Allocative Inefficiency under Heterogeneous Technology in Bolivian Agriculture

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Selected Paper prepared for presentation at the Southern Agricultural Economics Association (SAEA) Annual Meeting, Birmingham, Alabama, February 2-5, 2019

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ABSTRACT

Low utilization rates of apparently profitable agricultural inputs in developing countries have puzzled development economists for decades. This paper investigates whether farmers in Bolivia are using an optimal input mix, conditional on their target output level, the technology available to them, and their input price environment. It differs from prior work in that it estimates the entire production technology rather than using reduced form approaches. To estimate the cost function that is dual to the production function, this paper develops a new technique for estimating a censored system of equations with endogenous selection. A comparison of Bolivia's statistics of inorganic fertilizer use with neighboring countries appears to support the hypothesis of underuse of inorganic fertilizer. However, the econometric estimates indicate that fertilizer use levels are appropriate. Farmers do use an inefficient input mix, but the misallocation stems from overuse of other inputs. The misallocation raises farmers' costs, but the consequences are minor.

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This work benefited from comments from Bradford Barham, Jean-Paul Chavas, Jeremy Foltz, Cornelia Ilin, Matthew Klein, Daniel Phaneuf, and Emilia Tjernström. This version January 2019.

1 Introduction

If farmers in Bolivia were using inorganic fertilizer as intensively as their counterparts in other South American nations, they would be using ten times the amount they use today. Increasing fertilizer use could be a way of boosting agricultural productivity and providing a pathway out of poverty. But Bolivian farmers may already be doing the best they can, conditional on the prices and agroclimactic conditions they face.

Figure 1 indicates that fertilizer use in Bolivia is negligible compared to that of other nations in the Andean region. To answer the question of whether Bolivian farmers could raise their welfare by using more fertilizer, in this work I test whether their current input mix is minimizing cost. Traditional approaches to testing for cost minimization assume that firms use one homogenous technology. I relax this homogenous technology assumption. Farmers who use fertilizer may possess an unobserved productivity advantage over those who do not use fertilizer. Therefore, I use an estimation methodology that accounts for unobserved factors that encourage or discourage adoption of fertilizer.

Only 23 percent of farmers use any fertilizer at all. Furthermore, observed heterogeneity exerts a large influence on this extensive margin decision. Figure 2 displays the proportion of plots where fertilizer is used, broken down by crop and agroproductive zone. In the *valles* region – the middle of the country – fertilizer is employed on upwards of 30 percent of potato plots, while only 10 percent of potato plots in the *altiplano* region in the west use fertilizer. It is apparent that farmers select into fertilizer use based on observable characteristics. This fact leads to my hypothesis that there is also selection on characteristics that cannot be observed in the data, like farmer skill, or factors that are only imperfectly observed, such as soil quality.

My approach begins by estimating a cost function that permits the possibility of farmers optimizing to the "wrong" input prices - the shadow prices. If farmers optimize to the wrong prices, they are not minimizing cost and therefore are considered to be "allocatively inefficient".

To tackle unobserved heterogeneity, I exploit the fact that many farmers do not use fertilizer and estimate the first-stage choice to use a positive amount of fertilizer. The error terms of this discrete choice model can then be used to correct for the unobserved technological heterogeneity in the cost function. The data I use comes from a 2008 government survey of about 7,000 farmers. It asked questions about everything needed to estimate a cost function: crop output, input amounts, and their prices.

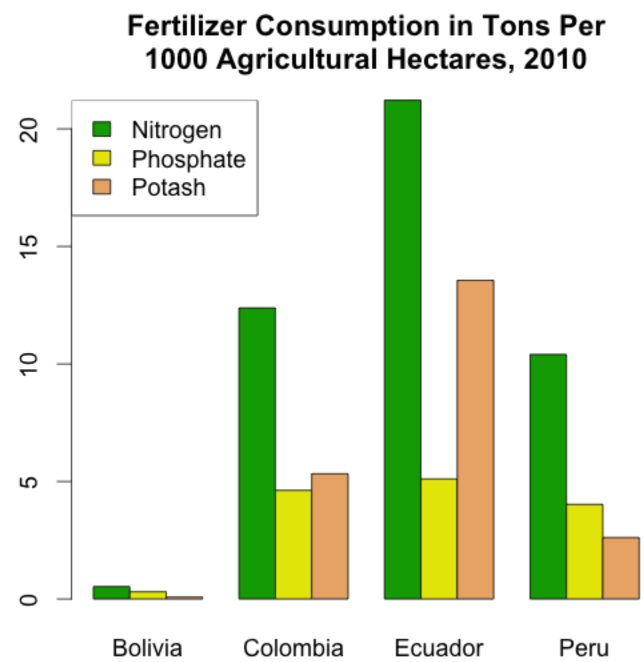
Myriad conditions can cause the low fertilizer use that we observe in Bolivia. My approach shrinks the space of possible reasons for low fertilizer use. First, it conditions on all input prices. "High" fertilizer prices may lead to low use, naturally, but what really matters is the ratio between fertilizer prices and other inputs. If labor, for example, is cheap, then substituting away from fertilizer and toward labor is optimal. Since

Bolivia is a developing country with cheap labor (and a low opportunity cost of household labor), this price ratio factor must be accounted for.

Second, credit constraints do not affect the ability of farmers to minimize cost. Therefore, my estimation can cleanly separate a failure to use an optimal input max and an institutional environment where farmers cannot obtain credit. A drawback is that I cannot separately estimate the effect of credit constraints; they are ignored.

I can also safely reject the lagging effect of learning as a possible source of low fertilizer use. Inorganic fertilizer has been available for decades and any learning about its effects should have taken place long ago. Empirically, we can see that there has not been the steep adoption rate over time that may indicate learning: in 1984, 16 percent of farmers used fertilizer, while in 2008, 23 percent of them did so, a rise in 7 percentage points over 24 years.

Figure 1: Fertilizer use in the Andean nations



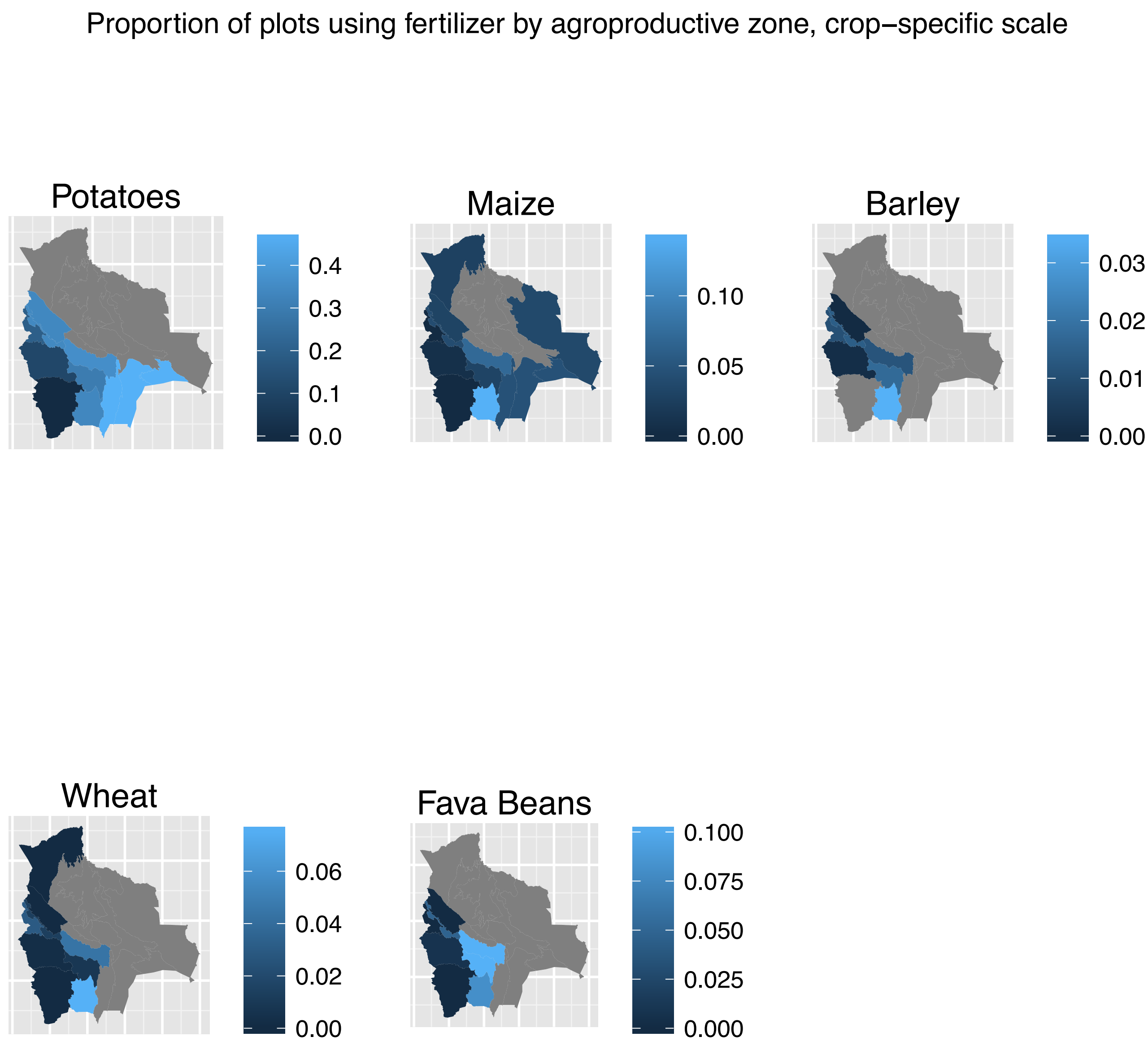
Source: Food & Agriculture Organization

2 Literature review

Most studies that have examined the determinants of fertilizer use in developing countries approach the question from a "reduced form" perspective. Typically, they use a double-hurdle model to explain fertilizer use. This model is similar to Tobit, but it allows the signs of the effects of the variables to differ for the extensive margin and intensive margin decisions.

Coady (1995) was one of the first studies to examine fertilizer demand with a double-hurdle model. In his sample of Pakistani farmers, 81 percent of respondents use at least some fertilizer. Access to credit is positively associated with the extensive margin decision. Conditional on choosing to use fertilizer, having slightly saline soil positively affects the amount of fertilizer purchased, while having irrigation negatively affects the amount purchased. Distance to the nearest town did not affect the amount purchased.

