standardized to 1. Then the derivative reduces to

$$\frac{\partial W(\underline{K})}{\partial \underline{K}/\underline{K}} = -G(\check{K})\left[h\underline{K} - pE^{*}(\underline{K})\right]. \tag{64}$$

This is the investing households' excess of capital costs over energy costs, and it can be understood as the dead-weight loss of efficiency policy.

Secondly, suppose that energy is supplied from a foreign country, and that policy-makers do not care about foreign welfare. Then $\lambda_E = 0$ while all other weights are 1. The derivative becomes

$$\frac{\partial W(\underline{K})}{\partial \underline{K}/\underline{K}} = -G(\check{K})\left[h\underline{K} - pE^{*}(\underline{K})\right] - \Omega_{p,\underline{K}}(\underline{K})p\Phi. \tag{65}$$

Thus, a marginal standard, which reduces the energy price, has a positive effect on national welfare. As the standard is made stricter, the cost increases and is paid for by the low-capital households who invest. However, note that energy supplied by the world market will usually imply that the energy-price elasticity ρ is high, such that the effect of the policy on the energy price will be low.

Thirdly, policy could possibly be pushed so far that it increases the energy price, but this would only be in the interest of energy suppliers. By contrast, a policy that increases the capital price by inducing additional capital demand and reducing energy demand is in the interest of non-investing households *and* capital suppliers, potentially creating political support. It is worthwhile to note the difference to a carbon tax. A carbon tax imposes costs on all households, including those who have chosen appliances that are optimal for lower energy prices. The cost of a minimum-efficiency standard, by contrast, is concentrated on low-capital households. This may be the outcome of a political preference for policies that do not impose a retroactive penalty on people who have recently invested.

Summarizing, we can note that, with positive but finite supply elasticites, a minimum-efficiency standard as described in Proposition 2 benefits households with efficient appliances – in particular those with appliances that are just efficient enough for them not to invest – and capital suppliers, while it disadvantages households with inefficient capital goods that want to invest, and energy suppliers. By Proposition 3, we know that at some point, a standard would minimize energy demand and only the capital suppliers would benefit from increasing the standard above this level.

5 Discussion

The present article analyzes the effects of a typical energy-efficiency policy – the introduction of a minimum-efficiency standard. This policy forces investing households to choose higher energy efficiency, and keeps others from investing. Therefore, a minimum-efficiency standard may have unintended consequences. Introducing a marginal minimum-efficiency standard – that is, a standard that enforces energy efficiency barely above what households would choose on their own – has the consequences one would expect:Increased demand for efficient capital and reduced energy demand. However, there is a critical value of the standard that minimizes energy demand, and increasing it further will increase energy demand again, while capital demand is still increased.

Heterogeneity with respect to household endowments implies that the described policy has different distributional effects on different households. If voters understand these effects, then the heterogeneity will influence their political preferences with respect to energy-efficiency policy. This implies that further analyzing the political economy of energy-efficiency policy, and the choice of this environmental-policy instrument over others, is a valuable direction for future research.[10] The empirical prediction of the present theoretical model is that households who have high-efficiency devices – like fuel-efficient cars, a refrigerator with a high efficiency class, an efficient space-heating system – or who at least have recently invested into such devices, favor minimum-efficiency standards, but they care less the more efficient their device is. Households with old, inefficient devices should oppose such policies.

However, such policy preferences require that voters understand equilibrium effects of energy efficiency. This may not be the case, given that they do not even understand their own private energy savings (Allcott, 2011; Allcott and Greenstone, 2012; Allcott et al., 2014). While the model presented in this article assumes that households correctly anticipate their costs and benefits of investing in energy efficiency, a large literature has recently analyzed the implications of households underestimating the benefits of such investments, which implies an "energy-efficiency gap".[11] A simple way to represent this in our framework is to modify the household's condition for investment, (23), to:

$$U_i(\underline{K}, p, h\underline{K}) - \mu \geq U_i(K_{i,0}, p, 0), \tag{66}$$

where U_i is the function stating the true utility that the household will receive ex-post,

[10] A different kind of a political-economy analysis of energy-efficiency investment is analyzed in Voss (2015), where a government tries to influence its successor's energy consumption by choosing the investment into energy efficiency strategically.

[11] For an extensive overview of different meanings of the term, see Gerarden et al. (2017).

but due to misperceptions or undervaluation of the future, the household underestimates this alternative by μ. The implication is that the household's capital endowment that would induce him to choose investment, $\check{K}$, will be lower. The effects of minimum-efficiency standards will be qualitatively the same as previously discussed: As they force investing households out of their *perceived* private optimum, they make investment less attractive.

Up to now, the distribution of the endowments before the investment decision – and before the introduction of the efficiency policy – is exogenous, and households only invest for the current period. A valuable direction for future research is extending the model to a fully dynamic setting.

Finally, environmental effects have not yet been incorporated into the model. In the current model, the efficiency standard is therefore not welfare-improving even if it reduces aggregate energy consumption. Integrating this aspect into the analysis suggests a comparison with Pigou taxes.

