QUANTIFYING LONG RUN AGRICULTURAL RISKS AND EVALUATING FARMER RESPONSES TO RISK

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A Discrete Stochastic Programming Formulation of the Sectoral Model under Uncertain Yields

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The value of mathematical programming models in partial equilibrium analyses of the agricultural sector has long been recognized. The conceptual basis was originally detailed in Samuelson, who identified parallels between sectoral modeling and spatial equilibrium models. Takayama and Judge later developed quadratic programing models for multiproduct equilibria based on Samuelson's work. Successive contributors to the literature are legion (see McCarl and Spreen for a review).

Validation of sectoral models proved troublesome, however, in that predicted quantities of various outputs often overestimated observed supplies. Hazell and Scandizzo (1974, 1975, 1977) included risk as an additional cost of production when farmers, as a group, react negatively to revenue variance. Incorporating risk as a production cost shifts the supply curve leftwards for the risky enterprises, thus generally resulting in lower supplies and higher market prices for the product.

Although descriptive aspects of sectoral models improved, problems remained. Incorporating mean yield into the objective resulted in equilibrium being reached at average yields. Product price is thus not affected by yield variability since product balance equations use average values. In addition, marketing decisions beyond the farm gate are based on average yield outcomes, rather than being sensitive to output distributions.

This paper extends the expected revenue maximization model developed by Hazell and Scandizzo (HS) to include distributional consequences of farmer decisions. Producers plan their enterprise activities in response to subjective yield and price distributions. Producers commit to production decisions before actual prices and yields are known. Total output of farm products results from resource allocations and stochastic yield events. Processing and other marketing decisions are made sequentially, based on farm output under discrete yield states of nature.

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This paper presents a model being developed under Southern Regional Project 232, dealing with aggregate representation of risk and uncertainty in agricultural production.

Aggregate production decisions significantly affect national markets for outputs from the sector. Thus, product prices are determined endogenously in the model. Although deterministic demand functions are assumed, sector output will be state specific. Equilibrium prices and quantities will thus be state sensitive. The objective function is the maximization of expected consumers' plus producers' surplus evaluated under each state of nature.

Modifications of The Hazell-Scandizzo Model

The assumptions of the HS sectoral model are: 1) yields are the sole source of risk (endogenous prices will be random due to random yields); 2) farmers operate in a competitive environment, form expectations of unit revenue, and maximize E,V or E,σ utility[2]; and 3) farmers make decisions before prices and yields are known. HS formalize their linear demand model as:

$$\text{Max} \quad CPS(X) = E[(\alpha - 0.5\ BNX)'\ NX] - C'X$$

subject to

$$AX \leq b$$

$$X \geq 0$$

where X is a vector of activity levels, N is a diagonal matrix of stochastic yields, α - BNX is a vector of price dependent demand equations, C is a vector of unit activity costs, A is a matrix of resource usage coefficients, and b is a vector of resource endowments.

Under the discrete stochastic programming formulation of the HS model, state dependent quantity-price activities are explicitly incorporated:

$$\text{Max} \quad CPS(X) = \Sigma_s \Pr(\Theta_s)\ [(\alpha - 0.5\ BQ_s)'\ Q_s] - C'X$$

subject to

$$Q_s - N_s X \leq 0$$

$$AX \leq b$$

$$X \geq 0$$

where there are S states of nature, $\Pr(\Theta_s)$ is the probability of the s^{th} state occurring, Q_s is a vector of quantities consumed in

2 Risk aversion as an explicit expression in the objective function is ignored (i.e., producers are risk neutral in aggregate) in the following presentation to focus attention on model differences resulting from consideration of discrete events in the calculation of consumers' plus producers' surplus.

s, N_s is a diagonal matrix of yields under state s, and other variables are as defined earlier.

A further modification of the HS sector model considered here is incorporation of sequential decision making. Thus, different information sets are available through time, usually being conditional upon the occurrence of prior events. As an example, let X_1 and X_2 be a partition of X corresponding to decisions in two periods. The maximand for the problem becomes:

$$\max_{X_1} \Sigma_{s1} Pr(\Theta_{s1}) \{ \max_{X_2} Pr(\Theta_{s2|s1}) (\alpha - 0.5 BQ_{s2|s1})' Q_{s2|s1}\} - C'X$$

where s1 is the state of nature realized after the first set of decisions is made and s2 is the state realized after all decisions have been made. Thus, decisions in period 1 are made under the assumption that decisions in period 2 will be optimal for each possible set of outcomes. As with the nonsequential model, there is a set of product balance rows associated with each possible state combination.

