Risk

THEORETICAL CRITERIA AND A PROPOSED EMPIRICAL METHOD FOR COMPUTING HISTORICAL RISK MEASURES

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<u>Title</u>: Theoretical Criteria and a Proposed Empirical Model For Computing Historical Risk Measures

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ABSTRACT

Suggested criteria for evaluating the appropriateness of historical risk measures are presented. A continuously adjusted weighted moving average (CAWMA) method for calculating historical risk is introduced, and compared to past historical risk computation procedures both in terms of the suggested criteria, and in the context of specific empirical examples.

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Introduction

The concept of risk as variability finds theoretical justification in the expected utility (EU) maximization behavioral decision model. The popular empirical practice of assuming that the decision maker's underlying utility function is quadratic or that profits are normally distributed reduces expected utility to a function of mean and variance only.

The objectives of this paper are to set forth a set of theoretical criteria and a proposed empirical method to compute objective variability indices from economic time series. The paper also examines the theoretical appropriateness and empirical performance of procedures that have been used in the literature to compute variability indices. This focus is not intended to deny the importance of higher moments of probability distributions for applications using the EU paradigm. The limitations of ignoring skewness and possibly higher moments are well known. Nor does this focus necessarily reject for certain applications alternative risk decision models such as the "safety first" or "minimax regret" models which emphasize different features of the probability distribution.

The focus of this paper receives practical justification from the widespread reporting and use of historical risk indices for farm enterprise prices, yields, and incomes. These indices have been frequently computed for direct extension purposes, for theoretical hypothesis testing, for enterprise diversification analysis, and as a necessary input for estimating E-V frontiers. In view of the popularity of historical variability measures, an evaluation of the theoretical validity and comparison of the empirical performance of alternative computational procedures would appear to be justified.

Suggested Criteria for Evaluating Historical Risk Measures

The variability concept of risk in abstract form is

(1) $\mathrm{Var}(X) = E(X-EX)^2$

Conceptually, there is no ambiguity as to what (1) represents for positive applications or tests of the expected utility maximization model; it is simply the second moment about the mean of the decision maker's current subjective probability distribution of X. However, if one desires to compute a variability index from a historical time series on X and also desires that the index be normatively relevant to decision makers during a current decision period, then these measures should be more than historical descriptive statistics. They should incorporate historical data in a way that decision makers might in formulating current subjective risk assessments. This increases the likelihood that "risk efficient" marketing and production plans indentified by E-V frontier or stochastic dominance analyses will indeed be considered risk efficient by their potential users. In accordance with this philosophy, the following seven criteria are set forth for evaluating the appropriateness of methods for evaluating historical risk.

Criterion 1. The variability measure (V) representing risk should be conceptualized as a weighted mean of squared forecast errors from a series of one-step ahead forecasts. Symbolically--

(2) $V = \sum_{t=1}^{n} b_t(X_t - \hat{X}_t)^2$

where n is the number of periods in the time series; b_t is the weight for period t; ($\sum_{t=1}^{n} b_t = 1$); X_t is the actual value; and $\hat{X}_t$ is the expectation of X_t generated in period t-1.

(3) $Va = (V)^{1/2}$

is the positive root mean square forecast error, referred to as absolute variability in this study.

(4) $Vr = (Va/B)100$,

where B is a specified "base period" value of X, is referred to as the relative variability index, expressed in percentage terms, in this study.

These expressions recognize that past forecast errors or "realized frustrations" are likely to be a dominant contributor to subjective risk perceptions. They relate to the magnitude of ex post disappointments due to the departure of actual outcomes from anticipated outcomes used for planning purposes.

Criterion 2. The expectation for period t should use only information available at the time the expectation is formed, i.e., only information from periods 1,..., t-1. This principle simply recognizes that decision makers can base subjective expectations only on past and present, not future, information.

Criterion 3. Procedures for computing the variability measure (V) and expected component (X_t) should incorporate information from a limited number of past periods. This criterion accomodates the reasonable principle that decision makers are likely to consider information from distant periods obsolete after a point.