A Appendix

A.1 Unconstrained Optimization: Calculations

Differentiating (9), taking (3) into account and making use of (5) yields:

$$\frac{\mathrm{d}U_i(K_i, p, hK_i)}{\mathrm{d}K_i} = \underbrace{\left[B'(S_i^*)K_i - p\right]}_{=0} \frac{\partial E^*(K_i, p)}{\partial K_i} - h + B'(S_i^*)E^*(K_i, p), \tag{A.1a}$$

$$\frac{\mathrm{d}^2U_i(K_i, p, hK_i)}{\mathrm{d}K_i^2} = B'(S_i^*)\frac{E^*(K_i, p)}{K_i}\left[\frac{1}{\beta(S_i^*)} - 2\right] < 0, \tag{A.1b}$$

where the sign stems from Assumption 1. By $\lim_{S\to 0} B'(S) = \infty$, $\varepsilon_{E,K}(K_i, p) < 0$, and $E^*(0, p) = 0$, it is always worthwhile to buy at least some capital: $K_i^\circ > 0$. The optimal amount depends on the capital price h and the energy price p, $K^\circ(h, p)$. Differentiating (10), we derive:

$$\varepsilon_{K^\circ,h}(h, p) \equiv \frac{\partial K^\circ(h, p)/K^\circ(h, p)}{\partial h/h} = -\frac{1}{2 + \varepsilon_{E,p}(K_i, p)}, \tag{A.2a}$$

$$\varepsilon_{K^\circ,p}(h, p) \equiv \frac{\partial K^\circ(h, p)/K^\circ(h, p)}{\partial p/p} = \frac{1 + \varepsilon_{E,p}(K_i, p)}{2 + \varepsilon_{E,p}(K_i, p)}. \tag{A.2b}$$

By Assumption 1, the denominators exceed unity, and the numerator of the second elasticity is positive. Thus, the household invests less if capital is more expensive, but more if energy is more expensive.

A.2 Minimum-Efficiency Standards: Calculations Without Assumptions About The Distribution Function

A.2.1 Price Changes on the Markets with Exogenous Price Changes on the Other Market

Differentiating (41) yields:

$$\begin{aligned}
\Phi'(p)\,\mathrm{d}p = &-g(\check{K})\Delta_E(\underline{K},\check{K},p)\frac{\partial \check{K}(h,p,\underline{K})}{\partial h}\,\mathrm{d}h \\
&+\left[G(\check{K})\frac{\partial E^*(\underline{K},p)}{\partial \underline{K}} - g(\check{K})\Delta_E(\underline{K},\check{K},p)\frac{\partial \check{K}(h,p,\underline{K})}{\partial \underline{K}}\right]\mathrm{d}\underline{K} \\
&+\left[G(\check{K})\frac{\partial E^*(\underline{K},p)}{\partial p} + \int_{\check{K}}^{\infty} g(K_{i,0})\frac{\partial E^*(K_{i,0},p)}{\partial p}\,\mathrm{d}K_{i,0}\right. \\
&\left.\quad - g(\check{K})\Delta_E(\underline{K},\check{K},p)\frac{\partial \check{K}(h,p,\underline{K})}{\partial p}\right]\mathrm{d}p, \qquad \text{(A.3a)}
\end{aligned}$$

$$\begin{aligned}
\kappa'(h)\,\mathrm{d}h = &\ G(\check{K}(h,p,\underline{K}))\,\mathrm{d}\underline{K} + g(\check{K}(h,p,\underline{K}))\underline{K}\frac{\partial \check{K}(h,p,\underline{K})}{\partial h}\,\mathrm{d}h \\
&+ g(\check{K}(h,p,\underline{K}))\underline{K}\frac{\partial \check{K}(h,p,\underline{K})}{\partial p}\,\mathrm{d}p + g(\check{K}(h,p,\underline{K}))\underline{K}\frac{\partial \check{K}(h,p,\underline{K})}{\partial \underline{K}}\,\mathrm{d}\underline{K}. \qquad \text{(A.3b)}
\end{aligned}$$

Substitute the demand-side elasticities from (5) and (26), and the supply-side elasticities and the definition of $\widetilde{\gamma}$ from (42), and collect terms:

$$\begin{aligned}
\frac{\rho(p)}{G(\check{K})}\frac{\Phi(p)}{E^*(\underline{K},p)}\widehat{p} = &\left[\varepsilon_{E,K}(\underline{K},p) - \varepsilon_{\check{K},\underline{K}}(h,p,\underline{K})\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K},p)}\right]\widehat{\underline{K}} \\
&+ (-\varepsilon_{\check{K},h}(h,p,\underline{K}))\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K},p)}\widehat{h} \\
&- \left[-\varepsilon_{E,p}(\underline{K},p) + \int_{\check{K}(h,p,\underline{K})}^{\infty}\frac{g(K_{i,0})}{G(\check{K})}(-\varepsilon_{E,p}(K_{i,0},p))\frac{E^*(K_{i,0},p)}{E^*(\underline{K},p)}\,\mathrm{d}K_{i,0}\right]\widehat{p} \\
&- \varepsilon_{\check{K},p}(h,p,\underline{K})\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K},p)}\widehat{p}, \qquad \text{(A.4a)}
\end{aligned}$$

