Pricing Exotic Basket Options for Milk Using Arbitrage-based Dynamic Conditional Correlation

Dr. Marin Bozic & Thomas Keller
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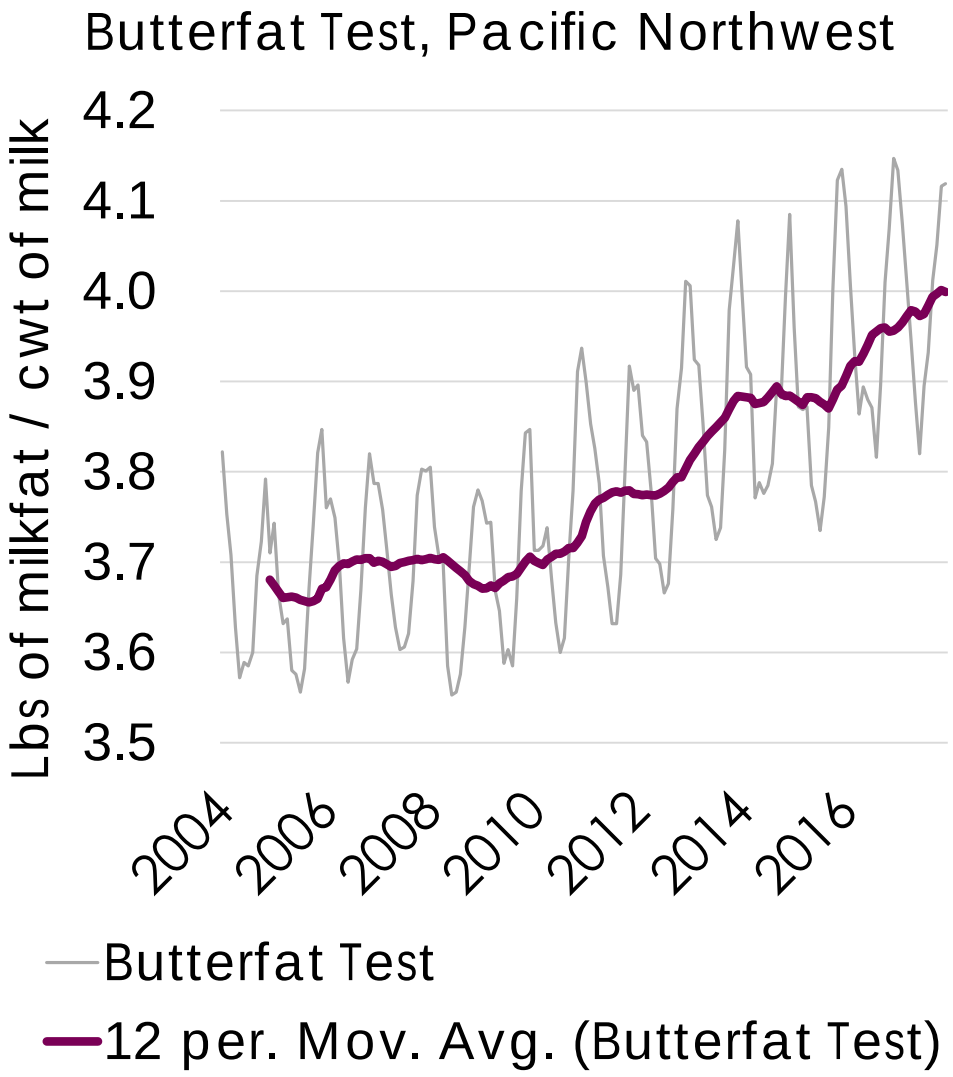
Motivation: Insurance for High-solids Milk

