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Estimation of Rational Risk Response Models for Storable Primary Commodities

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Abstract

Stochastic-dynamic programming and disequilibrium econometric methods are combined to obtain maximum likelihood estimates of a dynamic nonlinear rational expectations model of a market for a storable primary commodity. The structural model captures the essential processes governing the dynamics of primary commodity markets including: the nontrivial role of private speculative stockholding, the inherently nonlinear disequilibrium effects of government buffer stock intervention, and the complex roles of expectations and risk in private supply and stockholding decisions.

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Introduction

Markets for primary commodities are often characterized by production variability, typically arising from weather and other natural factors, and price inelastic short-run demands.[1] When combined, these features can give rise to severe price uncertainty. Under risk aversion, price uncertainty has been theoretically shown to reduce supply and lower societal welfare (Sandmo; Newbery and Stiglitz). For this reason, Economists have long been interested in empirically assessing the impact of risk on commodity supply decisions and the effects of market mechanisms and government intervention on commodity market stability.

A fundamental characteristic of most primary commodities is storability. Private speculative storage of commodities stabilizes supplies over time and imposes a strong dynamic structure on commodity prices. In normal times, intertemporal arbitrage by profit-seeking storer-speculators ensures that the current and expected future market prices of a commodity differ by the physical cost of storing the commodity, plus a normal rate of return and risk premium. In times of supply shortfalls, however, this intertemporal link between prices may be severed in a phenomenon known as backwardation. Backwardation arises when the price of a commodity rises sufficiently high that the difference between the current and expected future prices does not cover the cost of carrying the commodity. When this occurs, speculative stockholding of the commodity disappears and only small quantities of convenience or pipeline stocks continue to be held privately (Keynes).

Governments have historically regarded private storage as incapable of providing adequate commodity market stability and have often intervened through buffer stock stabilization programs. Buffer stock programs involve the storing of commodities in times of surplus and their subsequent release in times of scarcity by an authorized government agency. Under a typical buffer stock program, the government offers to purchase unlimited quantities of the commodity at a set support price and to sell any quantities in its possession at a set release price. Buffer stock programs form the cornerstone of agricultural stabilization programs in the United States; multilateral international buffer stock agreements have also played an important role in markets for rubber, cocoa, coffee, and tin (Behrman 1978, McNicol).

In this paper, we develop a method for estimating dynamic primary commodity market models that capture the complex structure of private speculative stockholding and the disequilibrium effects of government buffer stock intervention under risk aversion. Our estimation approach combines stochastic-dynamic programming and disequilibrium maximum likelihood techniques in an application of Fair and Taylor's iterative numerical strategy for estimating dynamic nonlinear rational expectations models. In the following section, past approaches to the empirical analysis of primary commodity markets are discussed. In the subsequent two sections, the commodity market model and the estimation method are presented. The remaining section is devoted to an empirical application to the U.S. soybean market.

Empirical Commodity Market Models

Historically, most econometric studies of primary commodity markets have employed adaptive expectation formulations to represent the expectation of a future price as weighted average of lagged prices (Nerlove 1972, 1979; Askari and Cummings; Labys). The autoregressive lag structure has also been used to represent price risk as a function of past deviations of observed prices from the expected price (Just 1974; Just 1975; Traill; Hurt and Garcia; Chavas and Holt). Alternatively, price risk has also been represented as a function of past deviations of observed prices from their simple moving mean (Behrman 1968; Lin; Ryan; Thraen and Hammond; Brorsen, Chavas, and Grant).

Although highly tractable, autoregressive expectation and risk formulations have come increasingly under attack in recent years. Lucas, in his seminal critique of econometric policy evaluation, argued that economic agents are capable of incorporating structural information into their decisions, particularly information regarding the current and future impacts of government interventions. Autoregressive price expectation and risk formulations only extrapolate past prices, ignoring most structural information, and thus cannot capture the adjustments made by future-regarding agents to exogenous changes in government policy.[2,3] As a consequence, models incorporating autoregressive expectations will give misleading results when used to analyze proposed changes in government policies or other significant shifts in market structure.

In response to Lucas's critique, many economists turned to Muthian rational expectations as an alternative model of expectation formation. Under rational expectations, market agents are assumed to form price expectations that are consistent with the underlying market structure and all contemporaneously available information. The work conducted by macroeconometricians such as Wallis, and Hansen and Sargent led to the development and widespread application of the linear rational expectations model in econometric estimation. The linear rational expectations model has been applied to commodity markets by Shonkwiler; Shonkwiler and Emerson; Goodwin and Sheffrin; Eckstein 1984, 1985; Gilbert and Palaskalas; and Ghosh, Gilbert, and Hughes-Hallet. The linear model has also been extended to incorporate rational risk response in commodity supply decisions by Antonovitz and Roe; Seale and Shonkwiler; Aradhyula and Holt; and Antonovitz and Green.