$$\begin{aligned}
\frac{\eta(h)}{G(\check{K})}\frac{\kappa(h)}{\underline{K}}\widehat{h} = &\left[1 + \varepsilon_{\check{K},\underline{K}}(h,p,\underline{K})\gamma(\check{K})\right]\widehat{\underline{K}} + \varepsilon_{\check{K},p}(h,p,\underline{K})\gamma(\check{K})\widehat{p} \\
&- (-\varepsilon_{\check{K},h}(h,p,\underline{K}))\gamma(\check{K})\widehat{h} \qquad \text{(A.4b)}
\end{aligned}$$

where, again, the hat notation represents relative changes. Now substituting $\Phi(p^\bullet)$ and $\kappa(h^\bullet)$ from (41) and isolating $\widehat{p}$ and $\widehat{h}$, respectively, yields (43).

A.2.2 The Equilibrium Elasticities with Isolelastic Functions

In the current section, I write down $\chi(\underline{K})$, as defined in (44), and the equilibrium elasticities for the case of isoelastic functions as laid out in Section 3.3. To do so, note that the isoelastic functions imply

$$\frac{E^*(K_{i,0},p)}{E^*(\underline{K},p)} = (\underline{K}/K_{i,0})^{\frac{\beta-1}{\beta}} \tag{A.5}$$

which in turn implies that (44) becomes

$$\chi(\underline{K}) = \left(\rho + \frac{1}{\beta}\right)\left[1 + \underline{K}^{\frac{\beta-1}{\beta}} \int_{\check{K}}^{\infty} \frac{g(K_{i,0})}{G(\check{K})} K_{i,0}^{-\frac{\beta-1}{\beta}} \, \mathrm{d}K_{i,0}\right]. \tag{A.6}$$

Now denote the numerator of (45a) by N_h, the numerator of (45b) by N_p, and their denominator by Θ, and use (36), (38), and (47). This yields:

$$\begin{aligned}
N_h(\underline{K}) &= \varepsilon_{\check{K},p}\gamma(\check{K})\left[\varepsilon_{E,K} + \frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right] + \chi(\underline{K})\left[1 + \varepsilon_{\check{K},\underline{K}}\gamma(\check{K})\right] \\
&= \left(\frac{\beta-1}{\beta}\right)^2 \frac{m(\underline{K})\left[m(\underline{K})-1\right]}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K}) + \chi(\underline{K})\left[1 - \frac{m(\underline{K})-1}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\right], \qquad \text{(A.7a)} \\
N_p(\underline{K}) &= -\varepsilon_{\check{K},h}\gamma(\check{K})\left[\varepsilon_{E,K} + \frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right] + \eta\left[\varepsilon_{E,K} - \varepsilon_{\check{K},\underline{K}}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right] \\
&= \frac{\beta-1}{\beta}\left[\frac{m(\underline{K})\left[m(\underline{K})-1\right]}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K}) - \eta\left[1 - \frac{m(\underline{K})\left[m(\underline{K})-1\right]}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\right]\right], \qquad \text{(A.7b)} \\
\Theta(\underline{K}) &= \chi(\underline{K})\eta - \chi(\underline{K})\varepsilon_{\check{K},h}\gamma(\check{K}) + \eta\varepsilon_{\check{K},p}\gamma(\check{K})\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})} \\
&= \chi(\underline{K})\eta + \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\right], \qquad \text{(A.7c)}
\end{aligned}$$

where $\chi(\underline{K})$ is stated in (A.6). Then,

$$\begin{aligned}
\Omega_{h,\underline{K}}(\underline{K}) &= \frac{\left(\frac{\beta-1}{\beta}\right)^2 \frac{m(\underline{K})[m(\underline{K})-1]}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K}) + \chi(\underline{K})\left[1 - \frac{m(\underline{K})-1}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\right]}{\chi(\underline{K})\eta + \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\right]}, \qquad \text{(A.8a)} \\
\Omega_{p,\underline{K}}(\underline{K}) &= \frac{\frac{\beta-1}{\beta}\left[\frac{m(\underline{K})[m(\underline{K})-1]}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K}) - \eta\left[1 - \frac{m(\underline{K})[m(\underline{K})-1]}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\right]\right]}{\chi(\underline{K})\eta + \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\right]}, \qquad \text{(A.8b)} \\
\Omega_{K,\underline{K}}(\underline{K}) &= \eta\Omega_{h,\underline{K}}(\underline{K}), \qquad \text{(A.8c)}
\end{aligned}$$

$$\Omega_{E,\underline{K}}(\underline{K}) = \rho \Omega_{p,\underline{K}}(\underline{K}). \tag{A.8d}$$

A.2.3 Proof of Lemma 1

For $m(\underline{K})$ from (47), such that we employ the elasticities from (45), we have

$$m'(\underline{K})\underline{K} = \left[\frac{2\beta-1}{\beta} + \Omega_{h,\underline{K}}(\underline{K}) - \frac{\beta-1}{\beta}\Omega_{p,\underline{K}}(\underline{K})\right] m(\underline{K}). \tag{A.9}$$

