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## **DOES FOOD AID REALLY DISCOURAGE FOOD PRODUCTION?**

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### **Does Food Aid Really Discourage Food Production?**

### Abstract

We examine whether food aid necessarily acts as a disincentive to food production in recipient economies. Since structural deficiencies of markets are a central reason why low-income agrarian economies receive food aid, we adopt a modeling framework that accommodates incomplete or imperfect markets. This simple, nonseparable, representative household model highlights the factor market effects of food aid overlooked in conventional, Schultzian analyses and, thus, the ambiguous effects of food aid on food production incentives in recipient economies.

### **Does Food Aid Really Discourage Food Production?**

Food aid has long been criticized as a potential disincentive to recipient country agricultural production. Schultz (1960) argued that food aid can drive down local food prices by increasing the domestic supply of food, thereby reducing incentives to recipient country food producers and potentially retarding economic development. The existence of partial equilibrium output price disincentives of this sort seems widely accepted in the literature (Maxwell and Singer 1979), although some analysts have wondered whether the potential income effects on food demand might mitigate or even offset these Schultzian disincentive effects (Fisher 1963).

However, the analytical food aid literature relies on implausible Arrow-Debreu models, even though the structural deficiencies of recipient country markets are a central reason why they receive food aid. Moreover, given that the literature on agricultural production in poor countries generally pays great attention to factor markets — not just to product markets — we find it puzzling that the literature on food aid generally ignores factor market (dis)incentives created by food aid shipments. This paper therefore revisits the issue of the incentives to food production in food aid recipient economies by employing a model that accommodates incomplete or imperfect markets, thereby enabling identification of potential indirect effects through factor markets. The net effect is that food aid generates ambiguous incentives for food producers in recipient economies.

#### I. A Nonseparable Representative Household Model of the Recipient Economy

This paper uses a nonseparable household model to analyze the impacts of exogenous food aid shocks on recipient economy food production volumes. Before presenting the model, let us briefly explain our model selection strategy. Food aid is a macroeconomic phenomenon, while output response is an inherently microeconomic one. General equilibrium modeling is commonly used to integrate macroeconomic and microeconomic concerns because it endogenizes nontradables' prices. Nonetheless, in this paper we adopt a partial equilibrium approach combined with sensitivity analysis on the principle of parsimony: it is the simplest model which generates results invariant to the introduction of greater complexity into the model. If the objective was to derive specific estimates of food aid's net effects in a particular setting, one could empirically implement our nonseparable household model by nesting it in computable general equilibrium (CGE) simulations, following DeJanvry et al. (1992). Our objective in this paper is, however, more modest: to establish the analytical ambiguity of food aid's incentive effects on recipient country food production and the need for empirical analysis, for which CGE modeling may offer one useful approach.

Almost a quarter of the world's population belongs to peasant households in low- and middle-income countries. Recent advances in the theory of household decision-making emphasize complex relationships between consumption, labor allocation, and production decisions in peasant households that consume a significant proportion of their own output (Singh et al. 1986; DeJanvry et al. 1991). These households are commonly found in villages where poor transportation and communications infrastructure — and hence high transactions costs — and low disposable incomes constrain market participation. Selective labor and financial market failures are consequently common. When (perhaps household-specific) market failures occur, household utility maximization no longer reduces production decisions to familiar profit-maximization choice rules. Rather, consumption, labor allocation, and production decisions become inextricable; these models are therefore often called "nonseparable" household models. Nonseparable models accommodate

selective market failures, e.g., for labor and/or finance, that condition producers' response to external shocks, such as the delivery of food aid. Given the weakness of markets in food aid recipient economies, a nonseparable representative household model could be useful in assessing analytically the effects of food aid on producer incentives.

Following DeJanvry et al. (1991), we therefore consider a representative household that owns a plot of land and produces agricultural commodities. Those factors and products that are tradable in the market are said to belong to the set T, those that are nontradable due to excessive transactions costs, risk, or other reasons belong to the set N. Production of these crops employs labor ( $Q_1$ ) and a purchased input ( $Q_x$ ) on a fixed amount of land (D) to produce cash crops ( $Q_c$ ) and food ( $Q_f$ ). The household maximizes utility defined over consumption of food ( $C_f$ ), a manufactured product traded in the market ( $C_m$ ), and leisure ( $C_1$ ). Assume the utility function is monotonic, twice differentiable, and concave in each of its arguments. The household faces a technology constraint, a budget constraint, and a time constraint. Its problem is thus