The Discrete Stochastic Programming Sectoral Model

Total farm output is stochastic and is represented by a multivariate distribution of discrete outcomes in the discrete stochastic programming (DSP) sectoral model. Processing activities beyond the farm gate are explicitly considered in the model. Allocation of farm output among alternative processing uses, as well as final demand, will be dependent upon output distributions. State dependent allocations assume profit maximization by users of farm output with production and demand relationships known at the time processing decisions are made.

By imposing linear demands, HS were able to represent the expectation of consumers' plus producers' surplus using only the first two moments of the multivariate yield distribution. Although linear demands are used in the following example, different distributional assumptions regarding farm yields influence the state dependent processing activities and, hence, the farm level decisions. Thus, a consequence of the DSP model is that decisions depend on the entire yield distribution, not just the first two moments.

Consider the following representation of the DSP model:

[1] $$\max_{Y,X} CPS_1 = \Sigma_s Pr(\Theta_s)(\Sigma_p Q_{ps}(\alpha_p - 0.5\beta_p Q_{ps})) - \Sigma_f \Sigma_b C_b X_{bf} - \Sigma_s Pr(\Theta_s) \Sigma_c D_c Y_{cs}$$

Subject to

[1a] $[\mu_{if}]$ $\Sigma_b a_{ibf} X_{bf} \leq B_{if}$ for all i, f

$$[1b]\ [\pi_{ps}] \qquad -\Sigma_b\, \Sigma_f\, e_{pbsf} X_{bf} + Q_{ps} - \Sigma_c\, d_{pc} Y_{cs} \leq 0$$

$$Q_{ps},\ X_{bf},\ Y_{cs} \geq 0$$

Dual variables associated with constraint rows are to the left of the equations. Variable definitions are given in Table 1.

Constraint row [1a] imposes resource constraints on farm activities. Each farm's endowment of land, labor, capital, etc. limits crop and livestock activities. Constraint [1b] is the product balance row. The coefficient e_{pbsf} relates farm activities b to output p. In the situation in which X_b is a net supplier of a good, such as corn acreage grown results in corn available for sale or further processing, e is a positive value. When an activity uses the output of a primary activity, such as the beef cow enterprise consuming a certain quantity of corn and hay, the coefficient is negative.

Processing use of farm output is captured through choice variable Y_{cs}. For example, total farm output of corn is determined by eX. Any corn that is used to produce feedgrain or is fed through the fed beef processing activity will be represented by a negative coefficient on Y_{cs} ($d_{pc}<0$). Output from the processing activities, such as fed beef or feedgrain, is added to the product balance row by a positive coefficient d_{pc}.

The variable Q_{ps} is the total quantity of commodity p consumed as a final good in state s. Q can be either a primary farm product, such as bushels of corn, or can result from the processing activities.

First order conditions resulting from [1] characterize farmer and processor decisions.

Individual Farm Production Levels (X_{bf}):

$$[2a]\ \delta L/\delta X_{bf} = \Sigma_s\, \Sigma_p\, \pi_{ps}\, e_{pbsf} - C_b - \Sigma_i\, \mu_{if} a_{ibf} \leq 0$$

$$[2b]\ (\delta L/\delta X_{bf})\, X_{bf} = 0$$

Processing Activity (Y_{cs}):

$$[2c]\ \delta L/\delta Y_{cs} = -Pr(\Theta_s)\, D_c + \Sigma_p\, \pi_{ps}\, d_{pc} \leq 0$$

$$[2d]\ (\delta L/\delta Y_{cs})\, Y_{cs} = 0$$

Aggregate Supply (Q_{ps}):

$$[2e]\ \delta L/\delta Q_{ps} = Pr(\Theta_s)\, (\alpha_p - \beta_p Q_{ps}) - \pi_{ps} \leq 0$$

$$[2f]\ (\delta L/\delta Q_{ps})\, Q_{ps} = 0$$

When output p is produced, such that Q_{ps} is positive, the shadow price on the product balance row for p will equal its price under state of nature s, weighted by the state's probability:

$$Price_{ps} = \alpha_p - \beta_p Q_{ps} = (1/Pr(\Theta_s))\ \pi_{ps}$$

Substituting into [2a], farm allocations to activity X_{bf} are increased until the expected marginal value product of X_b in all uses p equals cash and opportunity costs associated with the production activity.