Figure 2: Fertilizer use by agroproductive zone



Croppenstedt, Demeke, & Meschi (2003) use a double-hurdle model on Ethiopian data. Literacy, years of education, and household size increased the probability of purchasing any amount of fertilizer. Soil type also has an effect on the extensive margin decision. Age of the household head positively affected the amount purchased. Female headed households tended to purchase a lower amount.

Xu et al. (2009) investigated fertilizer demand in Malawi with a double hurdle model. Distance to the nearest district town negatively affected the probability of purchasing fertilizer. Household assets increased the amount of fertilizer purchased.

Zerfu & Larson (2010) investigate the determinants of fertilizer use in Ethiopia. Using data collected in 2004 and 2006, they estimate a panel selection model, which is closely related to double-hurdle model. Sixty-eight percent and 63 percent of their sample use fertilizer in 2004 and 2006, respectively. Years of education of the household head, access to credit and extension services, and historical precipitation are positively associated with the extensive margin decision to purchase fertilizer. Age of the household head is negatively associated with the extensive margin decision. Neither household size nor age of the household head has a statistically significant effect upon the intensive margin decision, but years of education of the household head is positively associated with more fertilizer use.

Ricker-Gilbert, Jayne, & Chirwa (2011) estimated a double hurdle model on Malawian data. Sixty-three percent of their sample purchased fertilizer. The household's distance to nearest paved road, age of the household head, and long run average rainfall negatively affected the probability of purchasing fertilizer. Access to credit positively affected it. Household assets and long run average rainfall increased the amount of fertilizer purchased.

The reduced form approaches above did not directly estimate the technology parameters of the firms being studied. Some of the studies included output price and fertilizer price as possible determinants of fertilizer use, but they did not consider the price of other inputs. My project is to estimate farmers' actual technology.

Unlike the reduced form studies, Sheahan, Black, & Jayne (2013) estimate the maize production function of Kenyan farmers to determine whether farmers are using an optimal amount of fertilizer. Once the production function is estimated, the authors use maize and fertilizer prices to calculate profitability. They find that over time Kenyan farmers have converged to the optimal fertilizer use level, with farmers in only a few areas not currently using the optimal fertilizer amount. Their approach differs from mine in that they test whether farmers achieve profit maximization, while I test whether farmers achieve cost minimization. Cost minimization is a necessary but not sufficient condition for profit maximization.

In another line of reasoning, Duflo, Kremer, & Robinson (2011) hypothesize that time-inconsistent preferences can explain low fertilizer use even when the returns to fertilizer are high. They conduct an experiment in Kenya where farmers are offered at harvest time a small discount on fertilizer delivery at planting time. They find that this small nudge can apparently overcome the propensity to procrastinate. However, people should procrastinate on buying all inputs, so my allocative inefficiency framework should be able to get around this issue without directly addressing it. Furthermore, the estimates of Sheahan, Black, & Jayne (2013) above contradict the premise of Duflo, Kremer, & Robinson that farmers in Kenya are not using enough fertilizer.

In a contribution to the understanding of the effects of heterogeneity upon fertilizer use, Marenya & Barrett (2009) use direct measurements of plots' soil quality in Kenya to investigate the profitability of fertilizer use. They estimate a maize production function that incorporates soil organic matter and nitrogen content. Then they use average maize and fertilizer prices to calculate the break-even point for profitability of fertilizer use. Fertilizer application to about one-third of plots in their sample would be unprofitable. This study illustrates that heterogeneity in soil quality can lead to low fertilizer use even if the average return to fertilizer use is high.

Suri (2011) finds that unobserved heterogeneity influences Kenyan farmers' decisions to adopt hybrid seeds. Average returns to hybrid seeds in Kenya are high, but not all farmers adopt the improved seed. She finds that farmers select into the technology based on their expected returns to adoption. Hence, hybrid seed users are observed as having high returns, but non-users would not see high returns if they were to adopt. In substance and methods, this paper is most closely related to mine since the author uses a sample selection model to investigate differences in agricultural output.

In addition to contributing to the literature on fertilizer use, I add to the methodological choices available for estimating allocative inefficiency. The literature on estimation of inefficiency under heterogenous technology focuses on technical efficiency rather than allocative inefficiency. A firm is technically efficient if, given a certain quantity of inputs, it maximizes the output of those inputs. Tsionas & Kumbhakar (2014), Tsionas (2002), and Orea & Kumbhakar (2004) develop techniques for estimation of technical inefficiency under technological heterogeneity. Tsionas & Tran (2014) use local maximum likelihood to estimate firm-specific technology parameters in an allocative inefficiency framework. However, their technique requires panel data, which is not available for Bolivian farms. My main contribution to the allocative inefficiency literature is account for heterogeneity while estimating allocative inefficiency with cross-section data. My minor contribution is related: handling cases where the non-negativity constraint on input quantity binds.

I have not encountered a study of allocative inefficiency where zero use of certain inputs is observed in the data.

I also contribute to the literature on the estimation of a censored system of equations with sample selection. I develop a new estimator that is consistent, produces parameter estimates with low mean squared error, computationally feasible even with many equations, and requires no assumptions on the distribution family of the disturbance terms of the censored equations. Prior work on estimators for censored systems of equations suffer from a number of shortcomings. An early attempt to solve the estimation problem the problem by Heien and Wessells (1990) was later found to be inconsistent (Chen & Yen 2005). Shonkwiler & Yen (1999) later proposed a consistent estimator of a censored system. However, the variance of its parameter estimates can be very large and it is inconsistent if the either of its assumptions of joint normality or homoskdasticity of the disturbance terms is incorrect (Tauchmann 2005; Sam & Zheng 2010). The estimator developed by Yen (2005) is consistent in the presence of endogenous selection, but it is computationally infeasible for even a moderate number of equations – perhaps more than three – owing to the need to perform numerical integration over multi-dimensional integrals. And Kasteridis & Yen (2012) solve the problem of computational infeasibility, but at the expense of not handling any selection issues.

3 Data

My data source is the microdata from Bolivia's National Agricultural Survey, collected in June-July of 2008. This data has been publicly released. There are 7,169 crop-producing farms in the sample that manage 23,321 plots of land. Since I intend to estimate crop-specific technologies, the effective sample size is defined by the number of plots. These sample sizes for the top five crops are reported in Table 1. Some farmers grew the same type of crop in multiple plots, so some – approximately ten percent – of the observations are not independent at the farm level.

Table 1: **Number of plots in sample**

Crop	**Plots**
Potatoes	4,058
Maize	3,440
Barley	2,048
Wheat	1,647
Fava Beans	1,484

Since I am estimating production technologies, I use the survey data on input and output quantities and input prices. The variable inputs are inorganic fertilizer, purchased seeds, hired labor hours, tractor hours, organic fertilizer, and pesticides. Organic fertilizer includes manure but also more potent organic fertilizer like guano. The fixed inputs are family labor (the number of family members listed as working on the farm), a binary irrigation variable, and plot land area. The National Statistical Institute, a government body, managed collection of the data. Farmers were asked to report data on inputs and outputs for the one-year period preceding June 30, 2008. To reduce the influence of outliers, I excluded from the analysis any observations that exceeded the 97.5th percentile of price or input quantity of any variable input.

Censoring

Table 2 illustrates the degree of censoring in the input levels. Even in potatoes, which is the crop that is most likely to use fertilizer, only 24 percent of plots actually use fertilizer. The extensive margin of use is low for other inputs as well. Table 3 shows that plots that use fertilizer are more likely to use other inputs as well. For example, about 60 percent of potato plots that used fertilizer also used pesticides while only about 19 percent of potato plots that did not use fertilizer used pesticides.

Table 2: Percentage of plots where each input is used, by crop

	Potatoes	Maize	Barley	Wheat	Fava Beans
Inorganic fert	24.0	6.2	0.9	2.0	3.7
Purchased seeds	36.9	25.5	32.5	11.9	23.6
Tractor	22.1	14.3	22.2	8.5	12.8
Plaguicidas	28.7	11.7	1.1	5.9	9.5
Hired labor	6.3	5.7	4.0	3.7	4.8
Organic fert	25.7	16.4	3.1	4.7	8.4

Table 3: Percentage of plots where each input is used, conditional on whether fertilizer was used

Input	Conditional on	Potatoes	Maize	Barley	Wheat	Fava Beans
Purchased seeds	Used fert	46.8	47.5	38.9	24.2	49.1
Purchased seeds	Did not use fert	33.9	24.1	32.4	11.6	22.6
Tractor	Used fert	25.2	26.9	27.8	12.1	18.2
Tractor	Did not use fert	21.0	13.5	22.1	8.4	12.6
Plaguicidas	Used fert	60.2	36.1	11.1	54.5	43.6
Plaguicidas	Did not use fert	18.8	10.1	1.0	4.9	8.1
Hired labor	Used fert	10.6	5.0	11.1	12.1	14.5
Hired labor	Did not use fert	5.0	5.8	3.9	3.5	4.5
Organic fert	Used fert	40.4	30.1	16.7	18.2	20
Organic fert	Did not use fert	21.1	15.5	3.0	4.4	8.0

Price imputation

Naturally, plots where farmers use zero quantity of a given input have missing data for input price. For the most part, I follow Deaton's (1997) suggestion for recovering unobserved prices: "In cases where prices are available, and where the items being consumed are similar to those that are sold nearby, imputation is not difficult, although there are often difficulties over the choice between buying and [selling] prices." In other words, I rely on the law of one price to generate price estimates for missing prices.

For each missing input price observation, I implement the following procedure. First, if the farmer purchased the input for use on another plot, I used that price. Next, I checked whether at least three plots in the "segment" census unit used the input. If there are at least three such plots, I used the median price of these. In the case that less than three plots fit this criteria, I repeated this process for sector, canton, section, province, department, and finally nationwide if necessary. This imputation could cause measurement error if the imputed values do not closely resemble the prices that people actually face.

Geographic-linked data

The survey data specifies the *canton* of each farm. The median *canton* is populated by 411 households, so it is a fairly small geographic unit. I link the geographic coordinates of each *canton* to several global data sources to generate three additional fixed inputs.

