Bayesian Model Averaging for Copulas: An Application to Dependence Between Crop Yields and Futures Prices

Ford Ramsey [1] Eunchun Park [2] Xiaofei Li [2]

[1] Department of Agricultural and Applied Economics
Virginia Polytechnic Institute and State University

[2] Department of Agricultural Economics
Mississippi State University

Revenue Insurance

Overview: To describe an interesting actuarial aspect of revenue insurance, propose a new method, and show through a simple simulation that the method has potential.

The loss on revenue insurance policies is

$$\text{loss} = \max\left(Y_g P_g \lambda - Y_r P_r, 0\right)$$

Distribution of revenue is necessary to rate the policy or a joint distribution across the realized yield and harvest price.

Already well established that named joint distributions are not suitable for this purpose.

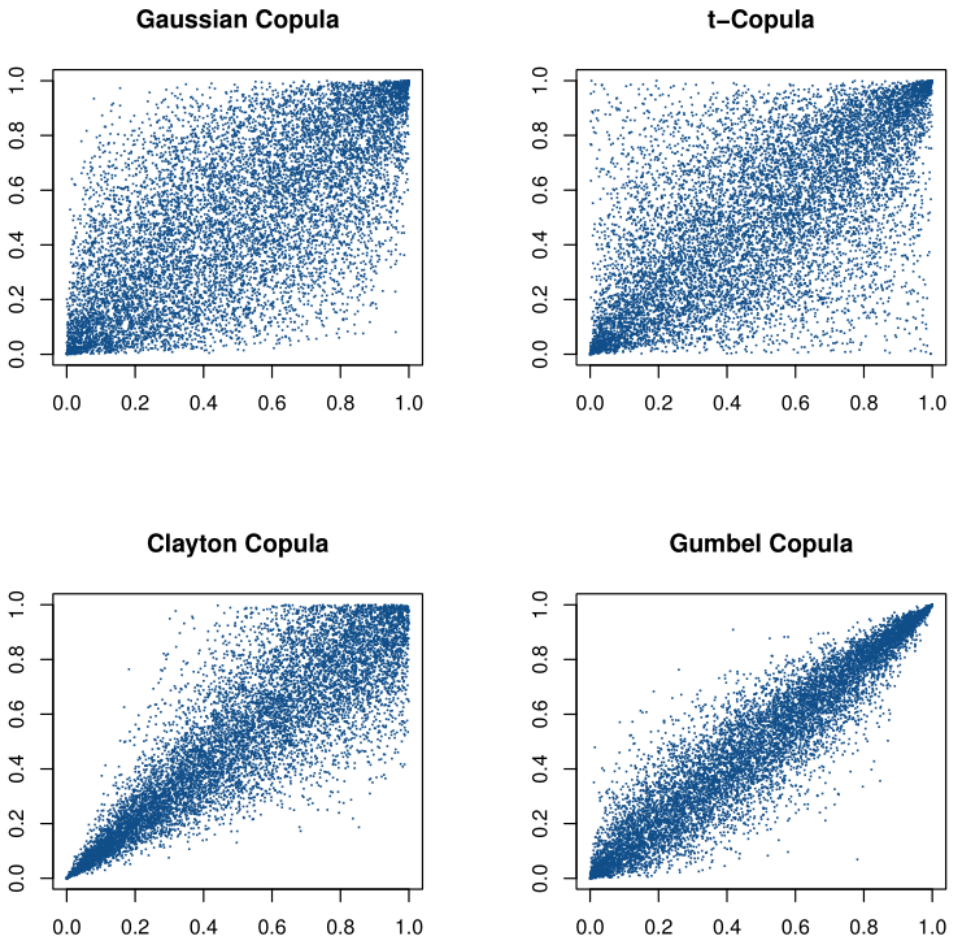
Copulas

Copulas have been applied in many areas including revenue and margin insurance (see articles in AJAE, AFR, etc.)

Sklar showed that any joint distribution can be decomposed into its marginals and the copula function.

$$F_{1,2}(x_1, x_2) = C(F_1(x_1), F_2(x_2))$$

If I know the marginal distributions of yields and prices, I can find the appropriate copula function and then form a joint distribution.