$$Max \quad U(\mathbf{C}) \tag{1}$$

s.t 
$$Z(\mathbf{Q}|\mathbf{D}) = 0$$
 (2)

$$\mathbf{P_c'C_t} \leq \mathbf{P_q'Q_t} + \mathbf{M} \qquad \forall \ t \in \mathbf{T}$$
(3)

$$\mathbf{C}_{n} + \mathbf{Q}_{n} \leq \mathbf{W}_{n} \qquad \forall n \in \mathbf{N}$$
(4)

where boldface type denotes a vector,  $\mathbf{P}_c$  is the subvector of prices associated with tradable consumption goods,  $\mathbf{C}_t$  is the subvector of consumption volumes for tradable goods,  $\mathbf{P}_q$  is the subvector of prices associated with tradable production netputs,  $\mathbf{Q}_t$  is the subvector of tradable production netput volumes, M represents exogenous income transfers, and  $\mathbf{W}_n$  is the vector of nontradable endowments (in particular, time). Food aid may comprise part of transfers when food is tradable, i.e.,  $\mathbf{M} = \mathbf{AP}_f + \mathbf{B}$ when  $f \in T$ , where A represents food aid volumes and B represents nonfood monetary transfers. If food is a nontradable, then M=B and food aid comprises the food endowment of the representative household, i.e.,  $W_f = A$ . Assuming U' > 0, constraints (3) and (4) will bind. The Lagrangian to this problem may be written as

$$\mathcal{G} = \mathbf{U}(\mathbf{C}) + \psi \mathbf{Z}(\mathbf{Q}) + (\mathbf{P}_{q}\mathbf{Q}_{t} - \mathbf{P}_{c}\mathbf{C}_{t} + \mathbf{M}) + (\mathbf{W} - \mathbf{Q} - \mathbf{C})$$
(5)

where  $\psi$  represents the marginal utility of technology improvement, is the marginal utility of income, can be regarded as the vector of marginal utilities of an extra unit of each nontradable good. and This formulation obviously accommodates transactions costs that drive a wedge between the purchase and sale prices of tradables, food that is either tradable or nontradable, multioutput technologies, etc., and implicitly reflects the absence of markets in land and finance; it is very generally applicable to low-income agrarian economies.

We treat labor as nontradable because in the recipient economies of interest, the vast majority of labor is engaged on the worker's own farm at a shadow wage that differs from any market wage (DeJanvry et al. 1991; Fafchamps 1993; Jacoby 1993; Skoufias 1994). Given that food can be reasonably classified as either tradable or nontradable in many low-income agrarian economies (Barrett and Carter 1997), we consider both cases: nontradable and tradable food. Since commercial inputs to food production and cash crops are almost always tradable, we treat them as belonging to the set T. Representing the shadow price of nontradables as  $P_n^* = n/n$ , and the price of tradable goods as  $P_i^* = P_i$ , we then have the standard first-order conditions for constrained utility maximization.

$$U_i = P_i^* \qquad \forall C_i \qquad (6)$$

$$\psi Z_{j} = -P_{j}^{*} \qquad \forall Q_{j} \qquad (7)$$

$$Z (\mathbf{Q}|\mathbf{D}) = 0 \qquad (8)$$

$$\mathbf{P_c} \cdot \mathbf{C_t} = \mathbf{P_q} \cdot \mathbf{Q_t} + \mathbf{M} \qquad \forall \mathbf{t} \in \mathbf{T}$$

$$\mathbf{C_n} + \mathbf{Q_n} = \mathbf{W_n} \qquad \forall \mathbf{n} \in \mathbf{N}$$
(10)

$$\mathbf{Q}_{n} = \mathbf{W}_{n} \qquad \forall \ n \in \mathbf{N}$$
(10)

Algebraic manipulation of these conditions yields a generalized profit function,  $*(\mathbf{P}_q^*) = \mathbf{P}_q^*, \mathbf{Q}$ , a system of factor demand and output supply functions,  $\mathbf{Q} = \mathbf{Q}(\mathbf{P}_q^*)$ , an expression for household full income,  $\mathbf{Y}^* = * + \mathbf{P}_1^* \mathbf{W}_1 + \mathbf{P}_f^* \mathbf{A} + \mathbf{B}$ , and a system of demand equations,  $\mathbf{C} = \mathbf{C}(\mathbf{P}_c^*, \mathbf{Y}^*)$ . One can also derive an equation for the endogenous shadow value of nontradables,  $\mathbf{P}_n^* = \mathbf{P}_n^* (\mathbf{P}_n^*, \mathbf{A}, \mathbf{B})$ , where  $\mathbf{P}_n^*$  is the shadow price vector,  $\mathbf{P}^*$ , excluding  $\mathbf{P}_n^*$ . The incentive effects of food aid come through its influence on  $\mathbf{Y}^*$  (via M in the case of tradable food, via  $\mathbf{W}_f$  in the case of nontradable food) and on the price vector,  $\mathbf{P}^*$ .