Criterion 4. More recent information should be given greater weight than more distant information in the computation of the expected components (X_t's) and variability levels (V's). This criterion reflects the reasonable principle that decision makers are likely to give greater weight to recent events in formulating subjective assessments because these events are judged better indicators of current and imminent changes in fundamental structure and trends. This criterion also reflects the intuitively appealing notion that memory of past events fades in strength as time passes.

Criterion 5. Expectation and variability values should be updated each period as new information becomes available. This recognizes the reasonable principle that decision makers update subjective probability assessments as new information becomes available.

Criterion 6. The functional specification and parameter values of the expected component model should be subject to revision in response to new information. This criterion reflects the principle that decision makers are likely to "learn by doing."

Criterion 7. The functional expressions for the expected component and variability index should be explicit and sufficiently simple to be plausible as subjective expectation formulation processes. Computational complexity should not be so great as to preclude their computation and use (including communication to clientele groups) by applied researchers and extensionists.

Evaluation of Past Procedures Used to Compute Historical Risk Indices

Most past procedures from the literature for computing risk indices from historical data have been deficient with respect to one or more of the criteria listed above. The following discussion briefly evaluates a number of expected component trend removal methods found in the literature with respect to the criteria specified in the previous section.

Use of the overall mean as the expected component, as in the study by Love, is revealed as a very naive process when viewed in a one-step-ahead expectation formulation perspective as suggested by criterion 1. It violates criterion 2 because information from the entire data series is used to compute the overall mean which is interpreted as the expected component for all data periods. Use of the overall mean as the expected component and the conventional variance of the entire series as the variability index violates criterion 4 calling for increased weights for more recent periods. When applied to a given historical data series, the use of the overall mean and computed variance does not incorporate updating of either the values of the expectation and variation themselves, nor the method used to generate the values, and thus violates both criteria 5 and 6. The method does satisfy the requirement of simplicity and explicitness of criterion 7.

Love, Jones, and Smith have computed variability as the standard error of regression about the overall OLS estimated linear time trend, $\hat{X}_t = a + bt$. It suffers the same limitations as the overall mean method with respect to criteria 2, 4, 5, and 6. Love and Jones also used first differences to isolate the "random" deviations about expected trend. This procedure is grouped together with the linear time trend procedure because first differencing will totally remove an expected component time trend that is linear.

The variate difference method has been extensively used by agricultural economists (Carter and Dean; Mathia; Yahya and Adams). Users of the method advocate it because it does not require explicit specification of the functional component (Carter and Dean; Yahya and Adams). In view of the arguments related to criterion 7, however, this property does not emerge as a strength. Knowledge of the explicit specification of the expected component can be very useful for forecasting applications, is necessary to judge the plausibility of the specification as a subjective expectation formulation process, and greatly enhances analysts' capacity to convincingly communicate the procedures to their potential users. If a historical time series has a relatively "smooth" underlying pattern, differencing will be highly successful in removing that pattern, but it does not seem automatic that the pattern, regardless of how complex, should be regarded as the "expected component." The variate difference method also reflects the same limitations as the overall mean and regression trend removal methods with respect to criteria 2, 4, 5 and 6. Knowledge of the entire data series is required to determine the appropriate order of differencing, consequently the implicit expected component for early parts of the series relies on information not available at that time. The method does not attach greater weight to recent differences as suggested by criterion 4.

Ibrahim and Williams, and Bessler have used ARIMA models to represent the expected component. The explicit forecasting perspective and flexibility of these models makes them attractive for this application. These models imply an adaptive expectations procedure that incorporates only lagged observations as independent variables, but the entire data series is used to identify the appropriate ARIMA specification, which constitutes a violation of criterion 2, as well as inherently violating 5 and 6. The explicitness and relative simplicity of ARIMA processes give this method a distinct advantage over the variate difference method with respect to criterion 7.