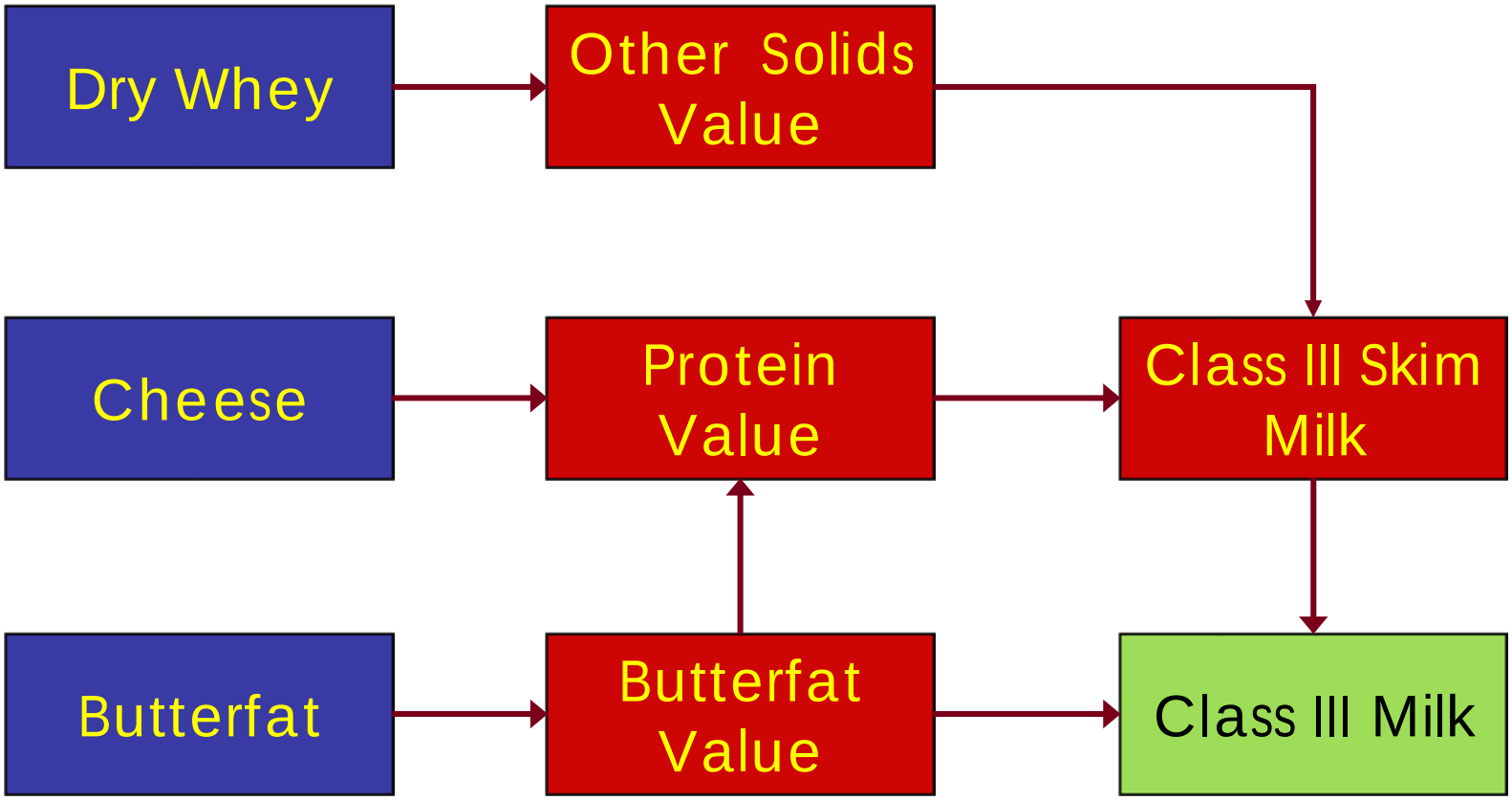


Dairy at CME

Featured | Grains and Oilseeds | Livestock | Dairy | Lumber | Softs | Biofuels

Product	Code	Contract		Last	Change	Chart	Open	High	Low	Globex Vol
Class III Milk Futures	DCK8	MAY 2018	OPT	14.71	+0.13		14.57	14.76	14.55	316
Class IV Milk Futures	GDKM8	JUN 2018	OPT	-	-		-	-	-	0
Non-fat Dry Milk Futures	GNFM8	JUN 2018	OPT	75.900	-0.900		76.400	76.875	75.750	27
Dry Whey Futures	DYN8	JUL 2018	OPT	29.000	-0.075		28.825	29.250	28.825	5
Cash-settled Butter Futures	CBM8	JUN 2018	OPT	234.000	+1.500		233.250	234.175	233.250	33
Cash-Settled Cheese Futures	CSCN8	JUL 2018	OPT	1.664	+0.007		1.660	1.672	1.657	37

Class III Milk Price



Class III Milk Price – a benchmark price for milk with 3.5% butterfat, 3.1% protein in skim milk, 5.9% other solids.

