Analysis of Economic Opportunity of Retaining Ownership of Cow-Calf Operations Under Three Production Systems: Grazing Experiment in Southeastern U.S.

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Abstract

This study seeks to analyze the economic opportunity of retaining the ownership of cow-calf operations under three production systems. We obtained data on animal performance from grazing experiment in Southeastern U.S. and introduce two improved livestock production systems and one extensive system. We used both discrete and stochastic infinite horizon models and Monte Carlo simulations to evaluate and estimate optimum value added of retaining ownership of cow-calf operation across different production systems and combination of systems for different seasons. Under different systems for different seasons, our results show that extensive system adds more value per acre in the cool season while the mixed forage system yields more value per acre to cow-calf operations in the warm season. Whereas, the combination of extensive and mixed forage grazing systems yields more value added than any other system combinations in cool seasons while the two improved systems yields more added value in the warm season. The optimum value of the retained ownership of the operation per acre for extensive production is \$281, \$352 per acre for improved-grass production system and \$376.5 for improved-mixed production system. The managerial flexibility of retaining ownership of cow-calf operations for an improved mixed legume-grass production system worth more in value extensive and improved grass system for the five years retained value. Overall, retaining ownership of cow-calf operation to the finishing stage creates an economic opportunity for high returns for all systems and provide a better evaluation of cow-calf operations than NPV.

keywords: Grazing, stochastic, Monte Carlo, improved, extensive, flexibility

JEL: B23, C61, D24, Q13

1 Introduction

The decision to retain calves, post- weaning, to maturity in order to sell in the future with the expectation of increase in livestock prices is synonymous to a purchaser of a future contract in an 'European Call Option' system. One of the few options for management of weaned calves is to background the operations with grazing systems stocked with high yield forages and diets. However, to successfully run a cow-calf operations there is the options to market calves at a heavier weight or option to retain the ownership of the operations till the finishing stage. Retaining ownership of the operations has numerous advantages. There is production strategies; this include ability to capture reward of improved genetics from a quality breeding strategies. There is also the marketing advantage; this is a situation where producers are able to examine the feasibility of increasing the performance of cattle with the aim of reaping the the market value of each cattle. Moreover, there is also an advantage of setting up and creating a breeding program. The breeding program provides useful information about production system and marketing strategies. Furthermore, health management program during the retention period is also another advantage. However, there are production risks and future price risks associated with the choice to retain ownership of cow-calf operations. In fact, unexpected post-weaning problem or variability may affect the performance of the cattle. To insulate and shield producer from these risks, producers and cattlemen can optimize their flexible management operations by establishing two different improved production systems to mitigate the production uncertainty associated with monoculture grazing dominated by native grass pasture. Dick et al (2015) analyze the extensive and improved production system of Southern Brazil, their study indicates that improved system results in less environmental impact than the extensive system but their result did not show whether the production system with low environmental impact also yield higher economic performance or result in higher investment evaluation. In this study, like Dick et al.(2015), we introduced two livestock production systems; the extensive and the improved but unlike them, our improved system has two components, the system with nitrogen fertilized grass based pastures-this is synonymous to Dick et al.(2015) improved system- and the mixed forage of grass and legume. Mixed forage pasture has proved to have numerous advantages over monocultures grazing. (i) it can fix atmospheric nitrogen, (ii) increase weaned cattle weight gain (Sanderson et al., 2013), (iii) less susceptible to devastation from adverse weather, (iv) attracts investment opportunities and prevent soil degradation, among others. Likewise, high yield in improved nitrogen fertilized grass gives a promising productivity in animal.

This study is motivated with the stylized fact of Brambilla et al.,(2012). The authors argue that livestock production responds linearly to the increased in nitrogen fertilized rate of ryegrass. For that reasons, an improved system could be dubbed to have more yield than extensive and thus, resulting in higher productivity. It become interesting to know whether increasing in animal performances will always result in higher returns and whether higher returns translate into higher value of cow-calf operations which could eventually influence the decision to retain ownership. In summary, it could be nice to evaluate the type of production systems that will enhance the investment opportunities in retain ownership. Sutherland et al., (1994) compares the economics variables and production associated with 112 days of post weaning grazing and feeding calves to evaluate marketing options. Their study show that strategies involving retaining ownership through feeding yields a higher returns than selling at weaning and they conclude that retained ownership of cow-calf operation results in an opportunity to maximize returns when compare to selling at weaning.

While returns to retaining the ownership of cow-calf operations has received a considerable attention in the literature (Greiner, 2010; Lawrence, 2005; Gillespie et al., 2004 and Lacy et al., 2003) studies are yet to consider the operational flexibility and investment opportunities of establishing and operating different production systems and their corresponding opportunity costs in retaining ownership. So, our study fill the gaps in previous studies in several ways. We use grazing experiment data in our studies, our experiment span from 2015 t0 2018 with different grazing days in the cool and warm seasons. Most studies have relied on secondary data or survey data from livestock farmers or producers. Enterprise budget estimates are extracted from the daily weigh gain performance of the animals and cost of establishment of each systems. Moreover, our study painstakingly evaluates the value added of each systems for each seasons and combination of two systems for each seasons, this has not been done before. Since different systems have different production season, so combining the systems can provide understanding of how producer can utilize information to minimize production risk and maximize market value of the cow-calf operations. Lastly, many of the studies have been done in static environment, mostly net return models, (Tang et al., 2017; Lewis et al., 2016) but our study provides stochastic solutions. We use both dynamic programming, binomial lattice model and Monte Carlos simulations to evaluate three livestock production system. To achieve our goal, we introduce an option to defer or postpone irreversible investment in a cow-calf operation system in any particular season analogous to an European call option theory. This investment strategy involves decision making process of creating a production capacity of the systems that can be postponed or defer to another season if it is not economically advisable to do so. Therefore, if production systems that need to be postponed, due to low investment returns in that season, was established, it may erodes the management flexibility value of a deferred or postponed investment strategy.

2 A Review of Previous Research

Franken et.al.,(2010) study cow-calf producer interest in retained ownership the authors argue that cow-calf producers who invest in quality registered cattle are and those incorporating feedlot data into herd management decisions are more interested in retain ownership than those who are not. For a vertical integration to be effective, producers must receive incentives that will make their investment in risky operations most rewarding. The risk appetite of producers also affect their decisions to retain ownership of cow-calf operation. Schroeder and Featherstone(1990) examine the dynamic marketing and optimal retention decisions for cow-calf producers using the expected utility maximization in discrete stochastic programming, their study shows how calf retention decisions depend on current profit, expected future profits distributions, pricing alternatives available and the cow-calf producer's risk aversion. In fact Pope et. al.,(2011) showed that risk aversion is an important factor affecting calf retention; the most risk averse producers tend to have more than 60% probability of selling calves at weaning while the most risk lover producers have less than 20% probability aof selling at weaning. Bohnert et al. (2013) study the late gestation supplementation of beef cows relative to the effects on cow and calf Performance, the authors found that the effect of the supplemental feeding program on the net returns to retained ownership of cow-calf operations depends on the cow's body condition score (BCS) and dried distillers grains with solubles (DDGS) at the time of pregnancy but no effect on developmental programming effect on feedlot performance. Studies on parametric estimations of animal characteristic in evaluating retained ownership of cow-calf operations have also received attention in the literature. Results from many of the parametric studies use net returns models to estimate factors affecting the profitability of retained ownership. For instance, Lewis et al.,(2016) examine the effect of animal characteristics and a supplemental prepartum feed program on a net returns for cow operations. They collected data on 160 steers in Tennessee which were finished in a feedlot. Their results show that supplemental prepartum feeding program decreased net returns of finished steers by about the cost of the supplemental feed but the same program had no impact on the quality grade of cattle retained. Their results however, show that an increase in the feed-to-gain ratio, average daily gain, and dressing percentage are the main animal characteristics that trigger profitability in net returns of finished steers and the probability of a steer grading Choice or higher(see also Mark et al.,2000). But the study on parametric estimations has yield mixed results. While it assumed that the supplemental prepartum feeding program should directly affects the profitability of the cow-calf operations given the fact that treatment herd should have a higher average daily gain, Bohnert et al., 2013 and Stalker et al., 2006 studies provide mixed impact results on supplemental prepartum feeding program. The volatility of cattle price is also a common factor that determines the profitability of retained ownership of cow-calf operations or investment (see Akande 2013). The more volatile the price of cattle is, the more uncertain the expected returns on the operation becomes and the opportunity to invest in such operations then depends on the risk appetite of the producers. Brown et al.,(2016) examine the opportunities inherent in cow-calf operations, the authors used a representative risk-averse producer and evaluate a decision set with seven possible marketing strategies for the optimal decision in Bayesian framework that allows for risk in price and production. They find that in many instances, retaining ownership of cow-calf operation appear to be a superior decision when combine with specific hedging strategies that utilizes options and future contracts. In all, retained ownership can has been proved to increase operation profits(see Fausti et al., 2003 and Lawrence, 2005) and thus add value to investment. Duration of retaining post weaned ownership could also attracts higher returns on the operations. The valuation of short period ownership should be different from long period and their risk level should vary inversely with period of retaining the operations. Lawrence(2002) and Carlberg and Brown (2001) independently show that comparing duration of retaining ownership beyond post grazing weaning is most profitable in most circumstances.