Klein initiated the moving autoregression model,

$$(5) \quad \hat{X}_t = \hat{\alpha}_t + \sum_{i=1}^{N_m} \hat{\beta}_{it} X_{t-i},$$

because he regarded Ibrahim and Williams' and other earlier measurement procedures as "fundamentally deficient because they use information...which is not, in fact, available to the individual until the end of the period." Both moving autoregression and moving time trend,

$$(6) \quad \hat{X}_t = \hat{\alpha}_t + \hat{\beta}_t t,$$

models are estimated entirely from observations prior to the predictive period. Each period, the equations are reestimated by dropping off the oldest observation and adding the newest observation. This periodic revision of the regression coefficients reflects the desired "learning by doing" emphasis of criterion 6, as well as updating expectation and variance values required by criterion 5.

Calvin has recently used variants of the weighted moving autoregression and moving linear time trend models which also satisfied criterion 4. These methods estimated the regression coefficients from seven past observations using weighted least squares with descending weights of $(0.5)^0$, $(0.5)^1$, $(0.5)^2$, ..., $(0.5)^6$ on the most recent to the most distant observation. Descending weights were also applied to the forecast errors in the weighted variability indices. Calvin employed the mean square forecast error concept of variability in accordance with criterion 1 rather than the standard error of regression as in Klein.

Simple moving average trends clearly meet criterion 2 requiring use only of past information. Use of a weighted average approach as in Persaud and Mapp also satisfies criterion 4. Continued updating of expectation and variance values are also accomplished in accordance with criterion 5, but criterion 6 is violated because the moving average weights remain the same each period.

The CAWMA Model For Computing Historical Risk Measures

In this section, we propose a constantly adjusted weighted moving average (CAWMA) trend model, recently used in Calvin's work, that satisfies all the criteria set forth earlier in this paper.

The CAWMA model's predictive equation for Y_t is

$$(7) \quad \hat{Y}_t = \hat{\beta}_{1t}Y_{t-1} + \hat{\beta}_{2t}Y_{t-2} + \hat{\beta}_{3t}Y_{t-3},$$

where $\hat{Y}_t$ = prediction for time period t; Y_{t-i} = observation in time period t-i, i = 1,2,3; and $\hat{\beta}_{it}$ = the moving average coefficient i for t'th period predictive equation. The moving average coefficients in the predictive equation were the coefficients that minimized the sum of weighted squared differences between actual observations and the three-year weighted moving average prediction for the previous seven years. Formally, finding the moving average coefficients was a constrained weighted regression problem minimizing

$$(8) \quad \sum_{t=0}^{N_0-1} (\alpha^t e_t)^2$$

where α was set equal to 0.5; N_0, the number of past observations used to estimate the coefficients, was set equal to seven; $e_t = Y_t - \hat{\beta}_1 Y_{t-1} - \hat{\beta}_2 Y_{t-2} - \hat{\beta}_3 Y_{t-3}$; the subscript t in (8) is interpreted such that t proceeds from the most recent to the most distant past observation as t increments from 0 to N_0-1; and the moving average coefficients were subject to the constraints:

$$(9) \quad \sum_{i=1}^{3} \hat{\beta}_i = 1$$

(10) $\hat{\beta}_1 \geq \hat{\beta}_2 \geq \hat{\beta}_3 \geq 0$

Constraint (10) satisfies the criterion that more recent events should have more influence in formulating expectations.

This constrained regression problem is in the quadratic form and was solved by a quadratic programming computer program. The α weights, number (N_m) of lagged values in the moving average, number (N_o) of observations used to estimate the β_i's in the CAWMA model and number (n) of forecast errors in the variability measures were all arbitrarily assigned in Calvin's work. Ideally these parameters (α, N_m, N_o, and n) should be based on input from the relevant users (farmers, lenders) and/or derived from a unified theoretical objective function. Work on these objectives is underway.

Empirical Comparison of Selected Risk Measurement Procedures

This section provides a brief comparison of the empirical results produced by eight different detrending procedures and four variability formulae. The eight detrending procedures compared in Table 1 vary in sophistication from the overall mean to the CAWMA. (See Young for more detail on specification of detrending procedures.)

