FORWARD AND FUTURES MARKETS AND
THE COMPETITIVE FIRM UNDER PRICE UNCERTAINTY

by Frances Antonovitz and Ray D. Nelson*

Introduction

Recent remedies for managing the output price risk faced by a competitive firm sometimes include the prescription of hedging. This practice usually entails combining spot market sales with trading opportunities in forward or futures markets. The forward hedge represents a risk free price. The futures hedge offers a risky alternative which arises because of basis, the variable relationship between the spot and futures quotations. Rather than treating forward and futures as mutually exclusive or as perfect substitutes, a competitive firm can carefully construct a portfolio which combines spot, forward, and futures positions.

Holthausen [1979] and Feder, Just, and Schmitz [1980] (hereafter FJS), initiate an extensive discussion of a risk-averse firm which uses futures contracts when faced with an uncertain output price but no basis risk. Both articles employ general utility and density functions to derive their results. Their conclusions include independence of the production decision from the probability density of spot price and the firm's degree of risk aversion. Extensions of these two articles usually focus either on the robustness of the separation conclusion to the addition of basis or the addition of production uncertainty to the models.

The risk free characteristic that Holthausen and FJS attribute to futures contracts really better describes a forward contract. Jarrow and Oldfield [1981], Paul et. al. [1976], and many others document the importance of recognizing the unique characteristics of these two different types of contracts. Batlin [1983] builds on the Holthausen and FJS foundation by adding basis risk to his model.

FJS explicitly qualify their model as applicable to only those commodities with little or no production uncertainty. Subsequent articles augment their analysis with the condition of stochastic production. Chavas and Pope [1982], Anderson and Danthine [1983], Marcus and Modest [1984], Ho [1984], and Grant [1985] all include production uncertainty in different permutations of the fundamental model. Those which simultaneously include both basis and production risk achieve analytical solutions by assuming specific utility or density functions.

Although many recognize the difference between forward and futures contacts, most do not allow decision makers to use both alternatives. Recent models which do consider the full range of trading opportunities include Hildreth [1984], Kawai and Zilcha [1986], and Paroush and Wolf [1986].

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Hildreth distinguishes his contribution by recognizing that significant illiquidity, prohibitive transactions costs, margin requirements, and lending institution constraints may also cause a firm to restrict forward and futures positions to hedging related sales.

This analysis also includes the full range of marketing alternatives in the feasible decision set of the competitive firm. The investigation employs a general utility and density function framework to consider optimal production and marketing decisions. The results contribute to the theory of the firm in the following two ways. First, the methodology decomposes the entire production and marketing decision problem into a sequence of more manageable modules. Second, the two models presented herein document the effects of different sets of restrictions on marketing and production decisions.

General Model Description

The general model postulates a risk-averse firm selling output from a deterministic production process in forward, futures, and spot markets. The firm views production and marketing decisions from a two period perspective. The production decision must occur at time t-1. At that stage, the firm can simultaneously assume buying and selling commitments in forward and futures markets. If neither forward or futures prices offer sufficiently attractive returns, the firm can elect to wait until period t and sell the entire output in the spot market.

The formal representation of this decision process begins by assuming the firm maximizes a von Neumann-Morgenstern utility function U defined on profit π. Consistent with the risk aversion assumptions, $U'(\pi) > 0$ and $U''(\pi) < 0$. The model postulates that the firm produces a positive output Q at cost C(Q). This cost function manifests the normal attributes of positive and increasing marginal cost, which requires that $C'(Q) > 0$ and $C''(Q) > 0$.

As mentioned previously, the decision makers can trade in three different markets. The first alternative arises in the forward market where the firm can buy or sell forward contracts R at the known price P^R in period t-1. This generates forward market revenues of $P^R \cdot R$.

In the second alternative, the firm engages in a combination of futures and spot market transactions. At time t-1, the firm can buy or sell a quantity F at the known futures price P^F_{t-1}. Associated with this original position are two different market transactions which occur at time t. Because delivery does not occur with the majority of futures contracts, futures trades imply offsetting transactions at the uncertain futures price P^F_t in period t. At this time, an associated transaction occurs in the spot market. This means an original futures market sale, $F > 0$, at time t-1 requires a subsequent futures purchase and spot market sale at time t. An original futures market purchase, $F < 0$ requires a corresponding futures sale and spot purchase. Thus, the unit revenue from the futures position is the futures price at t-1 plus the basis at time t. The notation which represents the revenue from this choice is $[P^F_{t-1} + (P^S_t - P^F_t)] * F$.

In the third alternative, the output not associated with forward and futures trades constitutes an open spot position. This quantity defined as $NRF = Q - F - R$ sells at the unknown spot price of P^S_t in period t. In the event that production exceeds forward and futures sales, the resulting positive value for NRF represents a spot market sale at the price P^S_t. When forward and futures sales exceed production, satisfaction of forward contract obligations at time period t may require spot market purchases signified by $NRF < 0$. This gives a revenue (cost) of $NRF * P^S_t$ in the spot market.