I use the Harmonized World Soil Database to approximate the soil conditions that farmers face. The FAO and various other agencies developed this database. By interpolating soil samples across the world, they developed a complete global map of soil characteristics at a resolution of one square kilometer. The high resolution gives the database high "precision", but since actual soil samples may be sparse across any given area of land, the accuracy may be low. Then, using estimates of Jaenicke and Lengnick (1999) for the impact of soil pH and water absorption on maize yield, I calculate a soil quality index.

I derived a rainfall measure from the "Terrestrial Precipitation: 1900-2010 Gridded Monthly Time Series" database developed by the University of Delaware's Department of Geography. The main source of the data is weather station readings. The resolution of this database is one-half degree, which is approximately squares of 50×50 kilometers. The main shortcoming of this database is that it cannot capture the microclimates that arise due to Bolivia's mountainous terrain. Satellite readings provide an alternative, but the advantage in their higher resolution is offset by the fact that the satellite only takes a snapshot of precipitation every twelve hours. Since estimation of a cost function is supposed to reflect input purchases that firms undertake based on anticipated conditions, I compute the average rainfall during the growing season in the five years before the survey period rather than the rainfall that actually occurred in the year of survey.

The final fixed input is elevation. The digital elevation model of the Shuttle Radar Topography Mission is used to to construct the elevation variable. The resolution of this data was 30 arc-seconds, or about one square kilometer.

4 Theoretical model

Allocative inefficiency framework

Any firm that fails to minimize costs for a target level of output is allocatively inefficient. Despite the behavioral assumption of cost minimization, a firm may not meet the usual condition of the marginal rate of technical substitution between any two variable inputs being equal to the ratio of their prices. It may face other constraints that prevent the achievement of unconditional cost minimization. These constraints may come in the form of government regulations, input costs not reflected in nominal price, or cognitive biases among firm owners that cause systematic mistakes in input choice. In the model, all firms in the market face the same set of constraints.

A firm facing such an environment must solve for the following cost function:

$$
\begin{aligned}
C\left(\mathbf{w}, y, \mathbf{q}\right) = \quad & \min_{\mathbf{x}} \left\{\mathbf{x} \cdot \mathbf{w}\right\} \\
& \text{s.t.} \\
& f\left(\mathbf{x}, \left[\mathbf{q}, U\right]\right) = y \\
& R^s\left(\mathbf{x}\right) = 0,\ s = 1, \ldots, S
\end{aligned}
$$

where $\mathbf{x}$ is the vector of N variable inputs for the farming technology, $\mathbf{w}$ is the vector of prices for these variable inputs, $\mathbf{q}$ is the vector of fixed inputs for the farming technology, y is the output, f is the agricultural technology, and R^s is the s'th constraint that causes allocative inefficiency.

Let f and R^s be quasi-concave in $\mathbf{x}$ and let f fulfill the Inada conditions. Hence there is a unique interior solution.

Form the Lagrangian:

$$\mathscr{L} = \mathbf{x} \cdot \mathbf{w} + \lambda \left[y - f\left(\mathbf{x}, \mathbf{q}, L_a\right)\right] + \sum_s^S \mu_s \left[R_s\left(\mathbf{x}\right)\right]$$

$$[x_k]\ w_k - \lambda \cdot f_{x_k}\left(\mathbf{x}, \mathbf{q}, L_a\right) + \sum_s^S \mu_s R^s_{x_k}\left(\mathbf{x}\right) = 0,\ \forall k$$

$$[\lambda]\ y - f\left(\mathbf{x}, \mathbf{q}, L_a\right) = 0$$

$[\mu_s]\ R_s(\mathbf{x}) = 0,\ \forall s$

Now, $[x_k]$ implies:

$\lambda \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) = w_k + \sum_s^S \mu_s R_{x_k}^s(\mathbf{x})$

$\frac{f_{x_k}(\mathbf{x}, \mathbf{q}, L_a)}{f_{x_j}(\mathbf{x}, \mathbf{q}, L_a)} = \frac{w_k + \sum_s^S \mu_s R_{x_k}^s(\mathbf{x})}{w_j + \sum_s^S \mu_s R_{x_j}^s(\mathbf{x})},\ \forall k \neq j$

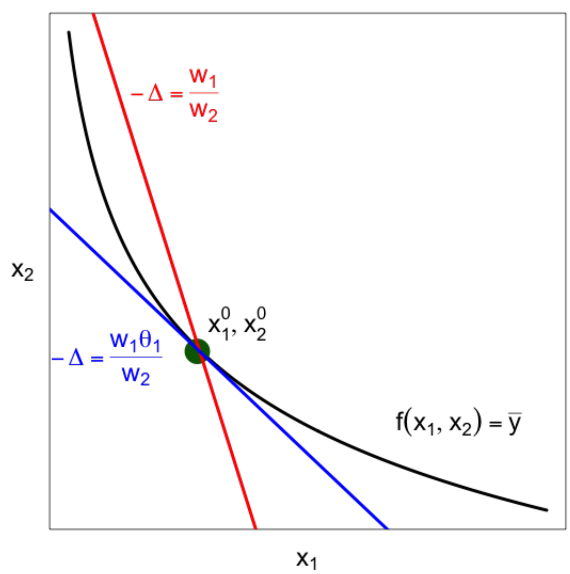
If we denote $w_k + \sum_s^S \mu_s R_{x_k}^s(\mathbf{x})$ as w_k^*, then:

$$\frac{f_{x_k}(\mathbf{x}, \mathbf{q}, L_a)}{f_{x_j}(\mathbf{x}, \mathbf{q}, L_a)} = \frac{w_k^*}{w_j^*},\ \forall k \neq j$$

w_k^* is the shadow price of the kth input. If $w_k^* \neq w_k$, then the firm is allocatively inefficient.

Figure 3 is a visual representation of the shadow prices approach to allocative inefficiency with two inputs.

Figure 3: Allocative inefficiency and shadow prices



Heterogeneous technologies

The motivation of the heterogenous technology framework starts with the firm's primal problem, even though parameters of the cost function are actually estimated in the empirical application. Let there be N variable inputs x and J fixed inputs q. For the purposes of this illustration, assume that the production function's functional form is quadratic. Then the firm's production function is:

$$f(\mathbf{x},\mathbf{q}) = \alpha + \sum_{i=1}^{N} \beta_i x_i + \sum_{i=1}^{J} \gamma_i q_i + \sum_{i=1}^{N}\sum_{j=1}^{N} \delta_{ij} x_i x_j + \sum_{i=1}^{J}\sum_{j=1}^{J} \omega_{ij} q_i q_j + \sum_{i=1}^{N}\sum_{i=1}^{J} \psi_{ij} x_i q_i$$

where α, β_i, γ_i, δ_{ij}, ω_{ij}, and ψ_{ij} are the technological parameters.

Let the J'th fixed factor be observable to the firm but not to the econometrician. Call it U. Rewrite the production function:

$$\begin{aligned} f(\mathbf{x},[\mathbf{q},U]) = \quad & \alpha + \sum_{i=1}^{N} \beta_i x_i + \sum_{i=1}^{J-1} \gamma_i q_i + \gamma_U U + \sum_{i=1}^{N}\sum_{j=1}^{N} \delta_{ij} x_i x_j + \sum_{i=1}^{J-1}\sum_{j=1}^{J-1} \omega_{ij} q_i q_j + 2\times \sum_{i=1}^{J-1} \omega_{iU} q_i U + \omega_{UU} U^2 + \\ & \sum_{i=1}^{N}\sum_{i=1}^{J-1} \psi_{ij} x_i q_i + \sum_{i=1}^{N} \psi_{iU} x_i U \end{aligned}$$

For simplicity, assume all technology parameters associated with U are zero except for ψ_{lU} for some l. Let $\psi_{lU} > 0$ and imagine that the lth variable input is fertilizer. Then the marginal product of fertilizer is

$$\frac{\partial f(\mathbf{x},[\mathbf{q},U])}{\partial x_l} = \beta_l + 2\times \sum_{i=1}^{N} \delta_{il} x_i + \sum_{i=1}^{J-1} \psi_{lj} q_i + \psi_{lU} U$$

Since $\psi_{lU} > 0$, the marginal product of fertilizer rises when U is higher, i.e. $\dfrac{\partial^2 f(\mathbf{x},[\mathbf{q},U])}{\partial x_l \partial U} > 0$

Now we proceed to the investigation of the firm's cost minimization problem under this condition.

The firm's problem is:

$$\begin{aligned} C(\mathbf{w}, y, [\mathbf{q},U]) = \quad & \min_{\mathbf{x}} \{\mathbf{x}\cdot\mathbf{w}\} \\ & \text{s.t.} \\ & f(\mathbf{x},[\mathbf{q},U]) = y \end{aligned}$$

The solution to this problem implies

$$\frac{f_{x_k}(\mathbf{x},[\mathbf{q},U])}{f_{x_j}(\mathbf{x},[\mathbf{q},U])} = \frac{w_k}{w_j}, \forall k \neq j$$

Let two firms m and p face the same $\mathbf{w}$ and the same q_i for every i, except for the Jth fixed input, which is U. Let the m'th firm face a lower U than the p'th firm and denote their sets of fixed inputs $[\mathbf{q}, U^m]$ and $[\mathbf{q}, U^p]$, respectively. Since $\psi_{lU} > 0$, we have $f_{x_l}(\mathbf{x},[\mathbf{q},U^m]) < f_{x_l}(\mathbf{x},[\mathbf{q},U^p])$ for a given value of $\mathbf{x}$. By cost minimization, the $\mathbf{x}$ choice of firm m must ensure that the marginal rate of technical substitution equals that of firm p:

$$\frac{w_l}{w_j} = \frac{f_{x_l}\left(\mathbf{x}, [\mathbf{q}, U^m]\right)}{f_{x_j}\left(\mathbf{x}, [\mathbf{q}, U^m]\right)} = \frac{f_{x_l}\left(\mathbf{x}, [\mathbf{q}, U^p]\right)}{f_{x_j}\left(\mathbf{x}, [\mathbf{q}, U^p]\right)}, \forall j$$

Since f is concave in its inputs $\mathbf{x}$, firm m must choose a lower x_l (fertilizer level) and/or a higher x_j to achieve this equilibrium condition. That is to say, the firm self-selects into a lower x_l intensity based on its lower draw of U. Relatedly, a low U draw will increase the likelihood that the firm does not use any fertilizer. Hence, this factor that is observed to the firm but not the econometrician creates a sample selection bias if the estimation does not account for this.

Combined model

My full proposed model combines the allocative inefficiency and heterogeneous technology models.

