QUANTIFYING LONG RUN AGRICULTURAL RISKS AND EVALUATING FARMER RESPONSES TO RISK

Proceedings of a Seminar sponsored by
Southern Regional Project S-232
"Quantifying Long Run Agricultural Risks and Evaluating
Farmer Responses to Risk"
Sanibel Island, Florida
January 28-31, 1990

Food and Resource Economics Department
University of Florida
Gainesville, Florida

April 1990

Aggregation, Heterogeneity and Risk: Adding Apples and Oranges, or Apples and Apples

Rulon D. Pope*

Abstract: Approaches to aggregation are reviewed. These consist of random parameters, random right hand side variables (Stoker's approach), and exact aggregation. Each approach utilizes different assumptions and has weaknesses. In a sense, exact aggregation is the nonparametric approach. The penultimate section of the paper shows that these concepts may be very important for risk analysis. A simulation model gives substantially biased estimates of regression parameters when exact aggregation procedures are not followed.

Key Words: Risk, Aggregation

Though agricultural economists have used methods that might be rationalized as having something to do with aggregation, there has been little attempt to explicitly "go after" aggregation. Programming work of Paris and later Onal and McCarl are the most direct look at aggregation issues of the heterogeneous firm variety using programming methods. To be sure, every market study attempts to model the social aggregation of decisions but these studies rarely focus on heterogeneity (A case of limited heterogeneity or variations in firms is found in Holt).

The sparse discussion of empirical aggregation procedures and issues in agricultural economics is surprising given the amount of activity this receives in other areas of economics. Either agricultural economists see their procedures as free from aggregation bias or they feel that there is little to be learned from studying aggregation.

The intention in this paper is to consider some micro/macro relationships as typical in the aggregation literature and then consider its relevance to risk taking. The emphasis is on heterogeneity among decision makers and procedures to account for this heterogeneity.

II. Background Discussion

The first issue is: How and to what extent does the market discipline micro expectations and decisions? That is, does the market force through entry and exit and learning a particular structure of beliefs or preferences? It seems clear that it may, but it is unclear how this occurs just as it is under certainty. The most popular view appears to be that micro expectations are similar to macro expectations. However, as Haltiwanger and Waldman (1989) point out, rational expectations at the market level and at the micro level are two very different things in general.

---

*Professor of Economics, Brigham Young University, currently, visiting Professor of Agricultural Economics, Texas A&M University.

A firm must expect what others in the market are doing in order to rationally expect market or aggregate price. This comes very close to completely unraveling competitive decentralized decision making. We often argue that the competitive firm under certainty need only know it's own cost structure and market price, it need not engage in speculation about other's behavior. In other words prices are sufficient statistics. Under uncertainty, without an infinitely repeated game, it appears that a firm whether risk averse or risk neutral must gather information and guess the impact of a large number of exogenous variables on aggregate behavior.

It will continue to be debated whether information is sufficiently costless to make rational or alternatively some kind of adaptive mechanism the best descriptor of short run experience. The work by the experimentalists seems to suggest that a great dispersion of expectations and behaviors occur in the day to day experience. Indeed, the stock market's very existence seems to suggest that individuals have heterogeneous beliefs.

Several possible approaches to aggregate risk analysis have been developed. The most common of concepts in agricultural economics is to model micro-decisions, and aggregate them up to form an aggregate supply and price distribution. This however requires a great deal of information and calculation. The most common practice is to merely model the aggregate using micro economic concepts. This generally implies that heterogeneity is not acknowledged.

This seems inappropriate for many agricultural commodities because there are quality, timing, and spatial differences across micro agents. To illustrate, there have been dozens of studies done on aggregate wheat response. Yet, the correlations of prices (annual averages) across states is not nearly unity. In the table below correlations for the prices of wheat for Oklahoma, Texas, Washington, California, and North Dakota are presented for years 1980-85 (Chambers and Pope):

	OK	TX	WA	CA	ND
OK	1.00	.92	.91	.89	.67
TX		1.00	.92	.81	.70
WA			1.00	.68	.62
CA				1.00	.70
ND					1.00

Timing and quality or fundamental grain differences cause very heterogeneous price regimes. Should each state be modeled as having the world market annual average price or does the market discipline these relationships among the various wheat qualities (states) in a systematic way. The former is surely not correct. Yet the latter requires that aggregate work take account of this heterogeneity. A necessary part of the puzzle is to explore how to use aggregate indices of prices (heterogeneity) to consistently aggregate micro prices (risks). Thus, we ask whether for example state level price and yield indices can be used to aggregate county (state) wide decision which are in different price/yield regimes. In general, approaches which are considered here look for ways to use indices, based upon micro-data for use in aggregate empirical work. This appears to be the best economists can hope for if consistent aggregation is to be achieved.

