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FOURIER SERIES ESTIMATION OF SPRING WHEAT PRICES

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WHEAT -- PRICES

ABSTRACT

Fourier analysis is used to forecast weekly hard red spring wheat prices 25 weeks into the future. This procedure applies sine and cosine functions fitted to historical price data. Projections are then made on the basis of past price movements continuing.

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In many price forecasting studies, a causal framework provides the basis for analysis and prediction. Wheat price movements, for example, have been attributed to changes in variables such as exports, domestic use, and production. While these supply and demand relationships are fundamentally correct, appropriate timing is an essential parameter for accurate forecasting. Since markets have tended to react to expected variable changes, the use of reported levels for explanatory variables has been relatively unsuccessful for purposes of prediction.

When the purpose is solely to predict, a structural (causal) framework is not required. Instead, predictions can be based solely on the past behavior of a specific variable and that variable alone. Methods developed by Box and Jenkins have frequently been used in price forecasting analyses. An alternative approach, Fourier analysis, is examined in this study with an application made to forecasting hard red spring (HRS) wheat prices.

Fourier analysis is a term used to describe any data analysis procedure that describes or measures the fluctuations in a time series by comparing them with sinusoids (Bloomfield, p. 2). Although the technique is widely used in engineering disciplines to estimate different cyclic patterns, it can be applied to less periodic data (i.e., grain prices) where an attempt is made to describe the tendency for oscillations to occur rather than actual oscillations themselves.

The mathematical background for this method was established by Joseph Fourier, a French physicist and mathematician in 1882 (Jenkins and Watts, p. 10). Fourier demonstrated that any periodic function which is finite, single-valued, and continuous may be represented by a series consisting of a constant term plus the sum of harmonically related sine and cosine terms (Figure 1).

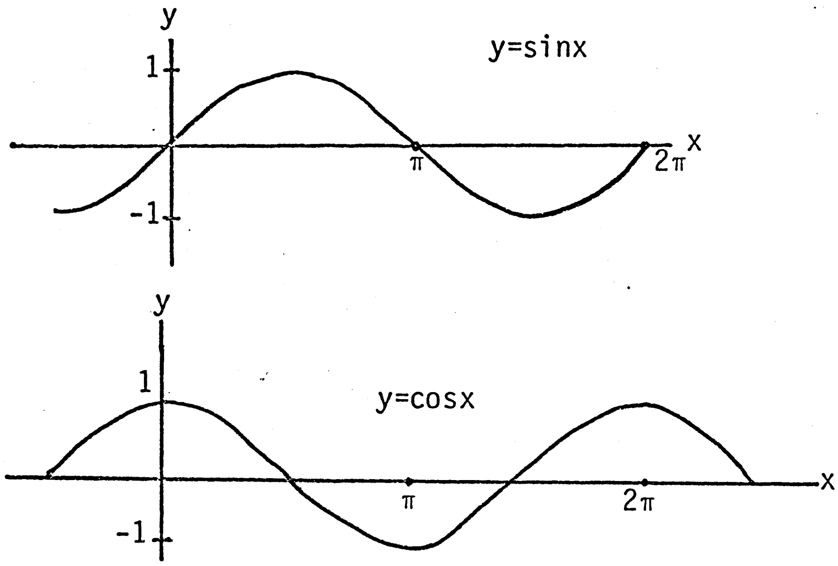

Figure 1. Sine and Cosine Functions

SOURCE: Fuller, Gordon, Plane Trigonometry, fourth edition, McGraw-Hill, St. Louis, Missouri, 1972, p. 101.

