# LAND VALUES AND CREDIT POLICIES IN AN IMPERFECT-RISKY MARKET

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## Introduction

U.S. agriculture is in the midst of two major crises. First is a general depression in which land values in the 7th Federal Reserve district which includes Iowa, Indiana, Illinois, Michigan, and Wisconsin have fallen by 60 percent in real terms from their 1981 peak (Benjamin). The second crisis is the destabilization of financial institutions which help farmers purchase land, particularly the Farm Credit System and the Farmers Home Administration (Bullock). Declining land values have eroded the security of these lenders and, in several cases, farmers have simply turned over farms and acreages rather than continue farming with small or even negative equity.

In the aftermath of this financial experience of weakened financial institutions and depressed land prices, the question is being asked: Are the two crises related? Has, for example, the ease of credit from the Farm Credit System and the sometimes subsidized credit from Farmers Home Administration allowed farmers to overbid the price of land in the late 70s and early 80s setting the stage for the major declines which occurred recently--and perhaps prolonging the economic recession?

Obviously, a number of factors, not just the availability and cost of credit, influenced the run-up of land values. Burt has convincing evidence from Illinois data that unrealistic income expectations led to the farm land market boom. Robison, Lins, and Venkataraman (RLV) tied the rise in 22 states to declines in the real interest rate, the demand for land for nonagricultural uses, taxes, as well as income expectations. More recently, Kelsey, Robison, and Koenig estimated the 1986 Tax Reform Act may have reduced real estate prices by as much as 20 percent.

Hughes et al. tested the hypothesis that Farmers Home Administration credit policies contributed to the run-up in farm land prices and concluded they did not. The Hughes et al. deductive framework did not include risk. It is believed that any analysis of credit should include risk because credit, an unused borrowing capacity, is a reserve against risk. The risk which credit insures against is the cost of liquidating fixed assets to meet cash flow obligations. Most assets cannot be converted to cash costlessly. Therefore, negative cash flow can most often be offset least expensively by borrowing or using some of the credit reserve. Only after exhausting that liquidity source are assets liquidated.

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Since the level of debt or the relationship between a firm's debt and equity signals the potential cash flow needs, it is also used by lenders to assess the ability of borrowers to repay their loans or to determine the credit reserve each firm possesses.

Restricting a firm's credit reserve limits its ability to purchase additional assets or in our case limits the demand for land which in turn restricts land price increases. Increasing credit reserves has the opposite effects. A larger credit reserve provides a larger cushion for borrowers. With greater credit availability they can borrow more because they can handle greater liquidation cost risks.

The goal of this paper is to show in a deductive sense how credit availability influences both individual firm borrowings and how the individual demand for borrowed funds influences the market price for land. We also intend to take the analysis farther. Farmers Home Administration offers their loans, often at subsidized rates, to selected customers. We intend to identify how selectively subsidizing borrowers with either lower interest rates or greater credit reserves benefit some and hurt others, and alter the equilibrium price of land. In addition, credit policies and changing market conditions differentiate against borrowers on the basis of when and at what price earlier purchases of land were made. This factor alone has been the most important determinant of those who have survived the most recent farm financial crisis. It will also determine which of the next generation of farmers succeeds.

The plan for presentation of the deductive framework linking financial institutions, the supply of credit, and land prices is as follows. In the next section, a generalized firm level model is presented which differentiates between firms on the basis of financial security. This model draws on the work of Robison, Barry, and Burghardt and Meyer and Robison. These earlier works are available elsewhere and are reviewed here only to the extent required to establish the deductive model for this paper.

The development of the firm model in the second section prepares the foundation for the aggregate results presented in the paper's third section. For the firm, the price of land, the major asset of the firm, is treated as an exogenous variable. In the third section, a market clearing condition is added that the sum of the acres owned by m firms equals some fixed quantity of land N.

Using the market clearing condition, the price of land is found which in turn determines how much land each firm owns. Thus, the price per unit of land V, while exogeneous to the firm, is endogenous to the market. The price of land now obtained by aggregating across m individuals becomes a function of the parameters affecting each individual as well as economy-wide parameters including interest rates and credit availability.

It would be unlikely that credit policies of such major lenders as the Farm Credit System and the Farmers Home Administration have not influenced land prices. Moreover, the effect of these credit policies would be selective--hurting some and helping others. Thus, the credit policies, not to mention aggregate fiscal and monetary policies, have a wealth redistributive effect which depends mostly on luck and timing than on carefully weighted judgments as to consequences.