3 Mathematical and Empirical Model

3.1 Real Option Theory

Option theories is best consider under two main approaches; the infinite time horizon approach and deterministic approach. This approach is necessary because it gives researchers or analyst the direction in evaluating either investment with a contingent claim or a dynamic programming solutions. Moreover, because model can become more complex, real option theories can become difficult to apply and thus, deriving a closed form solution to the set of differential equations describe by the model formation might be as well difficult. In this study, we use both infinite and the deterministic case to explain and analyze real option theories. We start from a simple deterministic approach, it is simple because we are excluding the stochastic representation of the model and then proceed to deterministic case with stochastic information and finally veered into the infinite time horizon case. These two approaches will be compared in providing solutions to the real option models.

3.2 Dynamic Programming

We use dynamic optimization to split the sequence of decisions, at each point of time, into the immediate and the future decisions. The dynamic programming equation is solved backward from the last decision to find the optimal sequence of decisions. An investor can make the best choice when there is no incentive to continue investment. Assume the state variable P_t at any period t remain the same as in deterministic case and ρ is the discount factor. This time, let assume the producer is able to make some choices that will affect the operation and the functionality of the asset and let this choice be the control variable, U_t. Assuming a constant discount rate ρ, the immediate profit flow is given as $\pi(P_t, U_t)$ and let $F_t(P_t)$ be the value of the asset and at time t, where the value of the asset must satisfy the Bellman equation;

$$F_t(P_t) = \max_{U_t} \left\{ \pi_t(P_t, U_t) + \frac{1}{1+\rho_t} \mathbb{E}_t[F_{t+1}(P_{t+1})] \right\} \tag{3.1}$$

We assume that $U_{t+1}, U_{t+2}, U_{t+3}......$, are the remaining choices that are optimal in their continuous value.The objective of the problem is to choose for the period t the optimal control variable U_t, which maximizes the sum of the two components in equation 3.1. If the state variable P_t follows continuous GBM defined by;

$$\frac{dP_t}{P_t} = \alpha dt + \sigma dz \tag{3.2}$$

Assuming also that the firm can either receive a cash flow $\pi(x,t)$ by continuing to wait or exercise the investment to obtain the payoff $\Omega(x,t)$ then the Bellman equation could be written as

$$F_t(P,t) = \max \left\{ \pi_t(X,U)dt + \frac{1}{1+\rho dt} \mathbb{E}_t[F(P+dt, t+dt)] \right\} \tag{3.3}$$

Since there is no profits at the initial stage, thus the investment project generates cash flows only at the time when the investment is undertaken, $\pi_t(P_t, U_t) = 0$ and profit is only realized in the second period with a probability. Therefore, the Bellman equation reduces to

$$\rho v(P,t)dt = \mathbb{E}[dv(P,t)] \tag{3.4}$$

Equation (3.4) states that the instantaneous return on the option to retain the ownership at anytime t is equal to its expected appreciation. Using the Ito's Lemma process and equate $\mathbb{E}[dv(P,t)] = 0$ we have,

$$\mathbb{E}[dv(P,t)] = \mathbb{E}\frac{dv(P,t)}{dp}(\alpha v(t)dt + \sigma P(t)dz(t)) + \frac{1}{2}\frac{d^2v(P,t)}{dp^2}(\alpha P(t)dt + \sigma P(t)dz(t))^2 + ... \tag{3.5}$$

but assuming that $\rho - \alpha > 0$

$$\Rightarrow \frac{dv(p,t)}{dp}(\rho - \delta)pdt + \frac{1}{2}\frac{d^2P(P,t)}{dp^2}\sigma^2 P^2 dt - \rho v(P,t)dt = 0 \tag{3.6}$$

The general solution to equation (3.13), Dixit and Pindyck (1994), can be given as

$$F(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2} \tag{3.7}$$

Condition to be satisfied assume $\beta_1 > 0, \beta_2 < 0$ although β_1 and β_2 are the 2 roots and both can be solved explicitly but for the fact $\beta_1 < 0$ as $P \to 0, A_2 = 0$, and $A_2 P^{\beta_2} \to \infty$ thus, equation (3.7) is reduced to

$$F(x) = A_1 P^{\beta_1} \tag{3.8}$$

But as

Assuming $F(P,t) = F(P)$. Equation (3.8) is a second order homogeneous nonlinear differential equation. To solve the equation we need boundary conditions. One boundary condition arises from the properties of stochastic processes, i.e. if $P = 0$, it will stay there due to the independent increments. Thus we have a boundary condition,

$$F(0) = 0 \tag{3.9}$$

.

There are also 2 optimal conditions for the solution:

$$F(P^*) = P^* - I \tag{3.10}$$

$$F'(P^*) = 1 \tag{3.11}$$

$$\frac{1}{2}\sigma^2\beta(\beta-1) + (\rho-\delta)\beta - \rho = 0 \tag{3.12}$$

Equation (3.10) states that the option to retain ownership of the operation becomes worthless as the returns on such operation tends to zero. We know that zero is an absorbing state for the Geometric Brownian Motion, hence, there will be no monetary and pecuniary incentive from either waiting to retain the ownership of the cow-calf operation or having an active one immediately. So, as an indication of stochastic process, if $P \to 0$ (3.10) the option to retain ownership will have no value.Ultimately, the optimal retaining ownership decisions depends on equation (3.10) and (3.11).

Equation (3.11) determines the valid payoff at the optimal stopping point and it is the value matching condition, which requires the value of the retention opportunity cost to equal the expected NPV at the optimal threshold price,P^*. The equation explains that the value lost from not exercising the option must equal the payoff from the investment at this trigger price. In other words, when investment is exercised, the producer receives $P^* - I$. There are useful investment information when it is written as $P^* - F(P^*) = I$, so when the producer retains ownership of cow-calf operation, it give up the opportunity cost or option to retain ownership at valued at $F(P)$ but get value P. Profit occurs when $P - F(P) > 0$. P^* is where the profit from the option to retain ownership equal the tangible cost of investment. If $P^* = I + F(P^*)$ which implies the value of the investment equal the total cost (direct cost plus opportunity cost) of making the entire cow calf operations. Equation (3.11)[1] determines the unique stopping point as other conditions contradict the definition of optimal point. This equation is commonly referred to as ”smooth pasting” condition.