Despite its mathematical tractability, however, the linear rational expectations model has proven to be incapable of adequately capturing the two essential processes that govern primary commodity market dynamics. First, private stockholding in the linear model is typically explained through an intertemporal arbitrage equation in which the current price and the expected future price differ by a constant cost of storage. This formulation not only fails to explain recurrent backwardation in primary commodity prices, it also implies that private stocks will be negative in times of short supplies. Second, the linear model cannot capture the inherently nonlinear disequilibrium distortions of the market price caused by government buffer stock intervention. Specifically, the linear model cannot adequately represent the distortions of the price distribution induced by government stock purchases and sales at set support and release prices. Because linear models fail to capture the profound effects of stockholding on commodity price variability, their usefulness in studying risk response in primary commodity markets is severely limited.

Holt and Johnson overcame one limitation of the linear model using a latent-dependent variable, disequilibrium formulation to capture the effects government buffer stock intervention on commodity price behavior.[4] Holt subsequently extended the formulation to allow for rational risk response in supply decisions. Unlike the linear model, the disequilibrium rational expectations model possesses no closed form solution and thus cannot be estimated using conventional time-series techniques. As an alternative to time-series methods, Holt and Johnson used an iterative numerical estimation strategy developed by Fair and Taylor for estimating nonlinear rational expectations models. Despite their significant methodological advancement, however, Holt and Johnson did not explicitly incorporate private storage into their model and thus fell short of providing a complete description of the fundamental factors affecting commodity price variability.

An alternative approach to modeling primary commodity market dynamics employs stochastic-dynamic programming methods to numerically simulate commodity market behavior under rational expectations. The stochastic-dynamic programming framework has successfully accommodated both private speculative storage and government buffer stock intervention. The initial studies employing the stochastic-dynamic programming approach examined the price stabilization effects of private speculative storage in the absence of government intervention (Gustafson; Gardner 1979; Wright and Williams 1982, 1984; Lowry et al.). Subsequent studies evaluated the interactive price stabilization effects of both government buffer stock intervention and private speculative storage (Miranda and Helmberger; Wright and Williams 1988; Glauber, Helmberger, and Miranda).

The stochastic-dynamic programming studies confirmed that both government and private storage have profound effects on commodity market dynamics and commodity price stability. A weakness of the stochastic-dynamic programming studies, however, was that the models were not estimated in a manner that simultaneously incorporated all of the structural restrictions implied by the theory. In all the cases cited, the models were parameterized either by judicious guesses made by the researcher or with estimates drawn from previously published empirical studies. In the sections that follow, we show how to econometrically estimate commodity market models that incorporate the structural relations commonly used in stochastic-dynamic programming models to capture the effects of private and government storage.

A Commodity Market Model With Risk Response and Private and Government Storage

Consider an annual model of an primary commodity market comprising consumers, producers, private storers, and a government buffer stock agency. The supply available in year t is composed of new production plus the preceding year's ending private and government stocks. Available supply is either consumed or stored, implying the material balance equation:

$$Q_t + S_{t-1} + G_{t-1} = C_t + S_t + G_t. \tag{1}$$

Here, new production is denoted by Q_t, consumption by C_t, ending private stocks by S_t, and ending government stocks by G_t. New production equals the acreage planted in the preceding year A_{t-1} times a random exogenous per-acre yield $\tilde{Y}_t$:

$$Q_t = A_{t-1} \cdot \tilde{Y}_t. \tag{2}$$

Consumption is a function of the current price P_t, a vector of exogenous variables X_t^C, and a random shock $\tilde{U}_t$:

$$C_t = C(P_t, X_t^C, \tilde{U}_t). \tag{3}$$

Acreage planted is a function the present value of the price expected the following year $\mathcal{E}_t \delta_t P_{t+1}$, the variance of the following year's discounted price conditional on contemporaneously available information $\mathcal{V}_t \delta_t P_{t+1}$, a vector of exogenous variables X_t^A, and a random shock $\tilde{W}_t$:

$$A_t = A(\mathcal{E}_t \delta_t P_{t+1}, \mathcal{V}_t \delta_t P_{t+1}, X_t^A, \tilde{W}_t). \tag{4}$$

Here, $\delta_t = \frac{1}{1+r_t}$ is the annual discount factor and r_t is the annual interest rate.

Private storage is undertaken by competitive profit-maximizing storers who equate the expected marginal revenue from storing to the marginal physical cost of storage adjusted for any marginal convenience yield and risk premium. Solving the equimarginality condition explicitly for private stocks gives the demand for private stocks:

$$S_t = S(\mathcal{E}_t \delta_t P_{t+1} - P_t, \mathcal{V}_t \delta_t P_{t+1}, X_t^S, \tilde{V}_t) \tag{5}$$

where X_t^S is a vector of exogenous variables, and $\tilde{V}_t$ is a random shock.

