Safety-First Models Based on Sample Statistics[1]

by

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Introduction

This paper presents a method whereby linear programming can be utilized to implement safety first decision rules with a discrete and finite population or sample. The method utilizes a stochastic inequality constructed with a lower partial moment. Should only a sample be available a statistical estimator of the lower partial moment is utilized which can be shown to be both unbiased and strongly convergent. A brief discussion of safety first and expected utility theory is followed by a presentation of the model with an empirical example.

Safety First and Expected Utility

Real world decisions must often be made in a setting of uncertainty when the outcomes of decisions are realized in future periods. Decision processes in such settings continue to stimulate considerable research efforts on the part of decision theorists and research economists. Several approaches to decision making are discussed in the literature. Included are expected utility theory, safety first, satisficing and game theory. In agricultural economics perhaps the most developed and accepted of the approaches is that of expected utility maximization. A rich literature field has developed dealing with the axiomatic foundations of utility theory, utility elicitation, stochastic dominance applications and other aspects of utility theory.[1/]

Expected utility theory has not been the only decision method discussed in the literature nor has it been free of criticism. The French school of utility, founded in the early 1950's by Allais and others, argues that expected utility maximization is not consistent with many observed behavioral phenomenon. They argue that the higher moments of utility (especially the second and third) are as important as mean utility in decision making. Expected utility maximization in this case might give relatively good approximations of behavior should the choices considered be in a sufficiently small subset of all possible choices. Methods to so constrain the feasible set of actions are not immediately apparent. A possible method might be to eliminate from consideration all distributions where the probability of failing to achieve some critical goal of the firm exceeds some threshold level. This concept is similar to certain safety first concepts discussed in following sections.

Several alternative approaches have been proposed. Included among these is the concept which has commonly been termed safety-first behavior. Safety-first behavior can be defined as behavior which is impacted or constrained by the probability of failing to achieve certain goals of the firm. This probability can be denoted as $Pr(x < g) \leq \lambda$ with g a goal of the firm and λ an acceptable limit on this probability. Various models of safety-first behavior have been discussed in the literature including those of Roy, Telser, Kataoka, and various chance constrained models. Roumasset presented a lexicographic system of safety first decision criteria for subsistence farmers in the Philippines.

While these models have been proposed and discussed since the 1950's, they have not gained widespread popularity among researchers, perhaps due to the common acceptance of expected utility theory. Many tend to feel that safety-first behavior is of questionable theoretical content or can be approximated by expected utility theory. Pyle and Turnovsky demonstrated that, with distributions uniquely defined by mean and variance (such as the normal), safety first solutions could also be obtained with properly specified expected utility models if borrowing and lending were excluded. If borrowing or risk free lending was allowed, the results were not consistent with expected utility. The methods utilized by Pyle and Turnovsky are applicable should the decision maker desire to select a portfolio from a set of investments with a multivariate normal distribution. Should the set of investments be non-normally distributed and dependent, then implementing safety-first models is much more difficult. In many cases, constructing probability statements over a large set of possible linear combinations of non normal, dependent random variables will be exceedingly difficult. As safety first models require probability information on these linear combinations, the ability to practically implement safety first models has been quite limited. One method which has been utilized is to use stochastic inequalities such as Chebychev's to generate sharp upper bounds on the probability. Chebychev's inequality is

$$\Pr(|x - \mu| \leq k\sigma) \leq (1/k)^2$$

The inequality places a sharp upper bound of $(1/k)^2$ on the probability of the random variable x falling more than k standard errors from the mean. Such upper bounds tend to be quite conservative. This paper presents an alternative method to implement safety first models should the decision maker face a discrete and finite set of possible state vectors. The method presented utilizes linear programming. A linear constraint guarantees that the probability concerns of the safety first model are satisfied. The linear constraint is constructed by utilizing a lower partial moment stochastic inequality.

