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Discontinuous Policy and Distorted Choices: The Case of Acreage Control

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Abstract

Past acreage response studies have failed to recognize that government control policies have typically introduced taxes and subsidies which are discontinuous over acreage planted. The nature of this discontinuity is reviewed and its impact on profit maximizing choice analyzed. A general model suitable for econometric estimation which would accommodate this discontinuity in policy is introduced. The effect of the discontinuity is to preclude ordinary least squares estimation. An extension of Tobin's maximum likelihood method is introduced to resolve the estimation complications.

Since Pigou [1951] economists have understood that taxes and subsidies are theoretically attractive policy instruments for nudging decisions toward social goals. However, in applications economists have also traditionally perceived a flaw in the lustre of such policies. This follows from the well-known result that the establishment of an optimal tax-subsidy scheme requires knowledge of the technologies faced by and behavioral objectives of the decision-makers to be taxed or subsidied, see Davis and Whinston [1966]. Such knowledge is of equal importance in the design of policies which introduce taxes or subsidies only over certain ranges of choices. Examples of such policies are AFDC, food stamps, and past as well as present acreage control policies.

Past studies of acreage response (e.g., Houck and Ryan [1973], Just [1974]) have attempted to introduce measures of policy instruments, however have failed to recognize that these instruments are continuously related to acreage and other production decisions only over a limited range. For example, under typical past acreage control programs, price support was paid on normal production so long as acreage planted did not exceed a target control level (e.g., the allotment or base acreage). If acreage was planted in excess of the target the entire structure of incentives[1] faced by the farmer changed. Under current legislation a similar discontinuity remains. This discontinuity in incentives implies that acreage and other production decisions will be discontinuously related to any particular set of incentives.

The purpose of this paper is to present a methodology which is capable of analyzing decisions made where policy is ~~discontinuous over~~

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[1] In addition to the structure of incentives, the uncertainty characterizing incentives would also change. However we shall overlook this effect for now.

the range of choices. First, a general model of decisions under discontinuous policy will be presented. This shall serve as the basis for emphasis of the importance of knowledge of technology and behavioral objectives to the policy maker hoping to design discontinuous policies. Next, the model will be translated into an empirically estimable form and maximum likelihood estimation procedures will be discussed.

Theoretical Framework

As a general example, suppose firms are profit maximizers choosing m net outputs (Y), n net variable inputs (X), and face a single regulated net input (L) and given flows from r fixed factors (θ). Technically efficient combinations of inputs and outputs shall be assumed to be related by a product transformation function which we may write in implicit form or, so long as the Jacobian is non-singular, in the explicit form:

$$1)\quad Y_1 = G(\hat{Y}, X, \theta, L)$$

where $\hat{Y}$ is the (M-1) 1 vector left after the deletion of Y_1.[1] As written, 1) relates net outputs to net inputs and fails to distinguish with which output particular input quantities are associated. To gain greater insight into the allocation of L among net outputs, we shall define A to be a M x 1 vector of output-specific utilizations of L, i.e., $A = (A_1, \ldots A_m)$. Policy targets for these allocations shall be indicated by $\bar{A}_i$. When L is fully utilized we re-write 1) as

$$2)\quad Y_1 = G(\hat{Y}, X, \theta, A_1, \ldots A_m).$$

Now suppose the objective of policy is to control A through a system of incentives (positive or negative) and in the absence of such policy the producer would face an m x 1 vector of market output prices (P), n x 1 vector of input prices r and suppose that L is fixed in the short-run but its allocation is unregulated. Further, we shall suppose the policy specifies that the producer receive a vector of subsidized prices $Pg = P + S$ so long as $A_i \leq \overline{A}_i$ for any $i = 1, \ldots m$. The implied discontinuity of such a policy is apparent when we write profit:

$$3) \quad \pi \equiv [Pg\, \alpha(A) + (I-\alpha(A))\, P]'Y - r'X - \lambda(\Sigma A_i - L)$$

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where $\alpha(A_i) = 1$ if $A_i \leq \overline{A}_i$ and $\alpha(A_i) = 0$ if $A_i > \overline{A}_i$ $\quad \forall\, i = 1, \ldots m$[2]

and $\alpha(A)$ is a m x m diagonal matrix with $\alpha(A_i)$ on the diagonal.