Private demand for storage is illustrated in Figure 1. For high levels of stocks, stocks are held predominantly for speculative purposes; except for the risk premium, the expected marginal revenue equals the marginal physical cost of storage, reflecting the elimination of speculative profit opportunities through competition among storers. For low levels of stocks, on the other hand, stocks are held mainly to smooth production and reduce adjustment costs in the commodity processing sector; the difference between the marginal physical cost of storage and the expected marginal revenue reflects the marginal convenience yield of pipeline stocks to commodity processors (Kaldor, Brennan). An increase in the variance of price raises the risk premium, causing private demand for stocks to shift downward.

Government demand for stocks, depicted in Figure 2, is governed by the provisions of the buffer stock program. If the market price P_t falls between the support price P_t^S and the release price P_t^R, the government neither acquires new stocks nor releases old stocks and simply carries out what it carried in:

$$P_t^R > P_t > P_t^S \Longrightarrow G_t = G_{t-1}. \tag{6}$$

More generally, if the market price exceeds the support price, the government acquires no new stocks:

$$P_t > P_t^S \Longrightarrow G_t \leq G_{t-1}; \tag{7}$$

and if the release price exceeds the market price, the government releases no old stocks:

$$P_t < P_t^R \Longrightarrow G_t \geq G_{t-1}. \tag{8}$$

The government's ability to acquire essentially unlimited stocks ensures that the market price will not fall below the support price:

$$P_t \geq P_t^S; \tag{9}$$

however, the market price can rise above the release price, though only after the government stockpile has been depleted:

$$P_t > P_t^R \Longrightarrow G_t = 0. \tag{10}$$

The market model is closed by assuming that private storers and producers form their price expectations rationally in the sense of Muth. That is, their price expectations and risk assessments are consistent with the underlying market structure and government policy.

Estimation Method

The commodity market model is estimated using a numerical strategy developed by Fair and Taylor for estimating nonlinear rational expectations models.[5] The procedure begins with judicious guesses in each period for the expectation and variance of the following period's price. The procedure then continues iteratively with each iteration consisting of two steps.[6] In the first step, ex-ante price expectations and variances are fixed and the model's parameters are conditionally estimated using full-information maximum likelihood methods. In the second step, the model's parameters are fixed and the model is solved numerically for the price expectations and variances implied by the model. The iterative process is repeated until the parameter estimates and the price expectations and variances converge. At convergence, the price expectations and variances and the estimated model are mutually consistent; that is, the parameter estimates observe the restrictions implied by the rationality assumption.

In order to describe the estimation procedure more fully, we must: first, posit an estimable model by specifying functional forms for the demand and supply equations; second, specify the econometric technique used to estimate the model parameters in the first step of each iteration; and third, specify the numerical technique used to solve for the price expectations and variances implied by the model in the second step of each iteration.

Estimation Model

For the purposes of estimation, the consumption demand equation (3), the private stock demand equation (5), and the acreage supply equation (4) are represented by flexible log-linear forms:

$$c_t = \alpha_c \cdot x_t^c + \beta_c \cdot p_t + \tilde{u}_t \tag{11}$$

$$s_t = \alpha_s \cdot x_t^s + \beta_s \cdot (p_t^e - p_t - r_t) + \gamma_s \cdot p_t^v + \tilde{v}_t \tag{12}$$

$$a_t = \alpha_a \cdot x_t^a + \beta_a \cdot (p_t^e - r_t) + \gamma_a \cdot p_t^v + \tilde{w}_t. \tag{13}$$

Here, c_t, s_t, and a_t are, respectively, the logarithms of consumption C_t, private stocks S_t, and acreage A_t; p_t and p_t^e are, respectively, the logarithms of the current price P_t and the expected future price $\mathcal{E}_t P_{t+1}$; p_t^v is the coefficient of variation (that is, the standard deviation divided by the expectation) of the future price P_{t+1}; r_t is the annual interest rate; and the vectors x_t^c, x_t^s, and x_t^a contain the logs of exogenous market variables and any constant and trend terms. The random variables $\tilde{u}_t$, $\tilde{v}_t$, and $\tilde{w}_t$ are assumed to be normally distributed and mutually and serially independent with zero mean and constant finite variances σ_u^2, σ_v^2, and σ_w^2.

The parameter $\beta_c < 0$ is the price elasticity of consumption demand; $\beta_s > 0$ is the elasticity of private stock demand with respect to the expected appreciation in the value of stocks; and $\beta_a > 0$ is the elasticity of acreage supply with respect to the expected price. The parameters $\gamma_s < 0$ and $\gamma_a < 0$ are, respectively, the elasticities of private stock demand and acreage supply with respect to price risk.

Econometric Estimation

In the first step of each iteration, the simultaneous equation system (1), (11)-(13) is estimated subject to the disequilibrium distortions (6)-(10) caused by government buffer stock intervention. Price expectations are treated as exogenous and are assigned the values computed in the second step of the preceding iteration. Because the acreage supply equation is block recursive with respect to the other equations, it may be estimated independently using ordinary least squares.