Lower partial moments are intimately related to stochastic dominance. Stochastic dominance concepts are attractive in that a partial ordering of distributions is often possible for individuals whose utility functions satisfy certain conditions. These conditions can be quite broad in which case stochastic dominance tests may eliminate only a small proportion of all possible outcomes. Imposing additional conditions on the utility function allow further reductions in the undominated set of possible outcomes. Commonly known forms of stochastic dominance include first order (F.S.D.), second order (S.S.D.) and third order stochastic dominance (T.S.D.). These forms will not be redefined here but will be referred to in the following sections. Other forms of stochastic dominance have been defined and have proven useful. Meyer's stochastic dominance with respect to a function allows the elimination of dominated distributions for all individuals whose risk aversion characteristics lie within certain bounds (see Meyer, King and Robison).

Porter first demonstrated the relationship between target semivariance and second order stochastic dominance. Target semivariance is defined as

$$(1) \quad \sigma^{2-} = \int_{-\infty}^{t} (t - x)^2 \, f(x) \, dx$$

Solutions which are mean-target semivariance efficient were shown by Porter to be members of the S.S.D. efficient set. Target semivariance is a special case of a lower partial moment (L.P.M.). Fishburn presented a general form of the lower partial moment which is defined as follows

$$(2) \quad \rho(\alpha,t) = \int_{-\infty}^{t} (t - x)^{\alpha} \, f(x) \, dx$$

Fishburn showed that models which examined mean-lower partial moment tradeoffs generated solutions which were S.S.D. efficient for all $\alpha \geq 1$ and T.S.D. efficient for all $\alpha \geq 2$. Thus Porter's target semivariance model actually generated subsets of the T.S.D. set.

Tauer recently reported similar results for the discrete case with $\alpha = 1$. McCamley and Kleibenstein likewise reported that, with $\alpha = 2$ and a discrete distribution, mean-target semivariance efficient solutions are elements of the T.S.D. efficient set. In addition to the properties discussed by Fishburn, L.P.M.'s are useful in a stochastic inequality which can be utilized in safety first programming.

Lower Partial Moments and Safety First

Berck and Hihn first presented a mean-semivariance stochastic inequality which generated considerably less conservative upper bounds than Chebychev's inequality. Atwood presented a general L.P.M. inequality and demonstrated the ability of alternative forms to provide less conservative upper probability bounds than the Chebychev or mean-semivariance inequality.

The general inequality is

(3) $Pr(x < g) = Pr(x < t - p\theta(\alpha,t)) \leq (1/p)^{\alpha}$

with g a goal of the firm as previously defined,

t a reference level of income,

α the power to which deviations are raised in Fishburn's L.P.M. $\rho(\alpha,t)$,

$\theta(\alpha,t)$ is the α'th positive root of $\rho(\alpha,t)$ i.e.

$\theta(\alpha,t) = [\rho(\alpha,t)]^{1/\alpha} \geq 0$, and

p is the number of $\theta(\alpha,t)$ units that g falls below t.[2/]

Utilizing inequality (3) it can be shown that enforcing the following constraint is sufficient to guarantee that $Pr(x < g) \leq \lambda$. The constraint is

(4) $t - q^{*}\theta(\alpha,t) \geq g$

with $(1/q^{*})^{\alpha} = \lambda$ or $q^{*} = (1/\lambda)^{(1/\alpha)}$

Should $\alpha = 1$ then (4) becomes

(5) $t - q^{*}\theta(1,t) \geq g$

with $q^{*} = 1/\lambda$

Constraint (5) requires that $\rho(1,t) = \theta(1,t)$ be known. With a finite discrete distribution this can be computed in a target-MOTAD model. Should the decision maker possess an independently and identically distributed sample of size n, the following statistic can be shown to be both unbiased and strongly convergent as an estimator of $\rho(\alpha,t)$. The statistic is

$$\hat{\rho}(\alpha,t) = \sum_{i=1}^{n} [(t - x_i)^{\alpha} \, I_{(-\infty,t]}(x_i)]$$

with x_i the i'th observation of the random variable and

$I_{(-\infty,t]}(x_i)$ is the indicator or zero-one function which

multiplies by 1 if $x_i \leq t$ or

0 if $x_i > t$

If the decision maker desires to select a portfolio of k activities which maximize expected aggregate income subject to a safety-first type constraint on aggregate income, the above inequality can be utilized as aggregate income in a univariate random variable. The sample in this case would consist of a set of vectors. Using $\hat{\rho}(1,t) = \hat{\theta}(1,t)$ as an estimator of $\rho(\alpha,t) = \theta(\alpha,t)$ this problem can be modeled by system