Note that

$$\begin{aligned} &1 + \Omega_{h,\underline{K}}(\underline{K}) - \frac{\beta-1}{\beta}\Omega_{p,\underline{K}}(\underline{K}) \\ &= \frac{\chi(\underline{K})\eta + \chi(\underline{K})\left[1 + \frac{\gamma(\check{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\right] + \eta\left(\frac{\beta-1}{\beta}\right)^2\left[1 + \frac{\gamma(\check{K})m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\right]}{\chi(\underline{K})\eta + \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\right]} > 0. \end{aligned} \tag{A.10}$$

Thus,

$$\begin{aligned} &m'(\underline{K})\underline{K} - \left[\frac{\beta-1}{\beta}m(\underline{K}) + 1\right] = \left[1 + \Omega_{h,\underline{K}}(\underline{K}) - \frac{\beta-1}{\beta}\Omega_{p,\underline{K}}(\underline{K})\right] m(\underline{K}) - 1 \\ &= \frac{(m(\underline{K})-1)\,\chi(\underline{K})\eta + m(\underline{K})\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2\right]}{\chi(\underline{K})\eta + \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\right]} > 0. \end{aligned} \tag{A.11}$$

The inequality follows from $m(K^\circ) = 1$ by (35) and from continuity for $\underline{K} > K^\circ$. This proves Lemma 1.

A.3 Isoelastic Distribution Function

We assume that household capital follows a isoelastic distribution (see Definition 2).[12] This assumption is apparently unrealistic if it is meant to represent a long-run distribution of capital because. The problem is not so much the upper bound of $K = X^{-1/\gamma}$ (we can standardize prices and the other functions to make that fit). However, we have seen that everybody owning less than a certain amount of capital would invest, and everybody would choose the same amount, such that after investment, we would always have a left-truncated distribution (no matter what its further properties look like) and a discontinuous spike. Even if immediately after investment there is some shock damaging all capital in a random manner, such that the distribution becomes

[12]The distribution is also called a Generalized Uniform Distribution (Lee, 2000), but other authors use the term differently (Balakrishnan and Nevzorov, 2003; Jayakumar and Sankaran, 2016), such that I use the term "isoelastic distribution" for clarity.

continuous, there will still be a bump. Nonetheless, for the moment I assume the isoelastic distribution for tractability. Its properties are as follows:

Lemma A.1 (Isoelastic Distribution: Properties)**.** *With an isoelastic distribution, the density function is*

$$g(K) = \frac{\partial G(K)}{\partial K} = \begin{cases} \gamma X K^{\gamma-1} & \textit{for } K \in \left[0, X^{-1/\gamma}\right], \\ 0 & \textit{else}, \end{cases} \tag{A.12a}$$

and the elasticity of the share of households with capital below K with respect to K is

$$\gamma(K) = \begin{cases} \gamma & \textit{for } K \in \left[0, X^{-1/\gamma}\right], \\ 0 & \textit{else.} \end{cases} \tag{A.12b}$$

The mean and the variance are

$$\mathcal{E}(K) = \int_0^1 g(K_{i,0}) K_{i,0} \,\mathrm{d}K_{i,0} = \frac{\gamma}{1+\gamma} X, \tag{A.12c}$$

$$\mathcal{V}(K) = \int_0^1 g(K_{i,0}) \left[K_{i,0} - \mathcal{E}(K)\right]^2 \mathrm{d}K_{i,0} = X\gamma \left(\frac{1}{2+\gamma} + X\gamma \frac{X-2}{(1+\gamma)^2} \right). \tag{A.12d}$$

Note that no other distribution function can be isoelastic:

Lemma A.2. *If $\gamma(K)$ is constant for $K \in [K_{\min}, K_{\max}]$, then the form of the distribution function of K must be the one from Definition 2.*

Proof. Solving $\frac{\partial G(K)}{\partial K} \frac{K}{G(K)} = \gamma$ (with a constant right-hand side) yields $G(K) = K^\gamma X$ for an arbitrary X. For a distribution function with bounds $K_{\min}, K_{\max}$, it must hold that $G(K_{\max}) = K_{\max}^\gamma X = 1$, which implies $K_{\max} = X^{-1/\gamma}$. Additionally, it must hold that $G(K_{\min}) = K_{\min}^\gamma X = 0$, which implies $K_{\min} = 0$. □

A.4 Minimum-Efficiency Standards: Calculations For The Isoelastic Distribution Function