In our minds, the only question is not: ''Do credit policies matter?'' but also ''To what extent have land values, the most significant source of wealth in the farm sector, been altered by credit policies?'' The answer to this question requires an empirical test which this paper does not provide. We suggest that this task be completed so that in the future credit policies by major lenders can be formed with a clearer view as to their consequences.

## The Model

Meyer has shown that EV models are consistent with expected utility models under more general conditions than normality or quadratic utility. One such EV model takes the popular form:

$$(1) \quad y_{CE} = E(\tilde{y}) - (\lambda/2)\, \sigma^2\, (\tilde{y})$$

where the certainty equivalent income $y_{CE}$ equals expected income $E(\tilde{y})$ minus one-half a risk coefficient $\lambda$ times the variance of income $\sigma^2(\tilde{y})$.

The model in (1) is of this same form used by Meyer and Robison to extend the traditional firm level results obtained by Sandmo. In Meyer and Robison's model, land was included as well as a variable input used to produce an output x.

They argued that if production functions were linearly homogeneous, then $\tilde{y}$ could be written as:

$$(2) \quad \tilde{y} = n[\tilde{p}x - C(x) - rV] - B + rE$$

where $\tilde{p}$ is a random output price, n is the acres of land, r is the interest rate, V is the current price of land, and B is a fixed cost. The final term is earnings from the firm's equity E.

The above model is linearly homogeneous in production but average cost is not constant. It can be solved using the EV model described in (1). The only requirement is to calculate $E(\tilde{y})$ and $\sigma^2(\tilde{y})$. These can be expressed for the $i^{th}$ firm as:

$$(3a) \quad E(\tilde{y}_i) = n_i[p_x - C(x) - rV] - B_i$$

and

$$(3b) \quad \sigma^2(\tilde{y}_i) = (n_i x)^2\, \sigma_p^2$$

where $\tilde{p} \sim (p, \sigma_p^2)$

The model above can be modified to include liquidity considerations deduced by Robison, Barry, and Burghardt. They separated firms into three categories. The first is the financially secure firm which faces no likelihood of liquidation or bankruptcy. For this firm, the shadow price of unused borrowing capacity called credit is zero.

The second category is the firm whose survival may, depending on the outcome of $\tilde{p}$, be forced to liquidate assets. The average cost of liquidation is assumed to be $\rho$ which limits the credit available to the firm. Credit constraints are typically expressed as leverage ratios equal to firm debt divided by firm equity. This ratio set by the lenders is $\alpha$ where:

(4) $\alpha = \alpha(\rho)$

For the financially insecure firm, borrowing is a less expensive alternative to liquidation. Thus credit is a valuable resource to the financially insecure firm.

The possibility of liquidation costs has two effects on the $i^{th}$ firm's output price distribution. First, it lowers the expected return on each unit of output sold from p to $p-\zeta_i(\alpha) > 0$ where $\zeta_i(\alpha) > 0$ is the expected liquidation cost per unit of x sold which depends on the credit available to the firm. Second, it adds a random element to the variance of $\tilde{p}$ so that the variance of returns per unit of x sold is $\sigma_p^2 + \delta_i(\alpha)$ where $\delta_i(\alpha) > 0$.

The third category of firms described by Robison, Barry, and Burghardt is the one facing bankruptcy. The possibility of bankruptcy with limited liability has two effects on the $i^{th}$ firm's output price distribution. First, it increases the expected return on each unit of output sold from p to $p-\zeta_i(\alpha) > 0$ where $\zeta_i(\alpha) > 0$ is the expected liquidation cost per unit of x sold which depends on the credit available to the firm. Second, it reduces the variance of $\tilde{p}$ so that the variance of returns per unit of x sold is $\sigma_p^2 + \delta_i(\alpha)$ where $\delta_i(\alpha) > 0$.

Credit and liquidation costs modify the expected returns and variance for the $i^{th}$ firm described in equations (3a) and (3b) as follows:

(5a) $E(\tilde{y}_i) = n_i[(\tilde{p}-\zeta_i(\alpha))x-C(x)] - B_i + r[E_i-n_iV]$

and

(5b) $\sigma^2(\tilde{y}) = (n_ix)^2\ [\sigma_p^2+\delta_i(\alpha)]$

where $\zeta_i(\alpha) \gtrless 0$ and $\delta_i(\alpha) \gtrless 0$ depending on the firm's financial position.