Maximizing 3) subject to 2) we have as necessary conditions:

$$4a) \quad P_1^* \frac{\partial G}{\partial Y_i} + P_i^* = 0 \qquad \forall\, i = 2, \ldots m$$

$$b) \quad P_1^* \frac{\partial G}{\partial X} - r = 0 \qquad \forall\, h = 1, \ldots n$$

$$c) \quad P_1^* \frac{\partial G}{\partial A_i} - \lambda = 0 \qquad \forall\, i = 1, \ldots m$$

$$d) \quad \sum_{i=1}^{m} A_i - L = 0$$

$$e) \quad Y_1 = G(\hat{Y}, X, \theta, A_1, \ldots A_m)$$

where $P_i^* = Pg_i\, \alpha(A_i) + P_i[1 - \alpha(A_i)] \qquad \forall\, i = 1, \ldots m$

If we assume the Hessian of G is negative definite, the system of equations 4) may be solved simultaneously for 2m + n + 1 choice functions relating optimal values of Y, X, A, and λ to P^*, r, θ and L:

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[2]For cross-compliance we would require $\alpha(A_i) = 1$ only if $A_i \leq \overline{A}_i \;\forall\, i = 1, \ldots m$.

5a) $Y_i^* = Y_i^*(P^*, r, \theta, L) \qquad i = 1, . . . m$

b) $X_h^* = X_h^*(P^*, r, \theta, L) \qquad h = 1, . . . n$

c) $A_i^* = A_i^*(P^*, r, \theta, L) \qquad i = 1, . . . m$

d) $\lambda^* = \lambda^*(P^*, r, \theta, L)$

By substitution of these choice functions into the profit definition 3), and employing 4d) we may define the profit function by composition:

e) $\pi^* = \pi(P^*, r, \theta, L)$.

Although the choice functions in 5) are continuous in P^* the discontinuity of P^* in Pg and P implies the choice functions are also discontinuous in Pg and P.

The nature of this discontinuity may be illustrated if for a particular $\overline{A}_i$, the point of discontinuity in incentives, we graph the resulting optimal profit π^* as a function of the acreage planted to A_i. Figures 1-3 present three possible cases which might occur depending upon the nature of technology and the level at which $\overline{A}_i$ is set.

The importance of knowledge of the discontinuous relation between choices and incentives to policy design should be clear from the above figures. In the absence of such knowledge $\overline{A}_i$ and Pg could be set at levels such that producers would be subsidized to plant $A_i^* \leq \overline{A}_i$, however as Figure 1 illustrates, they may have found such allocations optimal even in the absence of the policy. Alternatively, Figure 2 illustrates the case where despite the policy it remains optimal to choose $A_i^* > \overline{A}_i$.

