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by

Pan Liu, Dmitry Vedenov, and Gabriel J. Power

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[1] Liu is a Ph.D. candidate and Vedenov is an Associate Professor, Department of Agricultural Economics, Texas A&M University. Power is Associate Professor, Department of Finance, Insurance, and Real Estate, Faculty of Business Administration, Université Laval.
Correspondence to: Pan Liu, 2124 TAMU, College Station, TX 77843-2124. E-mail: liupan@tamu.edu

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Traditional approach to hedging crude oil refining margin (crack spread) adopts a fixed 3:2:1 ratio between the futures positions of crude oil, gasoline, and heating oil. However, hedging the latter in arbitrary proportions might be more effective under some conditions. The paper constructs optimal hedging strategies for both scenarios during the periods of relatively stable and volatile oil prices observed in recent years. Minimization of downside risk (LPM_2) and variance are used as alternative hedging objectives. The joint distribution of spot and futures price shocks is modeled using a kernel copula method. The hedging performance of the constructed strategies is compared using hedging effectiveness, expected profit, and expected shortfall. Results show that allowing for arbitrary proportions in sizes of futures positions generally achieves better hedging performance. The advantage becomes particularly important during periods characterized by a high volatility of the cross-dependence between the prices of individual commodities. In addition, using LPM_2 as a hedging criterion can help hedgers to better track downside risk as well as lead to higher expected profit and lower expected shortfall.

Key words: crack spread, optimal hedge ratio, kernel copula, downside risk.

Introduction

Between 2014 and 2016, crude oil prices have exhibited unusual behavior, dropping from over $100 per barrel to below $40 per barrel in less than 2 years. During the same time period, prices of both gasoline and heating oil almost halved. Facing such drastic changes in both input and output prices, oil refineries are presented with challenging risk management decisions.

A typical oil refinery's profit margin is tied directly to the price difference between crude oil and refined products, commonly called the crack spread. The most popular crack spread, which approximates the real-world output ratio from the refining process, adopts a 3:2:1 ratio, namely, 3 barrels of crude oil can be cracked into 2 barrels of gasoline and 1 barrel of heating oil (EIA, 2002). Oil refineries can reduce their risk exposures to volatile market prices by hedging the crack spread in the futures market. In 1994, NYMEX launched the crack spread contract, which bundles the purchase of three crude oil futures contract with the sale of two unleaded gasoline futures contract and one heating oil futures contract[2] (with delivery a month later) and makes them a single trade, thus lowering margin costs. A 3:2:1 crack spread futures position can also be created as a synthetic contract by directly trading futures on crude oil, gasolines and heating oil at a fixed 3:2:1 ratio. Even though the crack spread futures has a very low trading volume, data shows that the trading volume in the synthetic 3:2:1 crack spread is pretty high.

However, given the somewhat erratic behavior of spot prices in recent years, the question arises whether hedging individual commodities at a ratio other than 3:2:1 might be more

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[2] The heating oil contract traded on the New York Mercantile Exchange has been renamed ultra-low-sulfur-diesel (ULSD) futures after the 2013 April contract, but to keep the terminology and notation consistent, we will use the term heating oil (HO) when referring to both the heating oil contract before April 2013 and the ULSD contract after April 2013.

effective. Indeed, Kaminski (2014) explains (p.S4) that: "This [3:2:1 ratio] wasn't a perfect hedge by any definition … The decoupling of the WTI prices from the world prices reduced the efficiency of the 3:2:1 hedge and induced many hedgers to switch to Brent futures as the preferred hedging instrument…". Yet, compared with hedging crude oil, hedging the crack spread has received much less attention in the literature (Mahringer and Prokopczuk, 2015).

In this paper we address the above question by constructing optimal hedging strategies for both cases (fixed 3:2:1 ratio and arbitrary proportions) during periods of both relatively stable and volatile oil prices. The hedging performance of the constructed strategies is compared using several criteria. We find that allowing deviations from the fixed 3:2:1 ratio improves hedging performance regardless of the criterion used. Furthermore, it appears that the key factor affecting hedging effectiveness is the dependence structure between the spot and futures price shocks.

The rest of the paper is organized as follows. The second section discusses the relevant literature. The third section outlines the modeling methodology including the hedging framework, our approach to modeling the joint distribution of spot and futures prices, as well as risk measures used to evaluate hedging performance. Data and implementation details are presented in the fourth section followed by the discussion of the results in the fifth section. The last section provides concluding remarks.

Literature Review

Commodity processing activities always involve multiple commodities and thus exposure to price risk on both the input and output side. Soybean crushers buy soybeans and sell soybean oil and soybean meal, ethanol manufacturing involves purchasing corn and selling ethanol and other output products, oil refineries crack crude oil into petroleum products, and so on. Therefore, commodity processors have to implement multi-commodity hedging strategies.

The literature on hedging has traditionally focused on single-commodity hedging, which does not take into account price co-movements between the input and output commodities. However, Haigh and Holt (2002) point out that the assumption of price independence is unreasonable and often leads to optimal hedge ratios that are different from those suggested by the multi-commodity hedging models in which the covariation between the input and output prices is explicitly accounted for (see also Peterson and Leuthold, 1987). Several recent papers discuss hedging in a multi-commodity setting. Manfredo, Garcia and Leuthold (2000) study the hedging problem for a typical soybean crushing complex. They find that incorporating a time-varying covariance matrix into the joint price modeling can improve hedging effectiveness. Power and Vedenov (2010) and Power et al. (2013) analyze the multi-commodity hedging problem faced by a feedlot operator who buys feeder cattle, corn, and soybeans and sells fed cattle. The authors suggest that incorporating the dependence structure between commodity prices into the hedging model leads to hedging behavior that is more consistent with the one observed in the marketplace.

The crack spread hedging problem for oil refineries has attracted interest in recent years, partly due to the highly volatile oil market. The North American oil production and refining market has undergone major changes in recent years. According to Kaminski (2014, p. S3) "[the] increase in production of crude in locations such as The Bakken and Eagle Ford, which were a few years ago of marginal importance to the US oil industry … collided with the existing

transportation and refining infrastructure. Several congestion points emerged in the transportation grid and this, in turn, resulted in the breakdown of historical price relationships …"

Haigh and Holt (2002) show that accounting for time variation in the relationship between energy price series (crude oil, gasoline and heating oil) yields substantial rewards to hedgers in terms of risk reduction. Ji and Fan (2011) adopt a dynamic hedging approach for refineries and find that considering the interaction between different product markets as well as variation in price behavior over time can lead to a better multiproduct hedging strategy.

Various multivariate modeling methods as well as risk measures have been used to determine optimal hedging strategies and to analyze their performance. Variance of the effective net price or revenue continues to be the most commonly used measure of risk in the hedging literature, with variance minimization being the hedging objective. For example, Awudu et al. (2016) compare different hedging strategies for an ethanol producer using a Mean-VaR framework. However, Lien and Tse (2002) argue that a one-sided risk measure is closer to commodity hedgers' risk objective than the traditional variance measure in the sense that upside deviations and downside deviations are not equally undesirable in risk management. In that spirit, several recent papers use the second-order lower partial moment (LPM_2) as an alternative to variance (for example, Demirer and Lien, 2003; Turvey and Nayak, 2003; Mattos et al., 2008; Power and Vedenov, 2010).

Naturally, different risk measures lead to different hedging strategies. Mattos et al. (2008) find that when transaction costs and alternative investments are introduced, the adoption of a downside risk measure with low reference levels can lead to hedge ratios that differ substantially from the minimum-variance hedge ratios. Power and Vedenov (2010) find that minimizing the LPM_2 measure results in smaller optimal hedge ratios compared to the minimum variance hedge. Furthermore, they suggest that the optimal hedge ratios implied by the downside risk criterion are more consistent with the behavior observed in the marketplace. In order to account for possibility of different hedging objectives, in this paper, we construct optimal hedging strategies using both variance minimization (MV) and LPM_2 minimization as hedging criteria.