III. Notation, Definitions, and Selected Earlier Results

In this section, a brief review of some of the price and income aggregation literature is undertaken. It generally presumes that there is a single heterogeneous variable in a functional relationship. I will assume that decisions are represented by

$$(1)\ x_i = f_i(p, y_i)$$

where f_i is once (and sometimes twice) differentiable and monotonic in its arguments, y_i is the heterogeneous variable (e.g., wheat price expectations), and p is a vector of variables common to each firm.

Exact Aggregation

Under exact aggregation, there exists an aggregate index y_o such that

$$(2)\ x_o\,(y_o,\ p) = X(x, y, p),$$

where x and y are vectors of dimension N and p is of dimension m. That is, there are N individuals and m common prices or parameters p.

Equation 2 will be called an aggregation structure or rule, and y_o = $Y(y, p)$ is called the aggregate y index. If one interprets y as income, then y_o might be per capita incomes, a variance, or a Gini index.

The right side of (2) could simply be that

$$(3)\quad X(x, y, p) = X(x) = \sum_{i=1}^{N} x_i .$$

That is the aggregate relationship $x_o(y_o, p)$ yields the sum of micro decisions. Gorman considered such a case where y_o was per capita or average income.

A second example of the right side of (2) is Muellbauer's aggregation rule:

$$(4)\ X(x, y, p) = p \sum_{i=1}^{N} x_i \Big/ \sum_{i=1}^{N} y_i = \sum_{i=1}^{N}(y_i\ w_i)\Big/\sum_{i=1}^{N} y_i$$

where w_i is the budget share $p\ x_i/y_i$. The aggregation structures in (3) and (4) have been used extensively in empirical work. Letting y_o in (3) be the sum of y_i or per capita (average) income, yields the famous Gorman Polar form:

$$(5)\ x_i = \alpha_i(p) + \beta(p)\ y_i \qquad i = 1, \ldots, N,$$

as the micro decision which yields exact aggregation in (3). In the consumer case, where y_i is income, these income consumption curves are often described by quasi-homothetic preferences. That is, Engel curves are linear but do not necessarily go through the origin.

In the case of (4), the popular AIDS or PIGL demand system is obtained as a special case where,

$$(6)\quad x_i = \alpha_i(p) + \beta(p)\, y_i^{\lambda}. \qquad i = 1,\ldots,N$$

and all commodities in the system are of this form (Deaton and Muellbauer). When the parameter λ is one, Gorman's polar form is obtained. This form where x_i is interpreted as a budget share has had widespread and successful use independent of the aggregation issue.

In a series of recent papers, Robert Chambers and I have applied and generalized the work of Gorman, Muellbauer, Lau and developed results more applicable to the agricultural production case. The approach recognizes that the government variously calculates y_o. For example, if one considers y_i to be the price of wheat, the published aggregate U.S. price of wheat is of the Laspeyres type (Pope and Chambers):

$$(7)\quad y_o = \sum_{i=1}^{N} y_i\, x_i / \sum_{i=1}^{N} y_i$$

This will determine the set of decisions x_i which consistently aggregate (7). This form with structure (3) is:

$$(8)\quad x_i = \frac{\alpha_i(p)}{y_i - \beta(p)} \qquad i=1,\ldots,N.$$

Thus, firms vary with respect to α_i and y_i only. This greatly reduces the kind of heterogeneity allowed. Clearly, (8) can be nested in a more general form and then a test for consistent aggregation can be accomplished (Chambers and Pope). Further, it may well be that it will fit data well as the AIDS appears to have done. Preliminary work rejects this Laspeyres type aggregation rule (Chambers and Pope). The essential point is that if one picks an aggregation rule and an aggregate index, consistent aggregation yields a rather specific form of micro decision functions.