Using (3.12) and constraints (3.9)-(3.11), the boundary solution become;

$$P^* = \frac{\beta_1 I}{\beta_1 - 1} \tag{3.13}$$

[1]is a first-order condition for optimization that reflects the fact that the marginal benefit of waiting must equal the marginal cost of waiting at P^*

$$\frac{dP^*}{dI} = \frac{\beta_1}{\beta_1 - 1} > 0$$

As long as $\beta > 1$ equation(3.13) implies that a higher operational cost and lower future price will increase the trigger to retain ownership by $\frac{\beta_1}{\beta_1 - 1}$ factor but we expect $\beta_1 > 1$. $\frac{\beta_1}{\beta_1 - 1}$ is called a wedge in Dixit and Pindyck (1994) and it is expected to be greater than 1 since $\beta_1 > 1$. The value of parameter β_1 is determined by the uncertainty of the cow-calf operations, a higher uncertainty of the cow-calf operation expected returns, the lower the parameter value of the β_1 from positive numbers $\mathbb{R}^+ \infty$ to 1 and the wedge increases.

$$A_1 = \frac{(\beta_1 - 1)^{\beta - 1}}{\beta_1^{\beta_1} I^{\beta_1 - 1}} \tag{3.14}$$

and

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{\alpha}{\sigma^2} - \frac{1}{2})^2 + 2\frac{\rho}{\sigma^2}} > 1 \tag{3.15}$$

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{(\frac{\alpha}{\sigma^2} - \frac{1}{2})^2 + 2\frac{\rho}{\sigma^2}} < 0 \tag{3.16}$$

and the formula of the Infinite Real Option Value Pricing Model is given as

$$ROV = (P^* - I)(\frac{P}{P^*})\beta_1 \tag{3.17}$$

$\alpha = r - \delta$

where, P^* is the minimum value of the underlying operation that triggers the decision to retain the ownership, I is the initial cost of making the investment, P is the current value of the underlying investment, r is the risk-free rate of interest, δ is the cash flow yield (discount rate) and σ is the standard deviation of the rate of return of the underlying operations.

3.3 Binomial Tree Option

When the financial market problems can not be solved with the Black-Scholes assumptions then researchers resorted to numerical methods to solve real option valuation problems. Cox, Ross and Rubinstein(1979) among other authors independently derived a two-state real option pricing model. This model later became a popular tool for numerical solution for real option values. In the real option pricing model, the continuous time stochastic process model is replaced with a discrete state with probability, p, for up, u, and down, d, movement.

The up and down movement can respectively be represented as follows;

$$u = \exp^{\sigma\sqrt{\Delta t}} \tag{3.18}$$

$$d = 1/u \tag{3.19}$$

$$p = \frac{\exp^{r\Delta t - d}}{u - d} \tag{3.20}$$

From this models structure, a tree that show a time path of market values of an asset and market prices and their probabilities can be constructed. To solve the binomial tree lattice model for the value of the real option at different states, we use the recursive backward induction in a risk-neutral world, where all investors are risk neutral and where the expected return equals the risk free rate of interest. We also assume that there is no arbitrage opportunities. The first step in any valuation exercise under uncertainty is to describe the nature of the randomness and which variables it affects. Hence, we construct the binomial lattice (tree) for the project's value which we take to be a random variable. However, we wish to choose the up and down step sizes in such a manner as to conserve the given volatility of the underlying assets' value, be they physical plant, intellectual property, the present value of net operating cash flows (or, if you prefer, free cash flows, EBITDA, etc.). By a theoretical argument for financial options, the up and down states need to be chosen in such a way that $d < r < u$ where $r = e^{\Delta t}$ is the interest rate factor otherwise, the risk-neutral probabilities needed to calculate the value of the real option will be ill defined. These two factors generate the project's random (present value) of cash flows so that the original cash flow volatility is preserved.

Given that the producer has discretion over operation and production timing, then the optimal time period to retain ownership becomes an important question to ask. Assuming no operating costs and that the investment cost is I, the expected NPV from immediate operation in any time period t is simply $\max\{V_t - I, 0\}$. This expected NPV will be compared with the discounted expected value of waiting to retain ownership in a subsequent period, where ρ is the periodic discount rate. If $d(V_t, t)$ is the value of the retained ownership opportunity in period t when the value of operation is V_t , then we have the recursive relationship equation given as;

$$d(V_t, t) = \left\{ \max\{V_t - I, 0\}, \frac{1}{1 + \rho\Delta t} \mathbb{E}_{V_t}[d(V_{t+\Delta t}, t + \Delta t)] \right\} \tag{3.21}$$

$$V_{i-1,j-1} = \mathbb{E}_{V_t}[d(V_{t+\Delta t}, t + \Delta t)] = \left(pV^{up}_{i,j-1} + (1 - p)V^{down}_{i-1,j} \right) \tag{3.22}$$

The boundary condition with respect to which the recurrence is solved is given as

$$d(V_{T\Delta t}, T\Delta t) = \max\{V_{T\Delta t} - I, 0\} \tag{3.23}$$

3.4 Grazing Trials and Data Description

Because there is no real data from which inferences on production systems identified in this study can be made therefore, an experiment was conducted on forage pasture management and cattle are placed on each lot for period of time. The grazing trials started in 2015 with treatments replicated in three blocks in a randomized complete block design for a total of 9 paddocks of approximately 0.85 ha each. Treatments consist of two livestock production systems as follows:

- **The extensive grazing system** unfertilized bahiagrass pastures during the warm-season, overseeded with similar rye/oat grass/clover mixture + 34 kg N ha^{-1} during the cool-season (BG).

- **The improved grazing system**

  1. N-fertilized bahiagrass pasture (113 kg N ha^{-1}) during the warm-season but overseeded with a mixture (45 kg $ha^{-1}$of each) of FL 401 cereal rye (Secale cereale, L.) and RAM oat (Avena sativa, L.) and fertilized with 113 kg N ha^{-1} during the cool-season (BGN-grass).

2. Rhizome peanut/bahiagrass pastures during the warm-season, overseeded with similar rye/oat mixture fertilized with 34 kg N ha^{-1} plus a mixture of clovers (17 kg ha^{-1} of Dixie crimson, 6.7 kg ha^{-1} of Southern Belle red, and 3.3 kgha^{-1} of Ball clover) during the cool-season (BG-RP);

Each of the systems has off and on season. In the off-season, the forage goes dormant and the yields are low and during the on-season, yields are high. For instance, in the improved system during cool season- between the month of January and May, 162 days- Bahiagrass goes dormant but are fertilized and overseeded with mixture of cereal rye, oat etc whereas, in the warm season- between the month of May and October, 168 days- mixed perennial peanut with bahiagrass pasture is available in the warm season but overseeded with rye/oat mixture, clovers/ dixie crimson mixture in the cool season. In the extensive system, in the cool season the, as previously mentioned, it goes dormant in the cool season and therefore overseeded with similar rye/oat grass/clover mixture in the improved system.

Table 1: Grazing Experiment Trials

Cool season 2016 and 2017

Treatment	Stocking rate (steer/ha)	ADG (kg/hd/d)	Gain per area (kg/ha/season)*
BG	3.3 a	0.86 a	352 a
BGN	3.3 a	0.80 a	322 a
BG-RP	3.3 a	0.77 a	324 a
SE	0.2	0.07	40

*2016 had 126 days and 2017 had 105 days

Summary grazing trial

Warm season 2015, 2016, 2017

Treatment	Stocking rate (steer/ha)	ADG (kg/hd/d)	Gain per area (kg/ha/season)*
BG	4.0 b	0.31 b	140 b
BGN	4.5 a	0.35 b	170 ab
BG-RP	3.5 c	0.56 a	246 a
SE	0.1	0.07	39

+75%

*2015 had 84 days, 2016 had 168 days, and 2017 had 147 days; numbers are averaged across three seasons

Cattle are continuously stocked and two tester steers remain grazing on each pasture throughout the season and the two tester steers are used to quantify the average daily gain. Water, shade, and a mineral supplement mixture were also provided for cattle in each pasture. Cattle are weighed every 21 days after fasting (withdrawal from feed and water) for a minimum of 16 hours. Using the daily weight gain, we derived the market value of the cattle for each system for each season by multiplying the market price of beef per pound. Different market value for each system were obtained from the trials, this become relevant because each grazing system is design with an efficient capacity to in providing adequate nutrition from the grazing but the nutritional value of each system differs. For instance, Poor cow nutrition lowers cattle's performance, reduces conception rates and return to estrus in cattle. The nutritional value directly affects calf-weaning weights. Daniel et al (2012) study the effect of nitrogen fertilization on native pasture overseeded with ryegrass was assessed regarding production, pasture management and performance of beef calves. Their results show that average daily weight gain of the cattle responded linearly to nitrogen rates in 2007, and quadratically in 2008.Consequently, livestock production and composition of forage are improved by the use of nitrogen. Moreover, Peter et al (2010 show that high legume forage can provide live weight gains in cattle that are 70% greater than those from perennial based pastures. In this study years are limited to 3 and this yield 92 observations for each system between 2015 through2017 and Interestingly, the grazing data has enough requisite parameters for all the stated models this study aims to examine.