Another important issue in the analysis of multi-commodity hedging is how to model the joint distribution of prices/returns. Multivariate normality or log-normality is often assumed for reasons of convenience. However, the distributions of spot and futures prices are known to deviate from normality (e.g. Ederington, 2011; Lai, 2015). Several methods have been suggested in the literature to circumvent this issue. For example, Manfredo et al. (2000) estimate a time-varying covariance matrix and a MGARCH(1,1) model with a constant correlation matrix. Power and Vedenov (2010) use a kernel copula approach to model the joint distribution of spot and futures prices in a multi-commodity setting. The kernel copula methodology is nonparametric and imposes minimum assumptions about the underlying distribution. Tong et al. (2013) use thirteen parametric copula models with different underlying assumptions on the dependence structures to estimate the co-movement between crude oil and petroleum product prices. Power et al. (2013) propose a Nonparametric Copula-based Generalized Autoregressive Conditional Heteroskedastic (NPC-GARCH) dynamic hedging approach and find that it better captures lower tail risk than do other models such as GARCH-DCC or GARCH-BEKK. In this paper, we follow Power and Vedenov (2010) and use a kernel copula approach to model the joint distribution of

spot and futures price shocks. This allows us to move away from the multivariate normality assumption and better reproduce both the individual and joint behavior of price series.

Methodology

Hedging Framework
We follow the conceptual framework suggested by Ji and Fan (2011) and assume a two-stage hedging cycle that covers three weeks (15 trading days) in total. The first stage is the planning stage that covers two weeks ($t-3$ to $t-1$). On the first day of the planning stage, the hedger (refinery) opens a long position in crude oil futures at F_{t-3}^{CL} and short positions in gasoline and heating oil futures at F_{t-3}^{RB} and F_{t-3}^{HO}, respectively[3]. The second stage is the operational stage which covers one week ($t-1$ to t). On the first day of the operational stage, the hedger buys crude oil on the spot market at S_{t-1}^{CL} to start the cracking process and concurrently closes the long position in crude oil at F_{t-1}^{CL}. On the last day of the operational stage (after the cracking process is finished), gasoline and heating oil are sold on the spot market at S_t^{RB} and S_t^{HO}, respectively, and the corresponding short positions are closed at F_t^{RB} and F_t^{HO}, respectively. In order to simplify the notation, in the rest of the section the subscript 0 is used to denote prices on the day when the hedge is set and the subscript 1 is used to denote prices on the day when the hedge position is liquidated. Assuming a 3:2:1 production ratio, the hedged crack margin per barrel of crude oil can be then written as

$$\pi(\boldsymbol{h}) = -S_1^{CL} + \frac{2}{3}S_1^{RB} + \frac{1}{3}S_1^{HO} + h^{CL}(F_1^{CL} - F_0^{CL}) + \frac{2}{3}h^{RB}(F_0^{RB} - F_1^{RB}) + \frac{1}{3}h^{HO}(F_0^{HO} - F_1^{HO}) \tag{1}$$

where $\pi(\boldsymbol{h})$ is the hedged crack profit, $\boldsymbol{h} = (h^{CL}, h^{RB}, h^{HO})$ is a vector of hedge ratios, { F_0^{CL}, F_0^{RB}, F_0^{HO} } are observable initial futures prices, { S_1^{CL}, F_1^{CL} } are spot and futures prices of crude oil 10 trading days ahead, and { $S_1^{RB}, S_1^{HO}, F_1^{RB}, F_1^{HO}$ } are spot and futures prices of gasoline and heating oil 15 trading days ahead, respectively.

Two scenarios are considered in the paper. In the first scenario, the refinery hedges the entire crack spread in the fixed 3:2:1 proportion implying $h^{CL} = h^{RB} = h^{HO} = h$ (single hedge ratio). In the second scenario, the refinery can hedge each of the three commodities individually, thus allowing for separate and not necessarily equal hedge ratios $\{h^{CL}, h^{RB}, h^{HO}\}$ (vector hedge ratio). No hedging corresponds to $h^{CL} = h^{RB} = h^{HO} = 0$.

Hedging Objectives
Generally, minimum variance (MV) is the most commonly used criterion to determine the optimal hedge ratio. The optimal hedge ratio calculated under the MV criterion is

$$\boldsymbol{h}^* = \arg\min_{\boldsymbol{h}} \text{Var}(\pi(\boldsymbol{h})) \tag{2}$$

However, the fact that MV penalizes upside deviations and downside deviations equally is undesirable in risk management (e.g., Power and Vedenov, 2010). The LPM family is more suitable for the measurement of downside risk, which is of more interest for commodity hedgers.

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[3] The superscripts CL, RB and HO correspond to the futures contract symbols as listed by the CME Group.

In particular, the second order lower partial moment (LPM$_2$) has been increasingly used in the recent literature (Mattos et al., 2008; Power and Vedenov, 2010). The LPM$_2$ relative to a reference level $\bar{X}$ is defined as

$$LPM_2 = \int_{-\infty}^{\bar{X}} (\bar{X} - X)^2 dF(X) \tag{3}$$

where X is a random variable of interest and $F_X(X)$ is its cumulative distribution function.

For the hedging profit defined in (1), the reference level $\bar{\pi}$ can be set as the expected profit without hedging, i.e. $\bar{\pi} = E\pi(0)$. The optimal hedge ratio under the LPM$_2$ criterion can be then found as

$$\boldsymbol{h}^* = \arg \min_{\boldsymbol{h}} LPM_2(\boldsymbol{h}) = \arg \min_{\boldsymbol{h}} \int_{-\infty}^{\bar{\pi}} [\bar{\pi} - \pi(\boldsymbol{h})]^2 dF(\pi(\boldsymbol{h})) \tag{4}$$

The Joint Distribution of Spot and Futures Prices

The optimization problem in (4) does not have a closed-form solution, and therefore needs to be solved numerically. Monte Carlo simulation can be used to calculate the value of LPM$_2$ for any given vector of hedge ratios $\boldsymbol{h}$, and numerical optimization methods can be used to find the optimal hedge ratio $\boldsymbol{h}^*$.

In order to implement the Monte Carlo integration in (4), joint realizations of spot and futures prices $\{S_1^{CL}, S_1^{RB}, S_1^{HO}, F_1^{CL}, F_1^{RB}, F_1^{HO}\}$ in (1) need to be generated. The following approach is used to achieve this goal. First, historical spot and futures prices are log-differenced to calculate the multiplicative shocks $\{\varepsilon_1, ..., \varepsilon_6\}$, where $\varepsilon_1 = \ln(S_1^{CL}) - \ln(S_0^{CL})$, $\varepsilon_2 = \ln(S_1^{RB}) - \ln(S_0^{RB})$, etc.[4]. The joint distribution of shocks is then modeled using the copula approach. The latter decomposes the joint distribution into a product of marginal distributions of individual variables and their dependence structure, or copula density (Cherubini, Luciano, and Vecciato, 2004).

Marginal probability density functions $f_1^\varepsilon(\cdot), ..., f_6^\varepsilon(\cdot)$ are estimated using the kernel density method (Wand and Jones, 1995). The copula density $c(u_1, ..., u_6)$ implied by the historical realizations of the shocks is estimated using the mirror image kernel approach (Charpentier, Fermanian, and Scaillet, 2007).

Next, N Monte Carlo draws $\{u_1^i, ..., u_6^i\}_{i=1}^N$ from the copula density are generated following the conditional sampling approach outlined in Cherubini, Luciano, and Vecciato (2004). The generated draws are then transformed to draws from the joint distributions of shocks using the inverse marginal cumulative distribution functions. More specifically, for a given draw

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[4] Note that the lags used for log-differencing correspond to the duration of the appropriate stages of the hedging cycle as described in Section Hedging Framework For example, ε_1 is obtained by log-differencing spot prices of crude oil two weeks apart, ε_2 is obtained by log-differencing spot prices of gasoline three weeks apart, and so on.

$u_j^i, i = 1, \dots, N, j = 1, \dots, 6$, from the copula density, the corresponding shock ε_j^i is found from the condition

$$u_j^i = \int_{-\infty}^{\varepsilon_j^i} f_j^\varepsilon(\epsilon) d\epsilon, \tag{5}$$

which can be solved numerically using standard numerical integration and root-finding methods (e.g. Miranda and Fackler, 2002).

Lastly, the generated shocks are used to construct realizations of final spot and futures prices by applying them to (known) initial observations of the same, e.g. $\{S_1^{CL}\}_i = S_0^{CL} \cdot \exp \varepsilon_1^i$, $i = 1, \dots, N$, and so on. The constructed spot and futures prices can be used to calculate realizations $\{\pi^i(\boldsymbol{h})\}_{i=1}^N$ of net profit from hedging in (1) for any given vector of hedge ratios $\boldsymbol{h}$, and therefore determine the values of the hedging criteria in (2) and (4).