Pricing Insurance Products

General Template

1. Define the asset at risk
2. Assume structure of marginal distributions
3. Fit marginal distributions
4. Assume structure of copula
5. Fit copula
6. Simulate expected losses

Example: Livestock Gross Margin for Dairy Cattle

1. Milk – (Corn + Soymeal)
2. Lognormal
3. Futures & Option-implied volatility
4. Normal
5. Historical futures shocks
6. 5000 draws of milk-feed margin

Pricing Insurance Products: Best Practices & Challenges

- Rely on forward-looking measures as much as feasible
  - Using futures for expected prices, assuming zero risk premium
  - Using option-based implied volatility to fit marginal distributions

- Use historical data if must
  - Correlation between price shocks
  - Yields
  - Correlation between yield shocks and price shocks

- But we understand that historical data may be outdated
  - GARCH-style modelling of conditional variance
  - Practitioners base pricing on 30-day historical volatility

- **Challenge: can we reverse-engineer market-based, forward-looking copula?**

Pricing Insurance Products: High-solids Milk

Expected Price = Butterfat Test x Butterfat Price
+ Protein Test x Protein Price
+ Other Solids Test x Other Solids Price

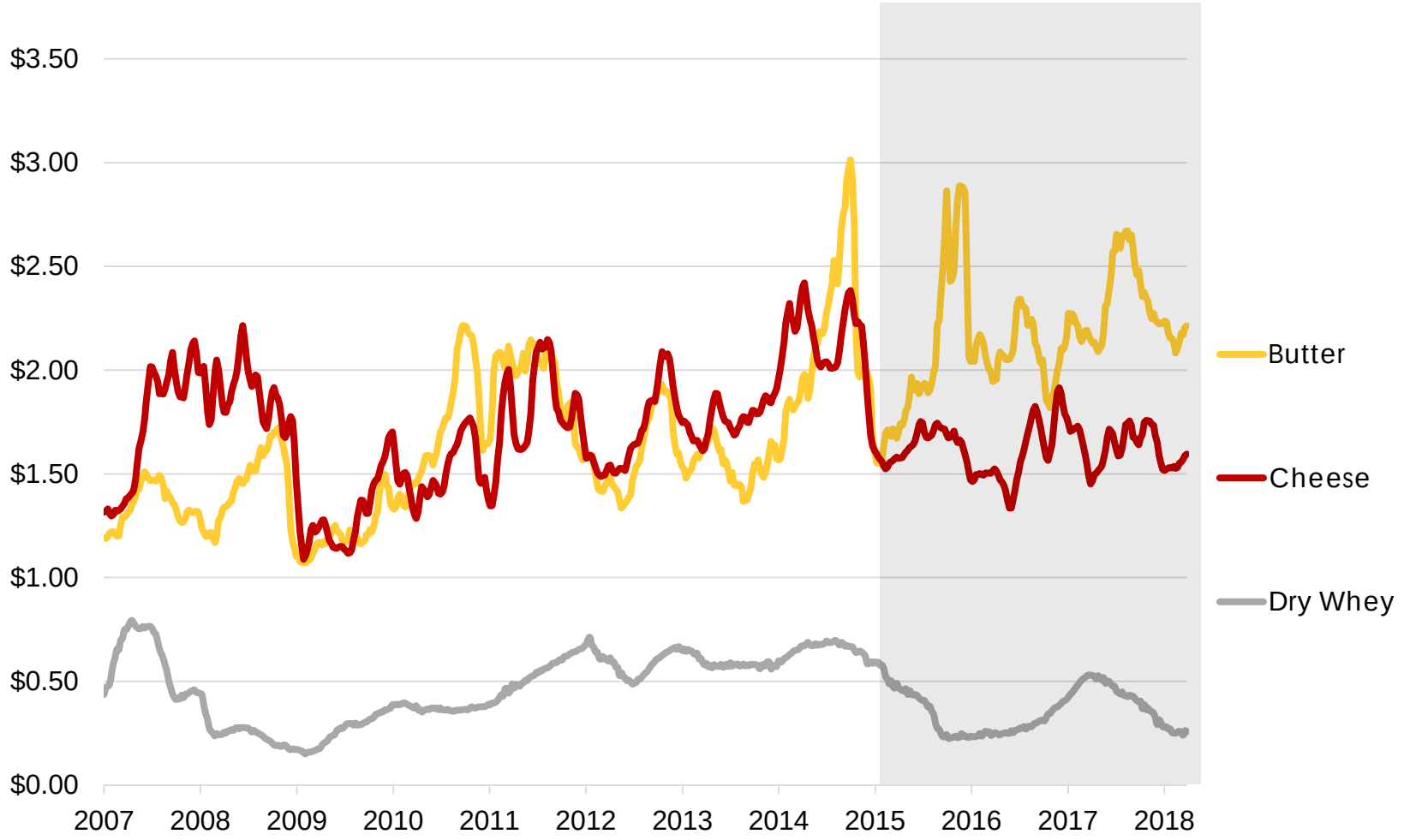
Measuring Risk

- Use implied volatility from butter, cheese and dry whey options

Measuring correlation among prices

- Historical correlations?

Structural Changes in Dairy Markets Post 2015



Historical Correlation of Futures Shocks

3-Months Out	2007-2014	2015-2018
Butter-Cheese	0.53	0.05
Butter-Whey	0.32	-0.18
Cheese-Whey	0.22	0.17

6-Months Out	2007-2014	2015-2018
Butter-Cheese	0.60	-0.29
Butter-Whey	0.34	-0.38
Cheese-Whey	0.13	0.41

8-Months Out	2007-2014	2015-2018