4 Numerical Results

Table 2 reveals the parameter estimates of the investment option in two livestock production systems in two seasons, cool and warm season. The table show the heterogeneous systems and homogeneous system. In the heterogeneous system, the improved has two improved systems in the cool and warm seasons, the system with nitrogen fertilized grass grazing (1) and mixed legume-grass grazing system(2). The extensive has only one grazing system-non fertilized grass (3) in the cool and warm season. All systems parameters is for the entire season. The cash flow and investment cost was obtained from the 2018 cow-calf enterprise budget prepared from the experiment grazing study. The standard deviation of the market value of the cattle performance, σ, varies across each system, this is expected because each system have different treatments with different expected animal performance. All the systems have volatility either about 50% or higher than 50%. The higher volatility strongly supports the real option value methodology and it shows the level of uncertainty surrounding the ownership of cow-calf operation.. This is because, with higher uncertainty or volatility and higher flexibility, it is better to retain when returns are low and sell in the future with higher returns. The risk-free rate is put at 3% and the time(years) of expiration of waiting is 1 year for seasonal-individual system but 5 years for all season-systems and all parameters are in per acre except for average daily weight gain of the animals which is used in calculating the mean(μ) and volatility (σ) are measured in kilograms. The purpose of seasonal-individual system is to determine the amount of seasonal added-value of retaining the ownership of a cow-calf operation.

Table 2: Investment Option Parameters

System	Improved				Extensive		All Systems		
Production Type	1		2		3		1	2	3
Season	Cool	Warm	Cool	Warm	Cool	Warm			
Cash Flow	$ 235	$ 123	$ 237	$ 179	$257	$ 103	$ 358	$416	$ 360
Investment cost	$161	$ 106	$ 134	$133	$134	$77	$ 267	$267	$ 211
Volatility(σ)	0.51	0.47	0.66	0.59	0.61	0.5	0.54	0.6	0.63
Risk-free rate	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Mean (μ)	0.78	0.31	0.76	0.54	0.82	0.25	0.48	0.46	0.62
Time (years)	1	1	1	1	1	1	5	5	5

Table 3 show the results of the binomial call option values for cool and warm season. The performance of the cattle was determined by the market value of daily weight gain of the growing cattle. The values reflect the per acre value added estimates of retaining the ownership of cow-calf operations. In the cool season, extensive system yields $127 real option value per acre compare to $36 in the warm season. In the cool season, improved system-nitrogen fertilized grass forage pasture-results in real option value of $90.3 and $36.6 in the warm season. While, the improved, mixed legume -grass forage pasture, system yields $114 real option value in the cool season and $70 real option value per acre in the warm season.

Table 3: Real Option Value

Production Systems	Season	
	Cool	Warm
Non-Fertilized Grass	$127	$36
N-Fertilized Grass	$90.3	$36.6
Mixed Forage	$114	$70.2

4.1 Option Value of the Subgame Nash Equilibrium of the Systems

In this study, we apply the subgame Nash equilibrium strategy to find the combination of production systems in any season that will yield maximum value added to operations. To do this, we assume that (1) producer has two strategies, produce in the cool or warm season, (2) there are two combination of production systems, improved-grass versus extensive or improved mixed forage versus extensive and improved-grass versus improved mixed forage, (3) There is complete information of the game and thus knows the choices available in advance, (4) we avoid implementing a different production system for different season but any production system suitable to maximize the value added condition of the operation in one season may still be found valuable in another season and (5) we can not entirely rely on a single system but rather which combination of systems yield the most suitable and profitable option in any particular season. The subgame of the production system is represented in table 4- 6.

Table 4: Simultaneous Investment Subgame

		Extensive System	
		Cool Season	Warm Season
Improved-Mixed	Cool Season	$114, $127	$114, $36
	Warm Season	$70.2, $127	$70.2, $36

Table 5: Simultaneous Investment Subgame

		Extensive System	
		Cool Season	Warm Season
Improved-Grass	Cool Season	$90.3, $127	$90.3, $36
	Warm Season	$36.6, $127	36.6, $36

Table 6: Simultaneous Investment Subgame

		Improved- N Grass	
		Cool Season	Warm Season
Improved-Mixed	Cool Season	$114, $90.3	$114, $36.6
	Warm Season	$70.2, $90.3	70.2, $36.6

Table 4- 6 presents one year sub-game analysis of the binomial real option value of the production systems. In table 4, livestock producer will add $241 value per acre if he retained the ownership of his cow-calf operation by using a combination of improved mixed legume-grass and extensive grass production system in the cool season. The incentive to wait and combine the two systems for production purposes in the cool season outweighs any other combination of real option value in any other season. That combination is actually a Nash equilibrium and a sub-game perfect Nash equilibrium in the livestock production system. If we compare table 4, 5 and 6, there is no incentive to deviate to another production plan where an optimum value can be added to the current operation asset. The producer has a higher incentive of waiting to execute this line of production systems in the cool season than in the warm season or operating each system at different season. But the second best option is available and that is extensive vs improved Nitrogen fertilized grass production systems. The second best option, the extensive system and fertilized grass systems, for cool season will add $217.3 value per acre to the operations so, if the best option of the system is not available, a producer can opt for the second best option. Warm season production plan is still between table 4, table 5 and table 6. The producer will be better off if he implement the two improved systems in the warm season. The producer will add $106.8 per acre to his cow-calf operations if he repeats the cool season production

systems in the warm season. While the second best option production systems, extensive and improved mixed system, add \$106.2 per acre.

4.1.1 Five Years Real Option Value

In the last period $t = T$ and our financial instrument's payoffs for a call option for five years evaluation for all systems with its corresponding costs $C_{t,i} = \max\{V_{t,i} - K_{t,i}, 0\}$ is reveal in table 7. Where t is the time period and i is the systems under the current study, $t = 1, 2, ...5$ and $i = 1, 2, 3$. Among the three, mixed legume-grass system, performs better than the improved fertilized and extensive-non fertilized grass grazing- systems for the entire periods. The reasons is due to the fact that the extensive system is has a higher and a better animal performance. The five years results of each system option value is depicted. table 7.

Table 7: Production System

	Real Option Value		
Time	Extensive	Improved-BGN	Improved-Mixed
1	\$167	\$133.1	\$180.2
2	\$201.7	\$167.4	\$220.5
3	\$225.9	\$191.9	\$248.7
4	\$244	\$211.2	\$270.5
5	\$259	\$226.9	\$288

Considering year 1 to 5, the mixed legume-grass system outperform all other systems followed by extensive system. The improved fertilized grass system perform poorly because of its high cost of production and lower performance in animal when compared to other systems.

For instance, if the cattleman were to retain ownership for one year for each system, the per acre value of the improved mixed-legume grazing system will be 35% higher than nitrogen fertilized grass and 8% greater than extensive system, dominated by non-fertilized grass. But retaining ownership for five years would have added 27% in value to improved mixed-legume grazing system over the nitrogen fertilized grass system and 11.2% value over extensive system.