Measures of Hedging Performance

In addition to the risk criteria used to determine the optimal hedge ratios (variance and LPM₂), we calculate three measures, namely hedging effectiveness, expected profit, and expected shortfall, which are commonly used in the literature to evaluate hedging performance.

Following Ederington (1979), hedging effectiveness is defined as the percentage reduction in risk criterion with hedging vs. without hedging. Specifically, the hedging effectiveness for minimum variance is determined as

$$HE_{MV} = 1 - \frac{\mathrm{Var}\big(\pi(\boldsymbol{h}^*)\big)}{\mathrm{Var}\big(\pi(0)\big)}. \tag{6}$$

Similarly, for LPM₂, hedging effectiveness can be determined as

$$HE_{MV} = 1 - \frac{LPM_2\big(\pi(\boldsymbol{h}^*)\big)}{LPM_2\big(\pi(0)\big)}. \tag{7}$$

Expected profit is calculated by averaging calculated realizations of net profit from hedging $\{\pi^i(\boldsymbol{h})\}_{i=1}^N$ over the Monte Carlo draws, i.e.

$$E\pi = \frac{1}{N} \sum_{i=1}^N \pi^i(\boldsymbol{h}) \tag{8}$$

Expected shortfall (ES) at $\alpha = A\%$ level measures the expected profit or loss in the worst $A\%$ of the cases. Expected shortfall belongs to the class of "coherent" risk measures (Acerbi and Tasche, 2002) and has been gaining popularity in financial risk management in recent years. For a continuous distribution with the probability density function $f(\cdot)$, the expected shortfall at the level α can be determined as

$$ES = -\frac{1}{\alpha} \int_{-\infty}^{x_\alpha} X f(X) dX, \quad \text{where} \quad \alpha = \int_{-\infty}^{x_\alpha} f(X) dX. \tag{9}$$

Data and Implementation

In order to implement the methodology described in the previous section, we use the moving window approach. Specifically, for a given day in the data set, we treat the previous 250 observations relative to that day as "historical" data and treat the spot and futures prices on that day as the "initial" spot and futures prices observed by a hedger. We then calculate the realizations of price shocks based on the "historical" data, use those to generate 10,000 Monte Carlo draws of shocks as outlined in Section 3.3, and apply the shocks to the "initial" spot and futures prices. Finally, the optimal hedge ratios in (2) and (4) are determined using numerical optimization methods.

Note that 250 trading days are approximately equal to one calendar year. Therefore, conceptually, we model a situation where on any given day a hedger uses one year's worth of historical data to estimate the distribution of the spot and futures prices at hedge liquidation and to determine the optimal hedge ratios that should be used to set up hedges on that day. The same steps are then repeated for all days for which data are available[5].

For the purposes of analysis, daily spot and futures prices for crude oil, gasoline, and heating oil were obtained from DataStream for the period between January 2011 and December 2015. Futures prices were obtained for all contracts traded on any given day. Continuous futures prices series were then constructed by collating prices of nearby contracts and switching to the next delivery month at contract expiration. Allowing for the 250-day length of the moving window, the optimal hedge ratios were calculated for each trading day between 1/2/2012 and 12/31/2015.

Spot and futures prices during this period are plotted in Figure 1 and Figure 2, respectively. The plots suggest that prices of all three commodities were relatively stable during 2012-2013 and the first half of 2014, sharply declined in the second half of 2014, and stabilized at a lower level during 2015.

[Figure 1 about here]

[Figure 2 about here]

The descriptive statistics of spot and futures prices of all three commodities are reported in Table 1 (separately for each year). The means of all series are similar in 2012-2013, then decrease somewhat in 2014 and drop dramatically in 2015. Variability is at its highest during 2014, followed by 2015. Futures and spot prices are highly correlated during the entire period. Crude oil and gasoline prices are skewed to the right except for 2014, when they are left-skewed. Prices of heating oil show negative skewness for the years 2012, 2013 and 2015, but are slightly skewed to the right during 2014. Kurtosis measures the degree of peakedness of a distribution relative to a normal distribution. From Table 1, the kurtosis of prices of all three commodities in 2012, 2013 and 2015 is less than 3, indicating fewer and less extreme outliers, while in 2014 the price series tend to have heavy tails, with kurtosis values larger than 3.

[Table 1 about here]

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[5] Since the first 250 observations are treated as "historical" data, the process effectively starts with the 251st observation in the sample.

Shocks were calculated for each day in the dataset by log-differencing spot and futures prices with appropriate lags (two weeks for crude oil and three weeks for gasoline and heating oil, respectively, as explained in Section 3.1). The shocks for futures prices were first calculated separately for each contract and then a continuous series of shocks was constructed by collating series of shocks from nearby contracts and switching to the next contract month at expiration. This was done in order to avoid potential discontinuities due to differencing futures prices for different contracts.

Augmented Dickey-Fuller tests were conducted for each series of shocks to test for unit roots. The results show that all six series of shocks are stationary and the null hypothesis of a unit root is always rejected. In addition, a discrete Fourier transform was used to assess the existence of seasonality in shocks. There is insufficient evidence to support cyclical behavior in the series of shocks.

Results

Variance-minimizing and LPM_2-minimizing hedge ratios were calculated for each trading day between January 1, 2012, and December 31, 2015, under two scenarios — a single hedge ratio used for all three commodities and separate hedge ratios (a vector hedge ratio) used for each individual commodity. The optimal hedge ratios implied by LPM_2 and MV criteria are plotted in Figure 3 and Figure 4, respectively.

[Figure 3 about here]

[Figure 4 about here]

The single hedge ratio (hedging the crack spread in the fixed 3:2:1 proportion) is relatively stable during the period considered, regardless of the criterion used. However, the vector hedge ratios (hedging individual commodities separately) show a lot variation over time. Furthermore, hedge ratios for individual commodities often diverge from each other and deviate from the single hedge ratio, with the most pronounced deviations observed during 2013 and then again during 2015 (more specifically, between October of 2012 and December of 2013 and between November of 2014 and December of 2015). The deviations are particularly dramatic for crude oil and heating oil, while the hedge ratio for gasoline tends to be more stable and closer to the single hedge ratio.

The results suggest that allowing for the separate hedging of individual commodities may lead to optimal hedges in proportions that are different from the conventional 3:2:1 crack spread. Furthermore, since the unconstrained minimum cannot be greater than the constrained one, it appears that hedging individual commodities separately may lead to a better hedging performance. In order to confirm this, we calculate three measures of hedging performance, as discussed in Section 3.4, viz. hedging effectiveness, expected profit, and expected shortfall at 5%.

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Reported in Table 2 through Table 4 are percent differences in each measure between the baseline case of using a single hedge ratio for all commodities, and the case where each commodity can be hedged separately[6].

Hedging individual commodities always yields a higher hedging effectiveness compared to hedging the crack spread in the fixed 3:2:1 proportion, regardless of the criteria used (Table 2). The improvement in hedging effectiveness is the greatest during 2013 and especially 2015. The periods during which the vector hedge ratios perform substantially better than the single hedge ratios are the same as when the vector hedge ratios deviate the most from single hedge ratios.

[Table 2 about here]

In terms of expected profit (Table 3), the LPM_2-minimizing vector hedge ratio outperforms the corresponding single hedge ratio most of the time (992 out of overall 1044 windows, or 95.0%). The improvement is less pronounced under the minimum variance criterion (600 out of overall 1044 windows, or 57.5%). A possible explanation is that the minimum variance criterion equally penalizes upside and downside deviations from the mean, thus reducing expected profit. Regardless of the criteria, the advantage of the vector hedge ratio is once again at its most pronounced during 2013 and 2015.

[Table 3 about here]

Lastly, we compare the performance of both hedging strategies from the perspective of tail risk. Percent differences in expected shortfall at 5% are reported in Table 4. Note that lower values of expected shortfall reflect lower tail risk, and therefore negative differences imply an improvement relative to the baseline. Once again, regardless of the criteria used, vector hedge ratios result in a better hedging performance most of the time (870 out of 1044 windows, or 83.3%, under LPM_2 and 748 out of 1044 windows, or 71.6% under MV), with the differences being most pronounced during 2013 and especially 2015. The improvement in hedging performance due to using vector hedge ratios are generally higher under the LPM_2 criterion, which by definition minimizes downside risk.