Expressing the revenues and costs generated by the above activities in an expected utility framework gives the following objective function[1]:

$$(1) \quad \max_{NRF,F,R} EU(\pi) = EU[P^S_t \bullet NRF + (P^F_{t-1} + P^S_t - P^F_t)F + P^R \bullet R - C(NRF + F + R)].$$

The following three assumptions define important stochastic relationships among the prices and profits in (1). First, because Just and Rausser [1981] show that futures prices generally outperform commercial models as price forecasters, the firm accepts P^F_{t-1} as the best predictor of P^F_t[2]. This means that $EP^F_t = P^F_{t-1}$. It also implies that the firm′s best forecast of the basis at time t is $P^F_{t-1} - EP^S_t$. Second, the firm regards the bivariate exponential family of density functions as representative of the stochastic relationships between π and P^S_t or P^F_t. Third, because spot and futures prices generally tend to fluctuate in the same directions, they are assumed to be positively, although imperfectly, correlated.

The futures revenue definition along with the assumption that $EP^F_t = P^F_{t-1}$ gives the futures position a special risk management function within the portfolio of within the portfolio of marketing alternatives. This risk management role arises because the expected revenue from the spot market exactly equals that of the futures position. The following instances illustrate the interaction which maintains equality between the expected revenues. First, an increase in the expected spot price causes an identical change in the expected basis part of the per unit futures revenue. Second, the expected futures price cannot increase without a corresponding increase in the current futures. The two effects net each other out which causes the per unit expected revenue generated by a futures position to remain the same. The spot and futures position therefore share a common level of expected revenue but differ in their levels of risk.

Two models are considered which utilize the above general framework. The first examines the unconstrained maximization of the objective function. In other words, the firm can buy or sell any number of forward and/or futures contracts. In a second model, the firm can only sell forward and futures contracts as long as the sum of these sales does not exceed production. This precludes the utilization of these contracts as instruments of speculation and insures their use in a hedging role.

Unconstrained Forward and Futures Marketing

In the first model, a well-capitalized firm sells output from a deterministic production process in forward, futures, and spot markets. No financial or institutional restrictions constrain the decision maker's marketing alternatives. Differentiating (1) gives the following first-order conditions:

$$\frac{\partial EU(\pi)}{\partial NRF} = E[U'(\pi)(P_t^S - C'(Q))] = 0 \tag{2}$$

$$\frac{\partial EU(\pi)}{\partial F} = E[U'(\pi)(P_{t-1}^F + P_t^S - P_t^F - C'(Q))] = 0 \tag{3}$$

$$\frac{\partial EU(\pi)}{\partial R} = E[U'(\pi)(P^R - C'(Q))] = 0. \tag{4}$$

These conditions allow the decomposition of decisions into three stages. In the first, the firm makes a production decision. In the second, the firm makes initial marketing decisions based on the relationship between the forward and expected spot prices. In the final stage, risk management considerations motivate the firm in determining the optimal open spot position.

Unrestricted Stage 1--Production Decision

As mentioned previously, Holthausen and FJS show that without basis risk, the production and marketing decisions separate. This means that the firm chooses that level of output for which marginal cost equals the certain futures price. As Batlin and others have subsequently shown, the existence of basis risk subverts separation. However, the inclusion of a forward market with its nonstochastic contract price produces separability once again which leads to the first conclusion:

Result 1

With no restrictions on forward and futures trading, the production and marketing decisions separate. The firm produces where the forward price equals the marginal cost of production.

This result comes directly from equation (4) which simplifies to $(P^R - C'(Q))EU'(\pi) = 0$ because P^R and $C(Q)$ are nonstochastic. Because marginal utility is assumed to always be positive, this equation holds when $P^R = C'(Q)$. Hence, the firm determines the optimal output Q^* independent of the degree of risk aversion or the probability distributions of uncertain prices. This matches the conclusions of similar derivations by Hildreth; Kawai and Zilcha; and Paroush and Wolf.

Unrestricted Stage II--Initial Forward and Futures Marketing Decisions

After determining optimal production, the firm makes its marketing decisions by choosing spot, forward, and futures positions. The size of these positions should depend on the relative magnitudes of the prices in the three alternative markets. This conclusion doesn't apply to the relationship between the current and expected futures prices, however, because of the assumption that $EP_t^F = P_{t-1}^F$. This means that the decision maker expects no futures trading profits and uses futures primarily as a risk management tool in the portfolio of marketing alternatives.

The three possible relationships between the forward and expected spot prices cause the firm to choose different marketing strategies. In the first relationship when the forward price exceeds the expected spot price, the firm should sell more in the forward market than it produces. Also, given that the unit expected revenues in both the spot and futures markets are low compared to the forward market, the firm not only becomes a net buyer in the spot market in period t in order to satisfy the demands of the forward contract, but also buys futures contracts.

In the second possible relationship where the expected spot just equals the forward price, the forward contract is more attractive because it offers the risk-averse decision maker a riskless marketing alternative with the same expected return as the risky alternative. Thus, the decision maker would trade no futures and would forward contract the entire planned production.

The final and most likely possibility occurs when the forward price falls below the expected spot price so that the spot and futures markets offer greater expected return but with an accompanying increase in risk. In this case, the firm attempts to gain the higher unit revenue offered by the futures market by selling futures. It may, however, reduce the added risk by covering a portion of production with forward contract sales or may even buy forward contracts.

Completeness encourages consideration of all three of the above price relationships. Arbitrage forces, however, would undoubtedly prevent the sustained existence of the first. The attractive forward price would encourage many producers to forward contract all their output and anticipate spot purchases at their projected delivery date. The forward sales and anticipated subsequent purchases would drive the forward price and expected spot price toward equality. No similar arbitrage forces exist for situation three.