Hence the problem is:

$$\begin{aligned} C\left(\mathbf{w}, y, [\mathbf{q}, U]\right) = \quad & \min_{\mathbf{x}} \left\{\mathbf{x} \cdot \mathbf{w}\right\} \\ & \text{s.t.} \\ & f\left(\mathbf{x}, [\mathbf{q}, U]\right) = y \\ & R^s\left(\mathbf{x}\right) = 0, \ s = 1, \ldots, S \end{aligned}$$

The solution to this problem implies

$$\frac{f_{x_k}\left(\mathbf{x}, [\mathbf{q}, U]\right)}{f_{x_j}\left(\mathbf{x}, [\mathbf{q}, U]\right)} = \frac{w_k + \sum_s^S \mu_s R^s_{x_k}\left(\mathbf{x}\right)}{w_j + \sum_s^S \mu_s R^s_{x_j}\left(\mathbf{x}\right)}, \forall k \neq j$$

5 Identification strategy and assumptions

First, I must put to rest some concerns that stem from the fact that farms are embedded in households. When households own the farm and some input markets are missing, households may not operate their farm as a profit maximizing enterprise. Household-specific utility characteristics in part determine the optimal production level. Household utility and production decisions are called "nonseparable" in this case.

The allocative inefficiency framework assumes that the firm's goal is cost minimization; any deviation from cost minimization is due to market distortions or systematic mistakes by firm owners. If nonseparability affected cost minimizing behavior, then the allocative inefficiency framework would not be appropriate since the true goal is utility maximization. I argue, via Proposition I, that cost minimization still occurs under these circumstances.

Proposition I[a]

Establish an agricultural household model with a homogenous agricultural good, a nonagricultural good, and a missing labor market for household labor. Assume standard regularity conditions on the utility function and production function so that the model has interior solutions to the utility maximization and cost minimization problems. Conditional on fixed inputs and the desired household labor contribution, households will minimize the cost of producing their target quantity of output.

[a]Proof in the appendix

Secondly, one hypothesized cause of low fertilizer use is inadequate credit markets. A feature of my model is that it permits the existence of credit constraints, but it does not mistake these constraints for allocative inefficiency. This is formalized in Proposition II.

Proposition II[a]

Establish the same agricultural household model as in Proposition I. Assume, in addition, that a household is prohibited from spending more than some value M on the sum of consumption goods and agricultural inputs. Conditional on fixed inputs and the desired household labor contribution, households will minimize the cost of producing their target quantity of output.

[a]Proof in the appendix

Any effect of credit constraints upon the choice of inputs is not captured within my model. Therefore, any misallocation that appears in my model is the result of factors beyond credit constraints.

Empirical approach

By Shepard's Lemma, the i'th firm with the cost function $C(\mathbf{w}_i, y_i, \mathbf{q}_i)$ has input demands

$$\frac{\partial C}{\partial w_{k,i}} = x_{k,i}(\mathbf{w}_i, y_i, \mathbf{q}_i), \; k = 1, \ldots, N$$

Due to measurement error in x_k and unobserved characteristics of the firm, there is an additive disturbance term ϵ

$$\frac{\partial C}{\partial w_{k,i}} = x_{k,i}(\mathbf{w}_i, y_i, \mathbf{q}_i) + \epsilon_i, \; k = 1, \ldots, N \tag{1}$$

Now let the equation governing whether $x_{k,i} = 0$ be

$$z_i = h_{k,i}\left(\mathbf{w}_i, y_i, \mathbf{q}_i\right) + \nu_i,\ k = 1, \ldots, N$$

Hence we have z_i reflects the influence of heterogenous technology.

$$\begin{aligned} \frac{\partial C}{\partial w_{k,i}} = \quad & x_{k,i}\left(\mathbf{w}_i, y_i, \mathbf{q}_i\right) + \epsilon_i \quad \text{if} \quad & z_i > h_{k,i}\left(\mathbf{w}_i, y_i, \mathbf{q}_i\right) + \nu_i \\ & & z_i \leq h_{k,i}\left(\mathbf{w}_i, y_i, \mathbf{q}_i\right) + \nu_i \\ & & k = 1, \ldots, N \end{aligned}$$

Let $\Omega_{\epsilon\nu}$ be the variance-covariance matrix of $[\boldsymbol{\epsilon}, \boldsymbol{\nu}]$. If $\Omega_{\epsilon\nu}$ is not block-diagonal (the elements corresponding to $\boldsymbol{\epsilon}$ and $\boldsymbol{\nu}$ being the two blocks) an endogeneity problem arises in the econometric model if we attempt to directly estimate the parameters of the equations in (1).

How can we solve the endogeneity problem and avoid inconsistent parameter estimates? The approach I use is based upon a maximum likelihood technique developed by Perraudin & Sørensen (2000). Their maximum likelihood method would be computationally infeasible for the 6 demand equations that I ultimately estimate, so below I describe how I modify their estimator.

They estimated U.S. consumers' Marshallian demand functions for asset portfolios. Importantly, their method allowed them to account for unobserved heterogeneity that influenced the probability of holding a certain portfolio of assets. It is a control function approach related to a Heckman selection correction. The possible portfolios are all combinations of money, stocks, and bonds. The general idea behind their approach is that the decision to choose among a set of possible portfolios reveals information about the underlying heterogeneity in demand for each portfolio. Now I turn to my own problem, which is estimation of a heterogeneous technology.

Imagine that the set of input choices can be divided into regimes. Let the inputs be enumerated $1, \ldots, N$ and denote the set of all inputs as $\mathbf{N}$. Then the set of regimes is the set of all possible combinations of use or nonuse of the inputs. This is the power set, $\mathcal{P}(\mathbf{N})$. For example, the regime $\{2, 4, 5\}$ denotes the use of the second, fourth, and fifth input. Let an element of this power set be denoted j.

Now let the i'th farmer's value function be defined as:

$V_i = G_i\left(y\right) - \min\left\{C_i^j : j \in \mathcal{P}(\mathbf{N})\right\}$

where y is crop output chosen by the farmer and G_i is some function translating output into money or utility. If farmers are profit maximizers, then $G_i\left(y\right) = p \cdot y$, where p is the price of the output.

C_i^j is the cost function $C(\mathbf{w}, y, \mathbf{q})$ under the j'th regime.[1] ($\mathbf{w}$ is the vector of input prices and $\mathbf{q}$ is the vector of fixed inputs.) In other words, C_i^j is the cost function where the inputs in the j'th set are used, while inputs not in the j'th set are not used. This merely describes the partitioning of the original cost function.

The technology partition transforms the firm's problem into a two-stage problem. The first stage is to choose among the input regimes. The second stage is to choose the optimal amount of each input.

Now let C_i^j be written as

$C_i^j = c_i^j + e_i^j$

where e_i^j is a regime-specific productivity term known to each farmer, but that is unobserved by the econometrician. Once again the conditional input demand functions may be derived from Shepard's lemma:

$$\frac{\partial C_i^j}{\partial w_k} = x_{k,i}^j(\mathbf{w}, y, \mathbf{q})$$

So every regime j has a separate demand function for every input k.

Unobserved heterogeneity

Now let there be some unobserved heterogeneity among farmers U. In the cost function, this unobserved heterogeneity can be handled as if it is another fixed input. That is to say, in the functional form of the cost function, U can be made to appear wherever an additional fixed cost may appear. So let $\tilde{\mathbf{q}} = \begin{bmatrix} \mathbf{q} & U \end{bmatrix}$.

How can the value of U be recovered? The choice of regime helps reveal U.

Holding in mind that C_i^j was re-written as $c_i^j + e_i^j$ above, a farm will choose a particular regime r if and only if for all other regimes j we have:

$c_i^r + e_i^r \leq c_i^j + e_i^j$, which is the same condition as $c_i^r - c_i^j \leq e_i^j - e_i^r$

Then the following probability can be defined:

$$\begin{aligned} P_i^j &\equiv Pr\left\{\text{farm facing prices } \mathbf{w} \text{ with target production } y \text{ and fixed inputs } \mathbf{q} \text{ chooses regime } j\right\} \\ &= Pr\left\{c_i^j - c_i^l \leq e_i^l - e_i^j \; \forall l \neq j\right\} \end{aligned}$$

If each e^j has a Type I extreme value distribution, then the above conditional probability can be modeled as a multinomial logit.

[1] $\left\{C_i^j : j \in \mathcal{P}(\mathbf{N})\right\}$ is just set builder notation for $\left\{\emptyset, C_i^{\{1\}}, C_i^{\{2\}}, \ldots, C_i^{\{1,2\}}, C_i^{\{1,3\}}, \ldots, C_i^{\{1,\ldots,N\}}\right\}$.

This logit form allows the recovery of the conditional expectation of U. Let each u^j be an independent, extreme-valued random variable such that $E\left[U_i|u_i^{\{1\}},\ldots,u_i^{\{1,\ldots,N\}}\right] = \sum_{j\in\mathcal{P}(\mathbf{N})} \lambda^j u_i^j$, where each λ^j is a constant parameter.

Then the expected value of U_i conditional on the chosen regime being j is proportional to:

$$E_j\left[U_i\right] \equiv E_j\left[U_i|j = \text{chosen regime}\right] = -\lambda^j \frac{\log P_i^j}{1-P_i^j} + \sum_{k\in\mathcal{P}(\mathbf{N})} \lambda^k \frac{\log P_i^k}{1-P_i^k}$$

The proportionality factor is unknown and can be normalized to one. Once $E_j\left[U_i\right]$ is in hand, it can stand in for the unobserved U in the $\tilde{\mathbf{q}}$ vector.

The cost function with unobserved heterogeneity is then $C^j\left(\mathbf{w}, y, \tilde{\mathbf{q}}\right)$. The associated input demand functions are $x_k^j\left(\mathbf{w}, y, \tilde{\mathbf{q}}\right)$.

Tauchmann (2010) shows that any estimator of a censored system of demand equations that uses a Heckman-like selection correction term such as $E_j\left[U_i\right]$ must condition on the entire pattern of positive binary variables for positive use of each input – i.e. the regimes – rather than just each binary variable equation-by-equation. An estimator that does not do this would be inconsistent. Therefore, the complex apparatus of estimating choice for each regime, though seemingly cumbersome, is necessary.