[Table 4 about here]

A cursory comparison of Figure 1 through Figure 4 suggests that the performance of vector hedge ratio relative to a single hedge ratio does not seem to be clearly related to the dynamics of the spot and futures prices. Indeed, the most substantial improvements in hedging performance are observed in 2013 and late 2014-2015. However, the first of these two periods is characterized by relatively stable levels of spot and futures prices, while the second is characterized by a stabilization on the tail end of a steep decline of price series. On the other hand, during 2014, vector hedge ratios moved together and were close to the single hedge ratios despite a sharp decline in spot and futures prices.

In the single-commodity hedging case, the correlation between spot and futures prices is the key determinant of the optimal hedge ratio, at least in case of variance minimization (e.g. Hull, 2008). However, during the period considered, the spot and futures prices of individual

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[6] For presentation purposes, the tables report the results summarized by calendar year. Specific results for each day in the sample are available upon request.

commodities moved closely together and the respective spot-futures price correlations remained fairly stable (see Table 1).

Therefore, we conjectured that, in the multi-commodity setting, cross-dependence between the prices of different commodities could matter. In order to verify this conjecture, Kendall's τ and Pearson correlation were used to measure the degree of dependence between the futures prices of different commodities. Kendall's τ measures rank dependence and is better at capturing tail dependence. Therefore, it appears to be a more appropriate measure to explain the behavior of LPM$_2$ hedge ratios. Pearson's correlation measures linear dependence, which is consistent with the underlying assumptions of the minimum variance method. Therefore, this measure seems appropriate to explain the behavior of the hedge ratios implied by the MV criterion. Both dependence measures were calculated for three pairs of futures price series using the same moving window approach as was used in calculating the optimal hedge ratios. The dynamics of Kendall's τ and Pearson's correlation coefficients between 2012 and 2015 are shown in Figure 5 and Figure 6, respectively.

[Figure 5 about here]

[Figure 6 about here]

All three Kendall's taus were relatively stable during 2012, declined during 2013, sharply rebounded by January of 2015 and then gradually decreased during 2015. The evolution of Pearson's correlation coefficients follows a similar pattern, except that the variation is larger during 2014. Note also that for a short period of time in the third quarter of 2014, the correlation between the gasoline and heating oil futures became negative, suggesting substantial changes in the pricing structure of the entire energy complex.

Thus the dynamics of the dependence measures is indeed more consistent with the behavior of the optimal hedge ratios than the dynamics of price levels. In particular, the periods of substantial deviations between vector hedge ratio and single hedge ratio seem to roughly coincide with the periods of decline in the dependence measures. In order to verify this result more formally, we ran a regression analysis of calculated optimal hedge ratios on the corresponding dependence measures (Kendall's τ for LMP$_2$ hedge ratios and Pearson's correlation for MV hedge ratios). The results of the regression are summarized in Table 5 for LPM$_2$ and in Table 6 for MV. Although the specific effect of any given dependence measure on a particular hedge ratio is somewhat hard to interpret, most of the dependence measures are significant in explaining the optimal hedge ratios. This seems to confirm our conjecture that the behavior of vector hedge ratios in the multi-commodity settings is driven by the cross-dependence between spot and futures prices of different commodities.

Conclusions

The objective of this study is to investigate the effectiveness of crack spread hedging strategies during a period of high volatility and changing patterns of dependence in the prices of crude oil and petroleum products. To that end, a moving window approach was used to calculate the optimal hedge ratios implied by the LPM$_2$ and minimum variance criteria for each trading day between January 1, 2012, and December 31, 2105. Two cases were considered — hedging all three commodities (crude oil, gasoline, and heating oil) in a fixed 3:2:1 proportion (single hedge

ratio) and allowing for separate hedge ratios for each commodity (vector hedge ratio). Hedging effectiveness, expected profit and expected shortfall at 5% were used to measure the hedging performance of the constructed hedging strategies.

The commonly used way of hedging the crack spread at the fixed 3:2:1 proportion is found to be generally less effective in reducing price risk than a strategy allowing for hedging individual commodities separately. This result is robust across several hedging criteria and measures of hedging performance used. Differences in hedge ratios and hedging performance are most pronounced during 2013 and 2015.

The deviations between the single and vector hedge ratios (and the corresponding improvements in hedging performance) seem to be unrelated to changes in the levels of spot and futures prices, nor are they related to pairwise correlations between the spot and futures prices of individual commodities. However, the cross-dependence structure between the futures prices of different commodities seems to explain the behavior of the optimal hedge ratios fairly well. When the measures of cross-dependence (Kendall's τ and Pearson's correlations) are relatively stable, the differences between the single and vector hedge ratios are relatively small, and so are the improvements in hedging performance. However, during periods of high variability in the cross-dependence structure between prices of different commodities, the strategy of hedging individual commodities separately substantially outperforms that of hedging the crack spread in a fixed proportion.

From a practical standpoint, these results suggest that refineries can generally achieve a better risk-reduction performance by hedging individual commodities than by hedging the crack spread in a fixed 3:2:1 proportion. The advantage of hedging commodities individually becomes particularly important during periods characterized by high volatility of the cross-dependence between the prices of individual commodities. Finally, using LPM_2 as a hedging criterion may not only help hedgers to better track downside risk, but also appears to lead to higher expected profit and a lower expected shortfall.

References

Acerbi, C., & Tasche, D. (2002). "Expected Shortfall: A Natural Coherent Alternative to Value at Risk" Economic Notes, 31, 2: 379-88.

Awudu, I., Wilson, W., & Dahl, B. (2016). Hedging strategy for ethanol processing with copula distributions. Energy Economics, 57, 59-65.

Charpentier, A., Fermanian, J.-D., & Scaillet, O. (2007). The Estimation of Copulas: Theory and Practice. In J. Rank (Ed.), Copulas: From Theory to Application in Finance (pp. 35-62). London: Risk Books.

Cherubini, U., Luciano, E., & Vecchiato, W. (2004). Copula Methods in Finance. Hoboken, NJ: John Wiley & Sons.

Demirer, R., & Lien, D. (2003). Downside risk for short and long hedgers. International Review of Economics & Finance, 12(1), 25-44.

Energy Information Administration.(2002). Derivatives and Risk Management in the Petroleum, Natural Gas, and Electricity Industries. U.S. Department of Energy, Washington, DC.

Ederington, L. H., Fernando, C. S., Lee, T. K., Linn, S. C., & May, A. D. (2011). Factors Influencing Oil Prices: A Survey of the Current State of Knowledge in the Context of the 2007-08 Oil Price Volatility. Unpublished manuscript.

Ederington, L. H. (1979). The hedging performance of the new futures markets. The Journal of Finance, *34*(1), 157-170.

Fackler, P. L., & McNew, K. P. (1993). Multiproduct hedging: Theory, estimation, and an application. Review of Agricultural Economics, 15(3), 521-535.

Haigh, M. S., & Holt, M. T. (2002). Crack spread hedging: accounting for time-varying volatility spillovers in the energy futures markets. Journal of Applied Econometrics, 17(3), 269-289.

Hull, J. (2008). Options, futures and other derivatives (7th ed.). Upper Saddle River, NJ: Prentice-Hall.

Ji, Q., & Fan, Y. (2011). A dynamic hedging approach for refineries in multiproduct oil markets. Energy, *36*(2), 881-887.

Kaminski, V. (2014). The microstructure of the North American oil market. Energy Economics, 46(S1), p. S1-S10.

Lai, J. Y. (2015). An Empirical Study of the Impact of Skewness and Kurtosis on Hedging Decisions. In: M. A. H. Dempster, & K. Tang (Eds.), Commodities (pp. 53-69). Chapman and Hall/CRC.

Lien, D., & Tse, Y. K. (2002). Some recent developments in futures hedging. Journal of Economic Surveys, *16*(3), 357-396.

Mahringer, S., & Prokopczuk, M. (2015). An empirical model comparison for valuing crack spread options. Energy Economics, 51, 177-187.

Manfredo, M. R., Garcia, P., & Leuthold, R. M. (2000,). Time-varying multiproduct hedge ratio estimation in the soybean complex: a simplified approach. In: *NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management* (pp. 17-18).