The following formal conclusions summarize these futures and forward commitments:

Result 2

(i) A long position is taken in futures and a short position exceeding production is taken in the forward market if and only if P^R exceeds EP^S_t.

(ii) No position is taken in futures and all production is covered by a short position in the forward market if and only if P^R equals EP^S_t.

(iii) A short position is taken in futures and a short position smaller than production or a long position is taken in the forward market if and only if EP^S_t exceeds P^R.

These results follow from the first-order conditions. Subtracting equation (2) from (3) and expanding the resulting expression gives:

$$EU'(\pi)\ E(P^F_{t-1} - P^F_t) - Cov(U'(\pi),\ P^F_t) = 0.$$

But by the assumption of equal current and expected futures prices, the covariance term equals zero. The assumed exponential family representation of the bivariate probability density of π and P^F_t allows use of Stein's Theorem[3] which can be applied to the covariance term to give:

$$Cov(U'(\pi),\ P^F_t) = EU''(\pi)\ [\sigma_{SF}(NRF + F) - \sigma^2_F\ F] = 0$$

where σ^2_F, σ^2_S, and σ_{SF} represent the variances and covariances of spot and futures prices. Since $EU''(\pi) < 0$, the expression in brackets must equal zero and can be rearranged to obtain:

$$\frac{F}{NRF + F} = \frac{\sigma_{SF}}{\sigma^2_F}. \qquad (5)$$

Subtracting equation (2) from (4) and expanding gives:

$$EU'(\pi)\ E(-P^S_t + P^R) - Cov(U'(\pi),\ P^S_t) = 0.$$

Rearranging and applying Stein's Theorem yields:

$$(6) \qquad P^R - EP_t^S = \frac{EU''(\pi)\ (\sigma_S^2\ (NRF + F) - \sigma_{SF}F)}{EU'(\pi)}.$$

Using equation (5) and substituting into (6) for (NRF + F) gives:

$$P^R - EP_t^S = \frac{EU''(\pi)}{EU'(\pi)}\ [F(\sigma_F^2\ \sigma_S^2 - \sigma_{SF}^2)].$$

By the Cauchy-Schwartz inequality, $\sigma_F^2\ \sigma_S^2 > \sigma_{SF}^2$. Since the first-order conditions are both necessary and sufficient, it can be concluded that:

$$P^R \gtrless EP_t^S \Longleftrightarrow F \lessgtr 0.$$

To determine the relationship between output and forward marketing positions, equation (5) can be used to substitute for F in equation (6):

$$P^R - EP_t^S = \frac{EU''(\pi)}{EU'(\pi)}\ (NRF + F)\ \frac{(\sigma_S^2\ \sigma_F^2 - \sigma_{SF}^2)}{\sigma_F^2}.$$

Noting that Q - R = NRF + F, it can be established that:

$$P^R \gtrless EP_t^S \Longleftrightarrow R \gtrless Q.$$

Unrestricted Stage 3--Determination of the Open Spot Position

In the third stage of the decision process, the firm must determine the optimal size of NRF, the open spot position. Because of the equality between the expected revenues from spot and futures positions, the relative risks from the respective positions contribute the key conditions for determining NRF.

The determination of the open spot position closely parallels the results of models without forward contracts specified in a linear mean-variance expected utility framework. Ederington (1979) uses a risk minimization argument to derive the following expression for the optimal hedge ratio:

$$(7) \qquad \frac{F}{Q} = \frac{\sigma_{SF}}{\sigma_F^2}\ .$$

Kahl (1983) points out that this result is equivalent to the optimal hedge ratio when the firm maximizes expected utility and expected profit from holding a futures position is assumed to equal zero ($P^F_{t-1} = EP^F_t$). Under these conditions, the firm would want to increase the futures position as long as the increase due to the variance of the futures price does not exceed the decrease in risk due to the covariance between futures and spot prices. Since the covariance usually does not attain the magnitude of the variance, the hedging ratio normally does not exceed one.

A comparison of expression (5) and (7) establishes the similarity between the results of the present general utility framework and a linear mean-variance model. Both equations represent the ratio of the futures position to the amount of production not committed through forward sales. Forward contracts are omitted from the models used to derive equation (7). Imposing a similar condition on equation (5) means that $R = 0$ and $NRF + F = (Q - R - F) + F = Q$. Thus, in the model including both forward and futures markets, the relative magnitudes of σ^2_F and σ_{SF} play an equally important in determining the size of NRF.

<u>Result 3</u>

Whenever the forward price exceeds the expected cash price,

(i) a net short spot position occurs if and only if σ_{SF} exceeds σ^2_F.

(ii) no open spot position occurs if and only if σ_{SF} equals σ^2_F.

(iii) a net long spot position occurs if and only if σ^2_F exceeds σ_{SF}.

Whenever the expected spot price equals the forward price, no open cash position occurs.

Whenever the expected spot price exceeds the forward price,

(i) a net long spot position occurs if and only if σ_{SF} exceeds σ^2_F.

(ii) no open spot position occurs if and only if σ_{SF} equals σ^2_F.

(iii) a net short spot position occurs if and only if σ^2_F exceeds σ_{SF}.

