April2019

FONDAZIONE ENI ENRICO MATTEI

Working Paper

008.2019

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Economics for Sustainability
Series Editor: Sergio Vergalli

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Summary

High dimensional composite index makes experts' preferences in setting weights a hard task. In the literature, one of the approaches to derive weights from a data set is Principal Component or Factor Analysis that, although conceptually different, they are similar in results when FA is based on Spectral Value Decomposition and rotation is not performed. This work motivates theoretical reasons to derive the weights of the elementary indicators in a composite index when multiple components are retained in the analysis. By Monte Carlo simulation it offers, moreover, the best strategy to identify the number of components to retain.

Keywords: Composite Index, Weighting, Correlation Matrix, Principal Component, Factor Analysis

JEL Classification: C38, C43, C15

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On the Use of Spectral Value Decomposition for the Construction of Composite Indices

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Abstract. High dimensional composite index makes experts' preferences in setting weights a hard task. In the literature, one of the approaches to derive weights from a data set is Principal Component or Factor Analysis that, although conceptually different, they are similar in results when FA is based on Spectral Value Decomposition and rotation is not performed. This works motivates theoretical reasons to derive the weights of the elementary indicators in a composite index when multiple components are retained in the analysis. By Monte Carlo simulation it offers, moreover, the best strategy to identify the number of components to retain.

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1 Introduction

Every composite index involves an initial effort by the developer to specify the phenomenon under analysis throughout the choice of the dimensions and underlying elementary indicators belonging to them. Apart this initial step, two polar typologies of composite index could be defined, based on the degree of involvement of one or more decision makers in its construction, namely in the data normalization and criteria weighting step: *fully unsupervised* and *fully supervised* composite index.

Most composite indices existing in the literature belong to the first category since that data driven normalization techniques (the most common min-max, or z-score), equal weighting or geometric mean or no weights (Mazziotta and Pareto, 2016) have been applied to construct such measure; fully supervised composite indices are strongly the minority; a non-exhaustive example can be found in Pinar et al. (2014) and Campagnolo et al. (2018).

There is no agreement on the best strategy to use for its construction despite the handbook of composite indicators developed by OECD in 2008 in collaboration with the Joint Research Centre COIN of the European Commission. Indeed, on one side some scholars criticize it because of its weak theoretically nature; they consider it an ideological statement rather than practically functional indicator; on the other side, there is a continuous effort by researchers in overcoming some implicit issues due to the techniques used to build them.

The weighting approach is one of the major disputes; at its root there is the debate between uniform and not uniform weights. In many cases there are not scientific reasons to choose equal weights, because communicating in a straightforward and easy

way the results of a composite index is the priority. Haq (2003) argued that, if there is insufficient reason to discriminate among the indices, equal weights should be used. Chowdhury and Squire (2006) referred to equal weighting as "obviously convenient but also universally considered to be wrong".
Weights, moreover, have a straightforward economic and social interpretation in linear aggregation functions. Ravallion (1997) for example, strongly criticized both the first edition of Human Development Index[1] and the Multidimensional Poverty Index (Akire and Foster, 2007) in 2011 because, focusing on the marginal rates of substitution (MRS) between the indices' dimensions, argued that they were questionable. MRS introduces a second debate on the use of compensatory approach in the construction of composite index. This issue can be solved using specific aggregation function such as the constant elasticity of substitution or the most advanced and non-additive technique, i.e. Choquet integral and fuzzy measures (Ishii and Sugeno, 1985; Grabisch, 1996, 1997, 2000; Marichal 2000a, 2000b; Meyer and Roubens, 2005), pushing the construction of composite indices towards the border between a fully supervised approach and modelling, namely Structural Equation Model (Kline, 1988).
However, high dimensional index makes experts' preferences in setting weights a hard task and obliges the developer to adopt a fully unsupervised approach. Referring again to Haq (2003) this paper argues that discriminating among indicators should be done whenever blocks of correlated indicators are detected and the latter are unevenly distributed among them. The reason is that the dimension with the highest number of indicators within it will have the highest influence on the composite index, not because it is explicitly more important than other dimensions, but implicitly for construction. This is hence an undesirable consequence and a valid reason to discriminate among variables. This work proposes the use of Spectral Value Decomposition (SVD) as a tool to differentiate weights. SVD is the fundamental mathematical properties of a covariance/correlation matrix and it is at the bases of Principal Component (PC) and Factor Analysis (FA). Since neither the components nor the factors are used to construct directly composite indices (for the reasons explained in section 3), but instead the rescaled eigenvectors or loadings, I voluntarily prefer citing the latter to avoid entering the field of formative/reflective models (see Simonetto, 2102 for a state of art) that is unnecessary and beyond the scope of this work.

