Simulating Crop Insurance Demand Under Prospect Theory

Thomas W. Sproul & Clayton P. Michaud
University of Rhode Island

SCC-76 Annual Meeting, April 7, 2018
Kansas City, MO

Why Prospect Theory?

Expected utility bad at predicting observed crop insurance buyup behavior.

- Babcock, Choi & Feinerman (JARE, 1993)

Prospect theory may do a better job.

- Babcock (AJAE, 2015):
  - "narrow framing" aspect drives most accurate demand predictions, where farmers view crop insurance as standalone investment or lottery (ignoring hedge).

CPT Overview

$$v(x) = (x - r)^{1-\sigma} \; if \; x \geq r, else - \lambda(r - x)^{(1-\sigma)}$$

$$\pi(p) = \frac{p^{\gamma}}{\left(p^{\gamma} + (1-p)^{\gamma}\right)^{\frac{1}{\gamma}}}$$

Kahneman & Tversky (1979)

$$\pi(p) = \exp\left(-(-\ln p)^{\alpha}\right)$$

Prelec (1998)

$$\pi(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma} + (1-p)^{\gamma}}$$

Goldstein & Einhorn (1987)

Crop Insurance and CPT

Key question: what is a loss?

Broad Framing: farmers recognize value of a hedge.

$$``x - r" = Y - E[Y] + I - p$$

Narrow framing: insurance is a standalone gamble.

$$``x - r" = I - p$$

The Value Function

- Insurance guarantee, G (e.g., $G = 0.75\ E[Y]$)
- Indemnity, $I = (G - Y)^+$
- Premium, $p(G)$
- Narrow framing, so insurance is **not** a hedge.

$$
\begin{aligned}
V(G) \; = \quad & w\big(1-F(G)\big)\cdot v^{-}\big(-p(G)\big) \\
& +\int_{y=G-p(G)}^{G} \frac{\partial w}{\partial F}\big(1-F(y)\big) f(y)\cdot v^{-}\big(G-p(G)-y\big)\mathrm{d}y \\
& +\int_{y=0}^{G-p(G)} \frac{\partial w}{\partial F}\big(F(y)\big) f(y)\cdot v^{+}\big(G-p(G)-y\big)\mathrm{d}y
\end{aligned}
$$

The Value Function, Detail

$$w\left(1-F(G)\right)\cdot v^{-}\left(-p(G)\right)$$

- Atomic point representing discrete probability of losing the full premium.
- Small, but the most extreme loss w/ narrow framing.
- Always underweighted, so long as $Pr(I = 0) > e^{-1}$.

The Value Function, Detail II

$$\int_{y=G-p(G)}^{G} \frac{\partial w}{\partial F}\big(1-F(y)\big) f(y)\cdot v^{-}\big(G-p(G)-y\big)\mathrm{d}y$$

- Range of small losses where $0 < I < p(G)$.
- Can be over/under-weighted depending on slope of w.
  - e.g., likely over-weighted if $Pr(I = 0) > 0.75$, since $w' > 1$ in that region.
- Risk-lovingness in loss domain (convexity of v^{-}) means this value lies above (less negative than):

$$v^{-}\Big(G-p(G)-E_w\big[y \,\big|\, 0 < I < p(G)\big]\Big)$$

The Value Function, Detail III

$$\int_{y=0}^{G-p(G)} \frac{\partial w}{\partial F}\big(F(y)\big) f(y) \cdot v^{+}\big(G - p(G) - y\big)\mathrm{d}y$$

- Range of gains from indemnity payoff, $I > p(G)$
- Over-weighted in the tails relative to higher probability, but smaller gains
- Induces bias towards lower coverage, e.g., $F(G) < 0.20$.
- Risk-aversion in gainsdomain (concavity of v^{+}) means this value lies below (less positive than):

$$v^{+}\Big(G - p(G) - E_{w}\big[y \big| I > p(G)\big]\Big)$$

Simulating Weights

Babcock (2015) introduces a simulation method similar to expected utility simulation, except cumulative weights accumulate separately, and from the extremes, for both losses and gains.