Managerial flexibility of retaining ownership is therefore reflected in the form of 'wait and see' through the resolution of expected future increase in cattle prices. This flexibility worth \$167 if ownership was retained for one year and \$259 per acre if retained for five years for an extensive systems. The improved nitrogen fertilized grass has a operation flexibility has an added value of \$133 if ownership was retained for one year and \$226.9 per acre if retained for five years. For mixed legume-grass operational flexibility, retaining ownership for one year will add \$180.2 per acre compare to \$288 if ownership were retained for five years. The results, therefore, indicate that post-weaned cow calves sale in order to understand how the operation environment evolves add more value to cow-calves operation. The producer comes in twice: first, he will postpone sale of cattle, learn and observe the position of the operations reflected in the net cash flow (investment) lattice and expect a higher future price. Second, the producer will decides in which states will determine the circumstances to go forward retaining the ownership of the cow-calf operations and the time he will abandon the operation by selling off the animal to the slaughter house. Incidentally, in this study we realize producer is at an advantage if he continue to retain the ownership of the weaned calf and sell in the later day. Doing so will add more value to the operations.

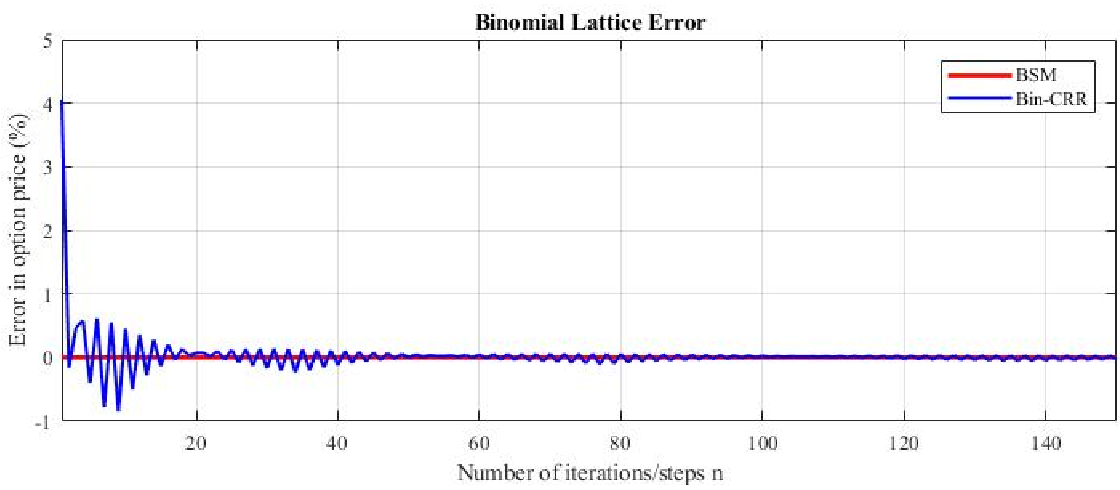
The next step is to calculate the value of the entire operations if decision to retain become plausible considering the no arbitrage opportunities assumption. We create two-step factors to complete the real option analysis of the extensive production system to determine the state, either up or down, the producer or investor is willing to be. The first determinant is the risk free rate, the rate must be chosen such that $d < r < u$,. Secondly, the risk-neutral probabilities needed to calculate the value of the real option are well defined and it is fully describe in binomial lattice in table A.1-A.3 and figure B.1-B.3 . These two factors generate the cow-calf operation's random (present value) of cash flows so that the original cash flow volatility is preserved. We replace the terminal values with the investment project's payoff in that state. Next, we calculate the value of the investment option for all states in the previous period and continue backward through the lattice until we get to the initial stage $(t = 0)$. Figure B.1-B.3 show the expected discounted investment option value for all available states taking into consideration that by waiting and then investing, a potential cattleman can lose revenue to the tune of 3% annually.

In appendix B, Figure B.1(A) gives the value in table A.1(A) in appendix A and it represents the lattice for per acre value of the extensive production system, it is used to derive the project value process while figure B.1(B) produces the value in table A.1(B)and it is the lattice used to obtain the investment option process. The initial underlying operation cash flow and the initial cost systems is respectively \$360 and \$211 for extensive, \$358 and \$267 for the improved fertilized system and \$416 and \$267 for improved mixed forage system. The values reveal different states that is profitable to retain the ownership and the states that will be unprofitable to do so. These investment and option values processes are discrete valuation of the cow-calf operation only.

4.1.2 Binomial Lattice Simulation

We divide the finite time horizon into 150 discrete time steps examine the convergence properties by running simulations with different grid sizes the simulation results are given in Figure 2.

Figure 1: Binomial Lattice Simulations



The value of the investment option (blue) using CRR model in comparison to the BSM solution error (red line) with different number of grid points. We notice two distinctive trends in the convergence; a sawtooth and oscillatory effect. That is, the convergence is oscillatory and non-monotonic. The sawtooth effect is commonly referred to as distribution error in the literature. This arises due to the fact that a discrete binomial probability is used to approximate the log-normal distribution of the stochastic diffusion process Figlewski and Gao (1999). The oscillatory effect, which is always referred to as the non-linearity error can be described as a periodic expansion of the investment option value that decreases as number of grid points increases. Therefore, the non-linearity error occurs due to large discontinuity in the terminal region between the current value and investment cost Figlewski and Gao (1999). We observe that the distribution error is not that bad because as the number of steps increases the error decreases.

4.2 Dynamic Programming Solutions

Cattleman or producer could immediately retain the ownership of the operations as long as $V > V^*$. V is the value of the retained ownership of the cow-calf operation while V^* is the optimal value at which the cow-calf operation can be retained. It is profitable to do so at this point since the choice will add more value to the operation. Otherwise, it may be better to avoid waiting to retain the ownership. So, in the stochastic infinite horizon case,we use dynamic programming to explain the boundary conditions in equation (3.9) and two optimal condition, which are matching conditions for the solutions in equation (3.10) and (3.11). Equation (3.9) is simple in that option to retain the ownership of the operation become worthless when the value is worthless or equal to zero.

Table 8: Retained-ownership Values in Fifth Year

	Extensive			Improved-Grass			Improved-Mixed		
State	Value	V_star	F(v)	Value	V_star	F(v)	Value	V_star	F(v)
1	2383	281.1	70.1	1809	352.1	85.1	2516.7	376.5	109.5
2	676	281.1	70.1	614.3	352.1	85.1	758	376.5	109.5
3	191.7	281.1	70.1	208.6	352.1	85.1	228.3	376.5	109.5
4	54.4	281.1	70.1	70.8	352.1	85.1	68.8	376.5	109.5
5	15.4	281.1	70.1	24.1	352.1	85.1	20.7	376.5	109.5

Table 8 shows the retain value of different states of the operation in the fifth year for all production system. For instance in the first state of the extensive production, non fertilized grass, system, the retain value of the operation is times eight greater than the optimal or the critical value of the cow-calf operation. The retain value of the improved systems are respectively, five times and 7 times greater than their optimal value of the retained operation. So, cattlemen should continue to retained the ownership for two years for all the systems since the value of retained ownership is still greater than its critical value. But this option become worthless and unprofitable in year three upward because the value to retain is lower than the optimal value. The solutions obtained from equation (3.19) and (3.20) reveal that the investment opportunity cost of retaining the ownership operations is the option price $(F(V))$ forgone in each system. For instance, the opportunity cost in the extensive system is \$70.1, \$85.1 for improved-grass and \$70.1 for improved-mixed systems. The higher the optimal value, V^*, the higher the opportunity cost. Since the, V^*, the optimal value of retained ownership is greater than the establishment cost for all the three systems, then Net Present Value (NPV) will not be accurate in evaluating the retained ownership operations. This confirms Dixit and Pindyck (1994) stochastic infinite horizon case of firm's decision under uncertainty. More information is available in appendix C.1-C.3.

4.3 Analytical and Numerical Solution Comparison

Figure 3-5 compare the solution with closed form , the binomial lattice solution, Monte Carlo solutions. Our goal is to show whether the binomial lattice solutions (discrete) converges quickly to the dynamic closed form(infinite time horizon) solutions and whether the Monte Carlos solutions converges at all to any of solutions. The closed form solutions of the systems are respectively, $281.1, $352.1 and $376.5 for extensive, improved nitrogen fertilized grass and improved mixed legume-grass.