The paper is organized as follows: section 2 describes the main approaches to weigh indicators; section 3 describes some existing procedures to derive weights by means of PC or FA and explain some theoretical reasons to derive the weights; Section 4 shows the results of Monte Carlo simulation for the special cases when blocks of independent indicators are generated. Section 6 concludes.

2 Weighting Techniques

Three macro typologies of weighting system exist: equal weighting, weighting based on experts' preferences and those based on the statistical properties of the data. There

[1] Human Development Report (1990)

is no weighting system above criticism (see Greco et al. 2013 for a review of issues on weighting). Each approach has its benefits and drawbacks. Probably there will be no end word on which approach is the best because most depends on the scope of the composite index and on the number of elementary indicators belonging to it.
This last point is the most binding one when weights are derived according to experts' preferences: when the number of indicators considered is relevant (see for example the list of indicators suggested by United Nations for measuring Sustainable Development Goals), approaches based on expert's preferences become a severe challenge, not only because it would be excessively time consuming for decision makers but also because the results could be biased by the complexity required. As mentioned in the introduction, the application of fuzzy measures and Choquet Integral would be the most powerful and brilliant approach to solve the problem of criteria weighting and data aggregation simultaneously, allowing to model both compensatory and non-compensatory preferences; indeed, they can exactly replicate the arithmetic mean, weighted average, min-max operator and Ordered Weighting Average (OWA) operator (Yager, 1988). The Achille's heel is the exponential complexity as the number of indicators used increases: the most simplified version, the so called 2-additive model (Grabisch, 1997) requires the elicitation of $n(n+1)/2$ parameters where n represents the number of indicators analysed. Other approaches based on experts' preferences are Budget Allocation Process, Analytic Hierarchy Process (Saaty, 1977) and its generalization Analytic Network Process (Saaty, 2004) and Conjoint analysis (Green et al. 2001; Wind and Green 2013); they allow for compensatory data aggregation only.

On weights derived by statistical properties of data, I mention *Correlation Analysis*, *Data Envelopment Analysis* (Charnes et al., 1978); *Principal Component* (Pearson, 1901; Hotelling, 1933) and *Factor Analysis* (Spearman, 1904) are the special focus of the next chapters.

3 Principal Component and Factor Analysis in Composite Indices

In Principal Components (PC) latent and orthogonal dimensions are linear function of observed variables; in Factor Analysis (FA) instead, observed variables are linear function of latent and orthogonal dimensions. Even though they are conceptually different, their utilization in the construction of composite indices is the same: nor in PC and in FA, indeed, the estimates of the components and factor themselves are used directly to compute the synthetic measure but, instead, as a device to derive the weights of the elementary indicators. This happens to avoid cases in which an indicator, adjusted for its polarity, has a direct impact that is discordant in sign with the synthetic measure we are computing. The results between PC and FA are identical whenever both eigenvectors and eigenvalues of the covariance or correlation matrix of the indicators (hence SVD) are used to estimate the linear parameters and factors are not rotated. Other techniques could be used in FA to extract factors, two of them still rely on SVD such as the Principal Factor method and Iterated Principal Factor method, the last is based on Maximum Likelihood method.

SVD is the fundamental property of symmetric matrices in general and of covariance and correlation matrix in statistics. The covariance matrix of a random vector $\mathbf{x}$ $(p \times 1)$, can be decomposed by SVD as $Cov(\mathbf{x}) \equiv \mathbf{\Sigma_x} = \mathbf{A\Lambda A'}$ where $\mathbf{A}$ is the eigenvector matrix and $\mathbf{\Lambda}$ the eigenvalue matrix. If we partition both the eigenvector matrix and the eigenvalue one into two submatrices representing the first k components and the last $(p-k)$, then $Cov(\mathbf{x}) = \mathbf{A}_1\mathbf{\Lambda}_1\mathbf{A'}_1 + \mathbf{A}_2\mathbf{\Lambda}_2\mathbf{A'}_2$. Both PC and FA (based on unrotated principal component) uses SVD to estimate the linear parameters of the first k components: in PC $\mathbf{z} = \mathbf{A'}_1\mathbf{x}$; in FA $\mathbf{x} = \mathbf{A}_1\mathbf{\Lambda}_1^{1/2}\mathbf{f} + \boldsymbol{\varepsilon}$. The coefficients of the FA are therefore proportional to those of PC and these makes identical the derived weights for composite indices.