Processing activity Y_{cs} will be state dependent. From [2c],

$$\Sigma_p\ \pi_{ps}\ d_{pc}^{+} \ \leq\ Pr(\Theta_s)\ D_c + \Sigma_p\ \pi_{ps}\ d_{pc}^{-}$$

where d^+ (i.e., d > 0) denotes products supplied by Y_{cs} and d^- (d < 0) denotes intermediate factors used in the processing activity. Marginal returns of the Y_c th processing activity will be equated to per unit cost and the marginal factor costs of factors used in the process. Marginal factor cost is state dependent, reflected by π_{ps}.

An Example

This section describes an example DSP model. Two regions were represented in the example, each having distinct farm yield characteristics and production coefficients. Processing activities were assumed to be national rather than specific to the two regions. Activities for the model are listed in Table 2.

Corn, wheat, soybeans, and hay yields were random. Per acre yields were generated using both normal and uniform marginal distributions. Parameters of the generated samples are listed in Table 3. In addition, procedures detailed in Richardson and Condra were used to approximate annual correlations among the series reported for 1976-88 (USDA). Sample correlation characteristics are also reported in Table 3.

Farm level allocations are reported in table 4 for regions 1 and 2 under both the normal and the uniform yield assumptions. An example of the output from the DSP formulation is reported in Table 5 for four of the twenty states of nature considered in the example.

Farm Prices

Even with the linear demand and supply functions specified in this model, the DSP formulation allows price distributions to differ markedly from yield distributions. Wheat prices, for example, do not mirror the underlying yield distribution (figure 1).

Changes in the proportions of output diverted to processing activities may result under different states of nature. For example, figure 2 shows how the proportions of wheat change between direct sales versus use in further processing.

Given the competitive structure of the model, marginal returns to an activity's output will equilibrate among all uses under each state of nature. Using wheat as an example again, returns to wheat accrue from various sources:

1. Quantity delivered to final demand: $\alpha - \beta Q_{ps}$

2. Intermediate uses of wheat in:

   a. the beef cow enterprise

   b. processing uses in three different ration mixtures yielding the commodity feedgrain

The interdependencies among the different activities thus transform the original distribution of wheat yields into a potentially very different distribution of, for example, wheat prices.

<u>Processing Activities</u>

Shares of the basic farm output going into various processing and final demand uses are state dependent, depending upon the marginal opportunity costs of the commodity represented in the left hand side of [2c].For example, figure 2 displays the proportion of wheat diverted to processing use versus direct sales to domestic demand. Although the relationships between yield and final price distributions are not exact, as discussed above, domestic price level is generally negatively correlated with low yield states of nature. Wheat's marginal contribution under low yield to value added processing activities is greater than direct sale.

Even under the simple example developed here, factor price relationships observed in the real world are reflected in the solutions. A good example is the proportion of nonfed versus fed beef sold to final demand. Feedgrain, which results from three alternative mixes of wheat and corn, is a major component in the feeding activity. Just as corn price is a major contributor to fed supply in econometric studies, a significant linear relationship is found between the state specific price of feedgrain and the ratio of nonfed to fed beef (figure 3).[3]

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3 One exception to the linear relationship occurred under a state of nature in which hay yields were near the minimum of their range. Under this state, all hay was used in nonfed beef production, yet supply was insufficient to produce nonfed beef beyond the level representative of the ratio observed under feedgrain price of $8.53.

Concluding Comments

Several distinguishing features of the DSP approach to sectoral modeling support its usefulness for analyzing sectoral response to exogenous factors. First, incorporation of discrete events provides distributional information on output, prices, and other measures in analyzing policy impacts. Chang et al. have already reported preliminary results using DSP to generate distributions of deficiency payments for different target prices.

Second, event outcomes are seen to affect processing activities beyond the farm gate. Even with the linear system of demands and supplies used here, knowledge of state dependent processing sector responses result in farm level decisions that depend upon the entire multivariate yield distribution, not just the first two moments. Sectoral models that include activities beyond the farm gate should thus consider distributional factors in modeling farm output as intermediate factors to value added activities.