Figure 2: Extensive System

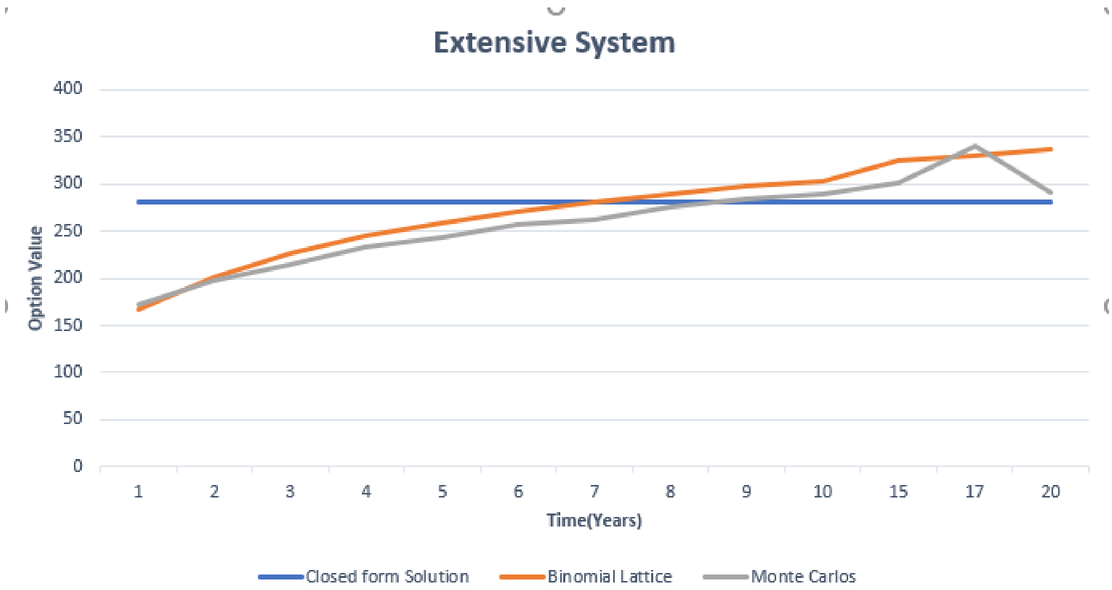


Figure 3: Improved Nitrogen Fertilized Grass System

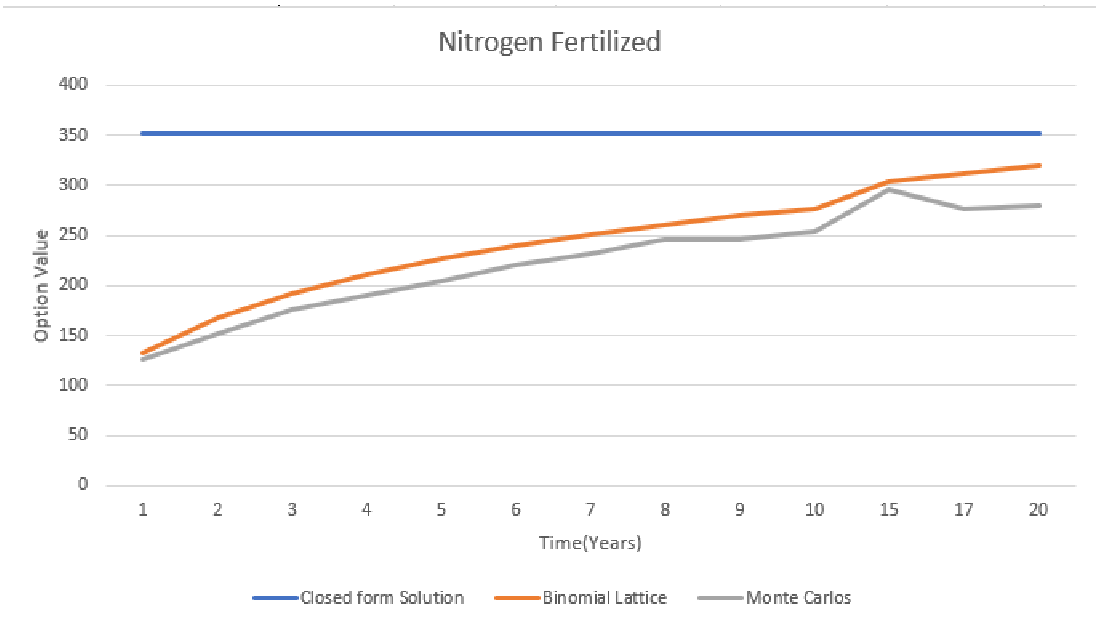
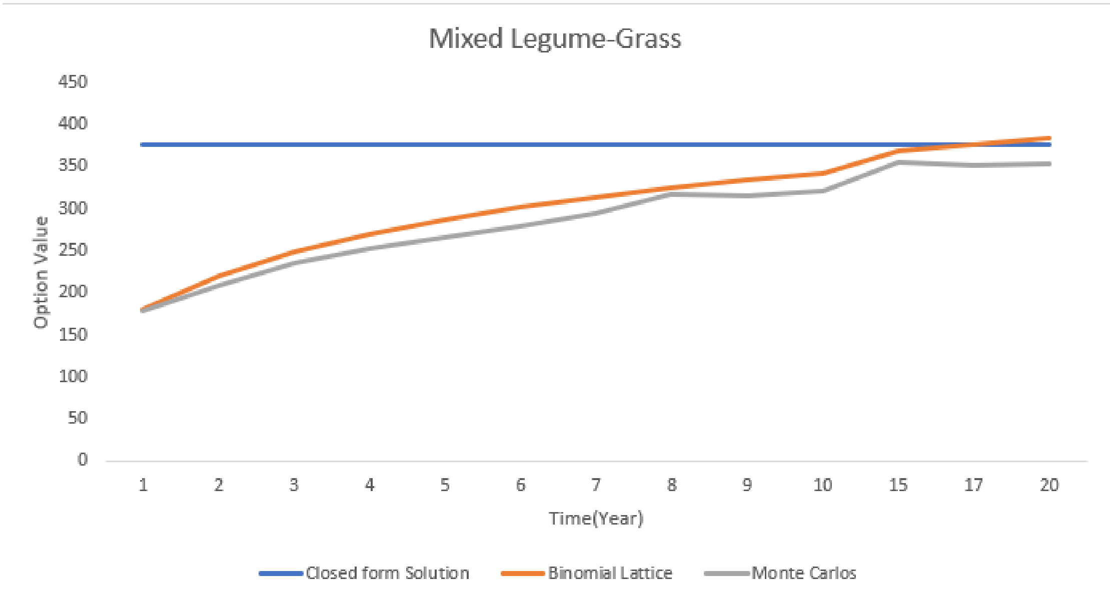


Figure 4: Improved Mixed Legume-Grass System



The extensive system result show that the binomial lattice converges quickly, in 7th year, than Monte Carlo solutions but the nitrogen fertilized results show otherwise. The Nitrogen fertilized system results indicate that binomial lattice converges slowly compares to Monte Carlo solution. From, the simulation results, the convergence of binomial to the closed form solutions take about 33 years while Monte Carlo results never converges but there is a thin line downward sloping error margin between binomial lattice and Monte Carlo solutions. The mixed legume-grass system results show that binomial solution converges at year 17 to its closed form solutions. The three figures show that the binomial solution and Monte Carlo solutions are approximately the same in the first year but moves parallel afterwards. Therefore, our results confirm the relevance of the numerical solution approximation and the analytical solutions of the methods employed in this study.