Many approaches exist in the literature to derive the weights and to construct a composite index by means of PC or FA, but other that, there is no consensus on which is the best one to use. Generally speaking these could be split into two main approaches: in the first one, the developer uses only the first component (the one explaining most of the variance in the data) to build an index; in the second one, the developer, conscious that the first component represents an unsatisfactory portion of the total variance, uses additional components and then merges the components weighting them according to the proportion of variance explained by each. To the first approach belong the studies of Ram (1982), Ayanso et al. (2011) and Nguefack et al. (2011) who rescaled the coefficients to sum one, because they were all concordant in sign. Other authors (Avanzini, 2011; Man et al., 2015) retained the first component and squared the coefficient to derive the weights; in this case the weights represent the proportion of variance explained for the first component. To the second approach, with different techniques, belong the studies of Berlage (1988), Dialga et. Al (2016) and Nicoletti et al. (2000); by means of FA, some of them used the squared loadings in the selected components, others kept only those with highest value among factors and them rescaled them. In all cases factors are weighted according to their proportion on explained variance.

In the following I will motivate why retaining only the first component could be wrong and why weighting components/factors according to their explained variance is, instead, always wrong; I will explain some theoretical remarks to give a rationality for a correct approach. On one side retaining only the first component could be unsatisfactory because other components would be discharged even if important. This is especially true when the covariance matrix and not the correlation one is used in SVD, because the first component will be formed essentially by the variables with highest variance. Moreover, suppose of having p statistically independent random variables with unit variance; each component will explain $1/p$ of the total variance. If we retain the first component, we would discharge $(p-1)$ equally important variables. Second; weighting components proportionally to their explained variance is conceptually wrong: this is very clear in FA since the columns of $\mathbf{A}_1\mathbf{\Lambda}_1^{1/2}$ are proportional to their eigenvalues. In doing that, we overweigh indicators that are correlated penalizing those that are independent, leading consequently to unbalanced composite index. The following example, although extreme, will better explain the above issue: suppose we are going to construct a composite index with three indicators, two of which are perfectly positive correlated, while the third is statistically independent from the others. One of the two correlated variable is redundant and could be dropped in the composite index.

If we impose equal weights among variables, the composite index is unbalanced because one dimension, formed by the two correlated variables, is weighted twice with respect the second one, formed by the independent variable. The reader can see that weighting each component according to the proportion of explained variance leads to the same undesirable result. In the construction of composite indices, it is not recommended the technique that fits the data best but, instead, it is to prefer the one that best fits the dimensions the data are explaining, in which indicators that are statistically independent are weighted proportionally more than those that are correlated. To this aim, when using PC/FA technique, each component/factor should have the same weight.

If we form, from the matrix $\mathbf{A}_1$, the matrix $\mathbf{B}$ where $b_{ij} = a_{ij}^2$ represents the portion of variance explained by variable i-th to the j-th component, then each column sums to 1; the optimal weight vector is:

$$\mathbf{w}^* = \frac{1}{k}\mathbf{1}'\mathbf{B} \tag{1}$$

where k represents the number of components retained. Hence the weight assigned to the variable i-th represents its average contribution on the total variance explained by the components. Using equation 1), in the above example the two correlated variables have weights $1/4$ and the third $1/2$.

Regarding the number of components to retain, some guidelines have been proposed (see Rencher, 2002); the leading one is to retain enough components to account for a specified percentage of the total variance, say 80% or those components whose eigenvalues are greater than the average. The above remarks suggest for merging the two: let λ_i the i-th eigenvalue and

$$C = \left\{d \middle| argmin_d f(d) = \left|\sum_{i=1}^{d} \lambda_i - 0.8p\right|\right\},\ d = \{1, \dots, p\} \tag{2}$$

$$D = \{v | \lambda_v \geq 1\},\ v = \{1, \dots, p\} \tag{3}$$

the optimal number of components to retain d^* is given by:

$$k^* = max(d, v) \tag{4}$$

Monte Carlo simulation (see Section 4) shows however that the best strategy, at least to identify potential blocks of independent variables in the composite index, is to retain components whose eigenvalue is greater than average.