This result can be most easily seen by rearranging equation (5) as:

$$\sigma_{SF}\ NRF = (\sigma^2_F - \sigma_{SF})\ F.$$

Under the assumption of positively correlated spot and futures prices, the following conclusions can be drawn:

(i) If $F > 0$, then $NRF \gtrless 0 \Leftrightarrow \sigma_{SF} \lessgtr \sigma_F^2$;

(ii) $F = 0 \Leftrightarrow NRF = 0$;

(iii) If $F < 0$, then $NRF \gtrless 0 \Leftrightarrow \sigma_{SF} \gtrless \sigma_F^2$.

Unrestricted Model--Decision Tree Summary

The diagram found in Figure 1 summarizes the flow of decisions in the unrestricted forward and futures trading model. The decision maker following this diagram would first make the production decision by equating the marginal cost of production with the forward price. The decision at the next stage hinges on a comparison of the forward and expected spot prices. Since arbitrage would usually preclude the forward quote from exceeding the expected cash price, the top branch would usually not represent a feasible branch for the firm to enter. If the expected cash exceeds the forward price, then a positive futures position results and forward sales do not exceed production. The third stage conditions compare the variance of futures with the covariance between futures and spot prices. Because the firm would usually find that the variance exceeds the covariance, the firm would normally maintain an open spot position.

Because causal empirical observation does not reveal substantial numbers of producers freely speculating in forward and futures markets, the next model considers the impact of restricting marketing alternatives to a hedging orientation.

Restricted Forward and Futures Trading

The possible illiquidity and high transactions costs of the forward market may cause the producer to limit forward contracts to only sales which do not exceed production. Trading restrictions specified in loan agreements, larger margin requirements for speculative positions, and personal trading preferences often cause decision makers to likewise constrain futures trading to hedging activities. This precludes buying either forward or futures contracts. It also proscribes Texas hedging or combined short commitments in excess of production.

Formal implementation of these restrictions in the final model requires nonnegative values for F, R, and NRF. Imposing these constraints on the objective function found in (1) gives the following Kuhn-Tucker first-order conditions:

$$(8) \quad \frac{\partial EU(\pi)}{\partial NRF} = E[U'(\pi)(P_t^S - C'(Q))] \leq 0 \qquad \frac{\partial EU(\pi)}{\partial NRF} \cdot NRF = 0$$

$$(9)\quad \frac{\partial EU(\pi)}{\partial F} = E[U'(\pi)(P^F_{t-1} + P^S_t - P^F_t - C'(Q))] \leq 0 \qquad \frac{\partial EU(\pi)}{\partial R} \bullet F = 0$$

$$(10)\quad \frac{\partial EU(\pi)}{\partial R} = E[U'(\pi)(P^R - C'(Q))] \leq 0 \qquad \frac{\partial EU(\pi)}{\partial R} \bullet R = 0.$$

For the optimal futures and open cash positions, the conclusions of this restricted model closely match those previously obtained from the unrestricted model. The imposition of constraints on marketing positions does, however, significantly alter the forward and production decisions. These constraints also change the sequence of the decision process.

The decomposition of the decision making process for the restricted model involves five principle stages. Separation between the production and marketing decisions now occurs only under specific conditions. This means that the firm cannot initially determine production and then make marketing decisions. In the first stage for the restricted model, the firm compares the forward and expected spot prices to determine the feasibility of trading futures contracts. If the forward price is at least as great as the expected spot price, all production will be forward contracted. When the expected spot exceeds the forward price, then the firm enters into the second stage of a more complex chain of decisions. In this second stage decision, the firm compares the variance of futures with the covariance between spot and futures to consider the desirability of an open spot position. After this judgment, the firm proceeds to the third stage where it makes preliminary marketing decisions by temporarily excluding forward trading. Since these proposed marketing commitments require a commensurate output, in the fourth stage the firm judges the feasibility of producing the quantities needed to satisfy the preliminary marketing plans. Finally, in the fifth stage, the firm revises its preliminary plans into final production and marketing commitments with the possibility of including forward trading.

Restricted Stage 1--Feasibility of Futures Trading

As in the case of the unconstrained model, the forward price serves as a riskless benchmark useful for making comparisons with risky alternatives. Once again the relationship between the forward and expected cash price implies different strategies for the firm. In the first possible relationship, the forward exceeds the expected cash price. In this instance the forward market offers a higher return with a smaller level of risk than all other marketing alternatives. The dominance of the forward opportunity converts the problem to a fundamental decision under certainty wherein the firm forward contracts to deliver an amount which exactly equates marginal cost of production with the known forward price. However, if expected cash price exceeds the forward price, the spot and futures markets offer greater expected returns and a position in the futures market may be attractive.

Result 4

i) A short position is taken in the futures market if and only if EP^S_t exceeds P^R.

ii) All production is forward contracted and the firm produces where the forward price equals the marginal cost of production if and only if P^R is at least as great as EP_t^S.