The $i^{th}$ firm described by the model above has two decisions to make: output per acre x and the number of acres $n_i$ on which to produce x. In effect, it becomes a portfolio decision problem with a solution similar to those obtained in portfolio theory.

It can be shown that all firms chose the same x, while $n_i$ depends on the firm's risk attitudes. The levels of x and $n_i$ are found so that the cost of increasing x, increasing the intensity of production, is just equal to the cost of increasing output by adding one more unit of $n_i$, an increase in the extensive production methods.

The solution for the $i^{th}$ firm is written as:

(6) $x = x(r,V)$

and

(7) $$n_i = \frac{(p-\zeta_i)x - C(x)-rV}{\lambda_i x^2(\sigma_p^2+\delta_i)}$$

The derivation for (6) and (7) is given in Appendix A to this paper. The comparative statics associated with (6) are:

$$\frac{dx}{dr} > 0,$$

and

$$\frac{dx}{dV} > 0.$$

These results state that an increase in the cost of employing one more unit of land motivates the firm to increase its output per acre.

The comparative statics associated with (7) are differentiated by the firm's financial conditions. Other factors being equal, the least secure firm ($\zeta_i$, $\delta_i < 0$) has the largest demand for $n_i$. Next largest is the financially secure firm ($\zeta_i$, $\delta_i = 0$). And the smallest demand for land $n_i$ is associated with the firm facing liquidation charges ($\zeta_i$, $\delta_i > 0$). All of the firms respond in the same direction to changes in p, r, V, $\lambda_i$, and $\sigma_p^2$.

$$\frac{dn_i}{dp} > 0,$$

$$\frac{dn_i}{dr} < 0,$$

$$\frac{dn_i}{dV} < 0,$$

$$\frac{dn_i}{d\lambda_i} < 0,$$

and

$$\frac{dn_i}{d\sigma_p^2} < 0.$$

However, should credit be increased, they may respond differently. Assuming that:

$$\text{(8a)} \quad \frac{d\zeta_i}{d\alpha} \lesseqgtr 0 \qquad \text{for } \zeta_i \gtrless 0$$

and

$$\text{(8b)} \quad \frac{d\delta_i}{d\alpha} \lesseqgtr 0 \qquad \text{as} \qquad \delta_i \gtrless 0.$$

Then depending on the financial position of the firm:

$$\frac{dn_i}{d\alpha} = 0$$

for the financially secure firm ($\zeta_i$, $\delta_i = 0$):

$$\frac{dn_i}{d\alpha} > 0$$

for both the firm facing liquidation costs ($\zeta_i$, $\delta_i > 0$) and the firm facing bankruptcy ($\zeta_i$, $\delta_i < 0$). Thus, the only firm not motivated to increase its holdings of land by an increase in credit is the financially secure firm.

## Aggregation Results

The actions of an individual firm are not likely to influence the price of land V. But the actions of all m buyers and sellers will determine the price of land. This collective effect on land prices can be found by summing $n_i$ over m buyers and sellers of land.

The sum of $n_i$ is limited by a physical supply equal to N. The result of summing over $n_i$ is:

$$\text{(9)} \quad N = \sum_{i=1}^{m} \left\{ \frac{(p-\zeta_i)x - C(x) - rV}{\lambda_i x^2(\sigma_p^2+\delta_i)} \right\}$$

Equation (9) alone, however, cannot determine V since x also depends on V. Thus the solution for V requires that (6) and (9) be solved simultaneously.

The primary focus of this paper is the effect of credit policies which change r, $\alpha$, or both. To find the effects of an increase in r and $\alpha$ as well as an increase in p and $\sigma_p^2$, the total derivatives of (6) and (9) are written as:

$$(10a) \quad dx = \frac{\partial x}{\partial V} dV + \frac{\partial x}{\partial r} dr$$

$$(10b) \quad - \left[ \sum_{i=1}^{m} \frac{r}{(..)} \right] dV + x \left[ \sum_{i=1}^{m} \frac{1}{(..)} \right] dp - \left[ \sum_{i=1}^{m} \frac{r}{(..)} \right] dr$$

$$\sum_{i=1}^{m} \left\{ \frac{[(p-\zeta_i) - C'(x)] - [(p-\zeta_i)x - C(x) - rV]}{(..)} \right\} dx - dN$$

$$- \left[ \sum_{i=1}^{m} \frac{(..)x \frac{\partial \zeta_i}{\partial \alpha} + (.)\ \lambda_i x^2 \frac{\partial \delta_i}{\partial \alpha}}{(..)^2} \right] d\alpha - \left[ \sum_{i=1}^{m} \frac{(.)\lambda_i x^2}{(..)^2} \right] d\sigma_p^2 = 0$$

where (.) is the numerator and (..) is the denominator in equation (9).