A.4.1 Substituting the Isoelastic Distribution Function

If additional to assuming isoelastic utility and supply functions we assume that distribution function is isoelastic as defined in Definition 2, (A.6) becomes

$$\chi(\underline{K}) = \left(\rho + \frac{1}{\beta}\right) \times \begin{cases} \left[1 + \gamma \left(\frac{\underline{K}}{\check{K}}\right)^\gamma \ln\left(X^{-\frac{1}{\gamma}}/\check{K}\right)\right] & \text{for } \gamma = \frac{\beta-1}{\beta}, \\ \left[1 + \frac{\gamma}{\gamma - \frac{\beta-1}{\beta}} \left(\frac{\underline{K}}{\check{K}}\right)^{\frac{\beta-1}{\beta}} \left[\left(X^{-\frac{1}{\gamma}}/\check{K}\right)^{\gamma - \frac{\beta-1}{\beta}} - 1\right]\right] & \text{for } \gamma \neq \frac{\beta-1}{\beta} \end{cases} \tag{A.13}$$

with

$$\left(\frac{\underline{K}}{\check{K}}\right)^{\frac{\beta-1}{\beta}} = \frac{\beta-1}{\beta}m(\underline{K})+1, \tag{A.14a}$$

$$\frac{X^{-\frac{1}{\gamma}}}{\check{K}} = \frac{X^{-\frac{1}{\gamma}}}{\underline{K}}\left[\frac{\beta-1}{\beta}m(\underline{K})+1\right]^{\frac{\beta}{\beta-1}} \tag{A.14b}$$

by (34). Note that $\check{K} < X^{-\frac{1}{\gamma}}$ by (54), because $X^{-\frac{1}{\gamma}}$ is the highest value of the distribution. Thus, it is immediately visible that $\chi(\underline{K}) > 0$ for $\gamma = \frac{\beta-1}{\beta}$. For $\gamma \neq \frac{\beta-1}{\beta}$, either $\gamma > \frac{\beta-1}{\beta}$, such that first term in the square-bracketed difference is greater than 1 and the fraction with which it is multiplied is positive, or $\gamma < \frac{\beta-1}{\beta}$, such that the opposite holds in both cases and the whole term is again positive.

The elasticities stated in (A.8) remain identical except that $\gamma(\check{K}) = \gamma$.

A.4.2 Proof of Lemma 3

Taking (36) and (47) into account, the equilibrium effect of a change of $\underline{K}$ on $\check{K}$ is:

$$\begin{aligned}\frac{\mathrm{d}\check{K}^{\bullet}(\underline{K})/\check{K}}{\widehat{\underline{K}}} &= \varepsilon_{\check{K},\underline{K}} + \varepsilon_{\check{K},h}\Omega_{h,\underline{K}}(\underline{K}) + \varepsilon_{\check{K},p}\Omega_{p,\underline{K}}(\underline{K}) \\ &= -\frac{\left[1+\Omega_{h,\underline{K}}(\underline{K}) - \frac{\beta-1}{\beta}\Omega_{p,\underline{K}}(\underline{K})\right]m(\underline{K})-1}{\frac{\beta-1}{\beta}m(\underline{K})+1} \\ &\left(= 1 - \frac{m'(\underline{K})\underline{K}}{\frac{\beta-1}{\beta}m(\underline{K})+1}\right) < 0\end{aligned} \tag{A.15}$$

by (A.10). Explicitly, inserting (A.10) into (A.15),

$$\frac{\mathrm{d}\check{K}^{\bullet}(\underline{K})/\check{K}}{\widehat{\underline{K}}} = -\frac{\frac{[m(\underline{K})-1]}{\frac{\beta-1}{\beta}m(\underline{K})+1}\chi(\underline{K})\eta + \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\left[\chi(\underline{K})+\eta\left(\frac{\beta-1}{\beta}\right)^2\right]}{\chi(\underline{K})\eta + \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma(\check{K})\left[\chi(\underline{K})+\eta\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\right]}. \tag{A.16}$$

The numerator is always positive. Thus, $\frac{\mathrm{d}\check{K}(h,p,\underline{K})/\check{K}}{\widehat{\underline{K}}}$ can only go to zero if the denominator goes to infinity. Note that

$$\frac{\mathrm{d}}{\mathrm{d}\underline{K}}\frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1} = \frac{m'(\underline{K})}{\left(\frac{\beta-1}{\beta}m(\underline{K})+1\right)^2} > 0, \tag{A.17a}$$

$$\frac{\mathrm{d}}{\mathrm{d}\underline{K}}\frac{m(\underline{K})-1}{\frac{\beta-1}{\beta}m(\underline{K})+1} = \frac{\left(1+\frac{\beta-1}{\beta}\right)m'(\underline{K})}{\left(\frac{\beta-1}{\beta}m(\underline{K})+1\right)^2} > 0, \tag{A.17b}$$

$$\lim_{\underline{K}\to\infty} \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1} = \lim_{\underline{K}\to\infty} \frac{m(\underline{K})-1}{\frac{\beta-1}{\beta}m(\underline{K})+1} = \frac{\beta}{\beta-1}. \tag{A.17c}$$

Employing an isoelastic distribution function ($\gamma(\check{K}) = \gamma$) and using Lemma 1,

$$\begin{aligned}\lim_{\underline{K}\to\infty} \frac{\mathrm{d}\check{K}^{\bullet}(\underline{K})/\check{K}}{\widehat{\underline{K}}} &= -\lim_{\underline{K}\to\infty} \frac{\frac{\beta}{\beta-1}\chi(\underline{K})\eta + \frac{\beta}{\beta-1}\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2\right]}{\chi(\underline{K})\eta + \frac{\beta}{\beta-1}\gamma\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\right]} \\ &= -\lim_{\underline{K}\to\infty} \frac{(1+\eta)\frac{\chi(\underline{K})}{m(\underline{K})}}{\frac{\chi(\underline{K})}{m(\underline{K})}\frac{\beta-1}{\beta}\eta + \gamma\left[\frac{\chi(\underline{K})}{m(\underline{K})} + \eta\left(\frac{\beta-1}{\beta}\right)^2\right]}.\end{aligned} \tag{A.18}$$