The model can be estimated in one step or two. Both approaches are consistent, but the one-step approach yields more precise parameter estimates. In a one-step approach, the parameters of the multinomial logit model that determine regime choice are estimated simultaneously with the parameters of the input demand equations x_k^j for each regime. In the two-step approach, the multinomial logit is first estimated, recovering $E_j\left[U_i\right]$, and the input demand equations are estimated with the $E_j\left[U_i\right]$ adjustment term. The two-step approach, which is the approach that I take, requires estimation of parameter estimates via bootstrapping since $E_j\left[U_i\right]$ is a generated regressor. In both approaches, the set of parameters for the input demand equations are different for the two regimes.

Adding allocative inefficiency

Inserting measurement of allocative inefficiency in this framework is straightforward. Estimation can proceed via the shadow price approach. This involves multiplying each price by a shadow price parameter. So define:

$w_k^* = \theta_k w_k$, $\forall k$, where θ_k is a shadow price parameter to be estimated.

Then the new cost function is $C^j\left(\mathbf{w}^*, y, \tilde{\mathbf{q}}\right)$ and the associated input demand functions are:

$$\frac{\partial C^j}{\partial w_k^*} = x_k^j\left(\mathbf{w}^*, y, \tilde{\mathbf{q}}\right)$$

Functional form

The cost function can be represented by many functional forms, but I argue that the Symmetric Generalized McFadden (SGM) cost function is preferred in this application (Pierani & Rizzi 2003). The main reason is that among linear flexible forms, the SGM is unusual in that it can be made globally concave in input prices via a relatively simple parametric restriction. Global concavity is important generally since a cost function only represents an underlying production function if it is concave. In the particular application of measuring allocative inefficiency, concavity takes on additional importance. Concavity of the cost function guarantees that being in an allocatively inefficient state costs a firm more than being in an allocatively efficient state. Without the enforcement of concavity, estimation results can imply that firms are better off being allocatively inefficient, which is nonsensical.

The SGM cost function is:

$$\begin{aligned}\frac{G}{y} = \quad & \frac{\sum_r^N \sum_k^N s_{rk} w_r^* w_k^*}{2\left(\sum_k^N \psi_k w_k^*\right)} + \sum_k^N b_{yk} w_k^* + \sum_k^N b_k \frac{w_k^*}{y} + \left(\sum_k^N \beta_k w_k^*\right) b_{yy} y + \\ & \sum_k^N \sum_j^J d_{kj} \frac{w_k^* q_j}{y} + \sum_j^J c_j q_j \left(\sum_k^N \delta_k w_k^*\right) + \frac{1}{2} \sum_j^J \sum_l^J c_{jl} \frac{q_j q_l}{y^2} \left(\sum_k^N \eta_k w_k^*\right)\end{aligned} \tag{2}$$

Where G is the sum of variable cost. Both sides of the equation have been divided by y to ease estimation.

The Greeks characters are not parameters to be estimated. Rather, they are all set to the average input quantity for the whole sample:

$\bar{x}_k = \psi_k = \beta_k = \delta_k = \eta_k, \forall k$

When the multinomial logit is used to estimate the choice of regime, the left side of the equation is an indicator for whether the fertilizer regime is chosen.

All the parameters of the cost function can be recovered simply by estimating (2) alone. This could be done via nonlinear least squares. The benefit of estimating the input demand equations with the cross-equation restrictions on the parameters is that statistical power is greatly enhanced. In a certain sense, by estimating the entire equation system, the effective number of observations in the sample is multiplied by the number of equations.

The SGM input demand function for the n'th input is:

$$\frac{x_n}{y} = \left\{ \frac{\sum_k^N s_{nk} w_k^*}{\sum_k^N \psi_k w_k^*} - \frac{\psi_n}{2} \frac{\sum_r^N \sum_k^N s_{rk} w_r^* w_k^*}{\left(\sum_k^N \psi_k w_k^*\right)^2} \right\} +$$

$$b_{yn} + \frac{b_n}{y} + \beta_n b_{yy} y + \sum_j^J d_{nj} \frac{q_j}{y} + \delta_n \sum_j^J c_j q_j + \frac{\eta_n}{2} \sum_j^J \sum_l^J c_{jl} \frac{q_j q_l}{y^2}$$

To ensure that the cost function is homogeneous of degree one, the restriction $\sum_k^N s_{rk} = 0 \; \forall r$ is imposed. Symmetry is also necessary: $c_{jl} = c_{lj}$ and $s_{rk} = s_{kr}$.

Enforcement of the concavity of the cost function in prices is achieved via the Cholesky decomposition: Let S be the square matrix composed of the s_{rk} parameters. Then estimation proceeds by ensuring that for some T, the following is satisfied:

$S = -TT'$, where T is a lower triangular matrix.

Key hypothesis tests

If the estimated shadow price parameters θ_k all equal 1 for each input, then there is no systematic allocative inefficiency. If this hypothesis is rejected, then Bolivian farms are systematically allocatively inefficient. Farms underuse fertilizer compared to another input j if the shadow price parameter for the jth input is less than the shadow price parameter for fertilizer. Finally, if the $\lambda^m = 0$, $\forall m$ hypothesis is rejected, then unobserved heterogeneity causes a selection into fertilizer use.

6 Estimation of the model

Estimating a censored demand system is challenging, and a number of approaches have been proposed. As a base framework I use the Generalized Maximum Entropy (GME) approach of Golan, Judge, & Perloff (1996) and Golan, Perloff, & Shen (2001). A complete reference to econometric estimation via entropy maximization is Golan, Judge, & Miller (1996).

The GME approach has a several of advantages over maximum likelihood techniques such as the ones outlined in Shonkwiler & Yen's (1999), Yen (2005), and Yen & Lin (2006). First, GME does not assume a particular distribution family for the error term, but only requires that the support be specified. Second, the computational demands of GME do not explode as the number of equations increases. With maximum likelihood approaches, additional equations require numerical integration over additional dimensions, making estimation infeasible in some cases. Finally, GME estimators tend to have low mean squared error. In simulations of a single censored equation, the GME estimator achieved lower mean squared error than a maximum likelihood estimator even when the error distribution was normal (Golan, Perloff, & Shen 2001). Recall that maximum likelihood estimators are only guaranteed to be asymptotically efficient, not necessarily efficient in finite samples. The GME estimator is described in the appendix.

In sum, I estimate a sample selection correction method developed by Perraudin & Sørensen (2000) via GME rather than maximum likelihood. I estimate two steps, the first being the multinomial logit on regime choice using Golan, Judge, & Perloff (1996), a consistent estimator. Then with the consistent estimate of $E_j [U_i]$ in hand, I use Golan, Perloff, & Shen's (2001) consistent estimator of a censored system of equations. Give that both steps are consistent, the combined two-step estimator is also consistent (Pagan 1986).

7 Results

The censored system of demand equations with selection correction was estimated for barley and potatoes. Since these crops have very different characteristics, concentrating offers a high contrast. Potatoes are viable as commercial product rather than just suitable as a subsistence crop, as barley is. The mean production quantity of potato farmers would earn 522 U.S. dollars in revenue at national market prices (Food & Agriculture Organization 2017). Mean barley output is such that the average barley output would generate only 107 U.S. dollars. A drawback of potato growing, however, is that they are more challenging to grow than barley. Barley can better tolerate weather shocks, so it can act as a backstop crop if other crops fail. The input profiles of these two crops reflect their contrasting roles. In terms of the extensive margin, potatoes have higher levels of use of inputs that almost every other crop, as Table 2 shows. Very low use of intermediate inputs characterizes barley farming, on the other hand.

To obtain correct standard errors with the two-step estimation method, about 130 bootstrap iterations were performed. Tables 4 and 5 display the shadow price parameters for the models with and without the selection correction. The price of organic fertilizer is set at one since it is arbitrarily chosen as the reference shadow price.

The overall hypothesis of $\theta_k = 1$ for all k can be strongly rejected for both crops. Therefore, the input mix used by farmers in the sample is allocatively inefficient. The estimates provide enough statistical power to examine individual shadow prices. The most striking result is the low estimate of the shadow price of

Table 4: Estimated shadow price parameters - Barley

Parameter	No selection correction	SE	t-stat	Selection correction	SE	t-stat
θ_{01}	1.12	0.18	0.64	1.20	0.50	0.39
θ_{02}	0.95	0.23	-0.23	0.60	0.31	-1.29
θ_{03}	0.91	0.26	-0.37	1.05	0.37	0.14
θ_{04}	0.96	0.18	-0.22	0.92	0.25	-0.33
θ_{05}	0.79	0.14	-1.45	0.60	0.28	-1.39

No selection correction: χ^2_5 Wald statistic on $\theta_k = 1$, $\forall$ k: 415.48 (p $<$ machine precision).
Selection correction: χ^2_5 Wald statistic on $\theta_k = 1$, $\forall$ k: 77.55 (p = 2.78e-15).
χ^2_5 Wald statistic on $\theta_k^{No\ correction} = \theta_k^{Correction}$, $\forall$ k: 3.09 (p = 0.686).

Table 5: Estimated shadow price parameters - Potatoes

Parameter	No selection correction	SE	t-stat	Selection correction	SE	t-stat
θ_{01}	0.74	0.25	-1.02	1.51	0.44	1.16
θ_{02}	1.63	0.26	2.45	0.08	0.57	-1.63
θ_{03}	0.39	0.23	-2.59	0.33	0.74	-0.90
θ_{04}	1.41	0.32	1.26	1.34	0.82	0.41
θ_{05}	0.57	0.14	-3.12	0.99	0.30	-0.04

No selection correction: χ^2_5 Wald statistic on $\theta_k = 1$, $\forall$ k: 620.33 (p $<$ machine precision).
Selection correction: χ^2_5 Wald statistic on $\theta_k = 1$, $\forall$ k: 42.77 (p = 4.11e-08).
χ^2_5 Wald statistic on $\theta_k^{No\ correction} = \theta_k^{Correction}$, $\forall$ k: 14.48 (p = 0.0129).

purchased seed for both crops. The parameter estimate of $\theta_{Seed} = 0.08$ for the selection-corrected potato model indicates farmers' choices would be allocatively efficient only if purchased seed was about 10 percent of the price of organic fertilizer, the numeraire input. Farmers are overusing purchased seed compared to most of the intermediate inputs.

The estimates for the shadow prices of inorganic fertilizer are not statistically different from one. This result suggests that the difference summary statistics of fertilizer use in Bolivia versus its Andean neighbors was a red herring.