Butter-Cheese	0.61	-0.20
Butter-Whey	0.37	-0.37
Cheese-Whey	0.20	0.44

Non-arbitrage Relationship

$$\begin{aligned} Class\ III = & 0.42379 \times Butter \\ & +9.63933 \times Cheese \\ & +5.86430 \times Dry\ \ Whey \\ & -3.17100 \end{aligned}$$

Identity Relationship Implication for Variance

$$m = -3.17100 + 0.42379 \times b + 9.63933 \times c + 5.86430 \times w$$

$$m = \alpha + \beta \times b + \gamma \times c + \delta \times w$$

$$\begin{aligned} Var(m) &= Var\left(\alpha + \beta \times b + \gamma \times c + \delta \times w\right) \\ &= \beta^2 Var\left(b\right) + \gamma^2 Var\left(c\right) + \delta^2 Var\left(w\right) \\ &\quad + 2 \times Cov\left(\beta \times b, \gamma \times c\right) \\ &\quad + 2 \times Cov\left(\beta \times b, \delta \times w\right) \\ &\quad + 2 \times Cov\left(\gamma \times c, \delta \times w\right) \end{aligned}$$

Identity Relationship Implication for Variance

$$Var(m) = \beta^2 Var(b) + \gamma^2 Var(c) + \delta^2 Var(w)$$
$$+2 \times \rho_{b,c} \beta \times \gamma \times \sqrt{Var(b) Var(c)}$$
$$+2 \times \rho_{b,w} \beta \times \delta \times \sqrt{Var(b) Var(w)}$$
$$+2 \times \rho_{c,w} \gamma \times \delta \times \sqrt{Var(c) Var(w)}$$

Identity Relationship Implication for Variance

$$Var(b) = \left[e^{\sigma_b^2} - 1 \right] e^{(2\mu_b + \sigma_b^2)}$$

$$\sigma_b^2 = IV_b^2 \tau$$

$$\mu_b = \ln(b) - 0.5 \times \sigma_b^2$$

Identity Relationship Implication for Variance

$$Var(m) = \beta^2 Var(b) + \gamma^2 Var(c) + \delta^2 Var(w)$$
$$+2 \times \rho_{b,c} \beta \times \gamma \times \sqrt{Var(b) Var(c)}$$
$$+2 \times \rho_{b,w} \beta \times \delta \times \sqrt{Var(b) Var(w)}$$
$$+2 \times \rho_{c,w} \gamma \times \delta \times \sqrt{Var(c) Var(w)}$$