In order to continue, we characterize the limit behavior of $\chi(\underline{K})/m(\underline{K})$. Firstly, we analyze the effect of a change of $\underline{K}$ on χ. Differentiating (A.13), using

$$\mathrm{d}\left(\left(\frac{\underline{K}}{\check{K}}\right)^{\frac{\beta-1}{\beta}}\right) = \frac{\beta-1}{\beta}\left(\frac{\underline{K}}{\check{K}}\right)^{\frac{\beta-1}{\beta}} \underbrace{\left(1 - \frac{\mathrm{d}\check{K}/\check{K}}{\widehat{\underline{K}}}\right)}_{>0} \widehat{\underline{K}}. \tag{A.19}$$

and simplifying, we obtain

$$\chi'(\underline{K})\underline{K} = \left(\rho + \frac{1}{\beta}\right) \times \gamma \times \left(\frac{\underline{K}}{\check{K}}\right)^{\frac{\beta-1}{\beta}} \times \begin{cases} \left[\gamma\left(1 - \frac{\mathrm{d}\check{K}/\check{K}}{\widehat{\underline{K}}}\right)\ln\left(X^{-\frac{1}{\gamma}}/\check{K}\right) - \frac{\mathrm{d}\check{K}/\check{K}}{\widehat{\underline{K}}}\right] & \text{for } \gamma = \frac{\beta-1}{\beta}, \\ \left[\frac{\frac{\beta-1}{\beta}}{\gamma - \frac{\beta-1}{\beta}}\left(1 - \frac{\mathrm{d}\check{K}/\check{K}}{\widehat{\underline{K}}}\right)\left[\left(X^{-\frac{1}{\gamma}}/\check{K}\right)^{\gamma - \frac{\beta-1}{\beta}} - 1\right] \right. & \\ \qquad \left. - \left(X^{-\frac{1}{\gamma}}/\check{K}\right)^{\gamma - \frac{\beta-1}{\beta}} \frac{\mathrm{d}\check{K}/\check{K}}{\widehat{\underline{K}}}\right] & \text{for } \gamma \neq \frac{\beta-1}{\beta}. \end{cases} \tag{A.20}$$

The sign for the first case is immediately clear by (A.15). For the second case, note that $\gamma > \frac{\beta-1}{\beta}$ implies, again, that the square-bracketed term is positive, and vice versa. Thus, the reasoning explaining the sign of (A.13) applies a forteriori. Using (A.14a), we can write

$$\chi'(\underline{K})\underline{K} = \left(\rho + \frac{1}{\beta}\right) \times \gamma \times m'(\underline{K})\underline{K} \times \begin{cases} \left[\gamma\ln\left(X^{-\frac{1}{\gamma}}/\check{K}\right) + 1 - \frac{\gamma m(\underline{K})+1}{m'(\underline{K})\underline{K}}\right] & \text{for } \gamma = \frac{\beta-1}{\beta}, \\ \left\{\frac{\frac{\beta-1}{\beta}}{\gamma - \frac{\beta-1}{\beta}}\left[\left(X^{-\frac{1}{\gamma}}/\check{K}\right)^{\gamma - \frac{\beta-1}{\beta}} - 1\right] + \left(X^{-\frac{1}{\gamma}}/\check{K}\right)^{\gamma - \frac{\beta-1}{\beta}}\left[1 - \frac{\frac{\beta-1}{\beta}m(\underline{K})+1}{m'(\underline{K})\underline{K}}\right]\right\} & \text{for } \gamma \neq \frac{\beta-1}{\beta}. \end{cases} \tag{A.21}$$

We derive a lower bound for this by substituting the limiting case from (48a), $m'(\underline{K})\underline{K} = \frac{\beta-1}{\beta}m(\underline{K}) + 1$. Substituting (A.14a) then yields:

$$\chi'(\underline{K})\underline{K} > \left(\rho + \frac{1}{\beta}\right)\frac{\beta-1}{\beta} \times \begin{cases} \gamma\left(\frac{\underline{K}}{\check{K}}\right)^{\gamma}\ln\left(X^{-\frac{1}{\gamma}}/\check{K}\right) & \text{for } \gamma = \frac{\beta-1}{\beta}, \\ \frac{\gamma}{\gamma-\frac{\beta-1}{\beta}}\left(\frac{\underline{K}}{\check{K}}\right)^{\frac{\beta-1}{\beta}}\left[\left(X^{-\frac{1}{\gamma}}/\check{K}\right)^{\gamma-\frac{\beta-1}{\beta}} - 1\right] & \text{for } \gamma \neq \frac{\beta-1}{\beta} \end{cases} \tag{A.22}$$

which is positive and increasing by (48a) and (A.15). Note that if $\chi'(\underline{K})\underline{K}$ were a positive constant, $\chi(\underline{K})$ would go to infinity as $\underline{K} \to \infty$. Thus, this holds for (A.21) a forteriori:

$$\lim_{\underline{K}\to\infty}\chi(\underline{K}) = \infty. \tag{A.23}$$

Using (A.14) and Lemma 1, we obtain

$$\lim_{\underline{K}\to\infty}\frac{\chi(\underline{K})}{m(\underline{K})} = \left(\rho + \frac{1}{\beta}\right) \times \frac{\beta-1}{\beta} \times \begin{cases} \gamma\lim_{\underline{K}\to\infty}\ln\left(X^{-\frac{1}{\gamma}}/\check{K}\right) & \text{for } \gamma = \frac{\beta-1}{\beta}, \\ \frac{\gamma}{\gamma-\frac{\beta-1}{\beta}}\lim_{\underline{K}\to\infty}\left(X^{-\frac{1}{\gamma}}/\check{K}\right)^{\gamma-\frac{\beta-1}{\beta}} & \text{for } \gamma \neq \frac{\beta-1}{\beta}. \end{cases} \tag{A.24}$$

Given the negative sign of (A.15), there are two seemingly possible cases. The first is that $\check{K}$ goes to some positive constant. Then $\lim_{\underline{K}\to\infty}\frac{\chi(\underline{K})}{m(\underline{K})} \equiv a$ is also some positive constant. But then,

$$\lim_{\underline{K}\to\infty}\frac{\mathrm{d}\check{K}^{\bullet}(\underline{K})/\check{K}}{\widehat{\underline{K}}} = -\frac{(1+\eta)a}{a\frac{\beta-1}{\beta}\eta + \gamma\left[a + \eta\left(\frac{\beta-1}{\beta}\right)^{2}\right]} \tag{A.25}$$

implies that $\lim_{\underline{K}\to\infty}\check{K} = 0$, which is a contradiction. Thus, consider $\lim_{\underline{K}\to\infty}\check{K} = 0$, which by (A.24) implies

$$\lim_{\underline{K}\to\infty}\frac{\chi(\underline{K})}{m(\underline{K})} = \infty. \tag{A.26}$$

Using this in (A.18) yields

$$\lim_{\underline{K}\to\infty}\frac{\mathrm{d}\check{K}^{\bullet}(\underline{K})/\check{K}}{\widehat{\underline{K}}} = -\frac{1+\eta}{\frac{\beta-1}{\beta}\eta + \gamma}. \tag{A.27}$$

Note that this is identical to the limit of the partial elasticity for $\eta \to \infty$; cf. (36c). This proves Lemma 3.

A.4.3 The Equilibrium Elasticities with Isolelastic Functions: Analysis

For a marginal standard, (A.8) reduces to

$$\Omega_{h,\underline{K}}(K^\circ) = \frac{\chi(K^\circ)}{\chi(K^\circ)\eta + \frac{\beta}{2\beta-1}\gamma(\check{K})\left[\chi(K^\circ) + \eta\left(\frac{\beta-1}{\beta}\right)^2\right]} \geq 0, \tag{A.28a}$$

$$\Omega_{p,\underline{K}}(K^\circ) = \frac{-\frac{\beta-1}{\beta}\eta}{\chi(K^\circ)\eta + \frac{\beta}{2\beta-1}\gamma(\check{K})\left[\chi(K^\circ) + \eta\left(\frac{\beta-1}{\beta}\right)^2\right]} \leq 0, \tag{A.28b}$$

$$\Omega_{K,\underline{K}}(K^\circ) = \eta\Omega_{h,\underline{K}}(K^\circ) \geq 0, \tag{A.28c}$$

$$\Omega_{E,\underline{K}}(K^\circ) = \rho\Omega_{p,\underline{K}}(K^\circ) \leq 0 \tag{A.28d}$$

where the inequalities are strict if supply elasticities are positive and finite. But this merely confirms Proposition 2. Turning to the analysis of stricter standards, I again assume an isoelastic distribution of capital endowments.

I first consider $\Omega_{h,\underline{K}}(\underline{K})$. As the derivatives of (A.7), even for $\gamma(\check{K}) = \gamma$, are quite involved, I focus on the limiting behavior. From (A.8a), we have

$$\begin{aligned}
\lim_{\underline{K}\to\infty} \Omega_{h,\underline{K}}(\underline{K}) &= \lim_{\underline{K}\to\infty} \frac{m(\underline{K})\frac{\beta-1}{\beta}\gamma + \chi(\underline{K})\left[1 - \frac{\beta}{\beta-1}\gamma\right]}{\chi(\underline{K})\eta + \frac{\beta}{\beta-1}\gamma\left[\chi(\underline{K}) + \eta\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\right]} \\
&= \lim_{\underline{K}\to\infty} \frac{\frac{\beta-1}{\beta}\gamma + \frac{\chi(\underline{K})}{m(\underline{K})}\left[1 - \frac{\beta}{\beta-1}\gamma\right]}{\frac{\chi(\underline{K})}{m(\underline{K})}\eta + \frac{\beta}{\beta-1}\gamma\left[\frac{\chi(\underline{K})}{m(\underline{K})} + \eta\left(\frac{\beta-1}{\beta}\right)^2\right]} \\
&= \frac{1 - \frac{\beta}{\beta-1}\gamma}{\eta + \frac{\beta}{\beta-1}\gamma} = \frac{\frac{\beta-1}{\beta} - \gamma}{\frac{\beta-1}{\beta}\eta + \gamma}
\end{aligned} \tag{A.29}$$