5 Discussion and Conclusion

Every competitive cow-calf producer is guided by the motive of maximizing profit from retaining the ownership of their operation and due to inherent risk and uncertainty involve managerial flexibility is required.This study focus on the analysis of the real option theory to retain cow-calf operations under three production system; an evidence from grazing experiment in Southeastern U.S. We introduce two improved livestock production systems and one extensive system. The two improved systems are the system with nitrogen fertilized grass based pastures and the mixed forage grass- legume. We compare the productivity of the three systems in both warm and cool season to determine the season a particular system combination yields the highest value-added to cow-caff operation. From the real option results, the livestock production with extensive-non fertilized grass-system yields more value added to cow-calf operations in cool than the improved system. The binomial real option value result of the system yields $127 per acre if a producer decides to retain the ownership of the cow-calf operation and sell off after post weaned its animals. The improved Nitrogen fertilized grass system and mixed forage-yield $90.3 and $114 per acre respectively in the cool season. This implies that the extensive system is more productive in the cool season provided producer is contemplating a single system approach. In the warm season, Mixed forage is the best production system producer can operate. The real option shows an added value of $70 per acre in the event of retaining ownership option was chosen. This value is more than the extensive system. However, the extensive and the nitrogen fertilized grass added about $36 value to cow-calf ownership after post weaned in the warm season. To minimize cost of switching production systems from one season to another, we consider combination of systems that may create a better option for producer to retain the ownership of their operations. We resorted to the simultaneous sub-game of system combinations of the operation. In the cool season, the combination of extensive and improved- mixed forage production system resulted in a Nash equilibrium in cool season. That implies that the best strategy is to combine the two systems in the cool . But the two improved systems will yield more added value to the retained operations. There is no incentive to deviate from these systems if optimum profit is the main goal of the retain ownership.

The managerial flexibility of retaining ownership of cow-calf operations for the three systems is reflected in the form of "wait and see" and the results show that waiting for five years would have added more value than if ownership was retain for a shorter period. Consequently, it pays off to wait to study the market and production environment before deciding on the timing to sell. The waiting period would have created require information to avoid future market and production risks. The optimum value of the operation per acre for extensive production is $281, $352 per acre for improved-grass production system and $376.5 for improved-mixed production system. So, in the long run, if option to retain ownership of cow-calf operation is adopted, improved-mixed forage production system yield the highest optimum returns than extensive and improved-grass systems.Considering two years of retained ownership, while all the three systems appear to be more profitable, mixed-forage still have higher productivity than other systems. The extensive system yield the lowest opportunity cost while the improved mixed yield the highest. The comparisons of the analytical and numerical solution of the system indicate that convergence of each of the approach. The results also show that NPV estimations are not accurate in estimating the value of the retained ownership. But in all, retaining ownership of cow-calf operation to the finishing stage creates an opportunity for high returns for all systems and add more value to operations than NPV. This conclusion confirms Sutherland et al., (1994); Carlberg and Brown, (2001); Lawrence,(2002) studies.

5.1 Limitations and Future Research

Lewis et al.,(2016) study was limited by a year data but we use three years grazing trials in this study to analyze our data. The paucity in experiment data could impact on the results of the study. Moreover, the value of our uncertainty, σ, is low and the low value might be due to lower experiment data. Also, the variance estimates is obtained from the performance of the steer in the grazing paddock and this might value may represents how volatile prices of cattle are.
Future research may consider a longer experimental data that will capture a good estimates of the variance. Future research could also incorporate a shock into the model to understand the health effect on the performance of the animal for each system.

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A Appendices

A.1 Binomial Lattice-Extensive System

A						
Lattice of the Investment Value						
	0	1	2	3	4	5
0	360	675.9	1269.2	2383	4474.3	8401
1		191.7	360	675.9	1269.2	2383
2			102.1	191.7	360	676
3				54.4	102.1	191.7
4					29	54.4
5						15.4

B						
Lattice for the Option Value						
	0	1	2	3	4	5
0	240	517	1080.7	2184.3	4269.5	8190
1		88.9	210.7	484.4	1064.4	2172
2			21.6	60	167	464.9
3				0	0	0
4					0	0
5						0

A.2 Binomial Lattice-Improved Mixed System

A						
Lattice of the Investment Value						
	0	1	2	3	4	5
0	416	758	1381	2516.7	4585.6	8355.6
1		228.3	416	758	1381.2	2516.7
2			125.3	228.3	416	758
3				68.8	125.3	228.3
4					37.7	68.8
5						20.7

B						
Lattice for the Option Value						
	0	1	2	3	4	5
0	265.6	560	1145.6	2265.2	4326.5	8088.6
1		99.5	231	520.6	1122.1	2249.7
2			24.3	66.1	180.2	491
3				0	0	0
4					0	0
5						0

A.3 Binomial Lattice-Improved Fertilized System

A

Lattice of Investment Value

	0	1	2	3	4	5
0	358	614.3	1054.2	1809	3104.3	5327
1		208.6	358	614.3	1054.2	1809
2			121.6	208.6	358	614.3
3				70.8	121.6	208.6
4					41.3	70.8
5						24.1

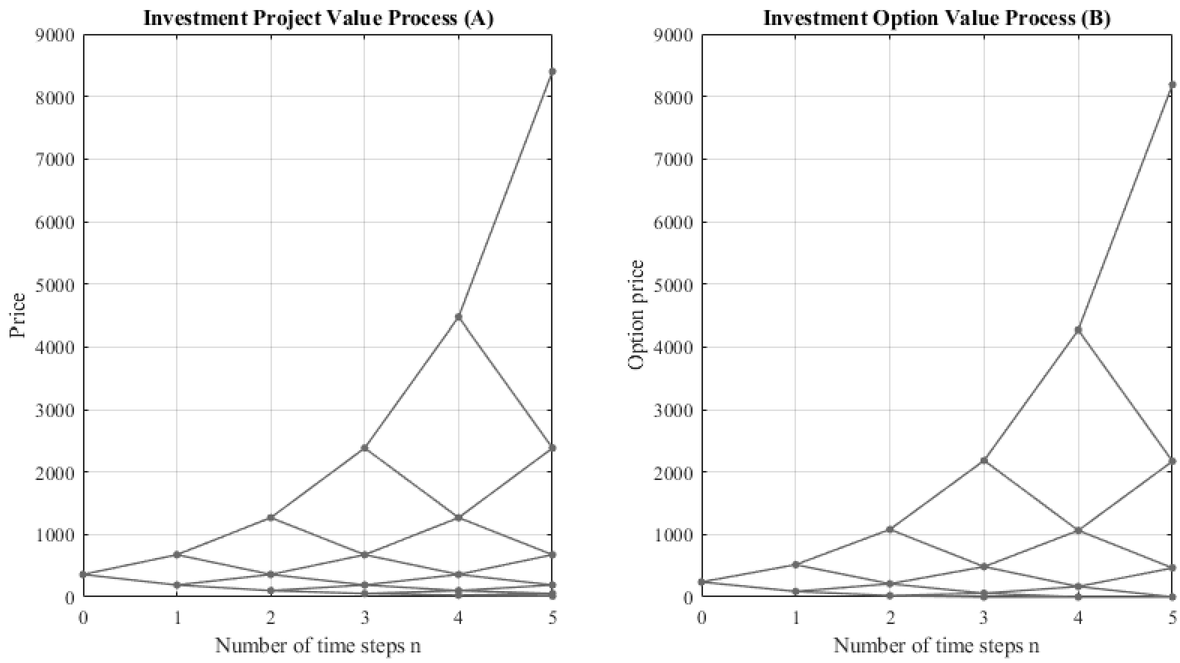
B

Lattice for the Option Value

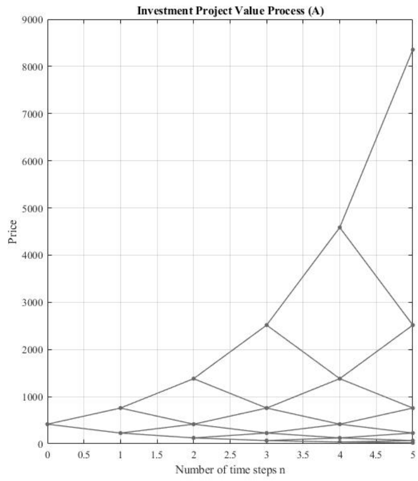
	0	1	2	3	4	5
0	207.2	418.9	822	1557.6	2845.2	5060
1		79.3	176.8	383	795.1	1542
2			19.6	51.1	133.2	347.3
3				0	0	0
4					0	0
5						0