The equation of the Fourier series is:

$$F_t = a_1 + a_2 \sin w_t + a_3 \cos w_t + a_4 \sin 2w_t + a_5 \cos 2w_t + a_6 \sin 3w_t + a_7 \cos 3w_t \ldots$$

where: F_t = the numerical value of the series computed at time t

a_1 = a constant term

$a_2, a_3 \ldots$ = coefficients defining the amplitude of the harmonics

$w_t = 2\pi/T$

T = the number of forecast intervals per period

The series is expressed as an infinite series because, in theory, an infinite number of terms are required to mathematically duplicate a given periodic function with complete accuracy (Buffa and Taubert, p. 55). Both single sine and cosine functions have amplitudes of one and periods of 2π, but when factors and coefficients are added to the functions, these measures change. A factor, such as $\sin 2w_t$, will reduce the period by 1/factor while a coefficient, such as $2 \sin w_t$, will increase the amplitude by the level of the coefficient itself. For example, the period and amplitude of $2\sin 2w_t$ are π and 4, respectively.

Extending the Fourier series method to price forecasting requires that an additional term be added to the previous equation to account for a possible trend in the data. With this addition, the forecasting model provides for three basic components of price analysis: average price, a trend factor, and seasonal patterns. The expanded Fourier series price forecasting model is:

$$F_t = a_1 + a_2 + a_3\sin w_t + a_4\cos w_t + a_5\sin 2w_t + a_6\cos 2w_t \ldots$$

The a_1 term represents the average price exclusive of seasonality, while the a_2 term represents the price trend. The remaining terms are fitted to the seasonal price patterns and provide a better fit of the model to historical price data.

The computer technique used for the Fourier series in this analysis was developed by R. G. Brown in 1967 (Brown, p. 125). The specific program was written by W. H. Taubert for a study on clothing sales in 1971 (Buffa and Taubert, p. 56). The program has a limit of 500 observations including forecasts and a variable range limit of 4 to 14. The model fitting process follows a procedure which involves the use of standard regression techniques to select the model coefficients so as to minimize the sum of squared deviations between the historical prices and the fitted values. Minimization of the sum of the squared forecast errors is used as the criteria of goodness of fit and, consequently, the distribution of forecast errors should be normally distributed with a mean of zero. Analysis of the distribution of forecast errors is also helpful in selecting an appropriate number of terms for the model.

Determining the number of terms in the model is a critical decision for the Fourier series procedure. Theoretically, the model's goodness of fit should increase as terms are added to the model. The minimum number of terms in the model, however, should be equal to two times the number of peaks in a seasonal cycle plus two (Buffa and Taubert, p. 56). The program also requires that only an even number of variables be used in the analysis to keep the trigonometric terms symetrical.

Application

Six different models with a varying number of terms were tested for their use in predicting wheat prices. Each model was fitted to two years of weekly average 13% protein HRS prices (Minneapolis quote). The models tested were:

4 term

$$F_t = a_1 + a_2 + a_3\sin w_t + a_4\cos w_t$$

6 term

$$F_t = a_1 + a_2 + a_3\sin w_t + a_4\cos w_t + a_5\sin 2w_t + a_6\cos 2w_t$$

8 term

$$F_t = a_1 + a_2 + a_3\sin w_t + a_4\cos w_t + a_5\sin 2w_t + a_6\cos 2w_t + a_7\sin 3w_t + a_8\cos 3w_t$$

10 term

$$F_t = a_1 + a_2 + a_3\sin w_t + a_4\cos w_t + a_5\sin 2w_t + a_6\cos 2w_t + a_7\sin 3w_t + a_8\cos 3w_t + a_9\sin 4w_t + a_{10}\cos 4w_t$$

12 term

$$F_t = a_1 + a_2 + a_3\sin w_t + a_4\cos w_t + a_5\sin 2w_t + a_6\cos 2w_t + a_7\sin 3w_t + a_8\cos 3w_t + a_9\sin 4w_t + a_{10}\cos 4w_t + a_{11}\sin 5w_t + a_{12}\cos 5w_t$$

14 term

$$F_t = a_1 + a_2 + a_3\sin w_t + a_4\cos w_t + a_5\sin 2w_t + a_6\cos 2w_t + a_7\sin 3w_t + a_8\cos 3w_t + a_9\sin 4w_t + a_{10}\cos 4w_t + a_{11}\sin 5w_t + a_{12}\cos 5w_t + a_{13}\sin 6w_t + a_{14}\cos 6w_t$$