The proof of part i) begins by considering that if $F^* > 0$, equation (9) holds with equality while (8) and (10) may not. Subtracting (9) from (10) and expanding using Stein's Theorem gives:

$$(11) \qquad P^R - EP_t^S \leq \frac{EU''(\pi)}{EU'(\pi)} [NRF\ (\sigma_S^2 - \sigma_{SF}) + F(\sigma_S^2 - 2\ \sigma_{SF} + \sigma_F^2)].$$

Because the expression $(\sigma_S^2 - 2\ \sigma_{SF} + \sigma_F^2)$ equals the variance of the difference between spot and futures prices, its value is always positive.[4] If $NRF^* = 0$, the right hand side of expression (11) will be negative since $EU'(\pi) > 0$ and $EU''(\pi) < 0$; and hence, $P^R < EP_t^S$. This same relationship holds when $NRF^* > 0$. In this instance, equations (8) and (9) are both equalities so that equation (5) again holds and can be used to substitute for NRF in (11) to give

$$(12) \qquad P^R - EP_t^S \leq \frac{EU''(\pi)}{EU'(\pi)} [F\ (\frac{\sigma_F^2\ \sigma_S^2 - \sigma_{SF}^2}{\sigma_{SF}})].$$

By the Cauchy-Schwartz inequality and the assumption of positive covariance, the right hand side of (12) is negative, so $P^R < P_t^S$. Because the Kuhn-Tucker first-order conditions are both necessary and sufficient, the following conclusion can be drawn: $F^* > 0 \Leftrightarrow P^R < P_t^S$. Clearly, if this relationship between forward and expected spot prices does not hold (i.e. $P^R \geq EP_t^S$), it cannot be true that $F > 0$ and hence, $F^* = 0$. Thus, $F^* = 0 \Leftrightarrow P^R \geq P_t^S$.

The proof of part ii) of Result 4 begins by considering the condition where $NRF^* > 0$. This makes equation (8) an equality while (9) and (10) may be inequalities. Simplifying and rearranging the difference between (9) from (8) yields:

$$(13) \qquad \sigma_{SF}\ NRF + (\sigma_{SF} - \sigma_F^2)\ F \leq 0.$$

However, if $F^* = 0$, σ_{SF} must be nonpositive which violates the assumption of a positive covariance. Hence, if $NRF^* > 0$, then $F^* > 0$. The contrapositive also holds so if $F^* = 0$, then $NRF^* = 0$.

The preceding results mean that the condition wherein the forward price exceeds or equals the expected spot price greatly simplifies production and marketing decisions. In this case both futures and spot markets offer inferior revenues with greater risk relative to the forward price. This allows the firm to safely exclude futures and spot marketing alternatives from its decision set. This exclusion, in conjunction with the assumption of some positive production, implies a positive R^*. Positive forward marketing in turn causes (10) to hold with equality which means that $C'(R^*) = P^R$. Thus, $P^R \geq EP_t^S$ generates separation between the production and marketing decisions. As explained previously, however, arbitrage forces will probably preclude such a simplification of decisions by preventing the forward price from exceeding the expected spot price.

In the more likely condition where expected spot price exceeds the forward price, the firm should assume a short futures position. This leads to the second stage of the restricted decision model.

Restricted Stage 2--Feasibility of Open Cash Position

The decisions of the second stage focus on the open spot position. Consistent with the intuition and discussion presented in conjunction with Result 3 for the unconstrained model, the relationship between the variance of futures price and the covariance between cash and futures prices strongly influences NRF.

Result 5

Whenever the expected spot price exceeds the forward price,

i) no open cash position occurs if and only if σ_{SF} equals or exceeds σ_F^2.

ii) a net short cash position occurs if and only if σ_F^2 exceeds σ_{SF}.

The proof of Result 4 establishes that when $NRF^* > 0$, then $F^* > 0$. Positive NRF^* and F^* require that equations (8) and (9) hold with equality. This in turn implies that (13) is an equality. The equality in (13) means that when NRF^* and F^* are positive, σ_{SF} must exceed σ_{SF}. Hence, $NRF^* > 0 \Leftrightarrow \sigma_{SF} < \sigma_F^2$. Because the first-order conditions are both necessary and sufficient, it can also be concluded that $NRF = 0 \Leftrightarrow \sigma_{SF} \geq \sigma_F^2$.

Restricted Stage 3--Exclusion of Forward Trading

In the third stage, the firm temporarily excludes forward trading from consideration. This simplification combined with the feasibility of an open spot position determined in Stage 1 allows the decision maker to enter two alternative preliminary marketing and production decision processes. In the first set of preliminary calculations, the variance of futures does not exceed the covariance between futures and spot. This means that the firm need only estimate its futures position $\tilde{F}$. The second set of preliminary calculations occurs when the excess of the variance of the futures over the covariance between the futures and spot prices prescribes positive values for both F and NRF. In this situation the firm determines preliminary estimates for futures $\hat{F}$ and open spot $\hat{NRF}$ positions.

Consider the first set of preliminary calculations when $F^* > 0$ and $NRF^* = 0$. In this instance, equation (9) holds with equality. By temporarily ignoring the forward market and setting R = 0, an estimate of the futures position $\tilde{F}$ can be determined by solving (9):

$$(14) \qquad EP_t^S - C'(\tilde{F}) = - \frac{EU''(\pi)}{EU'(\pi)} (\sigma_S^2 - 2\,\sigma_{SF} + \sigma_F^2)\,\tilde{F}.$$

In the second set of preliminary calculations when $F^* > 0$ and $NRF^* > 0$, equations (8) and (9) hold with equality. Temporary exclusion of the forward market by requiring that R = 0 generates estimates for futures sales $\hat{F}$ and open spot $\hat{NRF}$. These values are obtained by rearranging (8) and (9) and then solving the resulting equations:

$$(15) \qquad \begin{aligned} &\sigma_{SF}\,\hat{NRF} + (\sigma_{SF} - \sigma_F^2)\,\hat{F} = 0 \\ &EP_t^S - C'(\hat{NRF}+\hat{F}) = - \frac{EU''(\pi)}{EU'(\pi)} [\hat{NRF}\,(\sigma_S^2-\sigma_{SF}) + \hat{F}\,(\sigma_S^2-2\,\sigma_{SF}+\sigma_F^2)]. \end{aligned}$$

The preliminary estimates for F and NRF next allow the firm to evaluate the feasibility of forward contracting in the fourth stage of the decision process.