The consumption demand and private stock demand equations, on the other hand, exhibit simultanaity and are estimated jointly using disequilibrium full-information maximum likelihood methods (Maddala, Quandt). Observations are partitioned into four equilibrium-disequilibrium regimes (see Figure 2):

i. $P_t > P_t^R, \quad C_t + S_t = Q_t + G_{t-1} + S_{t-1},$

ii. $P_t = P_t^R, \quad Q_t + G_{t-1} + S_{t-1} \geq C_t + S_t \geq Q_t + S_{t-1},$

iii. $P_t^R > P_t > P_t^S, \quad C_t + S_t = Q_t + S_{t-1},$

iv. $P_t = P_t^S, \quad Q_t + S_{t-1} \geq C_t + S_t.$

For regimes (i) and (iii), price varies freely, but consumption and private stocks sum to a fixed pre-determined quantity. Using the material balance equation (1) to eliminate private stocks, the likelihood for these observations may be written $f(c_t, p_t)$ were f is derived from the joint density of $(\tilde{u}_t, \tilde{v}_t)$; the determinant of the Jacobian of the transformation $(\tilde{u}_t, \tilde{v}_t) \mapsto (c_t, p_t)$ is $-\beta_s - \beta_c \frac{C_t}{S_t}$. For regimes (ii) and (iv), price is fixed at the exogenously determined support or release price and consumption and private stocks vary independently. The likelihood for these observations may be written $f(c_t, s_t)$ were f is derived from the joint density of $(\tilde{u}_t, \tilde{v}_t)$; the determinant of the Jacobian of the transformation is unity. The likelihood function to be maximized is thus:

$$
\begin{aligned}
(14) \quad \mathcal{L} \quad = \quad & -n \log(2\pi) - \frac{n}{2} \log(\sigma_u^2 \cdot \sigma_v^2) + \sum_t \phi_t \log(-(\beta_s + \beta_c \cdot \frac{C_t}{S_t})) \\
& - \frac{1}{2\sigma_u^2} \sum_t \{c_t - \alpha_c \cdot x_t^c - \beta_c \cdot p_t\}^2 \\
& - \frac{1}{2\sigma_v^2} \sum_t \{s_t - \alpha_s \cdot x_t^s - \beta_s \cdot (p_t^e - p_t - r_t) - \gamma_s \cdot p_t^v\}^2.
\end{aligned}
$$

were n is the number of observations and

$$
(15) \qquad \phi_t = \begin{cases} 1 & \text{if regime i or iii was realized in year } t \\ 0 & \text{if regime ii or iv was realized in year } t. \end{cases}
$$

Computation of Expectations

In the second step of each iteration, the updated market model is solved for the one period ahead price expectations and variances. Due to its nonlinear dynamic structure, the market model possesses no closed form solution and must be solved numerically using recursive dynamic programming techniques. Below, the numerical solution strategy is discussed in general terms. A more extensive exposition of how to solve rational expectations models using dynamic programming is available elsewhere (Williams and Wright; Miranda).

Suppose we wish to compute $\mathcal{E}_t P_{t+1}$ and $\mathcal{V}_t P_{t+1}$, the expectation and variance of price in year $t+1$ conditional on the information available in year t. Select a time horizon T and assume zero carryout for that period. This assumption effectively severs the intertemporal link between period $t+T$ and subsequent periods, yielding a soluble finite horizon approximation to the infinite horizon model. As is typical of dynamic programming models, the price expectations and variances computed for the finite horizon model converge to the rational price expectations and variances of the infinite horizon model as the horizon T is increased.[7]

Under the finite horizon assumption, it is possible to solve the equation system (1)-(2), (6)-(13) for the price that will prevail in period $t+T$ as a function of the "state" realized in period $t+T$. The state vector consists of the preceding period's acreage planted, private stocks, and government stocks and the contemporaneous realization of the exogenous shocks $\tilde{\Theta}_{t+T} = (\tilde{U}_{t+T}, \tilde{V}_{t+T}, \tilde{W}_{t+T}, \tilde{Y}_{t+T})$:

$$P_{t+T} = P_{t+T}(A_{t+T-1}, S_{t+T-1}, G_{t+T-1}, \tilde{\Theta}_{t+T}). \tag{16}$$

Since the relation between the equilibrium price and the state variables is not expressible in closed form, the price function $P_{t+T}(\cdot)$ is approximated using an interpolative spline. This involves discretizing the state-space into a finite grid of points, solving for prices only at the grid points, and interpolating the grid prices whenever necessary (deBoor). The equation system (1)-(2), (6)-(13) is solved using a generalization of the Newton method for nonlinear complementarity systems (Josephy).