Mattos, F., Garcia, P., & Nelson, C. (2008). Relaxing standard hedging assumptions in the presence of downside risk. The Quarterly Review of Economics and Finance, *48*(1), 78-93.

Miranda, M.J., &Fackler, P.L. (2002). Applied computational economics and finance. Cambridge, MA: MIT Press.

Peterson, P. E., & Leuthold, R. M. (1987). A portfolio approach to optimal hedging for a commercial cattle feedlot. Journal of Futures Markets, 7(4), 443-457.

Power, G. J., & Vedenov, D. (2010). Dealing with downside risk in a multi-commodity setting: A case for a "Texas hedge"?. Journal of Futures Markets, *30*(3), 290-304.

Power, G. J., Vedenov, D. V., Anderson, D. P., & Klose, S. (2013). Market volatility and the dynamic hedging of multi-commodity price risk. Applied Economics, 45(27), 3891-3903.

Turvey, C. G., & Nayak, G. (2003). The semivariance-minimizing hedge ratio. Journal of Agricultural and Resource Economics, 100-115.

Tong, B., Wu, C., & Zhou, C. (2013). Modeling the co-movements between crude oil and refined petroleum markets. Energy Economics, *40*, 882-897.

Wand, M. P., & Jones, M. C. (1994). Kernel smoothing. Boca Raton, FL: Crc Press.

Table 1: Descriptive statistics of spot and futures prices for crude oil, gasoline, and heating oil between 01/01/2012 and 12/31/2015, in $/bbl

	Crude Oil (CL)		Gasoline (RB)		Heating Oil (HO)	
	Spot	Futures	Spot	Futures	Spot	Futures
2012						
Mean	93.776	93.8681	123.035	122.274	130.517	126.839
SD	7.685	7.6336	9.451	9.406	7.374	7.548
Skewness	0.159	0.1685	0.313	0.541	−0.906	−0.937
Kurtosis	1.944	2.518	3.059	1.964	2.388	3.208
Minimum	77.720	77.6900	102.451	107.104	110.830	106.063
Maximum	109.390	109.7700	153.909	143.497	141.595	139.268
Pearson's Correlation	0.9996		0.8350		0.9744	
Kendall's Tau	0.9847		0.6939		0.8257	
2013						
Mean	97.835	97.873	118.019	119.330	126.527	125.656
SD	5.434	5.404	5.645	6.932	4.726	4.215
Skewness	0.568	0.553	0.554	0.227	0.304	0.132
Kurtosis	2.258	2.497	2.979	2.265	2.088	2.609
Minimum	86.650	86.680	107.520	105.130	114.715	114.853
Maximum	110.620	110.530	133.270	134.547	139.075	136.013
Pearson's Correlation	0.9993		0.8535		0.9471	
Kendall's Tau	0.9762		0.6621		0.8361	
2014						
Mean	91.840	91.721	108.349	108.413	116.929	115.725
SD	13.519	13.456	15.185	18.127	13.852	12.993
Skewness	−1.499	−1.500	−1.609	−1.170	−0.894	−1.385
Kurtosis	4.3735	4.7476	3.7697	4.3832	3.4400	4.1627
Minimum	53.450	53.27	63.680	60.2826	77.755	77.557
Maximum	107.300	107.26	126.945	131.3634	149.995	137.735
Pearson's Correlation	0.9992		0.9754		0.9766	
Kendall's Tau	0.9700		0.8150		0.9373	
2015						
Mean	48.201	48.325	66.911	67.472	68.671	68.922
SD	6.822	6.779	10.282	12.460	11.146	10.155
Skewness	0.156	0.171	0.253	0.166	−0.281	−0.353
Kurtosis	2.2791	1.7911	2.7778	2.2966	1.4941	2.8102
Minimum	34.550	34.730	50.026	49.344	43.105	45.322
Maximum	61.360	61.430	88.095	90.149	97.705	96.554
Pearson's Correlation	0.9987		0.9770		0.9882	
Kendall's Tau	0.9785		0.8974		0.9519	

Table 2: Percent differences in hedging effectiveness using single hedge ratio (baseline) and vector hedge ratio under different criteria (higher values are better).

Year	2012	2013	2014	2015
	LPM$_2$			
Min	0.07%	0.96%	0.00%	4.35%
Max	11.52%	14.36%	26.98%	38.14%
Mean	1.18%	6.05%	0.96%	15.56%
% positive	100	100	100	100
	MV			
Min	0.37%	0.66%	0.03%	2.29%
Max	3.39%	2.88%	3.84%	10.25%
Mean	1.19%	1.60%	0.40%	5.38%
% positive	100	100	100	100

Table 3: Percent differences in expected profit using single hedge ratio (baseline) and vector hedge ratio under different criteria (higher values are better).

Year	2012	2013	2014	2015
	LPM$_2$			
Min	−0.08%	0.02%	−0.02%	0.16%
Max	0.57%	0.83%	0.80%	1.54%
Mean	0.12%	0.35%	0.10%	0.69%
% positive	85.8	100	94.3%	100%
	MV			
Min	−0.24%	−0.08%	−0.10%	0.13%
Max	0.15%	0.26%	0.24%	0.63%
Mean	−0.07%	0.05%	−0.01%	0.38%
% positive	18.0%	78.2%	33.7%	100

Table 4: Percent differences in expected shortfall at 5% using single hedge ratio (baseline) and vector hedge ratio under different criteria (lower values are better).

Year	2012	2013	2014	2015
	LPM_2			
Min	−0.50%	−0.57%	−1.49%	−1.94%
Max	0.11%	−0.05%	0.03%	0.11%
Mean	−0.04%	−0.26%	−0.12%	−0.66%
% negative	61.7%	100.0%	81.2%	93.4%
	MV			
Min	−0.53%	−0.40%	−0.76%	−1.19%
Max	0.31%	0.02%	0.21%	0.03%
Mean	−0.05%	−0.21%	0.00%	−0.58%
% negative	60.5%	98.5%	29.5%	98.1%

Table 5: Regression analysis of LPM$_2$ optimal hedge ratios on measures of dependence (Kendall's τ) between spot and futures prices.

	Hedge Ratio		
	Crude oil (CL)	**Gasoline (RB)**	**Heating Oil(HO)**
Intercept	13.499***	10.087***	−10.588***
Tau_CL.S_vs_RB.S	−0.620	−3.174	−30.642***
Tau_CL.S_vs_HO.S	−17.833*	−51.557***	53.948***
Tau_CL.S_vs_CL.F	−11.801***	−13.021***	8.807***
Tau_CL.S_vs_RB.F	2.992	−8.601*	49.321***
Tau_CL.S_vs_HO.F	26.055**	50.397***	−72.751***
Tau_RB.S_vs_HO.S	−6.692***	−4.241***	0.156
Tau_RB.S_vs_CL.F	−1.961	1.575	29.563***
Tau_RB.S_vs_RB.F	1.231***	1.815***	−1.265***
Tau_RB.S_vs_HO.F	6.912***	5.081***	1.040
Tau_HO.S_vs_CL.F	18.406*	49.721***	−58.231***
Tau_HO.S_vs_RB.F	6.755***	3.314***	1.397
Tau_HO.S_vs_HO.F	−2.473***	1.851***	3.541***
Tau_CL.F_vs_RB.F	−1.163	9.767**	−48.623***
Tau_CL.F_vs_HO.F	−25.398**	−47.536***	76.520***
Tau_RB.F_vs_HO.F	−8.335***	−5.152***	−1.131

Note: *** = significant at 0.001 level, ** = significant at 0.01 level, * = significant at 0.05 level

Table 6: Regression analysis of MV optimal hedge ratios on measures of dependence (Pearson correlations) between spot and futures prices.