Babcock's Method:

- Simulate the ECDF, yielding $N = m$ losses and n gains.
- For losses, $i = 1, \ldots, m$:

$$f(x_i) = w(Pr(x \leq x_i)) - w(Pr(x < x_i))$$
$$= w(i/N) - w(i/N - 1/N)$$

- For gains, $j = m + 1, \ldots, m + n$:

$$f(x_j) = w(Pr(x \geq x_j)) - w(Pr(x > x_j))$$
$$= w((N + 1 - j)/N) - w((N - j)/N)$$

Simulating Weights, II

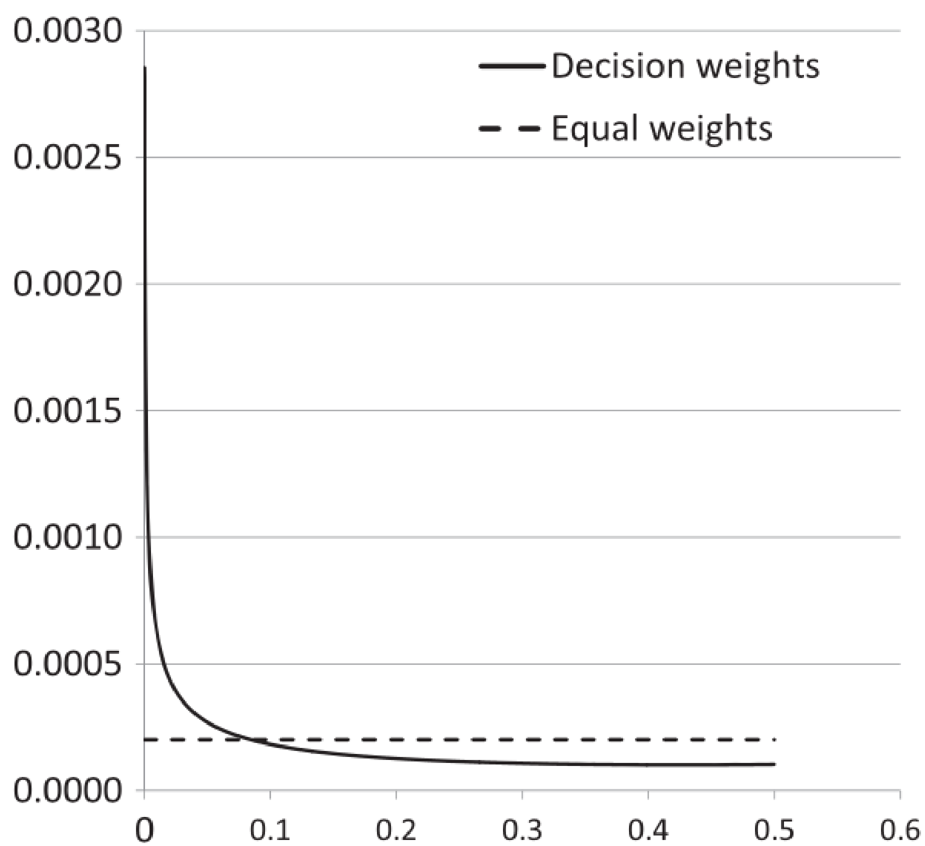
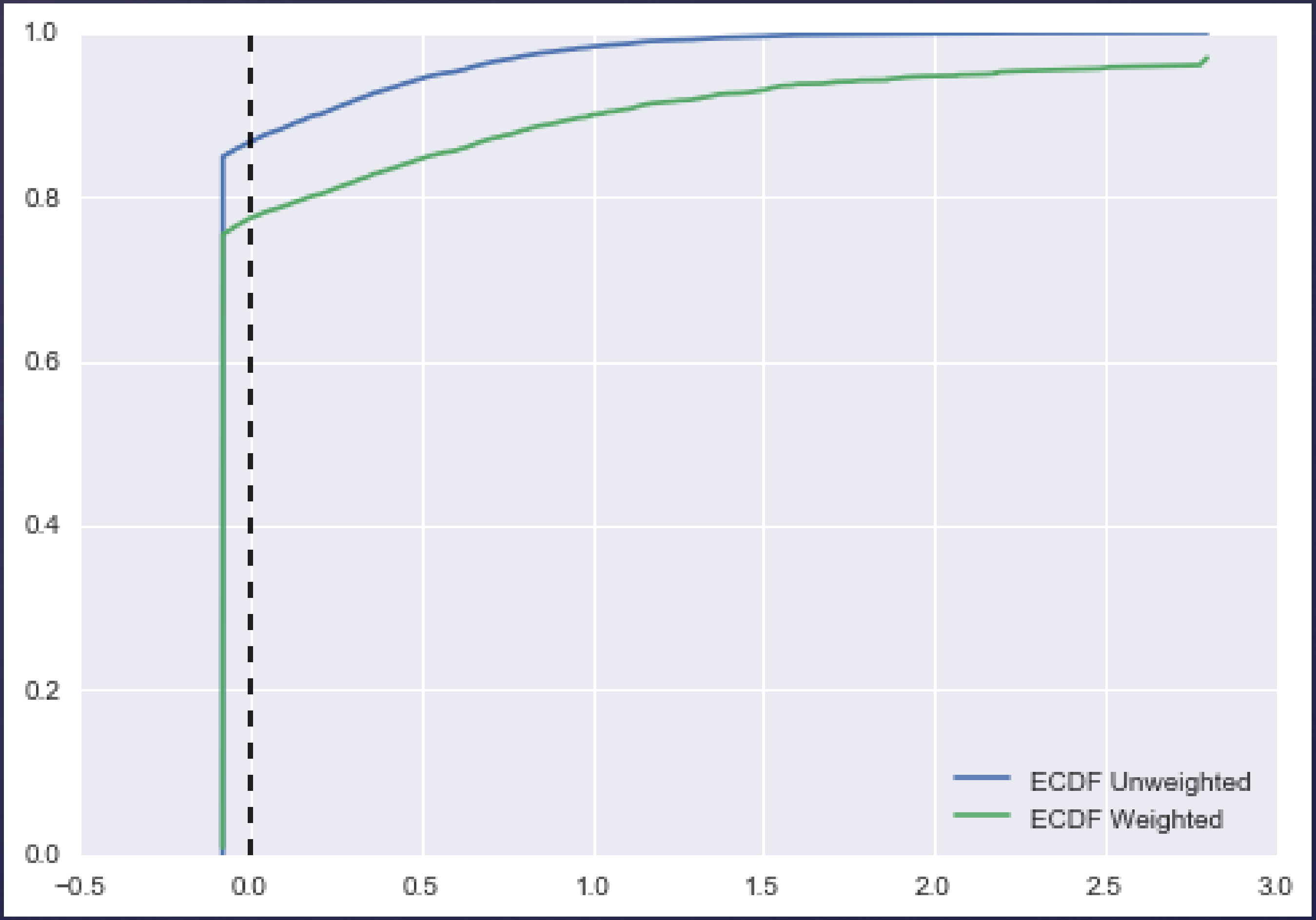