<u>Restricted Stage 4--Feasibility of Forward Trading</u>

The possible reintroduction of forward trading occurs by comparing the marginal cost of output required to satisfy the preliminary futures and open positions calculated in Stage 3. When the marginal cost exceeds the forward price, the firm should make no forward market commitment because the additional cost of greater production would exceed the return from the forward market. Result 6 formally states these conditions.

<u>Result 6</u>

The firm can exclude forward markets from its decision set and conclude that the preliminary estimates of F and NRF are optimal whenever:

i) the futures commitment is positive, the open spot position equals zero, and the marginal cost of producing $\tilde{F}$ exceeds the forward price.

ii) the futures commitment is positive, the open spot position exceeds zero, and the marginal cost of producing $\hat{NRF}$ and $\hat{F}$ exceeds the forward price.

The proof of part i) begins by remembering that the first-order conditions are both necessary and sufficient. If $\tilde{F}$ is indeed optimal, equation (10) must be satisfied at $\tilde{F}$. Hence, it can be concluded that

$$C'(\tilde{F}) \geq P^R \iff NRF^* = R^* = 0 \text{ and } F^* = \tilde{F}$$

$$C'(\tilde{F}) < P^R \iff F^* \neq \tilde{F} \text{ and } R^* > 0.$$

This result comes directly from the equality of equation (10) when $R^* > 0$. This equation becomes an inequality when $R^* = 0$ so that separability no longer holds and equation (14) generates the optimal value for F.

The second part of Result 6 also depends on the first-order conditions being both necessary and sufficient. Equation (10) must be satisfied when $\hat{F}$ and $\hat{NRF}$ are optimal. If the inequality of (10) does not hold at $\hat{F}$ and $\hat{NRF}$, these values are not optimal and, hence, $R^* > 0$. In summary,

$$C'(\hat{F} + \hat{NRF}) \geq P^R \iff F^* = \hat{F},\ NRF^* = \hat{NRF}, \text{ and } R^* = 0$$

$$C'(\hat{F} + \hat{NRF}) < P^R \iff F^* \neq \hat{F},\ NRF^* \neq \hat{NRF}, \text{ and } R^* > 0.$$

Restricted Stage 5--Final Production and Marketing Decisions

If the comparison of marginal cost with the forward price establishes the feasibility of forward trading, production and marketing decisions once again separate. Result 7 reports the procedure for determining the final production and marketing decisions with feasible forward trading.

Result 7

With a positive forward position, the production and marketing decisions separate. The production decision does not depend on the degree of risk aversion or subjective distributions of uncertain futures and spot prices. Risk aversion and probability distributions do, however, influence the optimal marketing combination.

In order to prove the generality in Result 7, the two different situations where $NRF^* = 0$ and $NRF^* > 0$ require consideration. First, when $NRF^* = 0$, $F^* > 0$, and $R^* > 0$, both equations (9) and (10) hold with equality so that optimal values of F and R can be determined by:

$$EP_t^S - C'(F^*) = - \frac{EU''(\pi)}{EU'(\pi)} [(\sigma_S^2 - 2\ \sigma_{SF} + \sigma_F^2)\ F^*$$

$$C'(R^* + F^*) = P^R.$$

If $NRF^* > 0$, $F^* > 0$, and $R^* > 0$, equations (9)-(11) hold with equality so that optimal values of F, NRF, and R can be determined by:

$$\sigma_{SF}\ NRF^* + (\sigma_{SF} - \sigma_F^2)\ F^* = 0$$

$$C'(R^* + NRF^* + F^*) = P^R$$

$$EP_t^S - C'(NRF^* + F^*) = - \frac{EU''(\pi)}{EU'(\pi)} [NRF^*(\sigma_S^2 - \sigma_{SF}) + F^*(\sigma_S^2 - 2\ \sigma_{SF} + \sigma_F^2)]$$

Restricted Model--Decision Tree Summary

The diagram in Figure 2 summarizes the flow of decisions in the restricted trading model. In the first stage, the firm compares the forward price with the expected spot price to evaluate the feasibility of trading futures contracts. When the forward price exceeds the expected spot price, the firm simply finds the quantity which equates marginal cost of production with the forward price. The firm then sells this entire output in the forward market. When the expected spot price exceeds the forward price, the firm concludes that a short futures position is optimal. This leads to the second decision stage wherein the firm must decide whether to leave some output unhedged. If the covariance between futures and spot prices exceeds the variance of futures prices, then the firm should hedge its entire output. Otherwise, the firm should maintain some output unhedged. In the third stage, the firm temporarily excludes forward markets from consideration and determines preliminary futures and open spot positions. In the fourth stage, the firm reintroduces the possibility of forward trading. It compares the marginal cost of producing the preliminary futures and open spot quantities. When the marginal cost of this output exceeds the forward price, the firm excludes the forward market from further consideration and simply produces and markets the preliminary quantities determined in Stage 3. In the case where the forward price exceeds the marginal cost of production, the firm then enters a Stage 5 decision. Feasible forward trading causes the marketing and production decisions to separate and the marketing decisions to depend on the firm's degree of risk aversion and beliefs regarding the probability densities of uncertain prices.