Estimates for the parameters a_1 to a_{14} in the six different models were calculated using the program developed by Taubert (Table 1). The chi-square statistic for the forecast errors indicated that each of the six models accurately fitted the HRS wheat price series (Table 2). The values for the chi-square static ranged from 2.19 to 1.98. The statistic for the 90% confidence level and two degrees of freedom is 4.61, which indicated that in all models the forecast errors were randomly distributed. As terms were added to the model, the standard error of the estimate constantly decreased. The standard error for the 4 term model was 12.2 cents per bushel, while the standard error for the 14 term model was 8.5 cents per bushel.

Obviously the smaller the standard error, the better the model's fit to historical data. The 14 term model provided the lowest standard error of estimate and it also provided the best fit with 70.2% of the forecast errors within ±1 standard error. The 14 term model was the one chosen for the forecasting analysis of HRS wheat prices.

Forecasting Results

The 14 term model was fitted to historical weekly data from April 1979 to August 1980 and then used to forecast prices 25 weeks into the future. Prices can be forecasted infinitely into the future, but limits must be established to retain reliability of the forecasts. Twenty-five weeks were chosen because of the relatively short length and the fact that most of the marketing year for HRS wheat is reflected by that time period.

TABLE 1. PARAMETER ESTIMATES FOR FOURIER SERIES ESTIMATION OF WEEKLY HRS PRICES

Number of Terms in the Model	Dependent Variable	Estimates for the Parameters													
		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}
4*	Price	.403	.655	.230	.969										
6	Price	.404	.636	.228	.959	-.737	.275								
8	Price	.404	.648	.236	.990	.770	.248	-.491	.654						
10	Price	.403	.666	.238	.962	.836	.215	-.503	.621	.248	.155				
12	Price	.401	.720	.248	.916	.131	.155	-.489	.549	.227	.841	.484	.538		
14	Price	.401	.723	.248	.913	.140	.155	-.469	.544	.253	.619	.499	.494	.100	.474

*Four-termed model is read: $F_t = .403 + .655 + .230 \sin w_t + .969 \cos w_t$

where: F_t = weekly HRS price at time t

$w_t = 2\pi/T$

T = number of forecast intervals per period

TABLE 2. COMPARISON OF GOODNESS OF FIT FOR FOURIER SERIES ESTIMATION MODELS

Number of Terms in Model	Standard Error (Cents/Bushel)	Chi-Square Statistic*	% of Errors Within ±1 Std. Error
4	12.2	2.19	63.3
6	12.1	2.09	68.4
8	10.6	2.14	66.7
10	10.4	1.98	70.0
12	9.2	2.18	63.3
14	8.5	1.98	70.2

*Chi-square statistic for .90 level is 4.61.

The model forecasted generally rising prices starting in September 1980 with a low of $4.65 per bushel to a high in March 1981 of $5.16 (Figure 2). An expected range was calculated for the 25 weeks of forecasted prices. The range broadens rapidly since it must account for the accumulation of the forecast errors in the observed series. The range for the first forecast is ±1 standard error, while the error for the second forecast is ±1 standard error (for itself) plus ±1 standard error for the first forecast, etc. In other words, it is a cumulative process (Pindyck and Rubinfield, p. 510).

Comparing the forecasted prices to the actual data revealed that the model was relatively accurate. Nearby forecasts, as can be expected, were more precise than forecasts further into the future. Because of this, a model similar to the Fourier series must be continually updated when employed to follow price movements.