#### II. The Opposing Effects of Food Aid on Peasant Producer Households

If food is nontradable, then food aid clearly has Schultzian (negative) effects on food prices, because it relaxes the availability constraint (4), thereby lowering <sub>r</sub> and thus P<sub>r</sub>\*. If food is tradable, then the recipient economy is a price taker on the international market. Nonetheless, food aid may impact recipient economy food prices through real exchange rate appreciation induced by the balance of payments effects caused by the substitution of food aid for commercial food imports. Despite the rhetoric of the international food aid convention, which requires maintenance of "usual marketing requirements" (UMRs) so that food aid is purely "additional" to commercial trade flows, the literature on food aid clearly shows that the additionality principle is commonly violated (Maxwell and Singer 1979; von Braun and Huddleston 1988). Food aid typically reduces recipients' commercial food imports, thereby relaxing balance of payments constraints. If that relaxation is considerable, it could induce real exchange rate appreciation, thereby reducing the price of tradables. If food aid has no effect on the real exchange rate and food is tradable, there will be no product price effect. Food aid should thus have nonpositive price effects on recipient economy food prices, irrespective of whether

food is tradable or nontradable.

Since the violation of additionality may induce exchange rate appreciation, leading to nonpositive effects on all tradables' prices, there may be factor market effects as well as output market effects. Since most low-income agrarian nations import a substantial portion of commercial agricultural inputs (e.g., inorganic fertilizer, machinery), relaxing the balance of payments constraint—the macroeconomic analog to the representative household's budget constraint—may stimulate food production in recipient economies. Just as incremental food aid shocks are expected to lead to nonpositive changes in the domestic market price of food,  $P_f^*$ , so would they lead to nonpositive changes in the price of tradable inputs,  $P_x^*$ . This is the root of the inherently ambiguous producer incentive effects of food aid. Factor and product market incentives move in opposing directions, so the impact of food aid on producer incentives and output volumes turns on the relative magnitudes of these effects; it is an inherently empirical question.

We can formalize this argument using the simple model of section I. The output response of food to an increase in food aid is

$$\frac{dQ_f}{dA} = \frac{\partial Q_f}{\partial P_f^*} \cdot \frac{dP_f^*}{dA} + \frac{\partial Q_f}{\partial P_l^*} \cdot \frac{dP_l^*}{dA} + \frac{\partial Q_f}{\partial P_x^*} \cdot \frac{dP_x^*}{dA}$$
(11)

The first term on the right-hand side of (11) is the Schultzian partial equilibrium supply response. This is nonpositive because  $\partial Q_f / \partial P_f^* > 0$ , from the output supply function, and  $dP_f^* / dA \le 0$  by the arguments of the preceding paragraph if food is tradable, while  $dP_f^* / dA = d(f_f) / dA < 0$  if food is nontradable. Two additional factor market effects must be considered. The third term will be nonnegative. It is positive if food aid relaxes the balance of payments constraint, prompting exchange rate appreciation,  $dP_x^* / dA < 0$ , and permitting additional intermediate imports at reduced domestic price. It is zero if there are no exchange rate effects (since then  $dP_x*/dA=0$ ). The output response to food aid is thus analytically ambiguous because of the opposing partial equilibrium effects in product and factor markets. This ambiguity is reinforced by the labor allocation incentive effects of food aid shown in the second term of (11).

Although  $\partial Q_f / \partial P_1^*$  is unambiguously negative by the convexity of the profit function, food aid has ambiguous effects on the shadow wage, as is evident by totally differentiating (10) and the expression for household full income, Y\*, then rearranging terms (see the appendix for a derivation). The response of the shadow wage to food aid depends on whether there is an induced fall in P<sub>f</sub>\*, on any increase in leisure demand stimulated by the transfer — whether in liquid form for tradable food, as captured in M, or in illiquid form for nontradable food, as captured in W<sub>f</sub> — and on the profit effect of induced reduction in P<sub>x</sub>\*. This is shown in expression (12).