Summary and Conclusions

By including a full range of marketing alternatives in the feasible decision set of the competitive firm, both unrestricted and restricted models show that forward and futures contracts should play an important role in the marketing and production decisions of a competitive risk averse firm. In both models, the forward price serves as a risk free benchmark which finds particular value in judging other marketing alternatives. Under a variety of conditions, the inclusion of the forward price in the model allows the separation of the production and marketing decisions. The addition of futures trading opportunities does not alter the expected profit of the firm but plays a significant risk management role. Few market conditions prescribe that futures positions be excluded from the firm's decision set.

Adding forward and futures trading opportunities to the model of a firm making decisions under uncertainty may seem an undue complication. However, the methodology allows decomposition of the entire production and marketing decision problem into a sequence of more manageable modules. The firm can follow a step by step procedure through the different branches of decision trees as it tests critical conditions.

Restraining marketing decisions to hedging leaves many of the conclusions from the unrestricted model remarkably intact. Although separation between production and marketing is not universal in the restricted model, it can still occur. The firm should still forward contract the entire production when the forward price exceeds the expected spot price. The relationship between the variance of futures and covariance of futures and spot prices still strongly influences the hedging decision in the restricted model. This matches the often quoted Ederington results and replicates the conditions for the unrestricted model.

All conclusions evolve from a general utility framework which assumes the exponential family of density functions adequately represents uncertain elements in the model. This degree of generality allows some new insights into the theory of the competitive firm under uncertainty. However, these result apply only to firms similar to those considered by FJS which experience at most insignificant production uncertainty. The addition of this dimension to the model represents a further and future contribution.

Footnotes

1. As stated in Hildreth [1984], the following conditions guarantee the strict convexity of $EU'(\pi)$. First, two continuous derivatives of $U(\pi)$ exist with $U'(\pi) > 0$, $U''(\pi) < 0$, and $\lim_{\pi \to \infty} U'(\pi) = 0$. Second, appropriate expectations exist which insure that $U(\pi)$ is twice differentiable under the expectation. Third, the cost function $C(Q)$ is strictly convex. These conditions mean that a unique maximum exists and first-order conditions are both necessary and sufficient.

2. This assumption does not imply that futures markets are efficient but that the firm views them as efficient. The firm simply decides that it cannot outguess the futures market.

3. Stein's theorem [1973], which was also derived independently by Rubinstein [1976], states that if X and Y follow a bivariate normal distribution and $g(Y)$ is a once-differentiable function of Y, then $Cov(g'(Y), X) = E(g'(Y)Cov(Y,X))$. The theorem was extended to variables belonging to the continuous exponential family by Hudson [1976].

4. Because it has been assumed that the spot and futures prices are not perfectly correlated, the distributions will not be identical and the variance of the difference will be positive.

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Figure 1 - Decision Tree for Unrestricted Model