The techniques for solving for dV and dx are well known. Their derivation is repeated in Appendix B to this paper. The solution satisfying the optimizing conditions of the firm and the equilibrium conditions of the market are:

$$\frac{dV}{d\alpha} > 0$$

$$\frac{dx}{d\alpha} > 0$$

and

$$\frac{dV}{dr} < 0$$

$$\frac{dx}{dr} < 0$$

The results are intuitive. Increasing credit reduces expected liquidation costs and increases the demand for land by the financially insecure firms. As the price of extensive production increases, the firms respond by intensifying production per acre and x increases.

If the price distribution is determined exogeneously, then total output and land price V increase simultaneously. However, policy makers could anticipate an increase in V and total output Nx and reduce output incentives by decreasing p or increasing $\sigma_p^2$.

If such were the result, then both V and x would fall:

$$\frac{dV}{dp} > 0$$

and

$$\frac{dx}{dp} > 0$$

Or if $\sigma_p^2$ were increased:

$$\frac{dV}{d\sigma_p^2} < 0$$

and

$$\frac{dx}{d\sigma_p^2} < 0$$

The net effect of a reduction in price supports and an increase in credit would be to transfer resources to the least financially secure firms. But providing the least secure firms more resources with decreased prices for their product may be no great benefit. It may simply expedite their exit since unfavorable prices and high leverage ratios combine to erode equity.

Thus it is that the precarious position of farmers with high leverage ratios generally dictates the opposite price policy. An increase in p and/or a reduction in $\sigma_p^2$. Although it is frequently accompanied by required reductions in Nx, the overall effect we assume is:

$$\frac{dV}{dp} > 0$$

and

$$\frac{dx}{dp} > 0$$

Or if $\sigma_p^2$ is reduced:

$$\frac{dV}{d\sigma_p^2} < 0$$

and

$$\frac{dx}{d\sigma_p^2} > 0$$

If the later scenario is adopted by policy makers, then an increase in p or a reduction in $\sigma_p^2$ simply serves to reinforce the credit effects. The credit program and the prices program are capitalized into land values benefitting the existing owners while increasing the demand for land by the least financially secure firms.

But higher land prices even with a price support program are unlikely to improve the lot of the financially insecure firm. The only real clear winners are those financially secure firms who owned land prior to the credit or income stabilization program.

An alternative policy response may be to restrict acreage under production. If such were the policy, then V and x change, according to our model, as follows:

$$\frac{dV}{dN} < 0$$

and

$$\frac{dx}{dN} < 0$$

Reducing N would produce the opposite results as the above derivatives would suggest. Thus, any effort at supply control through acreage reduction would be at least partially offset with increased output per acre.

The capitalization of credit programs into land values adds a serendipitious element into farm firm survival. The benefits to a firm from buying or selling land are dependent on when financial policy changes are made. If credit is reduced and interest rates are increasing following a purchase, land values will fall and the firm's equity will be reduced. Opposite results will occur when credit is increased and interest rates fall.

These deductive results are included in our model by adding two new variables: $n_{oi}$ and $V_o$ are, respectively, land owned by the $i^{th}$ firm and land prices in the previous period. Capital gains (or losses) for the $i^{th}$ firm can then be written as: $n_{oi}(V-V_{oi})$.

The addition of capital gains to our model requires that expected income and variance of income for the $i^{th}$ firm be rewritten as:

$$\text{(11a)} \quad E(\tilde{y}) = (n_{oi} + n_i) [(p-\zeta_i(\alpha)) x_i - C(x)] - B_i + r [E_i + n_{oi}(V-V_o) - n_i V]$$

(11b) $\sigma^2(\tilde{y}) = (n_{oi}+n_i)^2 \; x^2 \; [\sigma_p^2-\delta_i(\alpha)]$

The firm optimizes its expected utility by choosing x and $n_i$ so that the equations below are satisfied:

(12) $x = x(r,V)$

and

(13) $$n_i = \frac{(p-\zeta_i)x - C(x) - rV}{\lambda_i \; x^2(\sigma_p^2 + \delta_i)} - n_{oi}$$

Except for $n_{oi}$ subtracted from the right-hand side of (13), the solutions are those expressed earlier in (6) and (7). Hence, the comparative static results remain the same except for the addition of:

$$\frac{dn_i}{dn_{oi}} < 0$$

Moreover, if $n_i$ is summed over all m firms so that $\sum_{i=1}^{m} (n_i + n_{oi}) = N$, then the aggregate model is also the same as before.