Implications

Results of this study indicated:

1. A causal framework may not be necessary for accurate price forecasting.

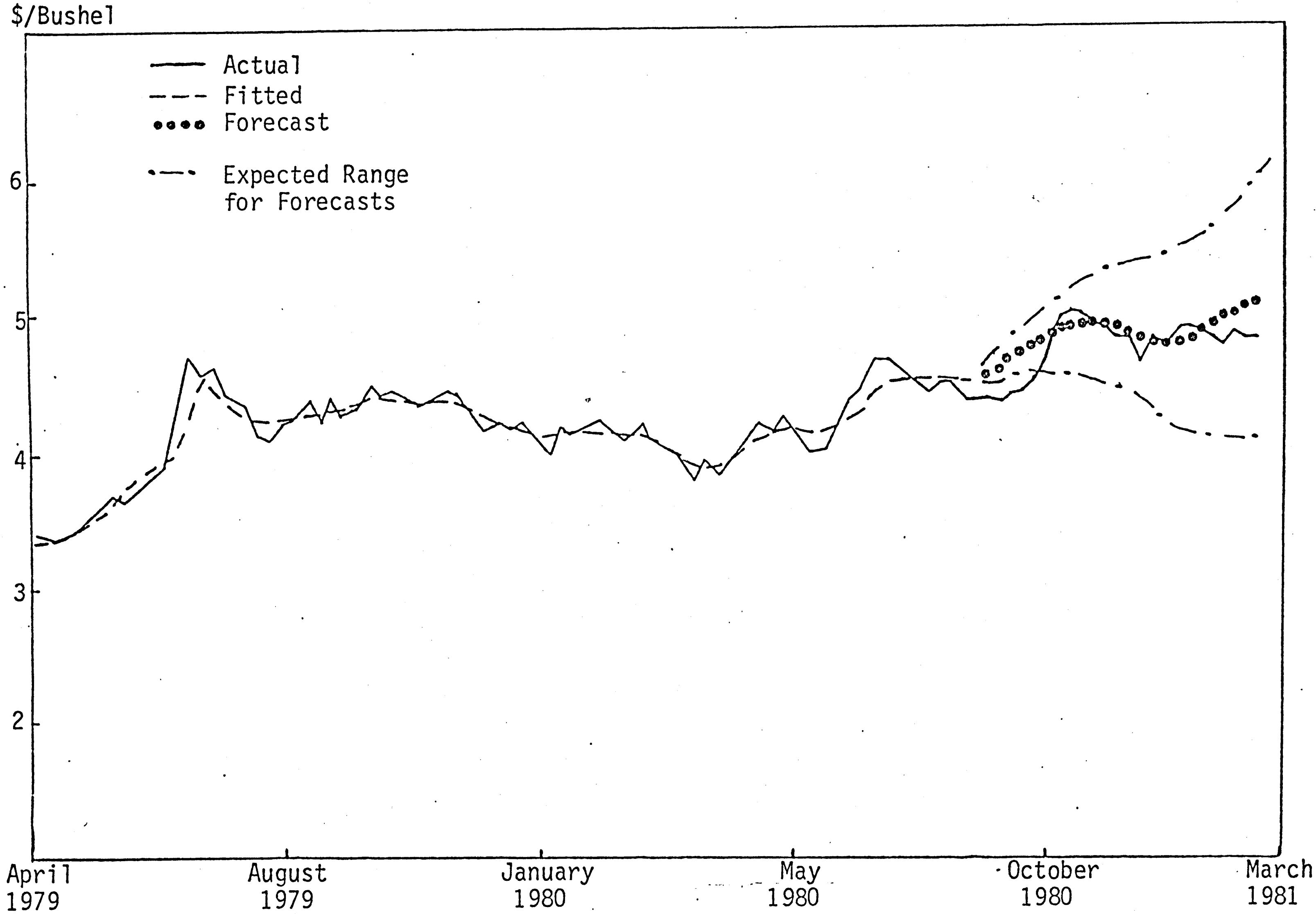

Figure 2. Fourier Series Model of Weekly HRS Wheat Prices (14 Terms)

2. Purely technical analysis may not be totally ignored in price analysis.

3. Fourier analysis offers an alternative technique to forecast agricultural price movements.

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