The absolute and relative variability indices in Table 1 were computed using equations (3) and (4) with b_t specified as follows for the equally weighted and declining weight indices

(11) Equally weighted: $b_t = 1/n$

(12) Declining weight: $b_t = .5^t / \sum_{t=1}^{n} .5^t$

as t increments from the most recent to the most distant past year.

The methods were applied to Washington state annual price series for green peas and lentils for the period 1960-1977. Lentils exhibited an erratic "zig zag" price trend while green peas have had relatively stable and generally

Table 1. Comparison of Absolute and Relative Risk Indices Computed Using Different Detrending Procedures and Variability Formulae: Green Pea and Lentil Prices, Washington State, 1960-77

	Green Processed Pea Prices ($/cwt)				Lentil Prices ($/cwt)			
	Absolute Variability		Relative Variability		Absolute Variability		Relative Variability	
Expected Component in Detrending Procedure	Equally Weighted	Declining Weight	Equally Weighted	Declining Weight	Equally Weighted	Declining Weight	Equally Weighted	Declining Weight
Overall mean	45.22	76.99	21.99	37.43	8.49	20.72	35.28	86.09
Overall linear time trend	26.19	27.09	12.73	13.17	6.48	14.22	26.93	59.06
Overall quadratic time trend	18.32	21.92	8.91	10.66	4.75	9.21	19.72	38.25
Variate difference method	16.73	---	8.13	---	3.56	---	14.78	---
Moving weighted 3rd order autoregression	45.43	1148.06	221.97	558.20	28.20	52.22	117.16	216.95
Moving weighted linear time trend	25.46	34.61	12.38	16.83	7.33	14.83	30.44	61.61
Equally weighted moving average of previous 3 years	28.95	35.44	14.08	17.23	7.31	18.06	30.39	75.02
Constantly adjusted weighted moving average of previous 3 years	22.14	23.38	10.77	11.37	7.21	18.09	29.95	75.17

smoothly upward adjusting prices except for a large increase in 1974 (Young).

The first four and second four detrending procedures in Table 1 are grouped by their violation and satisfaction, respectively, of criterion 2 on the use of only past information. Conceptually, all the variability measures represent projections of price risk levels for 1978. They use data through 1977. The base period for the relative variability index was the mean of the 1975-1977 actual prices.

A steady decline in the magnitude of the risk indices is observed over methods 1 through 4, due to the fact that the expected components increase in complexity over this progression. Despite their compliance with the specified theoretical criteria, the moving weighted autoregression and to a lesser extent, the moving weighted time trend models generated extremely erratic and unrealistic price projections. The erratic price predictions of the moving autoregression procedure resulted in very inflated and unrealistic variability indices. Interestingly, the moving time trend and the two moving average procedures produced variability indices with very similar numerical magnitudes for these two sample crops. Results computed by Calvin from prices, yields, and gross returns time series for 27 crops grown in Washington state revealed that the two moving average models especially yielded very similar risk indices.

Based on both theoretical and empirical criteria, it is our judgement that the two moving average models emerge as the superior procedures. The CAWMA model is somewhat more appealing theoretically, particularly with respect to criteria 4 and 6. In view of the numerical similarity of the results produced by the simple moving average procedure, however, a strong argument could be made in favor of it on the basis of its computational simplicity (Criterion 7).

Sensitivity Analysis of CAWMA Indices

The CAWMA risk indices reported in Table 1 were computed using arbitrarily assigned parameter values of $N_m = 3$, $n = 18$, $N_0 = 7$, $\alpha = 0.5$, and $b_t = .5^t / \sum_{t=1}^{n} .5^t$. In the absence of a secure empirical or theoretical basis for these values, it would be desirable to investigate the sensitivity of the variability indices to these parameters.