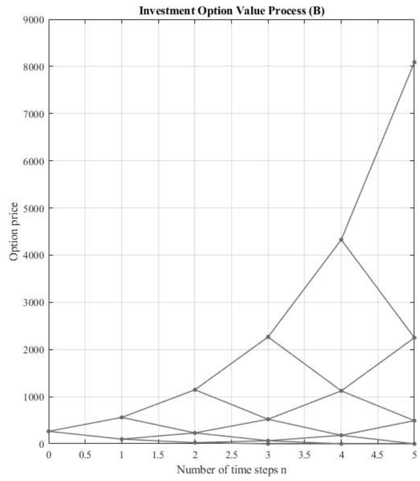
B Appendices

B.1 Binomial Lattice

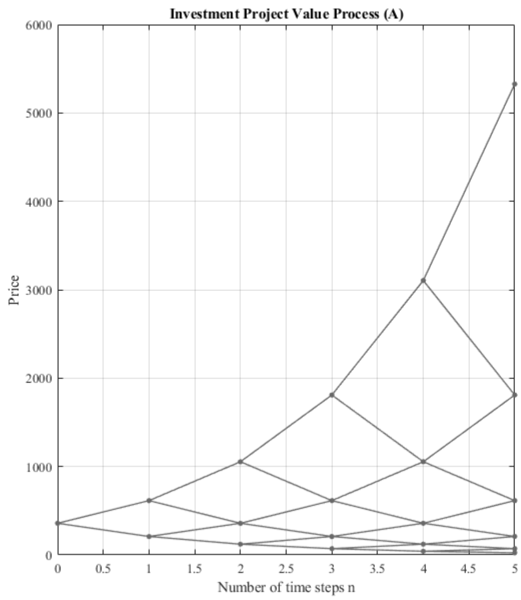


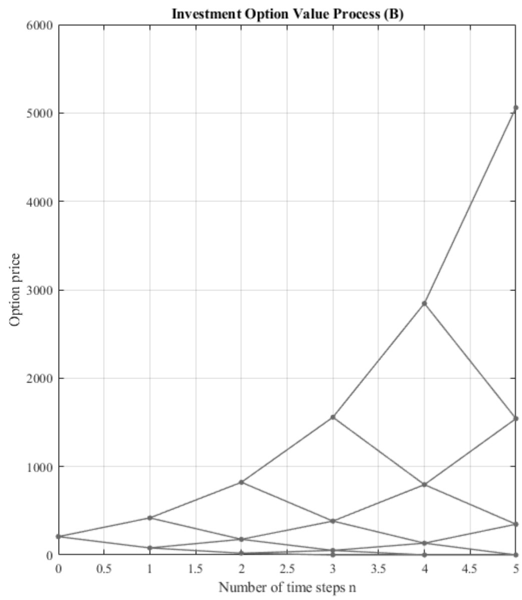
B.2 Binomial Lattice





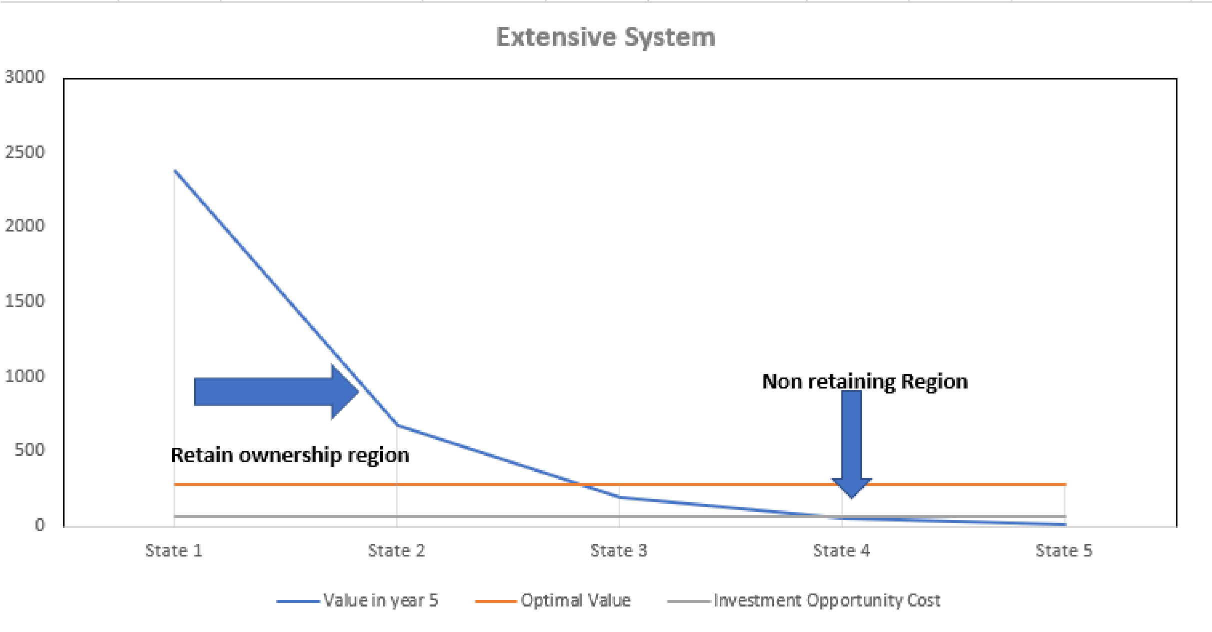
B.3 Binomial Lattice



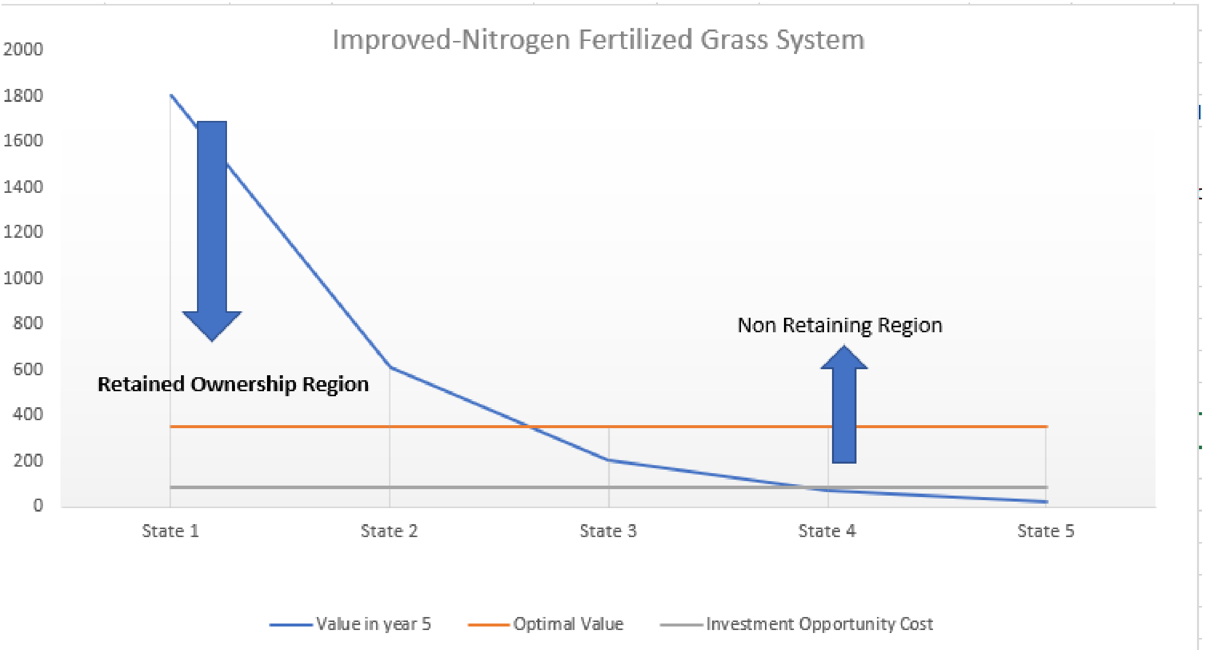


C Appendices

C.1 Extensive System



C.2 Improved Nitrogen Fertilized Grass System



C.3 Improved Mixed Legume-Grass System