(6) Max $\underline{\mu}^T \underline{c}$

Subject to

$A\underline{c} \leq \underline{b}$

$Y\underline{c} - \underline{1}\, t + I\underline{d} \geq \underline{0}$

$t - q^*(\underline{1/n})^T \, \underline{d} \geq g$

$\underline{c}, \underline{d}, \geq \underline{0}$

with $\underline{\mu}^T$ = transposed kx1 vector of sample means for the k activities,

$\underline{c}$ = kx1 choice vector of activity levels,

Y = $[\underline{y}_1, \underline{y}_2, \ldots \underline{y}_n]^T$ with $\underline{y}_i$ = a kx1 vector consisting of the i'th observation of the k activities' income levels,

$\underline{1}$ = nx1 vector of ones,

t = the reference level of income for the L.P.M.,

I = nxn identity matrix

$\underline{d}$ = nx1 vector with i'th element = $t - \underline{y}_i^T\underline{c}$ if $\underline{y}_i^T\underline{c} \leq t$
or = 0 if $\underline{y}_i^T\underline{c} > t$,

$\underline{0}$ = column vector of zeros,

q^* = $1/\lambda$,

$(\underline{1/n})$ = nx1 vector with all elements equal to 1/n, and

g = the safety first goal.

The above system is a modification of the model presented by Held, Watts, and Helmers. As constraint (5) is valid for all feasible levels for t, the optimization model endogenously selects the least constraining level of t. Should Y be a population or a subjectively estimated set of state vectors, the vector $(\underline{1/n})$ can be replaced with a probability vector $\underline{r}$ with r_i the probability of state $\underline{y}_i$. The above model then becomes a modified version of Tauer's Target-MOTAD.[3/]

In the following section an empirical example will be presented. The Y matrix is assumed to be a sample rather than a population. As such the statistical estimator $\hat{\theta}(1,t) = (\underline{1/n})^T\underline{d}$ will be utilized.

Empirical Model

The empirical example of this section assumes that the decision maker wishes to select a combination of activities which maximize expected income while satisfying certain safety goals of the firm. The decision maker can select from six activities subject to a set of linear technical constraints. Ten observations of the six activities are available. The assumption is made that each of the ten observations is from the same population of possible events that is currently anticipated by the decision maker. Table 1 present the sample mean, standard error, and coefficient of variation levels for the six activities. Table 2 presents the sample correlation coefficients. Note that while activity six has by far the highest coefficient of variation in Table 1, it is also the only activity which is negatively correlated to the others. Activity six can thus not be eliminated from consideration a priori.

The tableau for this problem is presented in Table 3. An additional row has been added to system 6 to allow separate computation of $\Theta = (\underline{1/n})^T\underline{d}$. The final row enforces constraint (5) while allowing the endogenous selection of the least constraining level for t. The tableau as presented maximizes expected income subject to $\Pr(\text{income} < \$90000) \leq .2 = 1/q^*$. This gives $q^* = 5$. The solution to this problem and for alternative levels of g and λ are presented in Table 4. Also reported in Table 4 are the actual number of times that income fell below the goal (i.e. $\underline{y}_i^T\underline{c} < g$) as well as the buffer between g and the smallest $\underline{y}_i^T\underline{c} \geq g$.

Several points should be noted when examining Table 4. The solutions for all levels of $\lambda \leq .1 = 1/n$ are identical. System (6) can not effectively discriminate at λ levels between 0 and 1/n. Note for λ levels of 0, .05, or .1 that no observations of income below either \$90000 or \$95000 occur. However, for each the smallest value of $\underline{y}_i^T\underline{c}$ equals g. Thus even though no observations actually occur below g, there may be one or more observations of $\underline{y}_i^T\underline{c}$ exactly equal to g. In this case, there is little room for specification or estimation error at levels of $\lambda \leq 1/n$. The same results hold if subjective probabilities $\underline{r}$ are utilized rather than $(\underline{1/n})$. The model will then not be able to discriminate at probability levels less than the smallest r_i value.