The hypothesis that there is no endogenous selection into input regimes cannot be rejected for barley. The Wald statistic for $\lambda^m = 0$ for all m is 4.0 on 19 degrees of freedom, which yields a p-value of almost one. The non-rejection of this hypothesis is corroborated by the similarity of the shadow price parameters for the corrected and uncorrected specifications as displayed in Tables 4 and 5. A formal Wald test fails to reject the hypothesis that the shadow price parameters are the same for both specifications.

One interpretation of this failure to reject endogenous selection into the various input regimes is that the inclusion of fixed inputs for environmental conditions – rainfall, elevation, and soil quality – in the model adequately capture the selection effects. The observed variables may capture all the factors that lead to heterogenous technology, and there are no remaining unobserved factors to correct for with the econometric selection technique.

In contrast, the hypothesis that there is no endogenous selection into input regimes is rejected for potatoes. The Wald statistic for $\lambda^m = 0$ for all m is 225.9 on 32 degrees of freedom, which yields a p-value of less than machine precision. The nonzero value for the λ parameters, in turn, affects the shadow price parameters: a Wald test rejects the hypothesis that the model without the selection correction has the same shadow price parameters as the model with the selection correction.

The technology parameters are displayed in the appendix. Given the numerous interaction terms, direct interpretation of the parameter estimates can be difficult. An exception is b_{yy}. Its negative sign indicates that the technology for both crops exhibits declining marginal cost in y, indicating increasing returns to scale. An important difference between the technologies is in how cost responds to elevation according to the first-order effects c_j. Higher elevation for barley translates into lower costs, whereas the opposite is true for potatoes. This result is consistent with the fact that barley is better adapted to high altitudes. The parameter estimates thus seem to confirm what may be our prior beliefs about the realities of the production technologies of the two crops. A word of caution is in order here, since the true marginal effects of the fixed inputs varies over the range of the data due to the interaction terms.

Excess cost due to misallocation

The consequences of misallocation cannot be judged from the shadow prices alone. The shape of the technology partly determines the difference between the attainable minimum cost and the cost that farmers actually incurred due to allocative inefficiency.

Table 6: Excess cost due to allocative inefficiency - Barley

Statistic	Mean	St. Dev.	Min	Median	Max
Inefficient expenditure	1,240.44	3,496.76	1.35	379.64	83,289.80
Efficient expenditure	1,235.48	3,490.54	1.22	377.70	83,253.53
Percent difference	0.49	0.74	0.0002	0.19	10.37

The 2008 exchange rate was about 7.4 Bolivianos per USD.

Input demand actually observed is $x_k(\mathbf{w}^*, y, \tilde{\mathbf{q}})$. Calculation of the allocatively-efficient counterfactual demand involves simply setting $\theta_k = 1$ for all k, but keeping the remaining technological parameters at their estimated values (Kumbhakar & Lovell 2000). The efficient input demand would then be $x_k(\mathbf{w}, y, \tilde{\mathbf{q}})$. Then total expenditure for each is simply $\sum_k^N w_k \cdot x_k(\mathbf{w}^*, y, \tilde{\mathbf{q}})$ and $\sum_k^N w_k \cdot x_k(\mathbf{w}, y, \tilde{\mathbf{q}})$. Since these quantities are data-dependent, they can be obtained by calculating the predicted input demand for each observation,

Table 7: Excess cost due to allocative inefficiency - Potatoes

Statistic	Mean	St. Dev.	Min	Median	Max
Inefficient expenditure	882.75	1,605.22	10.83	376.03	20,220.12
Efficient expenditure	845.66	1,543.48	1.93	361.90	19,005.68
Percent difference	7.89	130.22	0.0002	3.21	6,589.68

The 2008 exchange rate was about 7.4 Bolivianos per USD.

setting any negative demand values to zero, and summing to obtain total expenditure. Tables 6 and 7 display summary statistics for these inefficient and efficient cost calculations. The median barley farmer spends about 0.2 percent more due to using an inefficient input mix, a negligible amount. The median potato farmer, on the other hand, spends 3.2 percent, or 2 U.S. dollars, more due to misallocation. The low quantity in absolute terms reflects the overall low cash spending on inputs. The higher cost for potatoes could be due to the challenging nature of potato production as compared to barley. Farmers may have more difficulty in hitting the target optimal input mix for potato production.

In 2008, the mean per capita income of the poorest 40 percent of Bolivians (composed largely of rural dwellers) was $1,100 in 2011 U.S. dollars in purchasing power parity terms. The losses due to inefficiency therefore is not particularly harmful to farmers' welfare.

The computed inefficiency cost is fairly small despite the shadow price parameters being far from unity. The excess cost is governed by a subset of the technological parameters. The only terms in the SGM n'th input demand equation that are affected by setting $\theta_k = 1$ for all k are

$$\frac{\sum_k^N s_{nk} w_k^*}{\sum_k^N \psi_k w_k^*} - \frac{\psi_n}{2} \frac{\sum_r^N \sum_k^N s_{rk} w_r^* w_k^*}{\left(\sum_k^N \psi_k w_k^*\right)^2}$$

Only the s_{rk} parameters enter into consideration. These parameters determine the substitution patterns across the inputs. The elements of s_{rk} form the negative semi-definite matrix $\mathbf{S}$. If all the elements of $\mathbf{S}$ are zero, then nominal allocative inefficiency actually has no negative consequences for farmers in the form of excess cost. This fact drives home the point that structural estimation of the production technology is necessary to decide if allocative inefficiency matters for farmers.

8 Conclusion

Bolivian potato and barley farmer are using a suboptimal input mix, conditional on their input price environment, target output level, and the technology available to them. Despite summary statistics suggesting underuse of inorganic fertilizer compared to their Andean peers, Bolivian farmers' inefficiency does not stem from insufficient fertilizer use, but rather is mostly centered on an overuse of purchased seed The practical consequences for farmers are modest, however, being on the order of 3 percent excess cost due to inefficiency.

Most prior work that has investigated low use of agricultural inputs in developing countries has used reduced form approaches. In this paper I took a new structural approach to reveal the existence and magnitude inappropriate input mixes by estimating allocative inefficiency

Besides contributing to knowledge about agricultural input use, I developed a new estimator for estimating censored systems of equations that has desirable statistical properties. I did detected endogenous selection into positive levels of input use for potatoes, but not barley. This estimator holds promise for future use in other applications.

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Table 8: Technological parameters of cost function

	Barley			Potatoes		
Parameter	Estimate	SE	t-stat	Estimate	SE	t-stat
b01	0.341	1.43	0.239	7.11	2.34	3.04
b02	-8.82	3.74	-2.36	9.33	5.03	1.86
b03	-0.972	1.4	-0.693	-2.07	0.852	-2.43
b04	0.72	3.63	0.199	-3.47	2.72	-1.28
b05	0.222	2.01	0.11	-0.643	2.18	-0.295
b06	-0.951	1.35	-0.703	1.65	2.56	0.647
by01	-0.000947	0.00333	-0.284	0.00661	0.0136	0.485
by02	-0.64	0.114	-5.63	0.0335	0.0778	0.431
by03	-0.034	0.00753	-4.51	0.0018	0.000758	2.38
by04	-0.0259	0.0227	-1.14	0.0194	0.013	1.49
by05	-1.35	0.348	-3.88	-0.0141	0.122	-0.115
by06	-0.827	0.185	-4.48	-0.113	0.339	-0.333
byy	-2.85e-07	5.76e-08	-4.94	-6.2e-08	1.28e-08	-4.84
c01	0.000533	0.000151	3.53	0.000271	0.000117	2.32
c0101	-17.6	6.81	-2.59	-6.68	7.37	-0.906
c0102	-5.44	5.05	-1.08	6.19	6.06	1.02
c0103	0.0014	0.00614	0.228	-0.00949	0.00946	-1
c0104	-8.29	3.87	-2.14	2.16	8.25	0.261
c0105	1.9	1.76	1.08	-0.0846	2.3	-0.0368
c0106	-7.58	6.5	-1.17	-5.62	5.27	-1.07
c0202	-6.68e-05	0.000229	-0.292	0.000489	6.46e-05	7.56
c0203	-0.00598	0.00569	-1.05	-0.00357	0.00341	-1.05
c0204	6.48	4	1.62	14.5	8.2	1.77
c0205	-1.16	1.04	-1.11	-2.57	1.59	-1.61
c0206	-2.67	5.37	-0.496	-3.78	2.51	-1.5
c03	2.84e-07	2.06e-07	1.38	1.01e-07	9.72e-08	1.03
c0303	-2.6e-07	4.13e-06	-0.0629	4.11e-06	5.21e-06	0.789

c0304	0.00668	0.0207	0.323	0.00684	0.0196	0.349
c0305	-0.0014	0.00384	-0.365	-0.00124	0.00334	-0.37
c0306	0.00113	0.00389	0.291	-0.00371	0.00455	-0.815
c04	0.0368	0.00596	6.18	0.00216	0.00141	1.53
c0404	2.64	1.5	1.75	4.14	5.08	0.816
c0405	-0.0736	2.4	-0.0307	-0.921	3	-0.307
c0406	3.19	5.25	0.608	-7.34	8.44	-0.869
c05	0.000338	0.000226	1.5	-0.000342	5.4e-05	-6.34
c0505	-0.118	0.903	-0.13	-0.0114	1	-0.0114
c0506	0.256	1.36	0.188	1.92	1.6	1.2
c06	-0.00127	0.00043	-2.96	0.000113	8.65e-05	1.31
c0606	-8.07	4.8	-1.68	-0.241	3.53	-0.068
d0101	0.0761	0.242	0.314	0.406	0.858	0.474
d0102	0.191	0.417	0.458	-1.13	0.631	-1.78
d0103	-0.000114	0.000333	-0.342	0.00124	0.000733	1.69
d0104	-0.0398	1.39	-0.0287	-7.84	2.7	-2.91
d0105	-0.144	0.323	-0.446	-0.388	0.44	-0.882
d0106	0.148	0.168	0.88	0.802	0.781	1.03
d0201	7.97	2.56	3.12	19.1	5.19	3.69
d0202	2.06	2.02	1.02	2.35	2.86	0.82
d0203	-0.000166	0.00141	-0.118	0.00395	0.0071	0.557
d0204	7.76	4.87	1.59	14.8	4.62	3.21
d0205	0.22	1.04	0.212	-4.23	1.92	-2.2
d0206	3.99	2.74	1.46	-0.111	2.59	-0.043
d0301	0.387	0.189	2.04	0.636	0.128	4.96
d0302	0.101	0.127	0.793	-0.0389	0.0497	-0.784
d0303	-5.56e-05	0.000116	-0.481	1.32e-06	7.85e-05	0.0168
d0304	1.4	2.16	0.646	2.4	1.25	1.91
d0305	-0.02	0.0929	-0.215	0.0831	0.0435	1.91
d0306	0.0644	0.12	0.535	0.0142	0.0624	0.227
d0401	0.666	1.53	0.435	0.532	0.983	0.541