Case Study with Prices for Dec 2018 on April 5, 2018

Commodity	Variance	Contribution to Class III Variance			
		Zero Corr.	Hist.Corr '07-'14	Hist.Corr. '15-'18	Implied Corr.
Butter	0.10	0.02	0.02	0.02	0.02
Cheese	0.03	2.81	2.81	2.81	2.81
Dry Whey	0.00	0.06	0.06	0.06	0.06
Butter-Cheese		0.00			
Butter-Whey		0.00			
Cheese-Whey		0.00			
Calculated Class III Variance		2.88			
Actual Class III Variance	2.76	2.76	2.76	2.76	2.76
Difference		**+0.12**			

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Commodity	Variance	Contribution to Class III Variance			
		Zero Corr.	Hist.Corr '07-'14	Hist.Corr. '15-'18	Implied Corr.
Butter	0.10	0.02	0.02	0.02	0.02
Cheese	0.03	2.81	2.81	2.81	2.81
Dry Whey	0.00	0.06	0.06	0.06	0.06
Butter-Cheese		0.00	0.28	-0.09	
Butter-Whey		0.00	0.02	-0.02	
Cheese-Whey		0.00	0.16	0.35	
Calculated Class III Variance		2.88	3.34	3.12	
Actual Class III Variance	2.76	2.76	2.76	2.76	2.76
Difference		**+0.12**	**+0.58**	**+0.36**	

Case Study with Prices for Dec 18 on April 5, 2018

Case Study with Prices for Dec 2018 on April 5, 2018

Commodity	Variance	Contribution to Class III Variance			
		Zero Corr.	Hist.Corr '07-'14	Hist.Corr. '15-'18	Implied Corr.
Butter	0.10	0.02	0.02	0.02	0.02
Cheese	0.03	2.81	2.81	2.81	2.81
Dry Whey	0.00	0.06	0.06	0.06	0.06
Butter-Cheese		0.00	0.28	-0.09	-0.32
Butter-Whey		0.00	0.02	-0.02	0.01
Cheese-Whey		0.00	0.16	0.35	0.19
Calculated Class III Variance		2.88	3.34	3.12	2.76
Actual Class III Variance	2.76	2.76	2.76	2.76	2.76
Difference		**+0.12**	**+0.58**	**+0.36**	**0.00**

Historical vs. Implied Correlation

3-Months Out	2007-2014	2015-2018	Implied Corr
Butter-Cheese	0.53	0.05	**-0.63**
Butter-Whey	0.32	-0.18	0.15
Cheese-Whey	0.22	0.17	0.21

6-Months Out	2007-2014	2015-2018	Implied Corr
Butter-Cheese	0.60	-0.29	**-0.64**
Butter-Whey	0.34	-0.38	0.12
Cheese-Whey	0.13	0.41	0.18

8-Months Out	2007-2014	2015-2018	Implied Corr
Butter-Cheese	0.61	-0.20	**-0.70**
Butter-Whey	0.37	-0.37	0.13
Cheese-Whey	0.20	0.44	0.24

Open Issues

- **Dealing with partial identification?**
  - The no-arbitrage only restricts the degrees of freedom of the space of three correlation coefficients. How to optimally adjust the correlation coefficients?

- **Ensuring implied correlation matrix is positive semi-definite.**
  - Derive implied correlations
  - Project onto space of positive semidefinite matrices
  - Repeat until convergence

- **Trust implied correlation or implied volatility?**
  - Implied Correlations may not pass the common-sense test. If implied butter-cheese correlation needed to match variances is -0.9 would you still trust the results?
  - An alternative conclusion may be that implied Class III volatility is too low relative to implied vol. of butter and cheese. Speculative positions?

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Dr. Marin Bozic
mbozic@umn.edu
Department of Applied Economics
University of Minnesota-Twin Cities
317c Ruttan Hall
1994 Buford Avenue
St Paul, MN 55108

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