	Hedge Ratios		
	Crude oil (CL)	**Gasoline (RB)**	**Heating Oil(HO)**
Intercept	4.597***	−0.548	−2.927**
Corr_CL.S_vs_RB.S	0.755	3.665*	7.837**
Corr_CL.S_vs_HO.S	14.505***	15.730***	44.129***
Corr_CL.S_vs_CL.F	−2.045***	1.974***	0.995
Corr_CL.S_vs_RB.F	2.678	−2.442	3.110
Corr_CL.S_vs_HO.F	−16.121***	−18.879***	−48.107***
Corr_RB.S_vs_HO.S	−1.589***	1.060***	−0.390
Corr_RB.S_vs_CL.F	−1.165	−4.498**	−6.377*
Corr_RB.S_vs_RB.F	0.393***	1.197***	−0.264*
Corr_RB.S_vs_HO.F	1.110***	−1.310***	1.061**
Corr_HO.S_vs_CL.F	−12.726**	−14.463***	−47.161***
Corr_HO.S_vs_RB.F	1.670***	−0.741***	0.955*
Corr_HO.S_vs_HO.F	−2.270***	−1.551***	2.924***
Corr_CL.F_vs_RB.F	−2.544	3.049.	−4.011
Corr_CL.F_vs_HO.F	14.382***	17.840***	50.009***
Corr_RB.F_vs_HO.F	−1.561***	0.390*	−0.801*

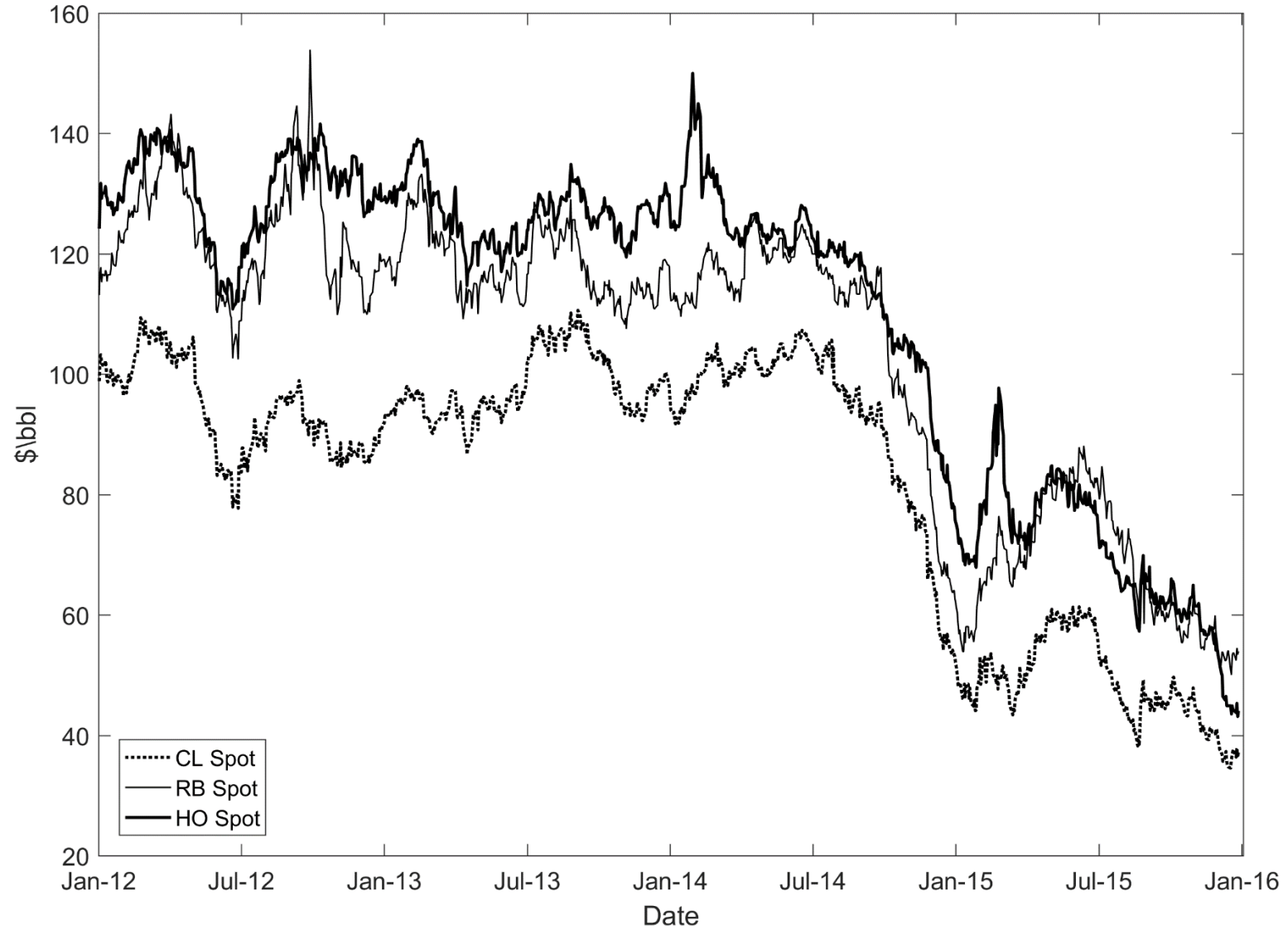*Note*: *** = significant at 0.001 level, ** = significant at 0.01 level, * = significant at 0.05 level