Next, take the expectation of (16) with respect to the random shocks $\tilde{\Theta}_{t+T}$ to obtain an approximate expression for the price expected in period $t+T-1$:

(17) $$\mathcal{E}_{t+T-1}P_{t+T} = f_{t+T-1}(A_{t+T-1}, S_{t+T-1}, G_{t+T-1}).$$

Similarly, calculate the second moment to obtain an approximate expression for the variance of price in period $t+T-1$:

(18) $$\mathcal{V}_{t+T-1}P_{t+T} = g_{t+T-1}(A_{t+T-1}, S_{t+T-1}, G_{t+T-1}).$$

Using expressions (17) and (18) as approximations for the rational price expectation and variance, solve the model for the price that will prevail in period $t+T-1$ as a function of the state variables, thus obtaining an expression similar to (16) for period $t+T-1$. As before, take expectations to obtain expressions similar to (17) and (18) for the price expectation and variance in period $t+T-2$. These expressions are then used to approximate the rational price expectation and variance for period $t+T-2$, which in turn are used to solve the model for period $t+T-2$, and so on. The procedure is repeated recursively backwards until period t is reached and an approximation for the rationally expected price $\mathcal{E}_tP_{t+1}$ and variance $\mathcal{V}_tP_{t+1}$ are obtained. The entire procedure is repeated for every period t over the estimation range.

In order to ensure that only current information is used in calculating future price expectations and variances, predicted rather than actual future values of exogenous variables are used in the recursion procedure. Specifically, since exogenous variables enter each structural equation only through a coefficient-weighted sum that collapses into a combined constant term, only the combined constant term is predicted. For example, the combined constant term in the consumption demand equation (11) is $k_t^c = \alpha_c \cdot x_t^c$; using the historically observed values of the exogenous variables contained in the vector series x_t^c, the univariate time series of constant terms k_t^c is generated and fitted to the first-order autoregressive relation $k_{t+1}^c = \bar{k}^c + \rho_c \cdot k_t^c + \tilde{\eta}$. When calculating the future price expectations and variances conditional on the information available in year t, only k_t^c is assumed known and predictions for k_{t+1}^c, k_{t+2}^c, and so on are generated using the autoregressive relation. The exogenous terms appearing the inventory demand and acreage supply equations are treated in a similar manner.

Application to U.S. Soybeans

The U.S. soybean market is nearly an ideal subject for an empirical application of the estimation techniques developed above.[8] Private and government stockholding have historically played an important role in the U.S. soybean market. Over the period 1960-88, end-of-year private stocks have varied from a low of 4% of annual consumption to a high of 22%; over the same period, government stocks have varied from nonexistent to 18% of annual consumption. The U.S. soybean market has also been largely free of other forms of government intervention, such as production controls and deficiency payments, that would undermine the validity of the model.

The U.S. soybean market model was estimated using annual 1960-1989 September to August marketing year data (U.S. Department of Agriculture). The consumption variable represents total domestic consumption, exports, and seed utilization over the marketing year. Government stocks represent Commodity Credit Corporation inventories at the end of the marketing year; remaining stocks were assumed to be privately owned. The price variable represents a marketing year average adjusted for inflation using the U.S. Consumer Price Index.

Exogenous variables in the consumption demand equation include the log of a domestic livestock-poultry population index measured in grain consuming animal equivalent units (GCAUs) and the log of the foreign exchange rate measured as the ratio of U.S. dollars to Standard Drawing Rights. The private stock demand equation includes an annual time trend and the interest rate, which is measured by the annualized 6-month commercial paper rate. The acreage supply equation includes the log of lagged acreage and the cost of production, measured by the inflation-adjusted producer price index. All three estimation equations include a constant term.

Table 1 reports the parameter and goodness of fit estimates for the U.S. soybean rational expectations market model; asymptotic standard errors for the parameter estimates appear in parentheses.[9] All the parameter estimates are of the expected sign and their magnitudes are comparable with those reported elsewhere. Preliminary estimates indicated significant autocorrelation in the error terms of all three estimation equations. The reported parameter estimates reflect a correction for autocorrelation using the standard Cochrane-Orcutt transformation.

The R-squared on the consumption demand equation exceeds 95%, indicating that the model explains a high proportion of the variation in consumption demand over the sample period. The high elasticities of the livestock-poultry index and foreign exchange reflect the importance of soybeans as a feed and as a commodity export; only the latter, however, was statistically significant at the 0.01 level. The price elasticity of consumption demand, -0.57, was significant at the 0.01 level.

The private stock demand equation has an R-square of 72%; all of the parameter estimates are significant at the 0.01 level. The elasticity of private stocks with respect to the expected appreciation in the value of stocks is 2.40, suggesting that private stocks are highly sensitive to changes in the current price, the expected future price, and the interest rate. The elasticity of private stocks with respect to price risk is -7.03, indicating that a one percent decrease in the coefficient of variation of price will increase private stock demand by about 7 percent. The significant price risk elasticity suggests that speculative storers are risk averse and confirms the existence of risk premiums in the demand for stocks. The annual exogenous rate of growth in private stocks is estimated to be 6.9%.