d0402	0.333	1.04	0.319	0.576	0.626		0.92
d0403	-0.000998	0.00379	-0.264	-0.000239	0.000885		-0.27
d0404	0.604	3.12	0.194	2.92	3.21		0.91
d0405	-0.421	1.99	-0.211	0.324	0.525		0.617
d0406	0.599	0.744	0.805	1.72	0.707		2.44
d0501	0.13	2.86	0.0455	8.61	2.67		3.22
d0502	1.03	2.88	0.358	2.48	4.24		0.585
d0503	-0.00535	0.00414	-1.29	0.0107	0.00889		1.21
d0504	0.176	1.49	0.118	-0.281	1.41		-0.2
d0505	0.146	1.27	0.115	1.22	0.997		1.23
d0506	5.89	5.34	1.1	-4.49	2.02		-2.22
d0601	-0.0285	4.44	-0.00642	-0.424	3.46		-0.123
d0602	7.21	3.48	2.07	14.6	6.08		2.4
d0603	0.0353	0.0164	2.15	-0.00954	0.0196		-0.488
d0604	-0.444	0.866	-0.513	1.31	1.84		0.714
d0605	-2.98	1.83	-1.63	3.38	2.06		1.64
d0606	-2.7	4.05	-0.666	1.02	5.44		0.188
s0202	-6.27	3.47	-1.81	-9.77	2.19		-4.46
s0203	-0.232	0.12	-1.94	-0.351	0.0739		-4.75
s0204	0.689	0.83	0.83	-2.33	0.6		-3.89
s0205	2.52	1.48	1.7	5.46	1.96		2.79
s0206	3.22	1.64	1.96	6.83	2.26		3.03
s0303	-0.0217	0.0126	-1.72	-0.0283	0.0168		-1.69
s0304	0.0326	0.0529	0.616	-0.05	0.0146		-3.41
s0305	-0.0911	0.0697	-1.31	0.394	0.15		2.63
s0306	0.322	0.157	2.06	0.0667	0.0973		0.686
s0404	-0.881	2.1	-0.418	-1.11	0.322		-3.44
s0405	0.274	0.899	0.305	-1.06	0.581		-1.82
s0406	-0.129	0.517	-0.25	4.99	1.09		4.6
s0505	-4.2	2.56	-1.64	-13.4	3.12		-4.3
s0506	1.63	1.18	1.38	10.5	2.77		3.8

s0606	-5.12	2.7	-1.9	-25.4	4.97			-5.12

See equation 2 for the model specification. Price w_i are 1 = inorganic fertilizer in Bolivianos/kg; 2 = purchased seed in Bs/kg; 3 = tractor Bs/hours; 4 = plaguicidas Bs/kg; 5 = hired labor Bs/hours; 6 = organic fertilizer Bs/kg. Fixed inputs q_i are 1 = hectares cultivated; 2 = has irrigation; 3 = family labor hours; 4 = soil quality; 6 = elevation in km; 5 = precipitation in cm/season.

Proof of Proposition I

First note the following fact about cost minimization:

The firm's cost minimization problem can be expressed as:

$\min_{\mathbf{x}} \{\mathbf{x} \cdot \mathbf{w}\}$

s.t. $y - f(\mathbf{x}, \mathbf{q}, L_a) = 0$

where

$\mathbf{x}$ is the vector of variable inputs for the farming technology

$\mathbf{w}$ is the vector of prices for these variable inputs

$\mathbf{q}$ is the vector of fixed inputs for the farming technology

L_a is the household's agricultural labor

Let λ be the Lagrange multiplier in this problem.

If a production technology $f(\mathbf{x}, \mathbf{q}, L_a)$ is quasi-concave in $\mathbf{x}$, then the $\mathbf{x}^*$ that satisfies

$$w_k - \lambda f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) = 0, \ \forall k$$

and

$$y - f(\mathbf{x}, \mathbf{q}, L_a) = 0$$

solves the firm's cost minimization problem, provided it is an interior solution.

Note finally that

$$w_k - \lambda f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) = 0 \iff \frac{f_{x_k}(\mathbf{x}, \mathbf{q}, L_a)}{f_{x_j}(\mathbf{x}, \mathbf{q}, L_a)} = \frac{w_k}{w_j}, \forall k \neq j$$

Recall also that cost minimization is a necessary but not sufficient condition for profit maximization. Therefore, the absence of profit maximization does not indicate the absence of cost minimization.

Let there be a household i. It is a unitary actor, so there is no intrahousehold bargaining. Where input and output markets exist, it is a price taker.

The household seeks to maximize its utility given by utility function $U(\cdot)$.

Arguments to its utility function are:

l, leisure

g_a, a homogenous agricultural good

g_m, a homogenous nonagricultural good

$\mathbf{z}$, Household-specific characteristics

$U(\cdot)$ is strictly increasing in l, g_a and g_m.

The household can directly produce g_a, but g_m must be purchased with cash.

There is an off-farm labor option for the household.

The household's problem is:

$$\max_{\{\mathbf{x},L_m,L_a,l,g_m,g_a\}} \{U(l, g_a, g_m, \mathbf{z})\}$$
s.t.

Agricultural technology: $f(\mathbf{x}, \mathbf{q}, L_a) \geq y$

Budget constraint: $L_m \cdot \omega + y \geq p_m g_m + p_a g_a + \mathbf{x} \cdot \mathbf{w}$

Time constraint: $L_m + L_a + l = 1$

Non-negativity constraints: $L_m, L_a, l, g_m, g_a, \mathbf{x} \geq 0$

where:

$\mathbf{x}$ is the vector of variable inputs for the farming technology

$\mathbf{w}$ is the vector of prices for these variable inputs

$\mathbf{q}$ is the vector of fixed inputs for the farming technology

L_a is household labor used in agricultural production

L_m is household labor used on off-farm activities

ω is the wage rate for off-farm activities

y is the farm output

p_m and p_a are the market prices of the nonagricultural and agricultural goods

The household's choice variables are: $\mathbf{x}$, L_m, L_a, l, g_m, and g_a

Notice that if agricultural production, y, is greater than desired agricultural consumption, g_a, then the household can sell y to allow more consumption of g_m. Notice also that we must have $\mathbf{x} \cdot \mathbf{w} \leq y$ since a household would not farm if the cost of production was higher than the value of the output. Assume that the household produces some output y since otherwise there is no problem is analyze. The budget constraint binds since the utility function is strictly increasing in goods g_m and g_a. In this setting, household and hired labor are not interchangeable due to the principal-agent problem, and the market for household labor is missing. Assume that the household is technically efficient in its farm production, so the technology constraint binds. Assume non-negativity constraints do not bind, by Inada conditions on the utility function and the production function.

Combine the constraints into one constraint:

$(1 - L_a - l) \cdot \omega + p_a \cdot f(\mathbf{x}, \mathbf{q}, L_a) = p_m \cdot g_m + p_a \cdot g_a + \mathbf{x} \cdot \mathbf{w}$

Form the Lagrangian:

$\mathscr{L} = U\left(l, g_a, g_m, \mathbf{z}\right) + \mu\left[(1 - L_a - l) \cdot \omega + p_a \cdot f(\mathbf{x}, \mathbf{q}, L_a) - p_m \cdot g_m - p_a \cdot g_a - \mathbf{x} \cdot \mathbf{w}\right]$

$[x_k]$ $p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) - w_k = 0, \forall k$

$[l]$ $U_l\left(l, g_a, g_m, \mathbf{z}\right) - \mu = 0$

$[L_a]$ $-\mu + p_a \cdot f_{L_a}(\mathbf{x}, \mathbf{q}, L_a) = 0$

$[g_m]$ $U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right) - \mu p_m = 0$

$[g_a]$ $U_{g_a}\left(l, g_a, g_m, \mathbf{z}\right) - \mu p_a = 0$

$[\mu]$ $(1 - L_a - l) \cdot \omega + p_a \cdot f(\mathbf{x}, \mathbf{q}, L_a) - p_m \cdot g_m - p_a \cdot g_a - \mathbf{x} \cdot \mathbf{w} = 0$

From $[x_k]$, we have:

$p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) = w_k$. Dividing this equality by any j equation yields:

$$\frac{p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a)}{p_a \cdot f_{x_j}(\mathbf{x}, \mathbf{q}, L_a)} = \frac{w_k}{w_j} \implies \frac{f_{x_k}(\mathbf{x}, \mathbf{q}, L_a)}{f_{x_j}(\mathbf{x}, \mathbf{q}, L_a)} = \frac{w_k}{w_j}, \forall k \neq j$$

This, along with the technical efficiency assumption of $f(\mathbf{x}, \mathbf{q}, L_a) = y$, proves that the household minimizes cost in agricultural production. ■

Proof of Proposition II

Assume that the total permitted expenditure by the household is M.

Off-farm and farm income come after the end of the period and so $L_m \cdot \omega + p_a \cdot y$ cannot help the household relieve this M liquidity constraint. Hence:

$p_m \cdot g_m + p_a \cdot g_a + \mathbf{x} \cdot \mathbf{w} \leq M$. Assume that this constraint binds.

The new problem is:

$$\max_{\{\mathbf{x}, L_m, L_a, l, g_m, g_a\}} \left\{U\left(l, g_a, g_m, \mathbf{z}\right)\right\}$$

s.t.