$$\frac{dP_{l}^{*}}{dA} = -\frac{+P_{f}^{*}\frac{\partial C_{l}}{\partial Y^{*}}}{\frac{\partial Q_{l}}{\partial P_{l}^{*}} + \frac{\partial C_{l}}{\partial P_{l}^{*}} + \frac{\partial C_{l}}{\partial Y}\left(\frac{\partial}{\partial P_{l}^{*}} + W_{l}\right)}$$
(12)

where 
$$= \frac{\partial P_{f}^{*}}{\partial A} \left[ \frac{\partial C_{l}}{\partial P_{f}^{*}} + \frac{\partial Q_{l}}{\partial P_{f}^{*}} + \frac{\partial C_{l}}{\partial Y^{*}} \left( A + \frac{\partial}{\partial P_{f}^{*}} \right) \right] + \frac{\partial P_{x}^{*}}{\partial A} \left[ \frac{\partial Q_{l}}{\partial P_{x}^{*}} + \frac{\partial C_{l}}{\partial Y^{*}} \cdot \frac{\partial}{\partial P_{x}^{*}} \right]$$
(13)

The denominator of (12) cannot be unambiguously signed since the bracketed term contains a positive term,  $W_1$ , and a negative one,  $\partial /\partial P_1^*$ . As reflected in (13), the first term in the numerator, , captures the responses of household labor demand and supply to changes (if any) in the output price,  $P_f^*$ , and the nonlabor input price,  $P_x^*$ , induced by increased food aid flows. If food is tradable and food aid has no effect on the real exchange rate, then =0. That is not sufficient, however, for food aid to have no effect on shadow wages. The second term in the numerator of (12) represents the

positive profit effect of an additional unit of food aid on leisure consumption. Thus, even when food aid has no impact on tradables prices, if labor markets are incomplete — as seems typical of low-income agrarian economies characterized by significant transactions costs — then food aid will have shadow wage effects that are nonzero but of ambiguous sign, reflecting the broader juxtaposition of product market disincentives and factor market incentives to food production. Despite widespread acceptance in the food aid literature, the simple but quite general model presented here demonstrates that negative Schultzian effects do not necessarily emerge in low-income agrarian economies characterized by incomplete markets — as is typical of food aid recipient nations. Rather, the incentive effects of food aid are analytically ambiguous, with countervailing factor and product market effects within the tradables sector and ambiguous effects on labor use patterns.

Moreover, a careful study of equations (11) and (12) reveals that it is not possible to predict easily whether factor market or product market effects will dominate. The balance of payments effects that stimulate output by reducing purchased input prices simultaneously exert upward pressure on the shadow wage, thereby discouraging agricultural employment at the margin. Conversely, the product market disincentive effects to food production exert downward pressure on the shadow wage, inducing countervailing employment effects at the margin. Schultz's prediction becomes analytically ambiguous once one considers the richness of incomplete markets and nonseparable household decision-making in poor agrarian economies. The economic effects of food aid are fundamentally an empirical question, one warranting further research.

#### III. Conclusion

This brief analytical note revisits the long-standing debate concerning the effects of food aid on food producer incentives in recipient economies. We explore the implications of nonseparable household decisions caused by widespread nonparticipation in labor, land, financial and/or food markets, as is typical of the low-income food aid recipient economies in which this issue is of greatest concern. The classic Schultzian findings do not hold under more general conditions. Rather, the selective market failures that permeate low-income, high-transactions-cost economies significantly complicate analysis of the effects of policy interventions, rendering analytically ambiguous the sign of the key relationships. A natural consequence is the cross-sectional and intertemporal heterogeneity of output responses to similar food aid disbursement histories, as evident in the literature.

Our model provides analytical support for common anecdotal accounts (Maxwell and Singer 1979). For example, because food aid is not wholly additional (i.e., it is to some degree a substitute for commercial food imports), relaxing recipients' balance of payments constraints may stimulate factor employment even as food prices fall in product markets. The classic theoretical work on food aid (Schultz 1960; Fisher 1963) and subsequent modeling efforts ignore these effects. Much as it is preferable to investigate the incentive effects of trade and exchange rate policies by studying effective rates of protection (which consider both factor and product market effects) rather than nominal protection coefficients (which are based only on product prices), so too is it important to emphasize the multiple, countervailing impacts of food aid across the full range of markets in which low-income food producers operate. Ascertaining the economic effects of food aid thus fundamentally requires empirical study.

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### Appendix

Begin with equation (10) and the expression for household full income derived from the first-order conditions (6)-(10).