Note also that at λ levels above 1/n, the solutions tend to be conservative in that the number of observations below g divided by n are less than the allowable λ levels. This results from the fact that inequality (3) generally provides conservative upper bounds for $Pr(x < g)$. An idea of the conservativeness of the solution can be gained by examining the value of the minimum value of $\underline{y}_i^T\underline{c} - g$ given that $\underline{y}_i^T\underline{c} \geq g$. This level represents the distance from g to the 'next highest' income level observed. The greater this number, the more conservative the solution can be said to be. For g = \$90000 and $\lambda = .20$, one observation of $\underline{y}_i^T\underline{c}$ was below \$90000 with the next lowest observation at \$90788. It can be seen that a certain buffer for specification error exists before the associated solution mix actually violates the condition $Pr(x < 90000) \leq .2$. The use of stochastic inequality (3) in safety-first models as opposed to exact probabilities is thus seen to result in a tradeoff. This tradeoff is between the conservativeness implicit to the use of stochastic inequalities and specification error protection.

As demonstrated by Atwood, the use of inequality (3) potentially results in less conservative upper bounds than Chebychev's or Berck and Hihn's inequality. However, by reducing the conservativeness of the upper bounds, the likelihood of underestimating $Pr(x < g)$ has increased should specification or sampling error exist. The seriousness of each type of error will depend upon the specific problem being analyzed.

Should the first type of error i.e. excess conservativeness be viewed as more serious by the decision maker, the use of system (6) or perhaps an even less conservative method may be warranted. Should the underestimation of $Pr(x < g)$ be viewed as more serious, the decision maker may wish to utilize Berck and Hihn's or Chebychev's inequality with a non-linear programming routine. Alternatively, system (6) could be utilized with a more conservative g or λ level.

A final point will be made concerning a comparison of the goals and the expected income levels of Table 4. As the income goal of concern was increased from \$90000 to \$95000, at a given λ level, the maximum possible mean income declined. No attempt will be made to rigorously prove why this occurs but an intuitively based explanation might be in order at this time. Maximizing expected net income with no probability restrictions yields an expected income of \$161088. The associated activity mix yields no observations of $\underline{y}_i^T\underline{c} <$ \$84721. Thus any probability restrictions on $g \leq$ \$84721 would be satisfied and the L.P. Solution would be optimal. As g is increased above \$84721, the activity mix may need to be modified depending on λ. This modification is likely to require a reduction in the expected income as the feasible set of solutions has now become more constrained. Increasing g further, given λ, constrains the model, resulting in previously attainable mean income levels being non-attainable. As g increases from \$90000 to \$95000 the model has become more constrained.

Summary and Conclusion

This paper has demonstrated a method to implement safety-first or probability constrained programming with linear programming. Probability bounds on linear combinations of nonnormal and dependent random variables can be constructed utilizing a linear lower partial moment (L.P.M.) inequality and a set of discrete state vectors. The inequality in general provides considerably less conservative upper bounds (and activity mixes) than other published inequalities.

If only a sample is available, an unbiased and strongly convergent estimator of the L.P.M. can be utilized in lieu of the actual parameter. (A subjective distribution can also be utilized.) As the solutions tend to be conservative, some level of specification or sampling error can exist with violating the probability constraint $Pr(x < g)$. The statistical properties of $\rho(\alpha,t)$ as an estimator of $\rho(\alpha,t)$ appear to merit further study.

The potential usefulness of linear probability constraints appears to be significant. All three safety-first criteria discussed by Pyle and Turnovsky can be modeled although only one criteria has been demonstrated in this paper. In addition, the possibility of expected utility maximization within a probability constrained space could be explored. Such a concept or approach might be more consistent with the views of the French school of utility. The solution to system (6) will be a member of the S.S.D. efficient set. Methods to generate additional stochastically efficient solutions within the probability constrained space would be useful. Such a procedure would reduce the F.S.D., S.S.D., or T.S.D. efficient sets, perhaps significantly.

The probability constrained random variable need not be aggregate income. The new method can thus be utilized to implement various forms of chance constraints. Examples would be chance constraints on various resources, internal flows, intermediate products or financial ratios if discrete potential outcomes can be listed or derived. Most previous applications of this type have utilized normality assumptions.