Figure 1: Spot prices of crude oil (CL), regular gasoline (RB), and heating oil (HO) between 01/01/2012 and 12/31/2015.
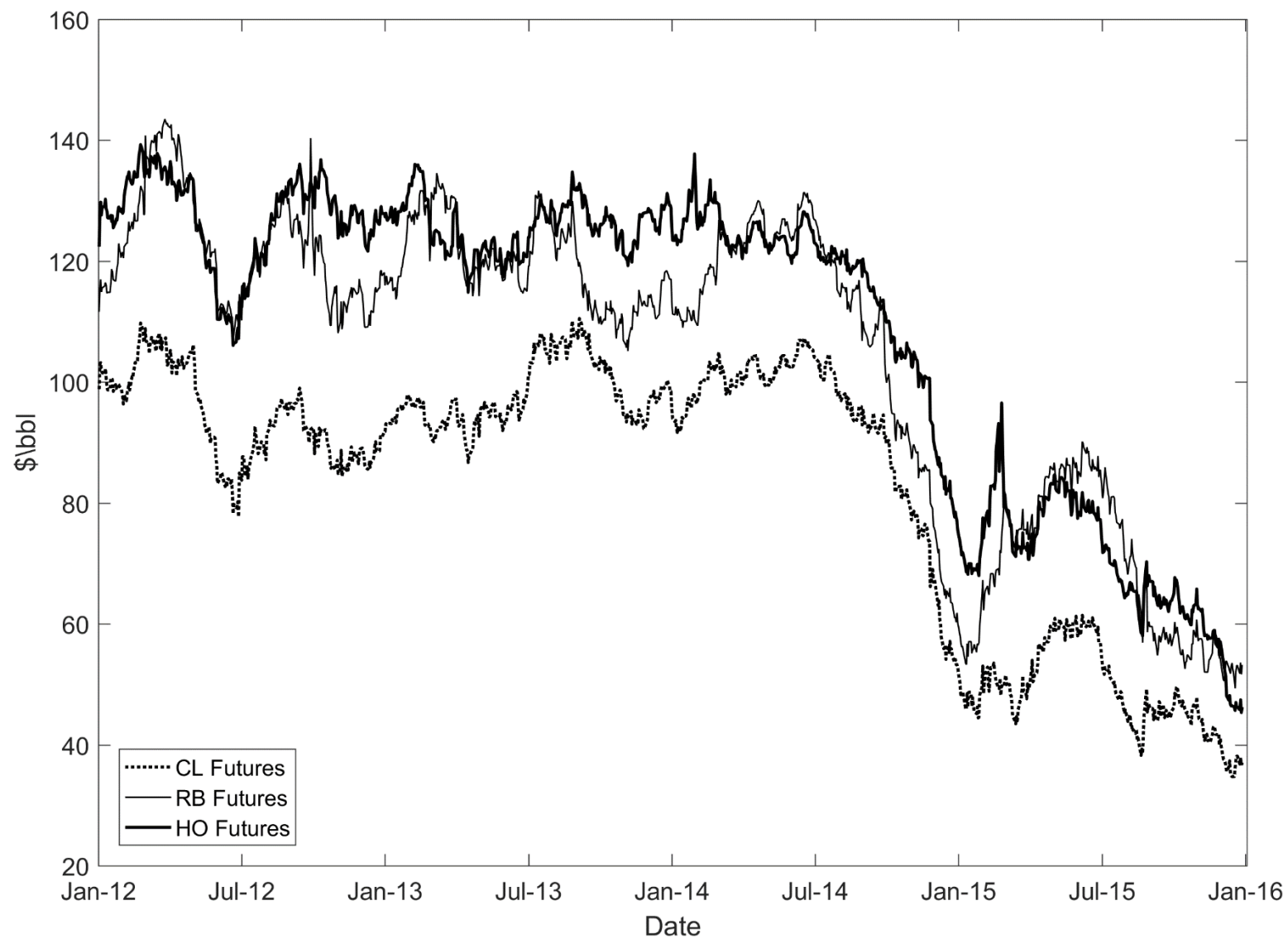
Figure 2: Futures prices of crude oil (CL), regular gasoline (RB), and heating oil (HO) between 01/01/2012 and 12/31/2015 (continuous series).
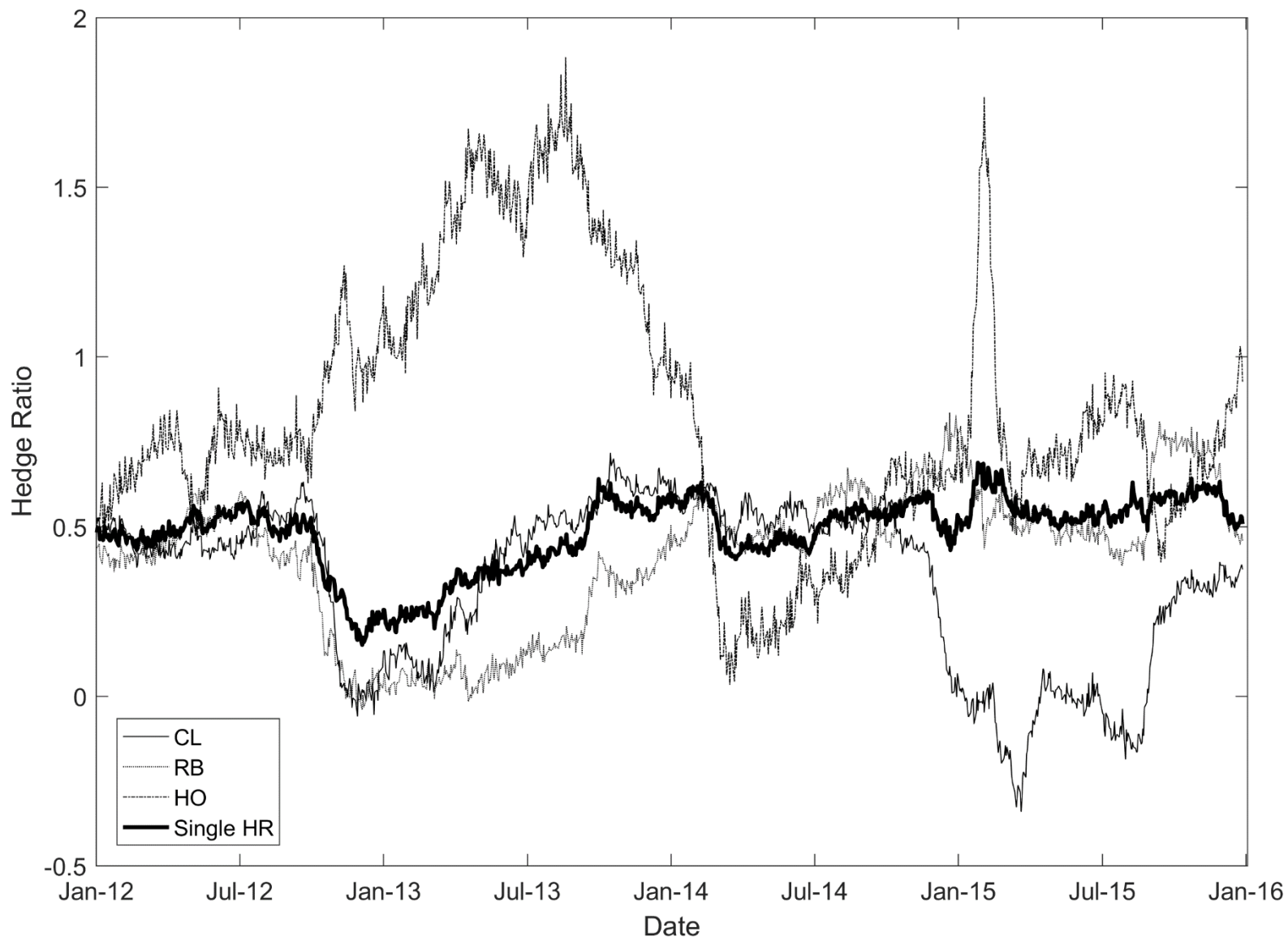
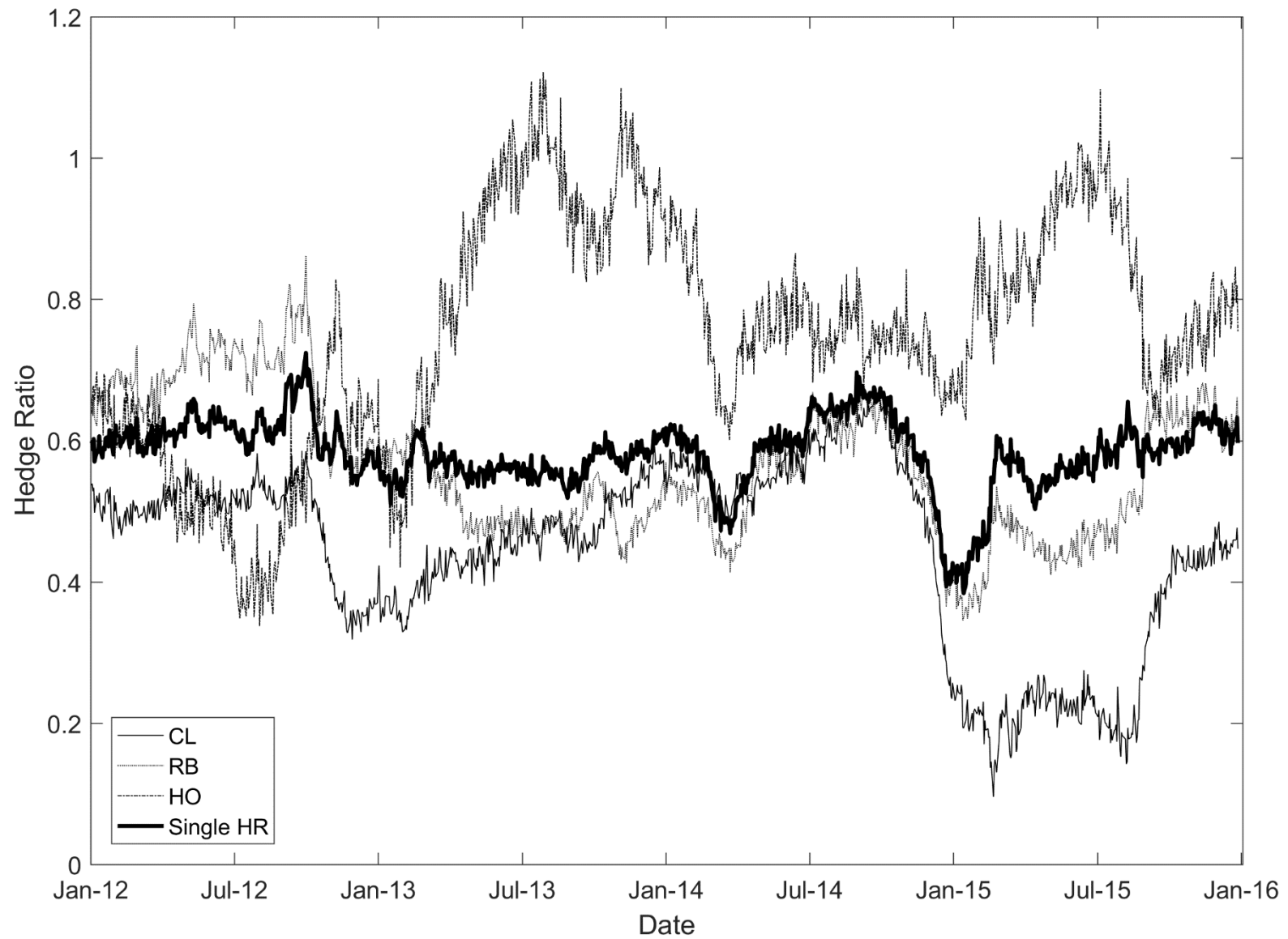Figure 3: Optimal hedge ratios under the LPM$_2$ criterion
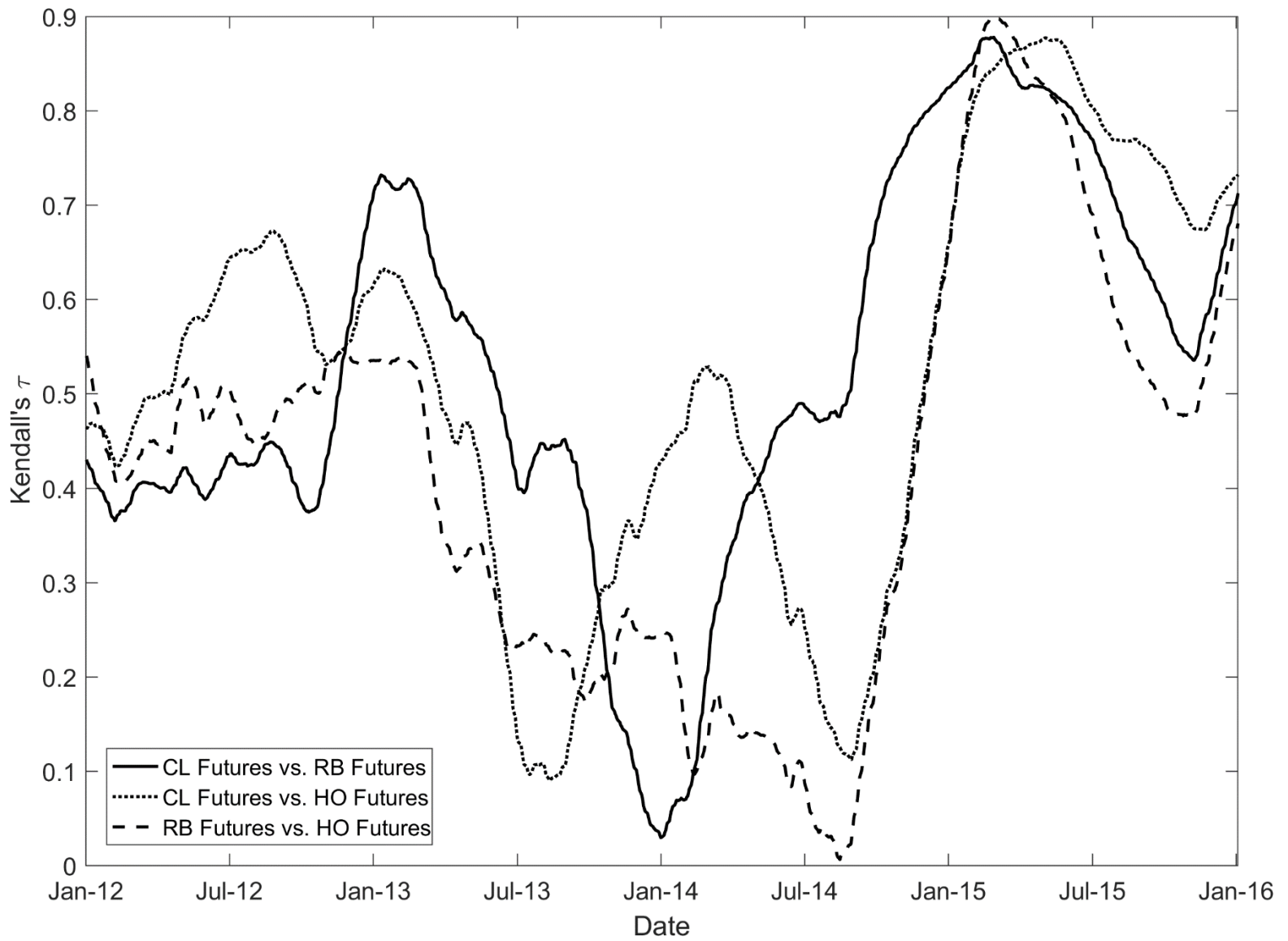
Figure 4: Optimal hedge ratios under the MV criterion