(i) 
$$C_1(P^*, Y^*) = W_1 - Q_1(P^*)$$

(ii) 
$$Y^* = B + P_f^* A + * P_l^* W_l$$

Next, totally differentiate both expressions.

(iii) 
$$\partial C_{f} / \partial P_{1}^{*} \cdot dP_{1}^{*} + \partial C_{f} / \partial P_{f}^{*} \cdot dP_{f}^{*} + \partial C_{f} / \partial Y^{*} \cdot dY^{*} = - \partial Q_{f} / \partial P_{1}^{*} \cdot dP_{1}^{*} - \partial Q_{f} / \partial P_{f}^{*} \cdot dP_{f}^{*} - \partial Q_{f} / \partial P_{x}^{*} \cdot dP_{x}^{*}$$
  
(iv)  $dY^{*} = P_{f}^{*} \cdot dA + A \cdot dP_{f}^{*} + \partial / \partial P_{1}^{*} \cdot dP_{1}^{*} + \partial / \partial P_{f}^{*} \cdot dP_{f}^{*} + \partial / \partial P_{x}^{*} \cdot dP_{x}^{*} + W_{1} \cdot dP_{1}^{*}$ 

Noting that

(v) 
$$dP_f^* = \partial P_f^* / \partial A \cdot dA$$

and

(vi) 
$$dP_x^* = \partial P_x^* / \partial A \cdot dA$$

substitute (v) and (vi) into (iv) and rearrange terms.

(vii) 
$$dY^* = (\partial /\partial P_1^* + W_1) dP_1^* + [P_f^* + \partial P_f^* /\partial A(A + \partial /\partial P_f^*) + \partial /\partial P_x^* \cdot \partial P_x^* /\partial A] dA$$

Now substitute (v), (vi) and (vii) into (iii).

$$(\text{viii}) \quad \partial C_{l} \partial P_{l}^{*} \cdot dP_{l}^{*} + \partial C_{l} \partial P_{f}^{*} \cdot \partial P_{f}^{*} / \partial A \cdot dA + \partial C_{l} \partial Y^{*} \{ (\partial /\partial P_{l}^{*} + W_{l}) dP_{l}^{*} + [P_{f}^{*} + \partial P_{f}^{*} / \partial A (A + \partial /\partial P_{f}^{*}) + \partial /\partial P_{x}^{*} \cdot \partial P_{x}^{*} / \partial A ] dA \} = - \partial Q_{l} / \partial P_{l}^{*} \cdot dP_{l}^{*} - (\partial Q_{l} / \partial P_{f}^{*} \cdot \partial P_{f}^{*} / \partial A + \partial Q_{l} / \partial P_{x}^{*} \cdot \partial P_{x}^{*} / \partial A) dA$$

Then rearrange terms.

$$(ix) \qquad \{\partial C_{l} / \partial P_{l}^{*} + \partial C_{l} / \partial Y^{*} (\partial / \partial P_{l}^{*} + W_{l}) + \partial Q_{l} / \partial P_{l}^{*} \} dP_{l}^{*} = -\{\partial C_{l} / \partial P_{f}^{*} / \partial A + \partial C_{l} / \partial Y^{*} [P_{f}^{*} + \partial P_{f}^{*} / \partial A (A + \partial / \partial P_{f}^{*}) + \partial / \partial P_{x}^{*} / \partial A] + \partial Q_{l} / \partial P_{f}^{*} / \partial A + \partial Q_{l} / \partial P_{x}^{*} / \partial A \} dA$$

Finally, divide through both sides.

$$\frac{dP_l^*}{dA} = -\frac{+P_f^* \frac{\partial C_l}{\partial Y^*}}{\frac{\partial Q_l}{\partial P_l^*} + \frac{\partial C_l}{\partial P_l^*} + \frac{\partial C_l}{\partial Y} \left(\frac{\partial}{\partial P_l^*} + W_l\right)}$$
(12)

where 
$$= \frac{\partial P_{f}^{*}}{\partial A} \left[ \frac{\partial C_{l}}{\partial P_{f}^{*}} + \frac{\partial Q_{l}}{\partial P_{f}^{*}} + \frac{\partial C_{l}}{\partial Y^{*}} \left( A + \frac{\partial}{\partial P_{f}^{*}} \right) \right] + \frac{\partial P_{x}^{*}}{\partial A} \left[ \frac{\partial Q_{l}}{\partial P_{x}^{*}} + \frac{\partial C_{l}}{\partial Y^{*}} \cdot \frac{\partial}{\partial P_{x}^{*}} \right]$$
(13)