In conclusion, the potential usefulness of lower partial moments for probability or safety constrained problems appears to be significant. The method may not be suited to all applications but should prove to be a useful tool for decision making under uncertainty.

Footnotes

1/ An interesting recent development in the area of stochastic dominance has been the relationship discovered between lower partial moments and stochastic dominance. This relationship will be briefly addressed in a following section of the paper.

2/ For a proof of the inequality (3) and constraint (4) see Atwood.

3/ Tauer demonstrated that solutions of the Target-MOTAD model were subsets of the S.S.D. efficient set. In this case the probability constrained solution to system (6) will also be a member of the S.S.D. efficient set if $t - q^* \underline{r}^T \underline{d} \geq g$ is constraining. Although the optimization process endogenously selects the level for t, constraint (5) effectively constrains $\Theta(1,t) = \underline{r}^T \underline{d}$ to be less than or equal to some level $M = (t - g)/q^*$ while maximizing expected income.

Table 1

Sample Mean, Standard Error, and Coefficients of Variation

Activity	μ_i	σ_i	σ_i/μ_i
C_1	538.64	238.48	.526
C_2	318.88	178.69	.560
C_3	260.78	65.24	.250
C_4	188.11	90.33	.480
C_5	123.04	44.94	.365
C_6	20.59	110.13	5.349

Table 2

Sample Correlation Coefficients for Example Problem

Activity	C_1	C_2	C_3	C_4	C_5	C_6
C_1	1	.877	.516	.838	.630	-.549
C_2		1	.297	.706	.467	-.419
C_3			1	.567	.709	-.404
C_4				1	.805	-.453
C_5					1	-.220
C_6						1

Table 3

Tableau for Empirical Example

ROW		C1	C2	C3	C4	C5	C6	T	D1	D2	D3	D4
OBJ FCN		538.6400	318.8800	260.7800	188.1100	123.0400	20.5900	.	.	.	.	.
R1	L	1.0000	1.0000	1.0000	1.0000	1.0000	.	.	.	.	.	.
R2	L	2.9700	1.7700	1.8200	1.8500	1.9000	0.4100	.	.	.	.	.
R3	L	1.0800	1.0900	1.2500	1.2800	0.2500	.	.	.	.	.	.
R4	L	2.8400	3.7100	3.0800	3.1400	0.9600	.	.	.	.	.	.
R5	L	2.3900	3.9100	4.4300	0.6400	1.2300	.	.	.	.	.	.
R6	L	5.6800	.	.	1.7400	.	0.2700	.	.	.	.	.
R7	L	2.7200	1.5600	1.6100	1.6300	0.6700	.	.	.	.	.	.
R8	L	1.0400	1.9900	1.2000	1.2200	0.0800	.	.	.	.	.	.
R9	L	0.5700	0.8800	0.8000	0.8300	0.3600	.	.	.	.	.	.
R10	L	0.1900	3.1500	3.6400	0.0800	0.1500	.	.	.	.	.	.
R11	L	5.3000	.	.	1.5800	.	.	.	.	.	.	.
R12	L	.	.	.	.	.	1.0000	.	.	.	.	.
R13	L	.	.	-1.0000	.	.	0.0780	.	.	.	.	.
R14	L	.	.	.	-1.0000	.	0.1010	.	.	.	.	.
R15	L	.	.	.	.	-0.8000	0.1010	.	.	.	.	.
Y1	G	516.5200	217.9900	296.5000	132.1400	106.2200	-50.1600	-1.0000	1.0000	.	.	.
Y2	G	781.5100	412.9500	343.0400	203.0800	126.1600	-92.1200	-1.0000	.	1.0000	.	.
Y3	G	420.0700	322.1800	213.4200	114.5300	111.5500	200.4900	-1.0000	.	.	1.0000	.
Y4	G	280.7700	139.0000	166.1400	105.5500	101.0900	141.8900	-1.0000	.	.	.	1.0000
Y5	G	332.2400	407.4100	198.0000	108.8800	65.7900	-9.6300	-1.0000	.	.	.	.
Y6	G	273.2500	117.7100	339.7200	174.3100	173.2600	62.7600	-1.0000	.	.	.	.
Y7	G	507.0200	274.6300	262.2600	273.9100	139.9700	-50.0200	-1.0000	.	.	.	.
Y8	G	1137.6000	669.9600	287.1900	348.8700	194.9000	-143.1700	-1.0000	.	.	.	.
Y9	G	801.7500	490.1000	313.9600	302.7000	158.4400	119.9300	-1.0000	.	.	.	.
Y10	G	335.6200	136.8900	187.5800	117.7300	53.5100	26.0700	-1.0000	.	.	.	.
THETA	L	.	.	.	.	.	.	.	0.1000	0.1000	0.1000	0.1000
SUFCONST	G	.	.	.	.	.	.	1.0000	.	.	.	.