Figure 1

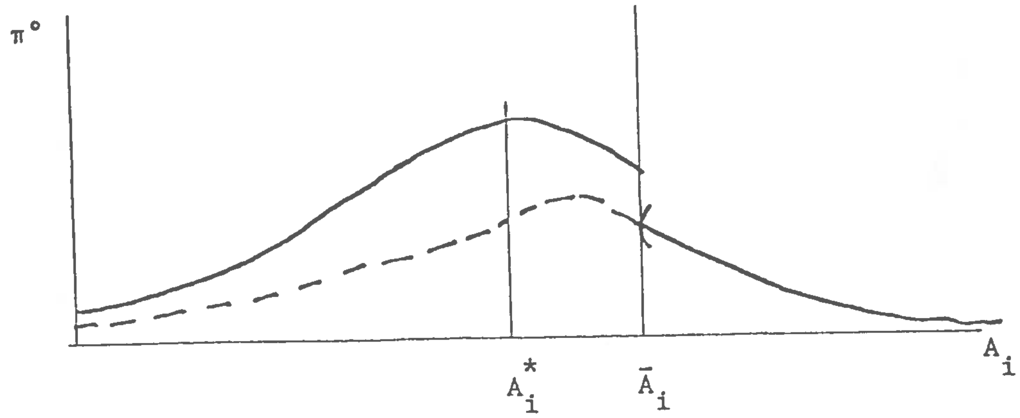


Figure 2

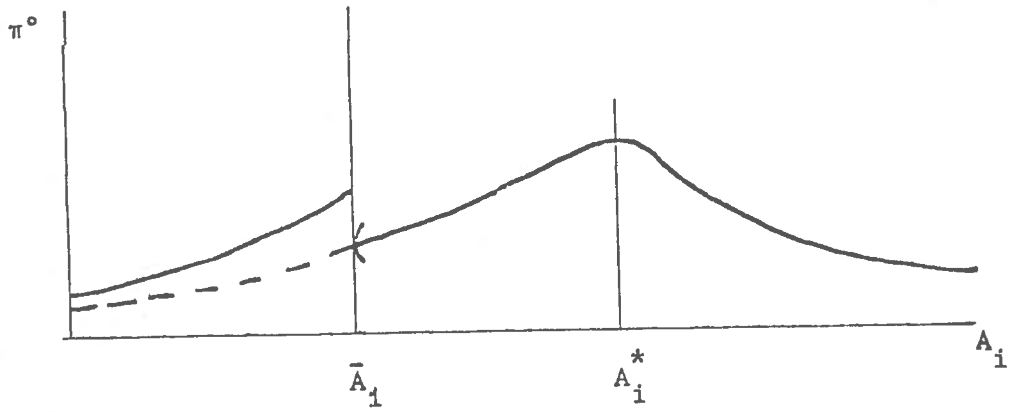
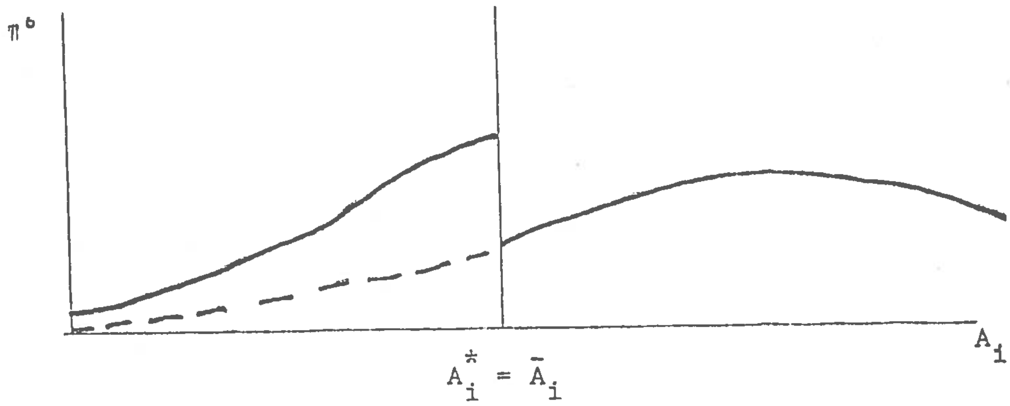


Figure 3



Finally, in Figure 3 the case of a binding policy is illustrated. In each of the above cases, the policy introduces a discontinuity into the decision objective and forces the producer to evaluate maximum profit in what amount to two different choice regimes. This same point may be seen by considering the discontinuity of the necessary conditions in 4).

Solution of the discontinous choice rules in 4) where p crop acreages are controlled, and cross compliance required results in the following discontinuous choice functions:

6) for $A_i^* < \bar{A}_i$ $\quad\forall\, i = 1, \ldots m$

$Y_i^* = Y_i(Pg, r, \theta, L)$ $\quad i = 1, \ldots m$

$X_h^* = X_h(Pg, r, \theta, L)$ $\quad h = 1, \ldots n$

$A_i^* = A_i(Pg, r, \theta, L)$ $\quad i = 1, \ldots m$

for $A_i^* = \bar{A}_i$ $\quad\forall\, i = 1, \ldots p,\ p < m$

$Y_i^* = Y_i(Pg, r, \theta, L)$ $\quad i = 1, \ldots m$

$X_h^* = X_h(Pg, r, \theta, L)$ $\quad h = 1, \ldots n$

$A_i^* = \bar{A}_i$ $\quad i = 1, \ldots p$

$A_i^* = A_i(P, r, \theta, L)$ $\quad i = p + 1, \ldots m$

for $A_i^* > \bar{A}_i$ for at least one i

$Y_i^* = Y_i(P, r, \theta, L)$ $\quad i = 1, \ldots m$

$X_h^* = X_h(P, r, \theta, L)$ $\quad h = 1, \ldots n$

$A_i^* = A_i(P, r, \theta, L)$ $\quad i = 1, \ldots m$

The framework presented is easily extended to incorporate diversion programs and other discontinuous policies, see Weaver [1978].