The impact of capital gains, then, would be of little or no consequence in the one time period model examined above. But in a dynamic model where the amount of credit extended to the firm depended on its liquidity measured by $\rho$ and its equity E, which includes capital gains from earlier periods, then the change in V has additional significance.

The importance is as follows. Suppose that in the current period there is an increase in $\alpha$, a decrease in r, an increase in p, or a decrease in $\sigma_p^2$, then V will increase the credit reserves of those fortuitous enough to have owned land in advance of land price increases.

Those most benefitted from the capital gains and increased credit would be, of course, those firms whose credit reserve is the most valued: the financially insecure firms. If these firms are the small family farms which policy is designed to protect, such may well be the result. If, on the other hand, the financially insecure firms are simply the poorly managed firms, then credit policies have the impact of increasing their ability to bid for resources at the expense of the well-managed firms.

Another extension of the model which might be considered is endogenizing p. Suppose that instead of p being a policy determined variable, that p depended on total output Nx. Then any change in V or r would also change x; for example, would decrease p which would in turn decrease V.

Thus a scenario could be: policy makers or credit institutions increase $\alpha$. This in turn increases the demand for land by the financially insecure firms which drives up V. An increase in V, in turn, increases the incentive to intensify production and x increases.

Increases in x and total output might then drive down p. But a decrease in p lowers V which has the result of offsetting the increase in V resulting from the original credit expansion. The overall effect may therefore be higher land prices and lower output prices, leaving neither the financially secure nor insecure firm necessarily better off.

These last results are examined deductively. This is achieved easily by adding a third equation, the market price equation, to (10a) and (10b). The market price equation is p = p(Nx), which is assumed to be signed as:

$$(14) \quad \frac{dp}{dx} = \frac{\partial p(Nx)}{\partial x} < 0$$

Combining (14) with (10a) and (10b), the results which are deduced in Appendix C are:

$$\frac{dV}{d\alpha} > 0,$$

$$\frac{dx}{d\alpha} > 0,$$

and

$$\frac{dp}{d\alpha} < 0.$$

The derivatives are as we predicted. An increase in credit results in an increase in land prices. An increase in land prices intensifies production and total output increases. Finally, increasing total output reduces product prices. The results of increased credit then may be mostly offset by a reduction in the output price.

One might wish to extend the scenario somewhat further. If output price decreases and output per acre, the cost of which is increasing, rises, then the financial condition of many firms may be destabilized. Moreover, with output and costs rising and firms facing financial difficulties, one might expect pressure to build on policy makers to stabilize or support output prices while at the same time attempting to restrict output, not realizing that at the heart of the problem was a generous credit program.

There is another result which could be inferred from our model. But this result requires an extension to our model. The extension recognizes that land is usually not traded in a monoculture-homogeneous market. The market for Iowa farm land is different from the Illinois, Michigan, or Wisconsin farm land markets. And these in turn are differentiated by quality of land, closeness to markets, and off-farm influences.

The question is, of course, which farm land market is the most vulnerable to swings in farm land prices resulting from either credit policy changes or price support programs? The answer provided by our deductive model is: the farm land market least diversified.

If diversification is motivated in part by portfolio effects which reduce risk, then the response to any credit or price support program targeted for a particular crop or output will be muted by the risk reduction benefits of diversification. Thus price support programs for corn in a widely diversified agricultural state like Michigan will have less effect on land prices than in an agriculturally less diversified state like Iowa.

## Summary and Conclusions

The effort in this paper was directed towards two tasks: (1) that of constructing an aggregate risk model; and (2) that of deducing implications from the model. These goals were realized by combining the modeling work completed in two earlier papers.

The deductive implications of the model were interesting. As one might expect upon reflection, credit extensions benefit most those who have the least. The second round effects of credit policies are less obvious and depend on whether or not prices are administered or market determined. If prices are administered, then increases in credit or lowering of interest rates earn capital gain for those fortunate to hold land and increase total output.