4 A Special Case – Blocks of Independent Variables

Consider for simplicity 5 indicators with zero mean; the first 3 belong to a first latent dimension; the other 2 to a second dimension independent from the first one. We can model the above setting in the following:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ a_2 & 0 \\ a_3 & 0 \\ 0 & a_4 \\ 0 & a_5 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{bmatrix} \tag{5}$$

or, compactly as:

$$\mathbf{x} = \mathbf{\Psi}'\mathbf{f} + \boldsymbol{\varepsilon} \tag{6}$$

with $Cov(\boldsymbol{\varepsilon}) = \mathbf{\Sigma_\varepsilon} = diag(\sigma^2_{\varepsilon_1}, \sigma^2_{\varepsilon_2}, \sigma^2_{\varepsilon_3}, \sigma^2_{\varepsilon_4}, \sigma^2_{\varepsilon_5})$ and $Cov(\mathbf{f}) = diag\left(\sigma^2_{f_1}, \sigma^2_{f_2}\right)$ for construction. Hence

$$Cov(\mathbf{x}) = \mathbf{\Sigma_x} = \mathbf{\Psi}'\mathbf{\Sigma_f}\mathbf{\Psi} + \mathbf{\Sigma_\varepsilon} \tag{7}$$

$$Cor(\mathbf{x}) = \mathbf{R_x} = \mathbf{D}\mathbf{\Sigma_x}\mathbf{D} \tag{8}$$

with $diag(D) = diag(\mathbf{\Sigma_x})^{-1/2}$. It follows for example that:

$$Cov(x_i, x_j) = a_i a_j \sigma^2_{f_1} \tag{9}$$

$$Cor(x_i, x_j) = \frac{a_i a_j \sigma^2_{f_1}}{\sqrt{a_i^2\sigma^2_{f_1} + \sigma^2_{\varepsilon_i}}\sqrt{a_j^2\sigma^2_{f_1} + \sigma^2_{\varepsilon_j}}} \tag{10}$$

$$Cov(x_i, y_j) = 0 \text{ for } i = 1,2,3 \text{ and } j = 1,2 \tag{11}$$

For each observation $i = 1, \dots n$ we have $\mathbf{x}_i{}' = \mathbf{f}_i{}'\mathbf{\Psi} + \boldsymbol{\varepsilon}_i{}'$ and:

$$\mathbf{X} = \mathbf{F}\,\mathbf{\Psi} + \mathbf{E} \tag{12}$$

Equation 12) allows us to simulate any set of cantered random variables with the properties given in equation 6), 7) or 8).
The following results have been found simulating 11 random variables; the first 6 belong to the fist dimension, the next 3 to a second one and the last two to a third dimension. In every simulation we allow some parameters to vary: $a_i \sim U[-1,1]$ with $i = 1, \dots, 11$; $f_j \sim N\left(0, \sigma^2_{f_j}\right)$ where $\sigma_{f_j} \sim U[1,6]$ with $j = 1, \dots, 3$. In all cases let $\varepsilon_i \sim N(0,1)$. Scope of the exercise is to check whether Spectral Value Decomposition can correctly identify the true structure of sample data and weigh the variables in such a way that the overall weight of each block is the same, hence one third.

Two techniques are compared to retain the correct number of components: Method A - many components as the number of eigenvalues greater than average; Method B - many components to ensure a total explained variance of 80%. The extraction is done using both the covariance (Table 1 to Table 3) and correlation matrix (Table 4 to Table 6).

Table 1 - Overall weight for each block on a given percentile. Extraction based on covariance matrix. Sample size n=50

	Method A			Method B		
Block	***5°***	***50°***	***95°***	***5°***	***50°***	***95°***
1°	0.290	0.489	0.995	0.048	0.496	0.996
2°	0.002	0.333	0.500	0.001	0.333	0.500
3°	0.001	0.326	0.497	0.001	0.251	0.497

Table 2 - Overall weight for each block on a given percentile. Extraction based on covariance matrix. Sample size n=100

	Method A			Method B		
Block	***5°***	***50°***	***95°***	***5°***	***50°***	***95°***
1°	0.324	0.495	0.998	0.031	0.497	0.998
2°	0.001	0.333	0.500	0.001	0.333	0.500
3°	0.000	0.329	0.498	0.000	0.255	0.498

Table 3 - Overall weight for each block on a given percentile. Extraction based on covariance matrix. Sample size n=300

	Method A			Method B		
Block	***5°***	***50°***	***95°***	***5°***	***50°***	***95°***
1°	0.329	0.498	0.999	0.010	0.499	0.999
2°	0.000	0.333	0.500	0.000	0.333	0.500
3°	0.000	0.332	0.499	0.000	0.260	0.499

Table 4 - Overall weight for each block on a given percentile. Extraction based on correlation matrix. Sample size n=50

	Method A			Method B		
Block	***5°***	***50°***	***95°***	***5°***	***50°***	***95°***
1°	0.332	0.381	0.546	0.293	0.427	0.595
2°	0.238	0.326	0.413	0.201	0.322	0.460
3°	0.188	0.279	0.332	0.158	0.248	0.341