Figure 5: Pairwise Kendall's τ for crude oil, gasoline, and heating oil futures prices, 250-day moving window
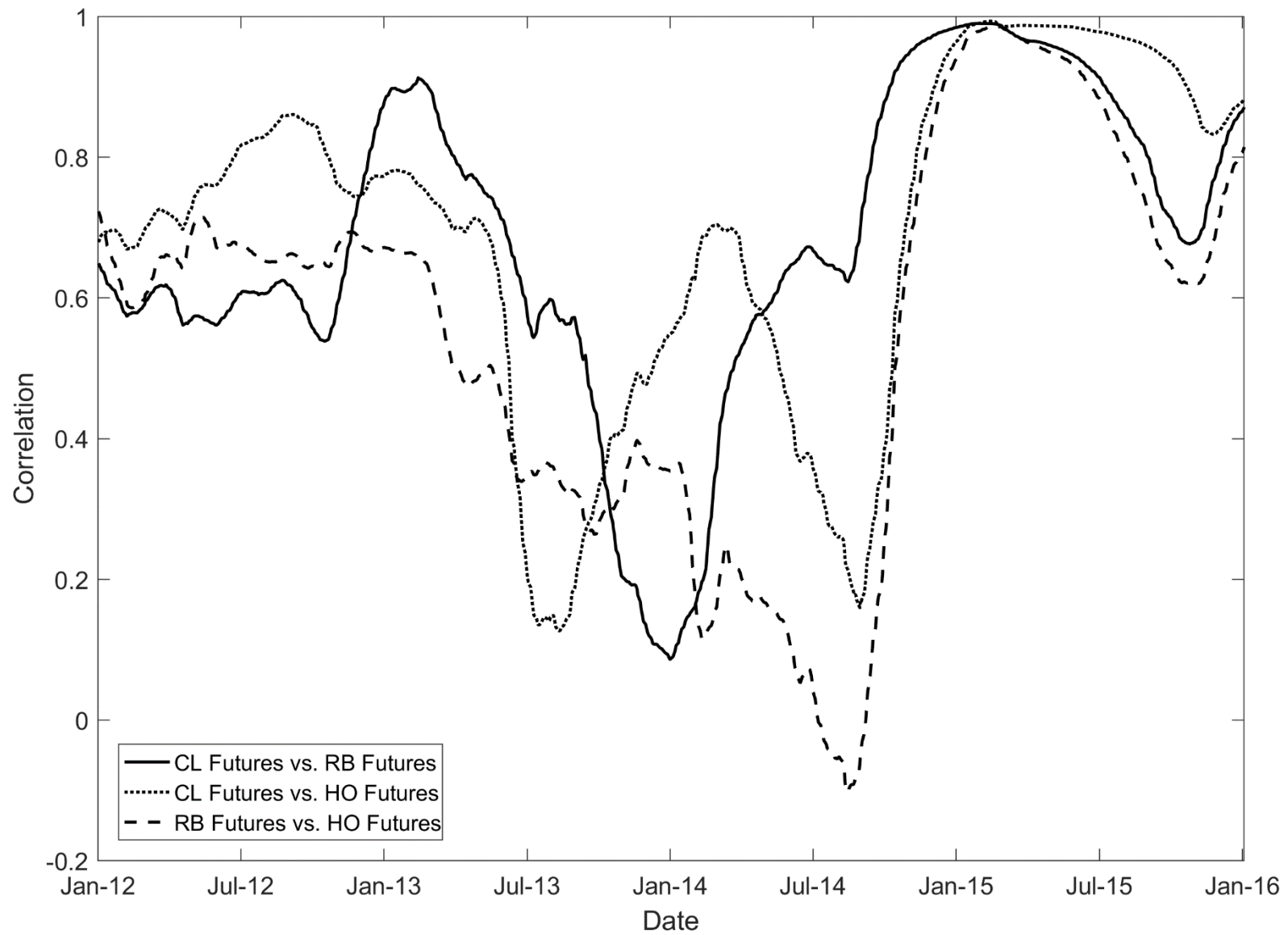
Figure 6: Pairwise correlations for crude oil, gasoline, and heating oil futures prices, 250-day moving window