If, on the other hand, product prices are determined in the market, then intensifying production as a result of land price increases may be offset by a reduction in the output price. Thus one might observe falling prices and increasing output, both of which might tend to intensify pressures on policy makers to stabilize prices and restrict output.

Finally, the model can be used to predict that land price sensitivity to changes in credit and interest rates will be determined by the diversity of land uses. A monocrop culture will be the most sensitive to changes in credit and price policies.

In 1971, the Farm Credit System allowed Federal land banks, the nation's largest farm real estate lender, increased credit as a percent of appraised valuation from 68 percent to 85 percent. This paper's result would suggest that this action and other similar credit policy changes could have had a significant effect. This hypothesis, however, needs an empirical test.

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# APPENDIX A

The first-order conditions to the firm's optimization problem are:

$$\text{(A-1)} \quad \frac{dy_{cE}}{dx_i} = [(p-\zeta_i) - C'(x)] - \lambda_i \; n_i x(\sigma_p^2 + \delta_i) = 0$$

and

$$\text{(A-2)} \quad \frac{dy_{cE}}{dn_i} = [(p-\zeta_i)x - C(x)] - rV - \lambda n_i x^2(\sigma_p^2 + \delta_i) = 0$$

Dividing the two equations after moving the variance term to the right-hand side of the equation results in the expression:

$$\text{(A-3)} \quad (p-\zeta_i) - C'(x) = [(p-\zeta_i)x - C(x) - rV]/x$$

from which is obtained:

$$\text{(A-4)} \quad x = x(r,v)$$

Substituting for x in (A-2) and solving for $n_i$ results in equation (7):

$$\text{(A-5)} \quad n_i = \frac{(p-\zeta_i)x - C(x) - rV}{\lambda_i x^2(\sigma_p^2 + \delta_i)}$$

The second-order conditions for equations (A-1) and (A-2) are guaranteed by the quadratic nature of the certainty equivalent expression and the assumption that marginal cost, C'(x), is increasing.

# APPENDIX B

The comparative static results from the aggregate model can be obtained by expressing (10a) and (10b) in matrix notation:

$$\begin{bmatrix} 1 & -\dfrac{\partial x}{\partial V} \\ \displaystyle\sum_{i=1}^{m} \left\{ \frac{[(p-\zeta_i)-C'(x)]-[(p-\zeta_i)x-C(x)-rV]}{(..)} \right\} & -\displaystyle\sum_{i=1}^{m} \frac{r}{(..)} \end{bmatrix} \begin{bmatrix} dx \\ dV \end{bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial r}\, dr \\ -x \left[ \displaystyle\sum_{i=1}^{m} \frac{1}{(..)} \right] dp + \left[ \displaystyle\sum_{i=1}^{m} \frac{r}{(..)} \right] dr + dN + \left[ \displaystyle\sum_{i=1}^{m} \frac{(..)x\frac{\partial \zeta_i}{\partial \alpha} + (.)\lambda_i x^2 \frac{\partial \delta_i}{\partial \alpha}}{(..)^2} \right] d\alpha + \left[ \displaystyle\sum_{i=1}^{m} \frac{(.)\lambda_i x^2}{(..)} \; d\sigma_p^2 \right] \end{bmatrix}$$

Comparative static results for the aggregate problem depend on the sign of the determinant $|D|$ which multiplies dx and dV. The terms in the right-hand column of the determinant are unambiguously negative. The sign of the lower left-hand element is inferred from (A-3) to be negative. The resulting sign of the determinant therefore is: negative or $|D| < 0$.

Cramer's rule can now be used to obtain comparative static results by setting all of the terms dr, dp, $d\alpha$, and $d\sigma_p^2$ but one equal to zero.

# APPENDIX C

The price for output can be endogenized by including the result that

$$dp = \frac{\partial p(N,x)}{\partial x} dx + \frac{\partial p(N,x)}{\partial N} dN$$

in the 2 x 2 matrix found in Appendix B. Including this result adds one row and column to the matrix in Appendix B. The determinant $|D|$ of the new 3 x 3 matrix can be shown to be negative for reasons given in Appendix B. Then by setting all but one of the terms dr, $d\alpha$, and $d\sigma_p^2$ equal to zero and solving for dx, dV, and dp one at a time using Cramer's rule, the comparative static results reported in the text are obtained.