Gaussian Copula
t–Copula
Clayton Copula
Gumbel Copula
0.0
0.2
0.4
0.6
0.8
1.0

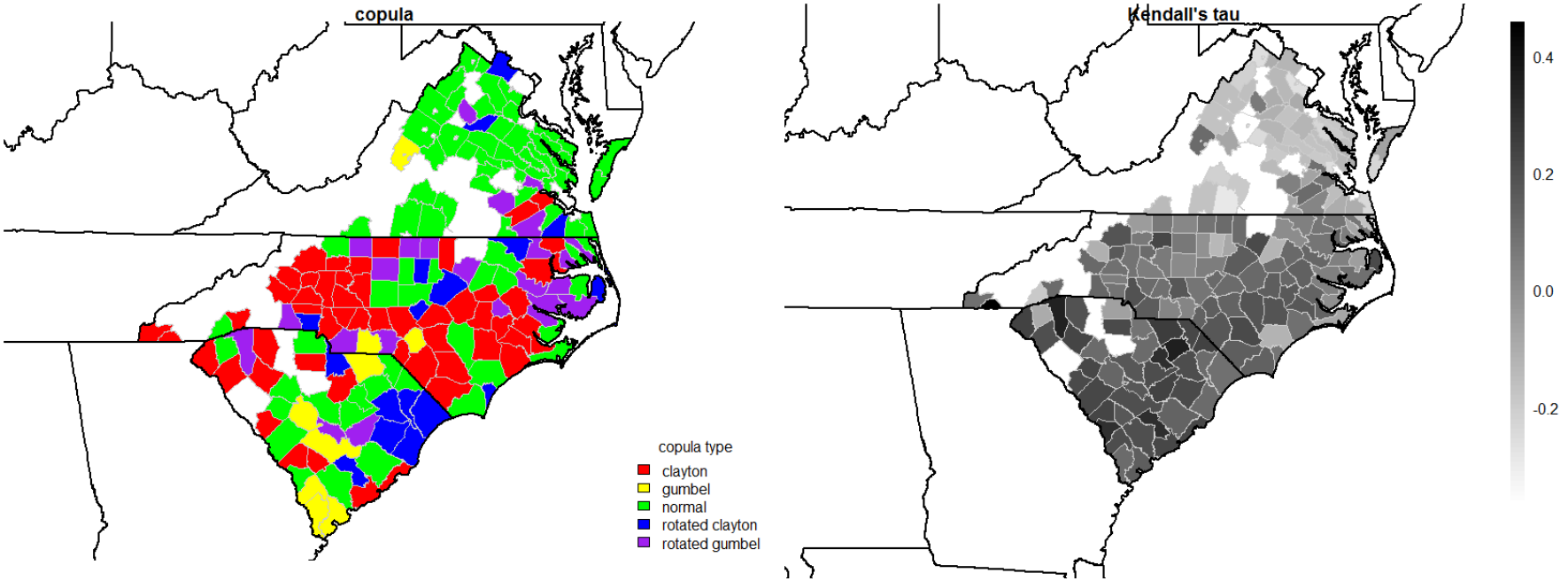
A Question of Assumptions

Always a question of **are we comfortable with the assumptions we're making?**

Actuarial approaches for revenue policies:

- Use Iman-Conover procedure which is equivalent to Gaussian copula
- Assume all counties within a state have same "dependence" between price and yield
- No pooling across counties

Spatial Variability in Copulas



Bayesian Model Averaging

Consider J models $M_1, \ldots, M_J$ where the analyst wishes to make inference or predict some quantity η. Given a set of data X, the posterior distribution of η given the data is

$$p(\eta|X) = \sum_{j=1}^{J} p(\eta|M_j, X)p(M_j|X)$$

Two issues arise in that the computation 1) becomes cumbersome as J increases and 2) requires integrated likelihood for each candidate model.

1 is not a problem if restricted to limited number of spatial locations. 2 is a problem.

Framework

Similar to Ker, Tolhurst, and Liu (2016) except we are averaging across locations and model types. More difficult task!

1. Estimate marginals using ECDF or given parametric distributions
2. Fit candidate copulas to data from each county. Select optimal copula using AIC, BIC, etc.
3. Construct BMA copula for county *i* as the weighted sum of optimal copula models for all other counties estimated on data from county *i*
   - Weights determined by integrated likelihood
4. Simulate from BMA copula and determine premium rates as normal

Two Simulated Scenarios

Sample Size	True Copula	MISE Individual	MISE BMA	Weight on Individual
n=25	Normal	3.28	3.22	0.44
	Clayton	1.25	1.56	0.99
	Gumbel	2.54	2.38	0.49
n=100	Normal	1.22	1.34	.77
	Clayton	1.01	1.01	1
	Gumbel	1.33	1.34	.99
n=500	Normal	0.87	0.87	1
	Clayton	0.58	0.58	1
	Gumbel	0.76	0.76	1
n=25	Normal	3.26	3.02	0.89
	Normal	3.16	3.10	0.45
	Normal	3.70	3.56	0.40
n=100	Normal	1.56	1.32	0.33
	Normal	1.62	1.24	0.33
	Normal	1.50	1.40	0.33
n=500	Normal	0.99	0.56	0.33
	Normal	0.78	0.68	0.33
	Normal	0.56	0.34	0.33

Conclusion

Bayesian Model Averaging leads to lower MISE and tends to select an appropriate weight as the sample size grows.

No way to get around likelihood integration. However, it's easy to deal with rotated copulas.

Two extensions come to mind. Could do BMA over the joint distribution of revenue or perhaps use EM or MCMC methods to average over the marginal distributions and the copula simultaneously.