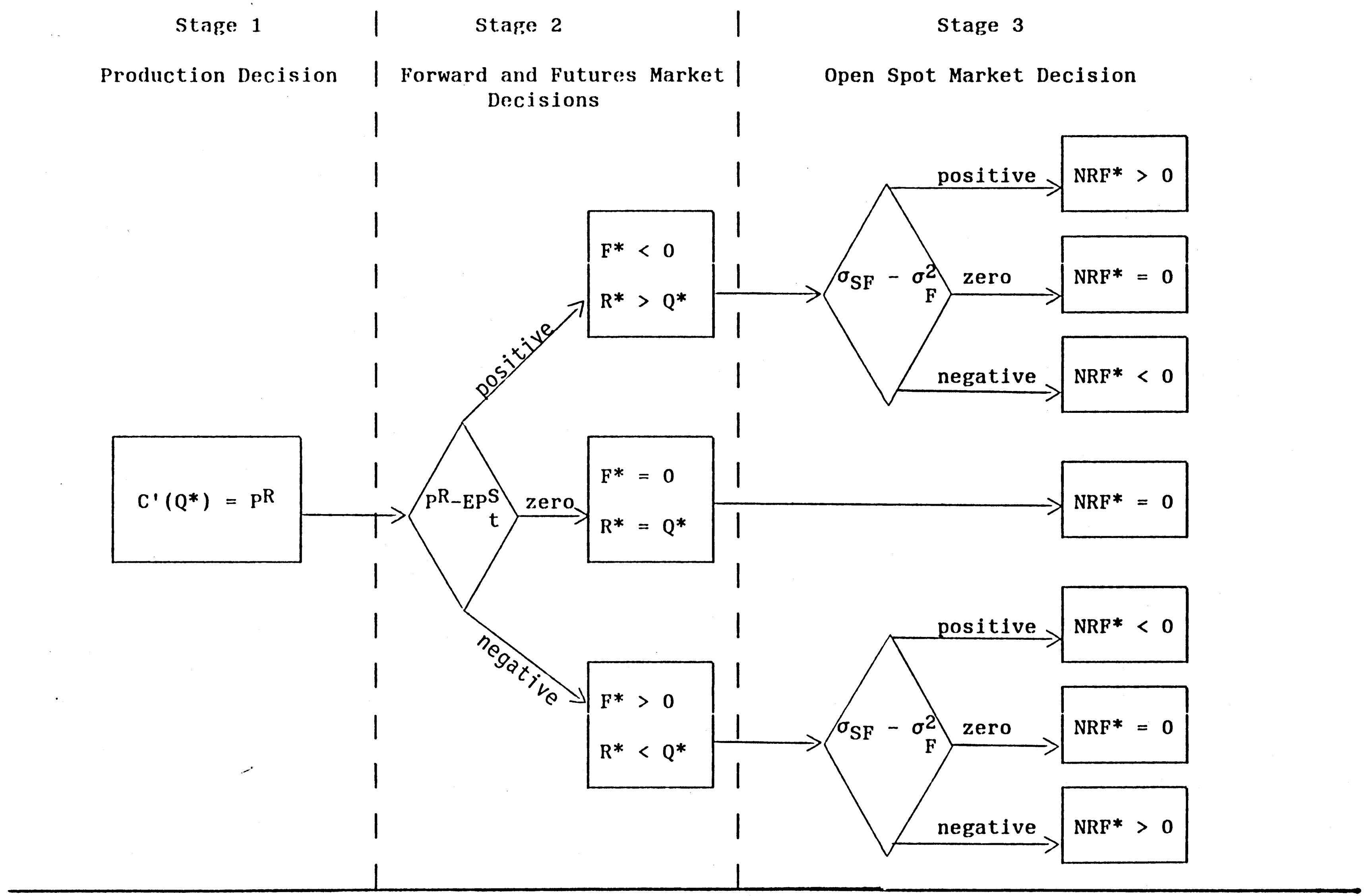

Figure 2 - Decision Tree for Restricted Model

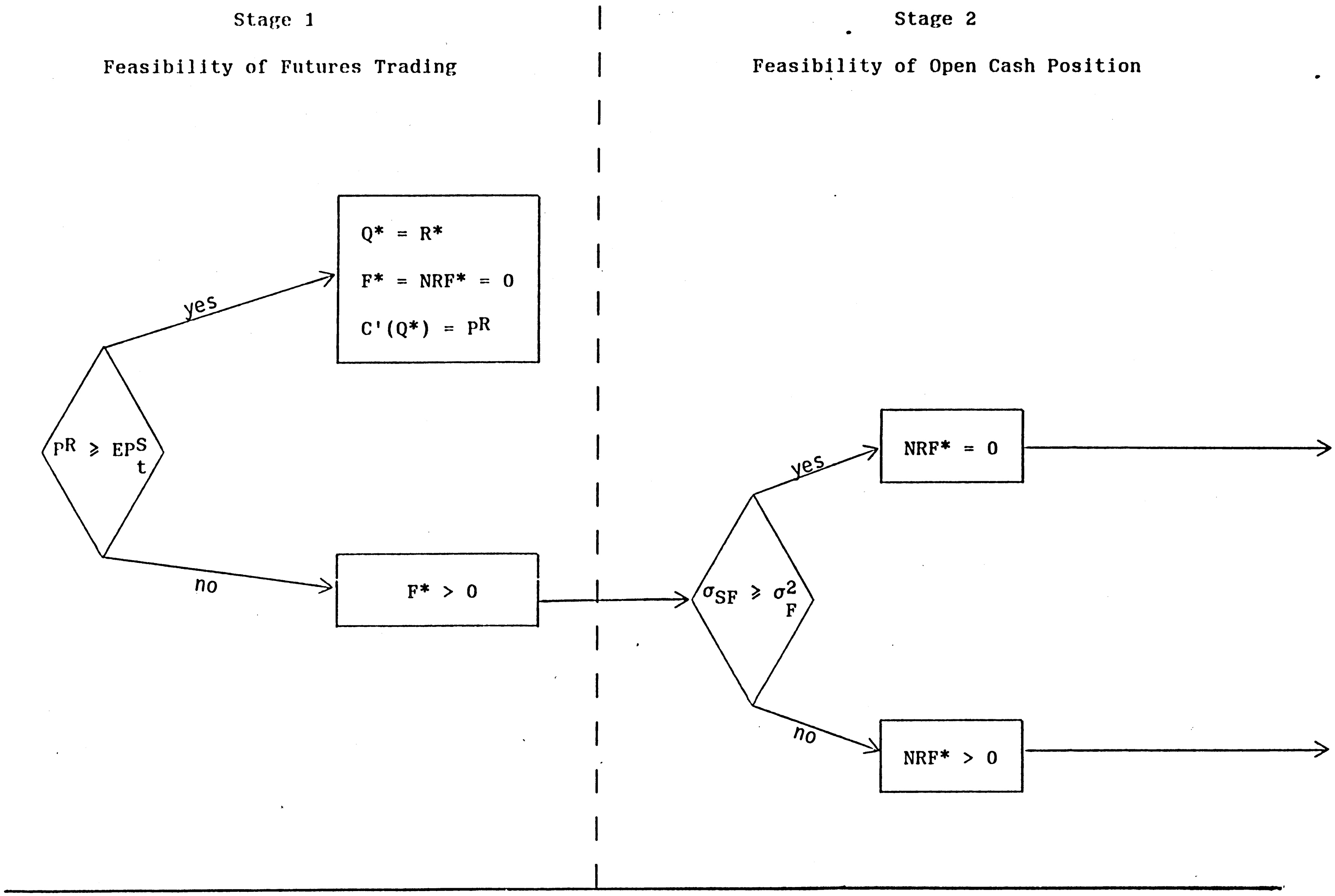

Figure 2 - Decision Tree for Restricted Model (continued)