ROW		D5	D6	D7	D8	D9	D10	T-THETA	R H S
OBJ FCN		.	.	.	.	.	.	.	########
R1	L	.	.	.	.	.	.	.	400.0000
R2	L	.	.	.	.	.	.	.	1084.0000
R3	L	.	.	.	.	.	.	.	1127.0000
R4	L	.	.	.	.	.	.	.	1611.0000
R5	L	.	.	.	.	.	.	.	1232.0000
R6	L	.	.	.	.	.	.	.	1084.0000
R7	L	.	.	.	.	.	.	.	805.0000
R8	L	.	.	.	.	.	.	.	768.0000
R9	L	.	.	.	.	.	.	.	1230.0000
R10	L	.	.	.	.	.	.	.	904.0000
R11	L	.	.	.	.	.	.	.	897.0000
R12	L	.	.	.	.	.	.	.	300.0000
R13	L	.	.	.	.	.	.	.	.
R14	L	.	.	.	.	.	.	.	.
R15	L	.	.	.	.	.	.	.	.
Y1	G	.	.	.	.	.	.	.	.
Y2	G	.	.	.	.	.	.	.	.
Y3	G	.	.	.	.	.	.	.	.
Y4	G	.	.	.	.	.	.	.	.
Y5	G	1.0000	.	.	.	.	.	.	.
Y6	G	.	1.0000	.	.	.	.	.	.
Y7	G	.	.	1.0000	.	.	.	.	.
Y8	G	.	.	.	1.0000	.	.	.	.
Y9	G	.	.	.	.	1.0000	.	.	.
Y10	G	.	.	.	.	.	1.0000	.	.
THETA	L	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	-1.0000	.
SUFCONST	G	.	.	.	.	.	.	-5.0000	90000.0000

Table 4

Safety First Solutions for Example Problem

Income Goal g	Probability Constraint λ	Constraint Coefficient q^*	Mean Income	Activity Levels C_1	C_2	C_3	C_4	C_5	C_6	Actual Number of $Y_i^T x < g$	Distance to Nearest $Y_i^T x \geq g$
90000	0	-	159621	164.9	173.6	28.8	14.5	18.2	144.0	0	0
	.05	20	159621	164.9	173.6	28.8	14.5	18.2	144.0	0	0
	.10	10	159621	164.9	173.6	28.8	14.5	18.2	144.0	0	0
	.15	6.67	159741	164.9	175.3	27.3	14.4	18.0	142.8	1	861
	.20	5	159840	165.0	176.8	26.0	14.3	18.0	141.7	1	788
	.25	4	160716	165.0	191.8	11.0	14.3	17.9	141.4	1	28
	.30	3.33	161088	165.4	195.4	10.0	13.0	16.2	128.4	2	6484
95000	0	-	154074	163.1	90.9	99.8	20.5	25.7	203.3	0	0
	.05	20	154074	163.1	90.96	99.8	20.5	25.7	203.3	0	0
	.10	10	154074	163.1	90.96	99.8	20.5	25.7	203.3	0	0
	.15	6.67	157032	164.1	135.0	61.9	17.3	21.7	171.7	1	4321
	.20	5	157531	164.2	142.4	55.6	16.8	21.0	166.4	1	2525
	.25	4	158564	162.7	171.1	16.9	21.9	27.4	217.0	2	13140
	.30	3.33	159373	163.6	178.9	14.7	19.0	23.8	188.5	2	9402
L.P. Solution	—	—	161088	165.4	195.4	10.0	13.0	16.2	128.4	—	—

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