Table 5 - Overall weight for each block on a given percentile. Extraction based on correlation matrix. Sample size n=100

	Method A			Method B		
Block	***5°***	***50°***	***95°***	***5°***	***50°***	***95°***
1°	0.332	0.357	0.523	0.263	0.421	0.602
2°	0.245	0.331	0.418	0.198	0.325	0.480
3°	0.200	0.305	0.333	0.164	0.250	0.372

Table 6 - Overall weight for each block on a given percentile. Extraction based on correlation matrix. Sample size n=300

	Method A			Method B		
Block	***5°***	***50°***	***95°***	***5°***	***50°***	***95°***
1°	0.333	0.340	0.504	0.254	0.406	0.621
2°	0.249	0.333	0.399	0.173	0.328	0.494
3°	0.229	0.326	0.333	0.165	0.250	0.391

As it can be seen from the results, latent dimension extraction based on the covariance matrix leads to biased results even in large sample cases. On the other hand, extraction based on correlation matrix and eigenvalues greater than 1 is the optimal solution guaranteeing unbiased and consistent estimates at a higher rate of convergence respect the method based on explained variance.

There is one more particularity on Method B that makes method A preferable. The following second simulation explains the issue. For each simulation, let $a_i = 1 \; \forall i = 1, \dots, 11$, $\sigma_{f_j}^2 = 1 \; \forall j = 1,2,3$ and we choose $\sigma_{\varepsilon_i}^2$ in such a way that the correlation among variables is fixed and equal for all combinations within blocks. Figure 1 shows, for a sample size $n = 50$, the overall block weight for any correlation values between 0.05 and 1. While with method A the trajectories converge to the equal weights case, with method B they converge only below a certain threshold and then explode to undesirable values. The reason is the following: for low level of correlations, more components are necessary to explain 80% of total variance; above a certain threshold instead, only few components are enough to explain the same amount of variance and the dimension formed by the fewest number of indicators is penalized.

A second characteristic is that the weights of the blocks converge at their desired values at a higher rate with method A respect method B as correlation increases. These results, at least for this special case, seems to slightly modify the conclusion made in equation 4 and opting for the eigenvalues criterion to retain components.

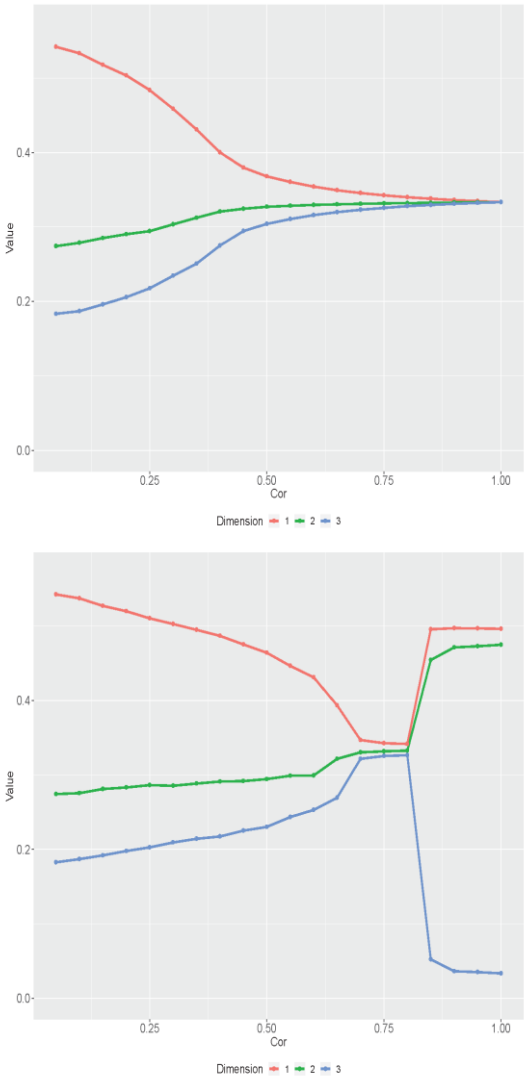

Figure 1 - Overall block weight given different correlation values; 50° percentile and sample size $n = 50$. Method A (top); Method B (bottom)

5 Conclusions

This works explains some theoretical reason to derive weights when multiple components or factors are retained in the analysis. Future works however are necessary to test

the validity of such approach when the variables among blocks are at least weakly correlated. However, most will depend on the strength of such blocks as the preliminary results of this work have shown: indeed, in the special case of independent variables among blocks, the higher the difference of correlation among blocks, the easier the identification of such structure and more reliable the derived weights as a consequence.

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