Econometrics of Choice under Discontinuous Policy

Econometric measurement of any theoretically derived relation undoubtedly relies upon accurate stochastic specification. Unfortunately, in the absence of empirical tests of hypothesized stochastic properties such hypotheses must be maintained, rendering results conditional upon their validity. To proceed, we shall employ additive, normally distributed, zero mean disturbances.

Focussing on the choice of any A_j, we have from 6) a discontinuous characterization of the choice of A_j:

7a) $A_j^* = A_j(Pg, r, \theta, L) \quad \text{if } A_j < \bar{A}_j$

b) $A_j^* = \bar{A}_j \quad \text{if } A_j = \bar{A}_j$

c) $A_j^* = A_j(P, r, \theta, L) \quad \text{if } A_j > \bar{A}_j$

To isolate the estimation problem, we represent these by a single function over the entire range of A_j:

8) $A_j^* = A_j(Pg, r, \theta, L) + A_j(P, r, \theta, L) + V_j$

where

$$V_j = \begin{cases} \varepsilon_j - A_j(P, r, \theta, L) & \text{if } A_j^* < \bar{A}_j \\ - A_j(Pg, r, \theta, L) - A_j(P, r, \theta, L) + \bar{A}_j & \text{if } A_j^* = \bar{A}_j \\ \varepsilon_j - A_j(Pg, r, \theta, L) & \text{if } A_j^* > \bar{A}_j \end{cases}$$

and $\varepsilon_j \sim N(0, \sigma_j^2)$.

Estimation of 8) by OLS would result in inconsistent estimates due to the correlation of V_j with hypothesized independent variables which also appear in its definition. Furthermore, we note $E(V_j) \neq 0$ and σ_{vj} is not constant over the sample. To proceed, we may recognize that estimation of 8) involves problems which are similar though more complex than those dealt with by Tobin [1958]. The root of the complication in Tobin's model as well as 8) lies in the discontinuity of the distribution of V_j. In Tobin's model V_j was truncated while in our model we have a more general type of discontinuity. Specifically, the specification of V_j is conditional upon the selection rules defined by the "if" statements in 8). In the case of Tobin's model these rules are expressed in terms the magnitude of the random variable ε_j relative to a limit.

The selection rules in 8) may be re-written in terms of ε_j using the following results:

$$9)\quad pr(A_j^* < \overline{A}_j) = pr(\varepsilon_j < N)$$

$$pr(A_j^* = \overline{A}_j) = pr(M \leq \varepsilon_j \leq N)$$

$$pr(A_j^* > \overline{A}_j) = pr(\varepsilon_j > M)$$

where $N = -A_j(Pg, r, \theta, L) + \overline{A}_j$

$M = -A_j(P, r, \theta, L) + \overline{A}_j$

$pr(\Sigma)$ indicates probability of event Σ.

Before proceeding, we shall specify a functional form for A_j^* in 7a) and 7c). For expository simplicity we shall assume the forms may be characterized by vectors of parameters β and α, respectively. Thus, we may insert these parameter vectors into the general forms of 7a) and 7c). Using this notation for the distrubutions of ε_j, for any sample $\Omega_t = (A_{jt}^*, Pg_t, P_t, r_t, \theta_t, L_t, \bar{A}_{jt})$ for t = 1, . . . T which we order such that for t = 1, . . . $\tau_1 A_j^* < \bar{A}_j$: for $t = \tau_1 + 1, \ldots \tau_2$, $A_j^* = \bar{A}_j$; and for $t = \tau_2 + 1, \ldots T$, $A_j^* > \bar{A}_j$: we may write the likelihood function as follows:

$$10) \quad L(\beta, \alpha, \Omega) = \prod_{t=1}^{\tau_1} f(\varepsilon_t) \prod_{\tau_1+1}^{\tau_2} \int_N^M f(\varepsilon_t) \prod_{\tau_2}^{T} F(\varepsilon_t)$$

where $N = -A_j(\beta, Pg, r, \theta, L) + \bar{A}_j$

$M = -A_j(\alpha, P, r, \theta, L) + \bar{A}_j$.

This likelihood function may be maximized by choice of β and α through grid-search techniques (e.g, see Thiel [1971]) or an adaptation of any of many Tobit estimation packages. Estimated parameters may then be employed to predict participation and its determinants (by the first two product terms involved in 10)) as well as the relation between determinants and acreage decisions. The methodology is currently being implemented by the author to investigate participation and acreage response at both the farm and state level.

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