Figure 3. Decision weights vs equal weights

A Simple Example

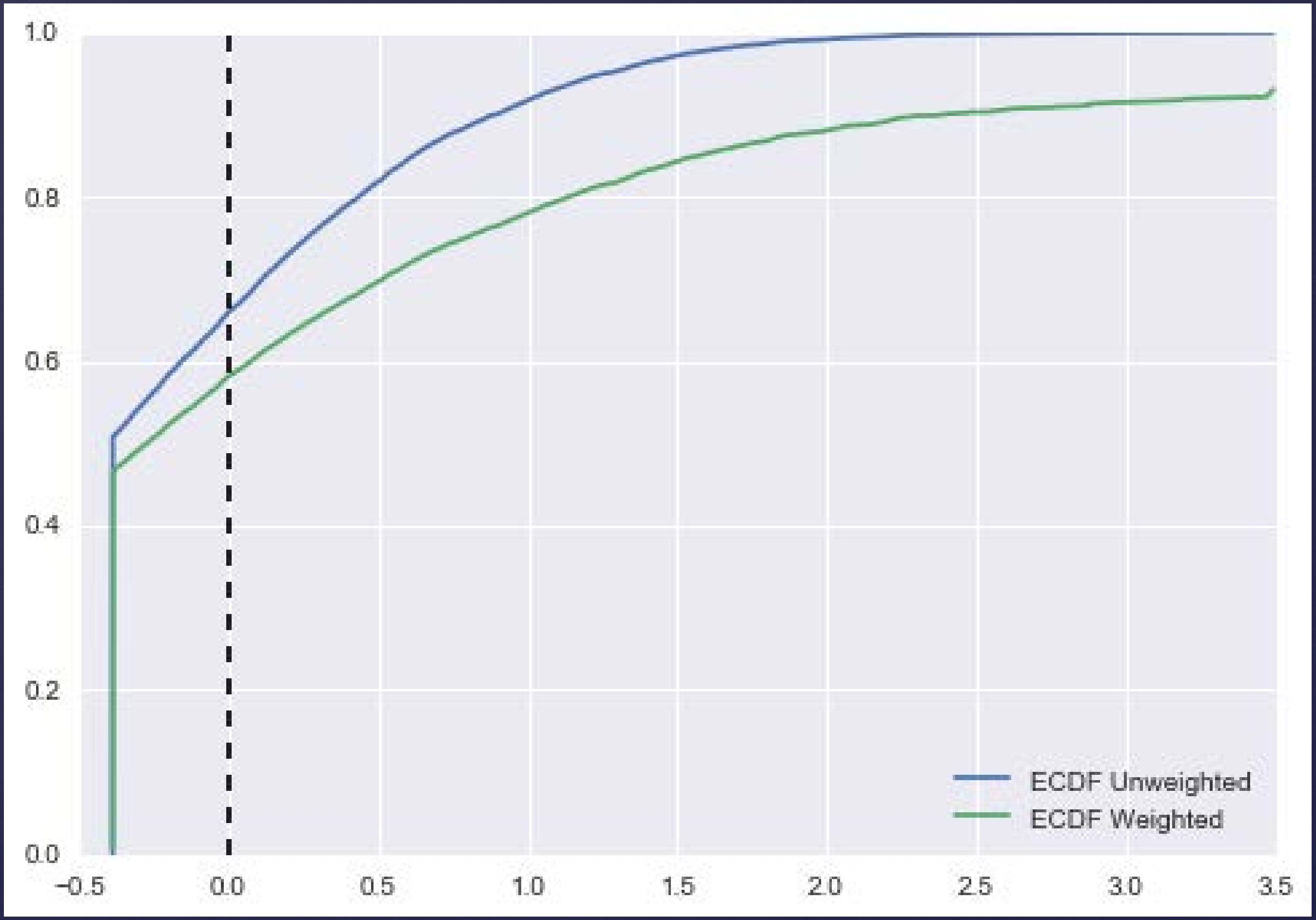
- Revenues ~ $N(4, 1)$
- Coverage = 75% (i.e., $G = 3$)
- Fair Premium
- Prelec weighting function, $a = 0.7$
- Narrow framing

Example ECDF



75% Coverage Level

Example ECDF



100% Coverage Level

Simulation Parameters

Table 1. Parameterizations and Premium Rates for Representative Farms

	Corn York Co, NE	Wheat Sumner Co, KS	Cotton Lubbock Co, TX
Type of Insurance	Revenue	Revenue	Yield
Expected Yield	190 bu/ac	33 bu/ac	650 lb/ac
Expected Price	\$4.40/bu	\$8.77/bu	\$0.55/lb
Price Volatility	37%	33%	25%
Price-Yield Correlation	0	−0.3	0
Premium Rate			
$\alpha = 0.50$	0.010	0.098	0.089
$\alpha = 0.55$	0.016	0.115	0.102
$\alpha = 0.60$	0.024	0.134	0.114
$\alpha = 0.65$	0.035	0.154	0.128
$\alpha = 0.70$	0.048	0.174	0.141
$\alpha = 0.75$	0.064	0.195	0.155
$\alpha = 0.80$	0.083	0.217	0.169
$\alpha = 0.85$	0.104	0.239	0.183
Yield Parameters			
Maximum	250	80	1338
Minimum	0	0	0
Shape1	9.340	1.938	1.363
Shape2	2.949	2.760	1.444

Note: Yields are assumed to follow a beta distribution and prices follow a log-normal distribution.