The acreage supply equation has an R-squared of 95%; all of the parameter estimates, with the exception of the price risk parameter, are significant at the 0.01 level. The elasticity of acreage supply with respect to lagged acreage, 0.95, indicates that a high proportion of the variation in acreage can be explained directly by lagged acreage, suggesting significant costs of adjustment in production. An elasticity of -0.71 with respect to input costs indicates that acreage supply decisions are also sensitive to variable costs. The elasticity of acreage supply with respect to expected price is 0.51. The estimates did not detect appreciable risk response in acreage supply.

For the purposes of comparison, the soybean market model was also estimated using an autoregressive expectation and variance formulation in place of the rationality assumption. Price expectations were modeled as a second order autoregressive process; price risk was represented by the three year moving coefficient of variation of price about its simple moving mean. As seen in the second column of Table 1, the fits provided for the consumption demand and acreage supply equations are roughly similar between the two models. The estimates of the private stock demand equation parameters, however, differ markedly. The rational expectations model explains the variation in private stocks demand (R-squared of 72%) significantly better than the autoregressive expectation model (R-squared of 47%). Moreover, the autoregressive model yields a positive elasticity of price risk, suggesting that storers are risk lovers; the elasticity, however, is statistically insignificant.

Finally, the rational expectations model's price forecasting ability was compared to that of the autoregressive model. For this purpose, a simple Fair-Shiller encompassing test was performed in which the realized market price was regressed against the ex-ante expectations implied by the rational and autoregressive expectations models. The parameter estimates for the Fair-Shiller auxiliary regression are presented in Table 2. The high significance and near unitary value on the rational expectation and the insignificant and near zero value on the autoregressive expectation strongly support the hypothesis that the rational expectations model captures substantially more information regarding future price movements than the autoregressive expectations model. The superiority of the rational expectations model is confirmed by a comparison of the mean-squared prediction errors, which are 1.578 for the rational expectations model and 2.223 for the autoregressive expectations model.

Conclusion

In this paper, we have developed a dynamic nonlinear rational expectations model of a commodity market characterized by private stockholding, government buffer stock intervention, and risk aversion. The model includes a nonlinear private stock demand equation that captures the recurrent backwardation associated with private speculative stockholding and uses complementarity relations rather than equations to explicitly capture the disequilibrium effects of government buffer stock intervention. An iterative numerical strategy that combines stochastic-dynamic programming and disequilibrium econometric methods was developed for generating maximum likelihood parameter estimates that observe all structural restrictions, including those implied by rational expectations.

In an empirical application to the U.S. soybean market, evidence of risk aversion was detected in private stockholding but not in acreage supply. A comparative estimation also established that the nonlinear rational expectations model outperforms the autoregressive expectations model both in the ability to explain private stockholding behavior and in the ability to forecast prices. The results confirm that private and government stockholding play an important role in primary commodity market dynamics and should not be ignored in empirical analysis. The model and method developed here should be applicable, with modifications, to markets for other domestically produced agricultural commodities, such as wheat or corn, and to internationally traded commodities regulated by buffer stock agreements such as cocoa, coffee, rubber, and tin.

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Footnotes

1. Primary commodities include agricultural field crops (e.g., corn, wheat, soybeans), agricultural tree crops (e.g., cocoa, coffee, rubber), and metals (e.g., aluminum, tin, copper).

2. Autoregressive and rational price expectations are equivalent only under the highly restrictive assumption of linearity. As we shall argue below, linear models are inherently inappropriate for commodities subject to either private or government storage.

3. Gardner (1976) suggested the use of observed futures prices in econometric estimation as an alternative to autoregressive and rational price expectations; Fackler and King suggest ways in which options data could be used to identify second and higher moments of the price distribution. The limitation of this approach, however, is that price expectations and variances would be exogenous to the model. Thus, the model could not be used to analyze counterfactual policy changes that would significantly alter the distribution of price.

4. An earlier application of disequilibrium methods to a commodity market model by Maddala and Shonkwiler assumed perfect foresight regarding the incidence of government intervention, not rational expectations.

5. Fair and Taylor's original application was specific to macroeconomic model that was free of many of the structural complexities, such as the disequilibrium effects of government buffer stock intervention, that arise in primary commodity markets.

6. We initiated the algorithm by setting the expected future price equal to the current price; in other words, we assumed naive expectations at the first iteration.

7. The parameter estimates obtained using a two-year horizon were found to differ only negligibly from those obtained using longer horizons. Accordingly, the two-year horizon was used in order to economize on computational effort.

8. Soybeans rival corn as the most important agricultural commodity produced in the United States, accounting for over $10 billion in farm-level revenues annually and nearly one-fifth of all U.S. agricultural commodity exports.