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FIGURE 1. Wheat Price Distributions under Uniform and Normal Yields

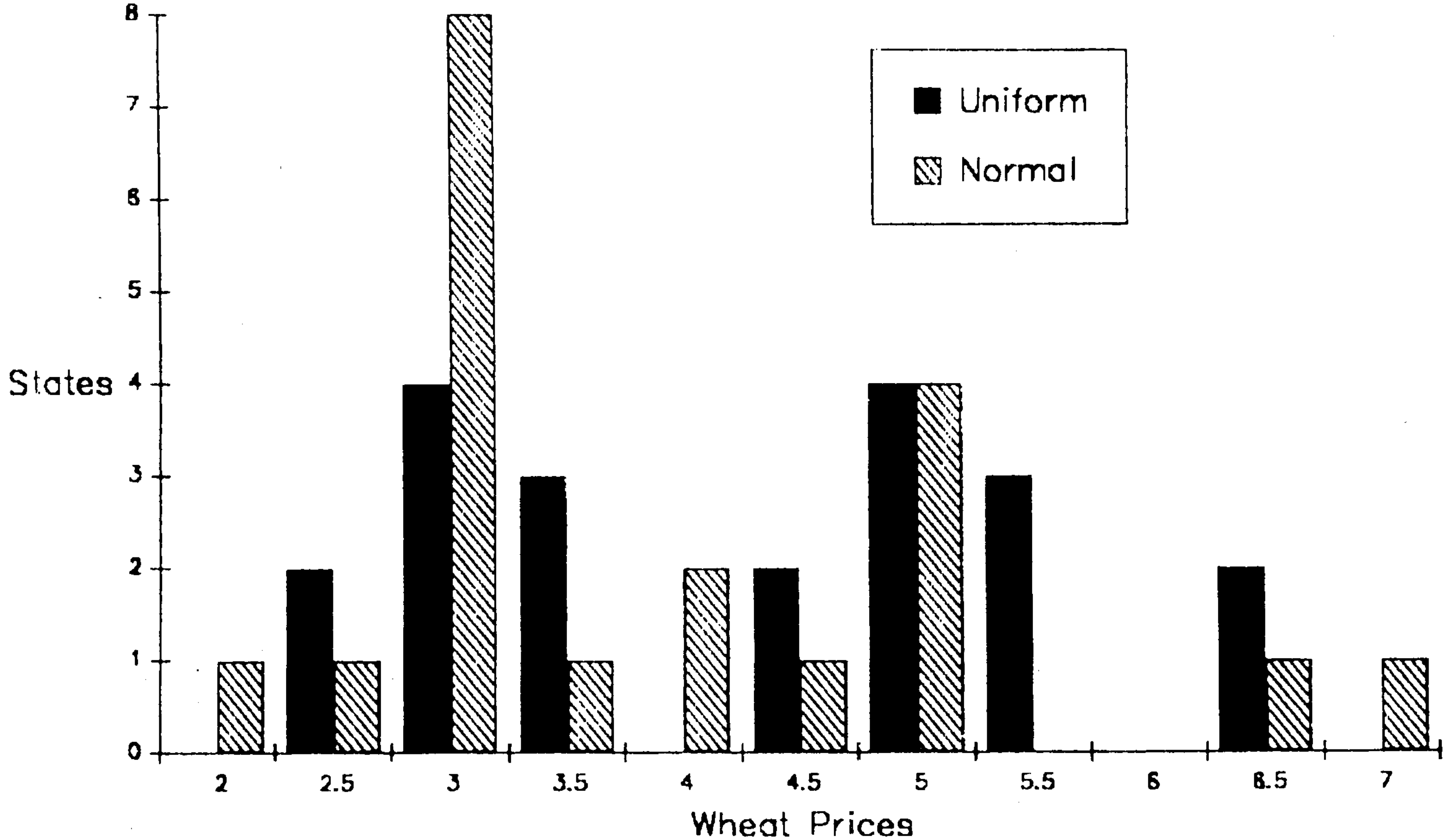

Figure 2. Ratio of Wheat in Processing Use to Domestic Use

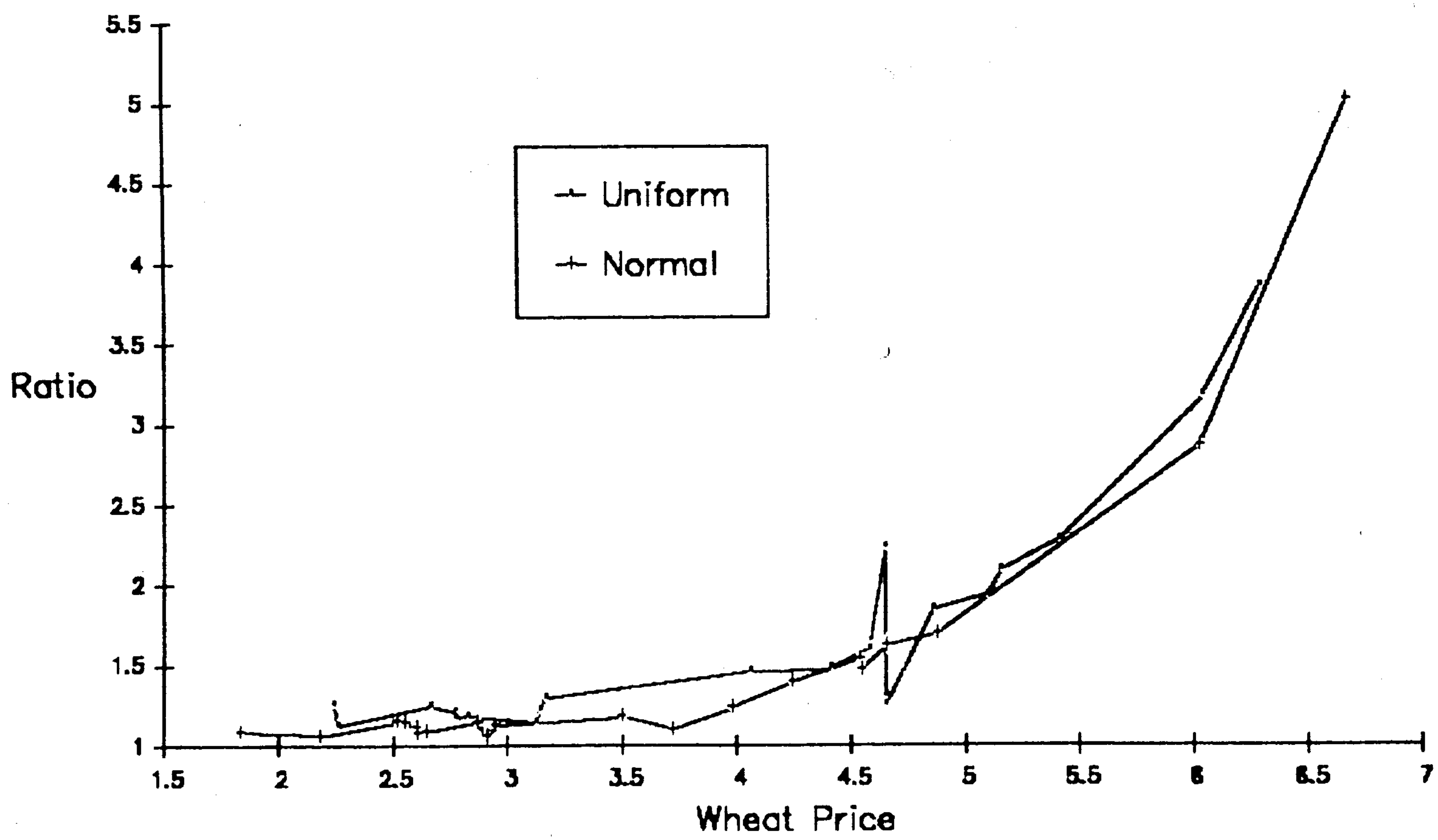

Figure 3. Ratio of Nonfed to Fed Beef as a Function of Feedgrain Price

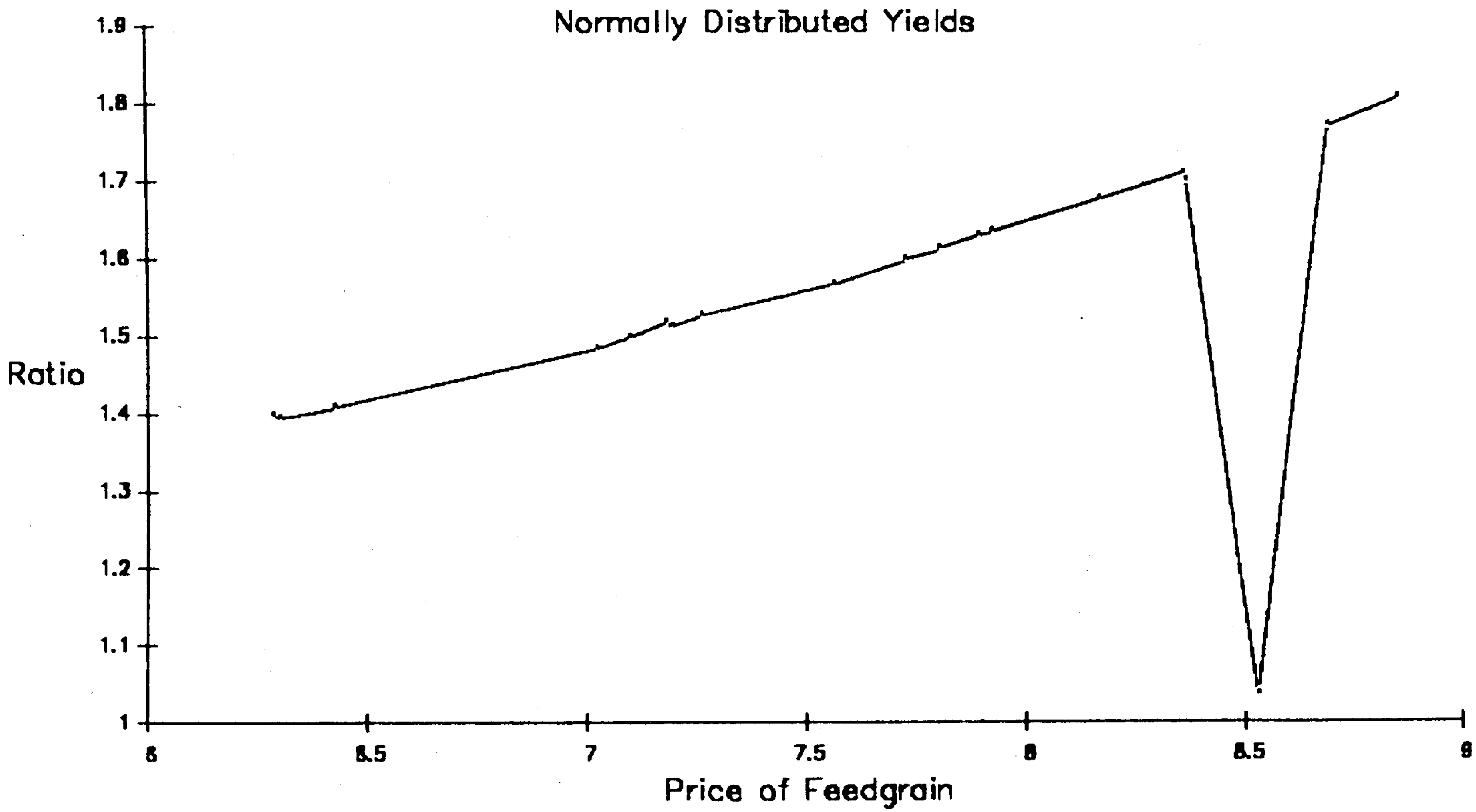

Table 1. Variable and Parameter Definitions for Example Problem

Endogenous Variables

CPS_1 - expected value of consumers plus producers surplus evaluated over all yield outcomes s

Q_{ps} - total quantity of product p consumed as a final good under state s

X_{bf} - farm level activity of b for farm f

Y_{cs} - processing activity level of c under state s

Parameters

$Pr(\Theta_s)$ - probability of state s occurring

α_p, β_p - intercept and slope, respectively, of inverse demand functions for output p

C_b - per unit cost of farm activity X_b

D_c - per unit processing cost for processing activity c

d_{pc} - per unit yield of commodity p from processing activity c

e_{pbsf} - production coefficient converting activity X_b to intermediate or final product p under state of nature s

a_{ibf} - farm activity X_b's use of fixed resource B_i

B_{if} - farm f's total endowment of resource i

Table 2. Example Model Components

Regional Farm Production Activities -
Corn, Wheat, Soybeans, Beefcowen, Hay

National Processing Activities -
Soycrush1, Soycrush2, Grain1, Grain2, Grain3, Cow-to-Fedbeef, Cow-to-Non-Fedbeef