where the last line follows from (A.26). Thus, the elasticity remains positive if $\frac{\beta-1}{\beta} > \gamma$, goes to zero if $\frac{\beta-1}{\beta} = \gamma$, and goes to a negative value if $\frac{\beta-1}{\beta} < \gamma$. In the last case, it crosses zero at which (A.7a) is zero, which can be written as

$$\left(\frac{\beta-1}{\beta}\right)^2 m(\underline{K})\left[m(\underline{K}) - 1\right]\gamma + \left[\left(\frac{\beta-1}{\beta} - \gamma\right) m(\underline{K}) + 1 + \gamma\right]\chi(\underline{K}) = 0. \tag{A.30}$$

The characterization of $\Omega_{K,\underline{K}}(\underline{K})$ then follows from its definition in (45).

I now consider $\Omega_{p,\underline{K}}(\underline{K})$. By (A.28b), it is negative for $\underline{K} = K^\circ$. Note that by (A.7) and (A.17), we have $N_p'(\underline{K}) > 0$, which is necessary for the elasticity to become positive. I calculate the critical value of $\underline{K}$ at which this is indeed the case. From (A.7b), $N_p(\underline{K}) = 0$ for the value given in (57). Thus, for $\eta > 0$, $\Omega_{p,\underline{K}}(\underline{K})$ is negative for a marginal standard, decreases in absolute terms as the standard is made stricter, and becomes zero for some value of $\underline{K}$ (that accordingly minimizes the energy price and/or the energy consumption). Finally, it asymptotically goes to zero for $\underline{K} \to \infty$: From (A.8b), we have, by (A.24),

$$\lim_{\underline{K}\to\infty} \Omega_{p,\underline{K}}(\underline{K}) = \lim_{\underline{K}\to\infty} \frac{\frac{\beta-1}{\beta}(1+\eta)\frac{m(\underline{K})-1}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma}{\frac{\chi(\underline{K})}{m(\underline{K})}\eta + \frac{m(\underline{K})}{\frac{\beta-1}{\beta}m(\underline{K})+1}\gamma\left[\frac{\chi(\underline{K})}{m(\underline{K})} + \eta\left(\frac{\beta-1}{\beta}\right)^2\right]}$$
$$= \lim_{\underline{K}\to\infty} \frac{(1+\eta)\gamma}{\frac{\chi(\underline{K})}{m(\underline{K})}\left[\eta + \frac{\beta}{\beta-1}\gamma\right]} = 0. \tag{A.31}$$

The characterization of $\Omega_{E,\underline{K}}(\underline{K})$ then follows from its definition in (45).

For the last sentence of the proposition, note that rearranging (56) yields:

$$\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})} = -\frac{\varepsilon_{\check{K},h}\gamma - \eta}{\gamma\left(\varepsilon_{\check{K},h} + \eta\varepsilon_{\check{K},\underline{K}}\right)}\varepsilon_{E,K} \tag{A.32}$$

Substituting into the left-hand side of (55) yields:

$$\varepsilon_{\check{K},p}\gamma\left[\varepsilon_{E,K} - \frac{\varepsilon_{\check{K},h}\gamma - \eta}{\gamma\left(\varepsilon_{\check{K},h} + \eta\varepsilon_{\check{K},\underline{K}}\right)}\varepsilon_{E,K}\right] + \chi(\underline{K})\left[1 + \varepsilon_{\check{K},\underline{K}}\gamma\right]$$
$$= \left(\frac{\eta\varepsilon_{\check{K},p}\varepsilon_{E,K}}{\varepsilon_{\check{K},h} + \eta\varepsilon_{\check{K},\underline{K}}} + \chi(\underline{K})\right)\left[1 + \varepsilon_{\check{K},\underline{K}}\gamma\right] \tag{A.33}$$

In the fraction, both elasticities in the denominator and $\varepsilon_{E,K}$ are negative. Thus, the round-bracketed term is positive. By $\chi(\underline{K}) > 0$, the sign of the whole elasticity at this point depends on the square-bracketed term. Using (56), we write:

$$\operatorname{sgn}\left[1 + \varepsilon_{\check{K},\underline{K}}\gamma\right]\Big|_{\Omega_{p,\underline{K}}(\underline{K})=\Omega_{E,\underline{K}}(\underline{K})=0} = \operatorname{sgn}\left[\frac{-\varepsilon_{\check{K},h}\gamma + \eta}{\eta\frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}}\left[\varepsilon_{E,K} + \frac{\Delta_E(\underline{K},\check{K},p)}{E^*(\underline{K})}\right]\right] > 0, \tag{A.34}$$

where the sign follows from Lemma 2.

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Fondazione Eni Enrico Mattei
Corso Magenta 63, Milano – Italia

Tel. +39 02.520.36934
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