Agricultural technology: $f(\mathbf{x}, \mathbf{q}, L_a) = y$

Budget constraint: $L_m \cdot \omega + p_a \cdot y \geq p_m \cdot g_m + p_a \cdot g_a + \mathbf{x} \cdot \mathbf{w}$

Time constraint: $L_m + L_a + l = 1$

Liquidity constraint: $p_m \cdot g_m + p_a \cdot g_a + \mathbf{x} \cdot \mathbf{w} = M$

Reformulate constraints:

$(1 - L_a - l) \cdot \omega + p_a \cdot f(\mathbf{x}, \mathbf{q}, L_a) \geq p_m \cdot g_m + p_a \cdot g_a + \mathbf{x} \cdot \mathbf{w}$

$p_m \cdot g_m + p_a \cdot g_a + \mathbf{x} \cdot \mathbf{w} = M$

Form the Lagrangian:

$\mathscr{L} = U\left(l, g_a, g_m, \mathbf{z}\right) + \mu\left[(1 - L_a - l) \cdot \omega + p_a \cdot f(\mathbf{x}, \mathbf{q}, L_a) - p_m \cdot g_m - p_a \cdot g_a - \mathbf{x} \cdot \mathbf{w}\right] -$

$-\lambda\left[p_m \cdot g_m + p_a \cdot g_a + \mathbf{x} \cdot \mathbf{w} - M\right]$

$[x_k]$ $\mu\left[p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) - w_k\right] - \lambda w_k = 0$

$[l]$ $U_l\left(l, g_a, g_m, \mathbf{z}\right) - \mu = 0$

$[L_a]$ $-\mu + p_a \cdot f_{L_a}(\mathbf{x}, \mathbf{q}, L_a) = 0$

$[g_m]$ $U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right) - \mu p_m - \lambda p_m = 0$

$[g_a]$ $U_{g_a}\left(l, g_a, g_m, \mathbf{z}\right) - \mu p_a - \lambda p_a = 0$

$[\mu]$ $(1 - L_a - l) \cdot \omega + p_a \cdot f(\mathbf{x}, \mathbf{q}, L_a) - p_m \cdot g_m - p_a \cdot g_a - \mathbf{x} \cdot \mathbf{w} = 0$

$[\lambda]$ $p_m \cdot g_m + p_a \cdot g_a + \mathbf{x} \cdot \mathbf{w} - M = 0$

From $[l]$, we have $U_l\left(l, g_a, g_m, \mathbf{z}\right) = \mu$

Hence from $[x_k]$:

$U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot \left[p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) - w_k\right] - \lambda w_k = 0$

Now simplify $[g_m]$:

$U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right) - U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot p_m = \lambda p_m$

$U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right)/p_m - U_l\left(l, g_a, g_m, \mathbf{z}\right) = \lambda$

Now plug the above expression for λ into $[x_k]$:

$U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot \left[p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) - w_k\right] - \left[U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right)/p_m - U_l\left(l, g_a, g_m, \mathbf{z}\right)\right] w_k = 0$

$U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot \left[p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) - w_k\right] + \left[U_l\left(l, g_a, g_m, \mathbf{z}\right) - U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right)/p_m\right] w_k = 0$

$U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) - w_k \cdot U_l\left(l, g_a, g_m, \mathbf{z}\right) + w_k \cdot U_l\left(l, g_a, g_m, \mathbf{z}\right) - w_k \cdot U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right)/p_m = 0$

$U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) - w_k \cdot U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right)/p_m = 0$

$U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a) = w_k \cdot U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right)/p_m$

This implies:

$$\frac{U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot p_a \cdot f_{x_k}(\mathbf{x}, \mathbf{q}, L_a)}{U_l\left(l, g_a, g_m, \mathbf{z}\right) \cdot p_a \cdot f_{x_j}(\mathbf{x}, \mathbf{q}, L_a)} = \frac{w_k \cdot U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right)/p_m}{w_j \cdot U_{g_m}\left(l, g_a, g_m, \mathbf{z}\right)/p_m}$$

and therefore:

$$\frac{f_{x_k}(\mathbf{x}, \mathbf{q}, L_a)}{f_{x_j}(\mathbf{x}, \mathbf{q}, L_a)} = \frac{w_k}{w_j}, \forall k \neq j$$

This, along with the technical efficiency assumption of $f(\mathbf{x}, \mathbf{q}, L_a) = y$, proves that the household minimizes cost in agricultural production. ■

Generalized Maximum Entropy Estimator

GME estimation requires choosing the support space for the parameters values and error terms. Define these support spaces as

$$\mathbf{z} = \begin{bmatrix} z_1 & z_2 & \dots & z_H \end{bmatrix} \text{ for parameters}$$

$$\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \dots & v_D \end{bmatrix} \text{ for error terms}$$

The endpoints of these support spaces define the minimum and maximum possible values of the parameters and error terms. To reduce computational complexity, typical these support vectors have just three elements and are symmetric about zero. Then corresponding weight vectors are the choice variables.

GME takes the form of the following constrained maximization problem:

$$\max_{\mathbf{p},\mathbf{w}} \left\{ -\sum_h^H p_{hk} \cdot \ln p_{hk} - \sum_d^D w_{dit} \cdot \ln w_{dit} \right\}$$

Subject to:

the product of the support space and the weight vectors add to the parameters and error terms:

$\sum_h^H p_{hk} z_h = \hat{\beta}_k$, where $\hat{\beta}_k$ stands in for every parameter to be estimated

$\sum_d^D w_{dit} v_d = \hat{\epsilon}_{it}$, where $\hat{\epsilon}_{it}$ is the estimated residual of the i'th observation and t'th equation the weight vectors add to one so as to be proper probabilities:

$$\sum_{d}^{D} w_d = 1$$

$$\sum_{h}^{H} p_h = 1$$

the weight vectors have non-negative elements so as to be proper probabilities:

$$w_d > 0,\ \forall d;\ p_h > 0,\ \forall h$$

and the estimated parameters and residuals are consistent with the data and the model:

$$\begin{aligned} &y_{it} = f_t\left(\mathbf{x_{it}}, \hat{\beta}\right) + \hat{\epsilon}_{it} \text{ when } y_{it} > 0 \\ &0 > f_t\left(\mathbf{x_{it}}, \hat{\beta}\right) + \hat{\epsilon}_{it} \text{ when } y_{it} = 0 \end{aligned}$$

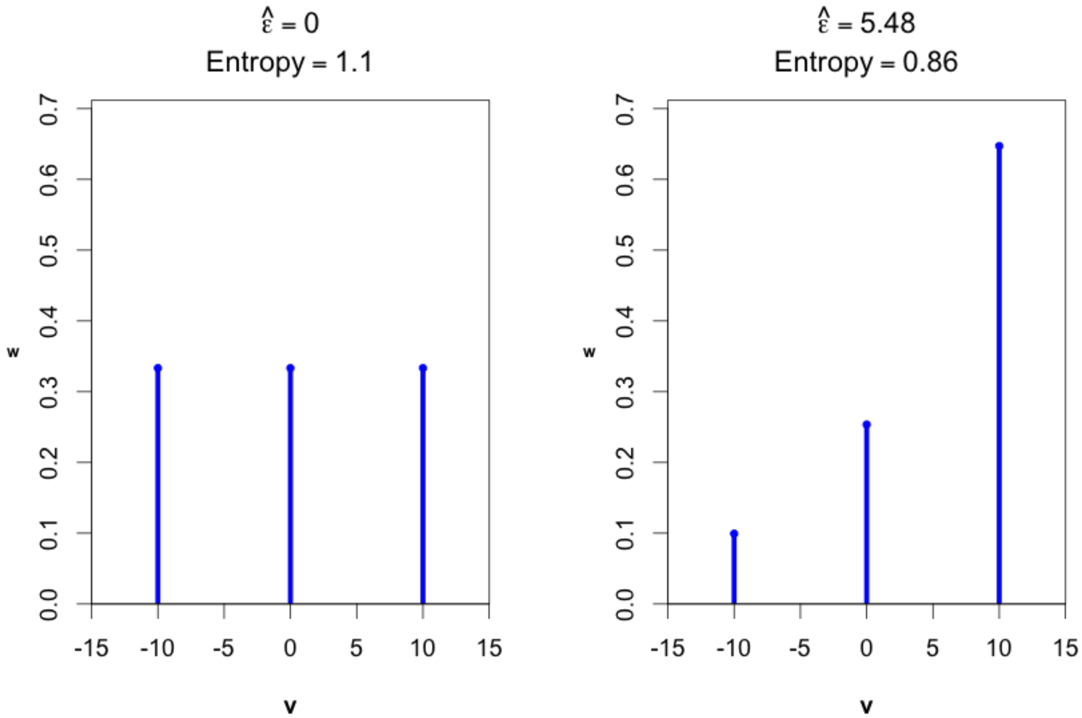
$f_t\left(\mathbf{x_{it}}, \hat{\beta}\right)$ is the model for the t'th equation. In my case these are the cost function and the six input demand equations. To estimate the parameters of the multinomial logit, I use a GME technique developed by Golan, Judge, & Perloff (1996).

The general idea behind is approach is that there are two opposing forces at work. Both the weight vectors associated with the parameters and the weight vectors associated with the residuals would "prefer" to force the parameters and residuals to equal the means of their support vectors. For residuals this mean is zero. For parameters, I have specified the mean to be zero for all parameters except that the support space vectors for the shadow price parameters have mean of unity. This is meant to reflect the idea that the estimation should prefer the null hypothesis if the data is uninformative.

In the case that the model is "uninformative", i.e. has poor fit, GME allows the estimated residuals to deviate far from zero while making the estimated parameters zero. If the model is informative, the maximization process forces the residuals toward zero by shifting the values of the parameters away from the mean of their support spaces. GME is thus a type of shrinkage estimator. It is biased in finite samples, but it gains lower variance in return for its biasedness.

Figure 4 displays an example support space and weight vector for a single residual. The support space is $\begin{bmatrix} -10 & 0 & 10 \end{bmatrix}$. As the left panel shows, subject to $\hat{\epsilon} = 0$, entropy is maximized when all three values of the weight vector $\mathbf{w}$ are $1/3$. If model consistency requires that $\hat{\epsilon} = 5.48$, entropy is maximized when the values of the weight vector are shifted toward the right end of the support, as shown in the right panel. When $\hat{\epsilon}$ changes from 0 to 5.48, entropy falls from 1.1 to 0.86, which illustrate the fact that the maximization

Figure 4: Example of residual entropy



process forces $\hat{\epsilon}$ toward zero if it is permitted by the model consistency constraints. In effect, this particular $\hat{\epsilon}$ competes with the other weight vectors. This behavior of the optimization algorithm is somewhat analogous to the objective in OLS of minimizing the sum of squared errors.