Production Input-Output Items -
Corn, Wheat, Soybeans, Hay, Feedgrain, Beanmeal, Fedbeef, Non-Fedbeef, Yearlings, Land, Fertilizer, Planting-Labor, Harvest-Labor, Plowing-Labor, Planting-Cost, Harvest-Cost, Plowing-Cost, Beefcowen-Cost

Commodities-Outputs -
Corn, Wheat, Soybeans, Hay, Feedgrain, Beanmeal, Fedbeef, Non-Fedbeef

Table 3. Sample Yield Characteristics

	Region 1	Region2
Corn		
Mean	82.77	112.77
Std Dev	6.40	6.40
Wheat		
Mean	49.33	28.59
Std Dev	10.09	4.48
Soybeans		
Mean	34.60	47.00
Std Dev	5.29	6.62
Hay		
Mean	1.64	2.85
Std Dev	0.32	0.43

Sample Correlation Matrix

Normal	Corn	Wheat	Soy	Hay
Corn	1.00			
Wheat	0.26	1.00		
Soy	0.86	-0.01	1.00	
Hay	0.70	0.56	0.56	1.00

Uniform	Corn	Wheat	Soy	Hay
Corn	1.00			
Wheat	0.30	1.00		
Soy	0.80	-0.03	1.00	
Hay	0.66	0.60	0.44	1.00

Table 4. Farm Activity Level Results

	Uniform Acres		Normal Acres	
Corn	2,386		2,334	
Wheat	1,783		1,784	
Soybeans	2,061		2,113	
Beefcowen	2,412	(Hd)	2,104	(Hd)
Hay	10,858		11,167	

Table 5. Results for Four States of Nature - Normally Distributed Yields

		State of Nature 1	2	3	4
PRICE	.CORN ($/bu)	2.20	2.55	2.70	2.47
PRICE	.WHEAT($/bu)	4.65	2.55	4.54	4.24
PRICE	.SOY ($/bu)	4.70	6.67	6.76	4.57
PRICE	.HAY ($/ton)	33.76	38.07	74.41	35.36
PRICE	.FEDBEEF ($/hd)	562.97	554.10	569.97	564.46
PRICE	.NFEDBEEF ($/hd)	482.40	481.74	485.85	482.66
PRICE	.FEEDGRAIN ($/bu)	7.81	7.10	8.39	7.93
PRICE	.BEANMEAL ($)	16.16	19.46	19.61	15.95
PRODUCTION	.CORN (bu)	252479	240835	230622	242301
PRODUCTION	.WHEAT (bu)	74486	105793	73046	78031
PRODUCTION	.SOY (bu)	82466	66631	65910	83490
PRODUCTION	.HAY (ton)	32844	31768	22677	32443
PROC-USE	.CORN (bu)	99189	99122	93487	97983
PROC-USE	.WHEAT (bu)	46000	56335	43491	45469
PROC-USE	.SOY (bu)	28824	21966	21654	29267
PROC-USE	.HAY (ton)	18784	18785	18781	18784
PROC-USE	.FEDBEEF (hd)	2490	2687	2334	2456
PROC-USE	.NFEDBEEF (hd	4014	4024	3962	4010
PROC-USE	.FEEDGRAIN (bu)	112458	120639	105880	111072
PROC-USE	.BEANMEAL (unit)	17294	13180	12992	17560
DOM-USE	.CORN (bu)	153289	141713	137136	144318
DOM-USE	.WHEAT (bu)	28485	49459	29555	32562
DOM-USE	.SOY (bu)	53642	44665	44256	54222
DOM-USE	.HAY (ton)	14059	12983	13896	13659
DOM-USE	.FEDBEEF (hd)	2490	2687	2334	2456
DOM-USE	.NFEDBEEF (hd)	4014	4024	3962	4010
DOM-USE	.FEEDGRAIN (bu)	39513	42739	36965	38969
DOM-USE	.BEANMEAL (unit)	17294	13180	12992	17560

Nonstate Dependent Activity Levels

PRICE	.YRLNG ($/hd)	347.30
PRICE	.FERTILIZER ($/lb)	0.65
PRODUCTION	.YRLNG (hd)	8417
ACRES	.CORN	2332
ACRES	.WHEAT	1784
ACRES	.SOY	2113
ACRES	.HAY	11167
ACRES	.BFCOWEN PASTURE	2104