Based on further consideration of the theoretical criteria discussed earlier in the this paper, it was concluded that consistent decision makers would consider the same number of forecast errors in formulating variability assessments as in formulating expected prices. Consequently, n was set equal to N_0 in the sensitivity analysis. Their common value was varied over the range of 3, 4, ..., 10. The b_t weights were specified equal to $\alpha^t / \sum_{t=1}^{n} \alpha^t$ and α was varied over the range 0.1, 0.2., ..., 0.9. N_m was held constant at three throughout.

The impact of these variations in n and α on the relative declining weight variability (VRW) indices for green pea and lentil prices is illustrated in Figures 1 and 2. The relationship between the variability index and the CAWMA parameters clearly depends upon the particular structure of the time series being examined, as is clearly exhibited by the markedly different variability response curves in Figures 1 and 2. Green pea prices have exhibited less variability in recent years. Consequently, higher α's, which increase the weight on more distant forecast errors, are associated with higher VRW's. This is reflected in the positively sloped curves in Figure 1. Recent increases in price volatility for lentils produced the opposite pattern in Figure 2.

The large increment in the green pea response curves from n = 3 to n = 4 was due to a sharp price increase for green peas in 1974 that lead to a very large forecast error for that year. Whenever this forecast error was

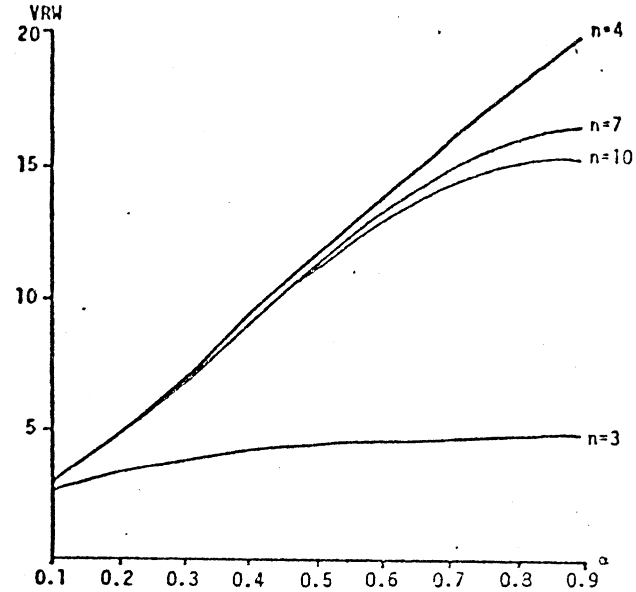

Fig. 1. Sensitivity of weighted relative variability (VRW) to CAWMA α and n parameters for green pea prices.

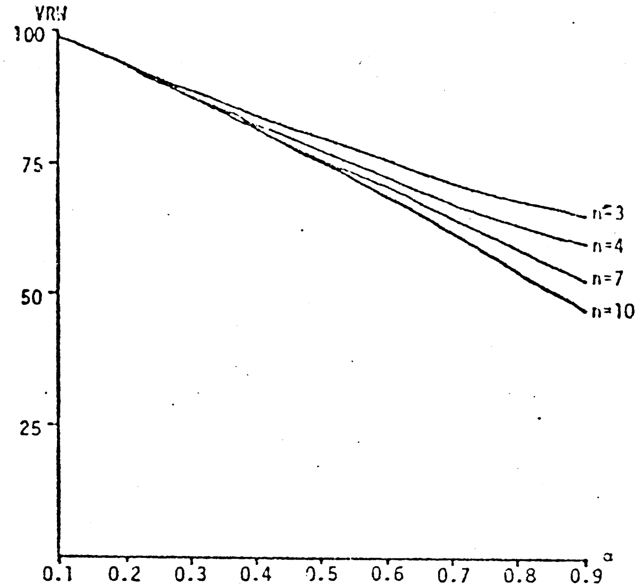

Fig. 2. Sensitivity of weighted relative variability (VRW) to CAWMA α and n parameters for lentil prices.

included, it substantially increased the variability index.

The marked sensitivity of the CAWMA risk indices to model parameters observed in this analysis underscores the need to devote attention to eliciting these parameters from farmers or other user groups of the ultimate risk measures.

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