9. Estimates for the standard errors were obtained from the diagonal entries of the Hessian of the likelihood function evaluated at its maximum.

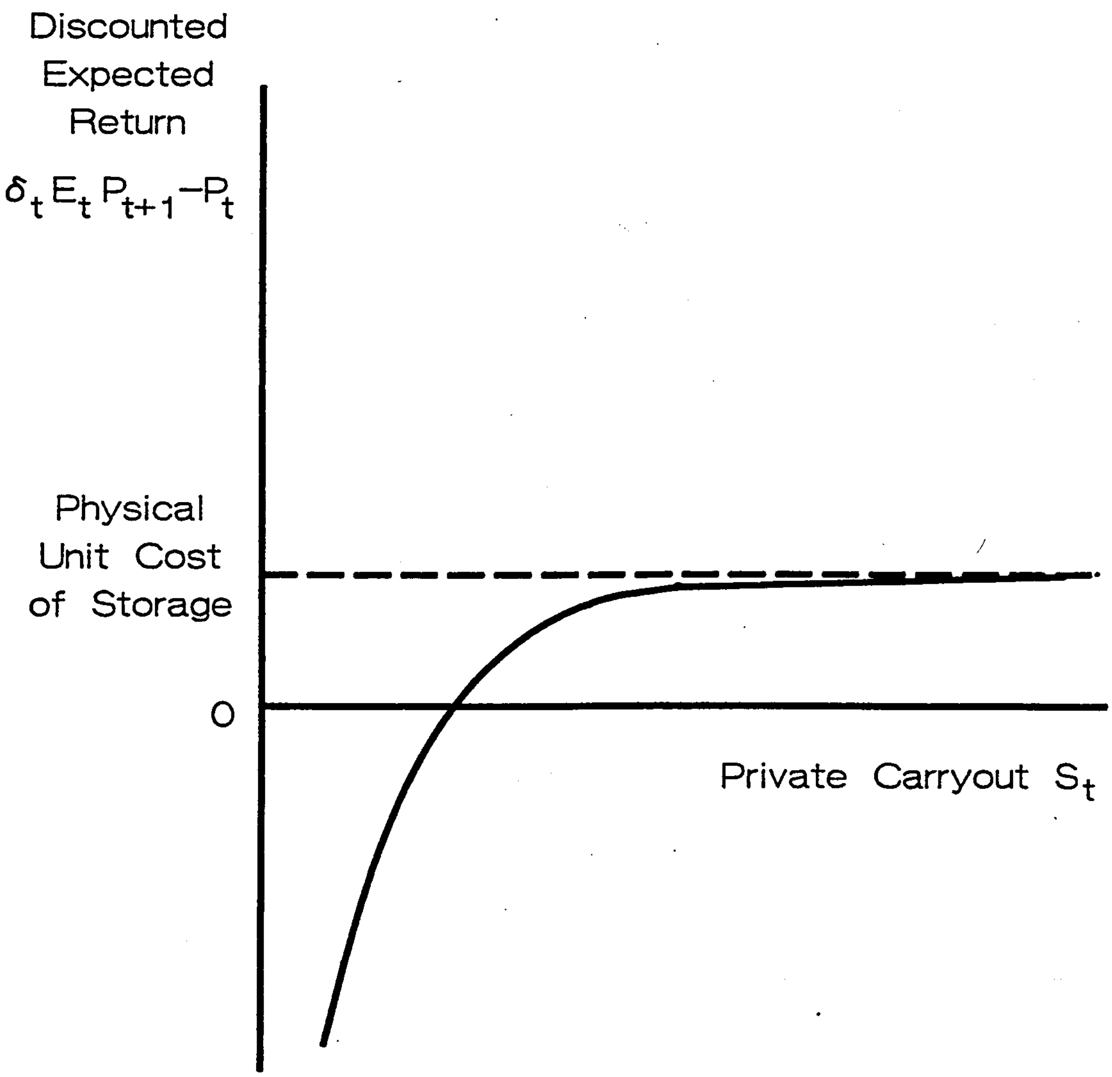

Figure 1. Private Demand for Stocks

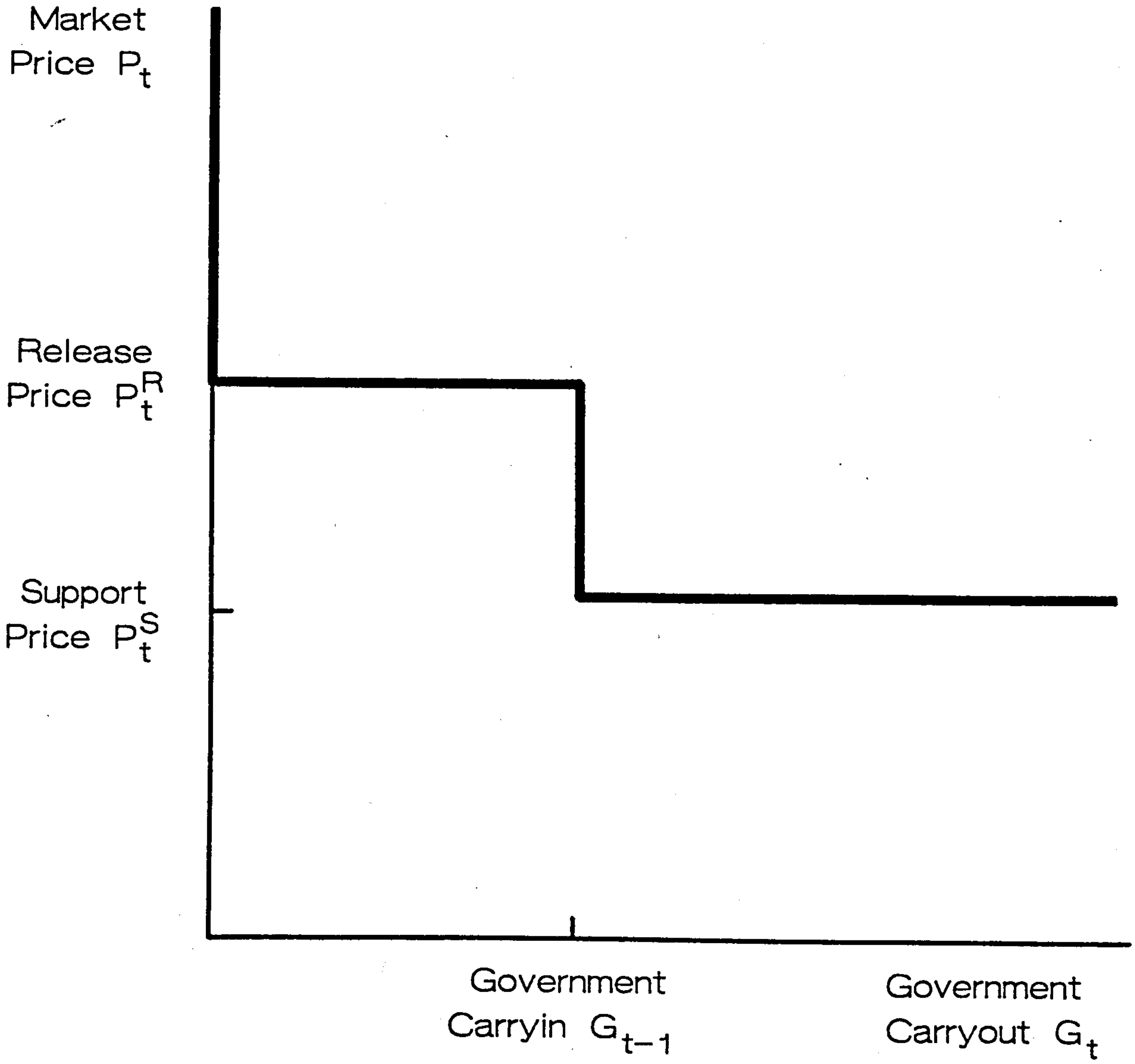

Figure 2. Government Demand for Stocks

TABLE 1. Parameter Estimates for the Rational Expectations and Autoregressive Expectations U.S. Soybean Market Models.*

Equation	Variable	Rational Expectations	Autoregressive Expectations
Consumption Demand	Constant	-0.304 (5.423)	-1.711 (4.804)
	Livestock-Poultry	0.779 (0.480)	0.882 (0.426)
	Exchange Rate	-0.881 (0.345)	-0.700 (0.306)
	Price	-0.574 (0.109)	-0.423 (0.089)
	R-squared	0.952	0.961
Private Stock Demand	Constant	4.913 (0.647)	4.891 (0.768)
	Trend	0.069 (0.028)	0.011 (0.041)
	Expected Appreciation	2.397 (0.659)	13.333 (3.388)
	Price Risk	-7.025 (2.927)	2.186 (2.431)
	R-squared	0.715	0.472
Acreage Supply	Constant	-0.621 (0.420)	-0.270 (0.155)
	Lagged Acres	0.953 (0.036)	0.881 (0.035)
	Cost of Production	-0.712 (0.333)	-0.701 (0.154)
	Expected Price	0.513 (0.220)	0.443 (0.080)
	Price Risk	-0.783 (0.618)	-0.289 (0.173)
	R-squared	0.959	0.975

*Asymptotic Standard Errors in Parentheses.

TABLE 2. Single-Period Price Forecasting Ability of the Rational and Autoregressive Expectations Models: Fair-Schiller Encompassing Test and Mean-Squared Error.

	Rational Expectations	Autoregressive Expectations
Encompassing Parameters*	1.015 (0.314)	-0.009 (0.318)
Mean-Squared Prediction Error	1.578	2.223

* Standard errors in parentheses.