There is another group of literature that considers multiple indices of a heterogeneous variable y. Probably the most pertinent of these is due to Gorman and Lau. Lau does not impose *a prior* linearity of aggregate demand in the indices as does Gorman. However, he does assume that the aggregate indices are independent of p and symmetric. In such case, the micro functions which exactly satisfy (3) are of the form[1/]:

$$(9)\quad x_i = \alpha_i(p) + \sum_{k=1}^{K} \beta^k(p)\, g^k(y_i) \qquad i=1,\ldots,N.$$

The aggregate function is:

$$(10)\quad x_o = F(p, \sum_{i=1}^{N} g^1(y_i),\ldots,\sum_{i=1}^{N} g^K(y_i)) = \sum_{i=1}^{N} x_i.$$

Thus symmetry rules out aggregators (aggregate y indices)which are not additive functions. This rules out Gini coefficients, fractiles, and apparently sample variances as aggregate indices. Though sample variances are symmetric, they aren't additive functions in the y_i's.

Random Coefficients-Parameter Heterogeneity

One of the oldest and perhaps most well known approaches to heterogeneity is given in Theil (1954). Let each agent have a linear decision form:

$$(11)\ x_i = \alpha_i + B_i y_i + U_i\ ,\ E(U_i) = 0$$

Summing and dividing by N (and multiplying by $(\Sigma y_i / \Sigma y_i)$)yields

$$(11)\ \bar{x} = \alpha + \frac{\sum_{i=1}^{N} B_i y_i}{\Sigma_i y_i}\ \bar{y} + \bar{U},$$

where the bars denote sample averages. Letting $B_i = B + \epsilon_i$ and y_i be fixed $E\ \bar{y} = \alpha + B\ \bar{Y}$. Thus if B can be consistently estimated, the mean of the micro parameters can be estimated. This is generally referred to as the macro parameter. This idea could be extended to any random coefficient model.

Indeed the idea appears to be more general. Letting $x_i = f(\beta_i, y_i)$ and let the density of β_i be described by $g(\beta_i | \bar{\beta}_i, \delta_i)$ where $\bar{\beta}_i = E\beta_i$ and δ_i represents other parameters or data. Then the conditional mean of y_i is $\int f_i(y_i, \beta_i)\ g(\beta_i | \bar{\beta}_i, \delta_i) d\beta_i = h(\bar{\beta}_i, \delta_i, y_i)$. Generally, data requires that $\bar{\beta}_i = \beta$ for all i. This idea of heterogeneous parameters in a regression is used extensively in labor economics (Butler and McDonald).

This random coefficient approach has been used throughout its development in agricultural economics. Sometimes aggregation may be the motivation for these models and other times its' just an attempt to obtain a particular form of heteroskedasticity to the basic model or to obtain a random component to firm level effects (Swamy, et al).

Random Exogenous Variables

Stoker looks at the aggregation issue from another point of view. The micro-model consists of a form assumed constant across individuals except that each has different random realizations of these variables. The macro function would be given be $E(x_i)$ in terms of moments of the distribution of y_i and α and β. Stoker develops a number of theorems about identification and estimation of the macro/micro relationships. The rather striking flavor of these results can be obtained by considering again the cross-sectional linear regression

$$(12)\ x_i = \alpha + \beta y_i + \mu_i \qquad\qquad i = 1, \ldots, N.$$

Assume that y_i is distributed lognormally with mean μ. Stoker shows that a consistent estimate of $\partial E(x_i)/\partial\mu$ is obtained by estimating the above linear regression with $\ln y_i$ as an instrument. This derivative identifies the micro-response when the distribution of y_i is from a family possessing completeness and sufficiency as does the lognormal to any exponential family. Thus, by specifying a distribution for either y_i or β_i, one can use the Theil or Stoker approach to obtain a consistent estimate of an average response.

Let me conclude this section with a few comments about the three approaches listed above. The exact aggregation approach is nonparametric in the sense that no distributional assumptions hold. For each possible value of y_i, the aggregation rule holds. However, it does substantially restrict the micro functional form and hence the aggregate distribution.

The random coefficient approach essentially requires that errors in the coefficients be uncorrelated with decision or measurement errors in the dependent variable. (The Stoker approach requires a similar assumption.) This seems unlikely to be the case. To my knowledge, little progress has been made on random coefficient nonlinear econometric models.

With regard to Stoker's approach, it is unknown how it will perform in small samples. All results are for large samples. This approach can subsume Theil's approach so that the micro function to be aggregated can be derived from "expecting" a random coefficient behavioral model. That is conditional expectation of the random coefficient model can yield the micro model which is to consistently aggregate. Finally, one must have a good idea of the distribution of the exogenous variables which must belong to a family admitting sufficient statistics. To my knowledge, no agricultural applications have been made of Stoker's approach.

There is much to commend each of the three approaches and at present there is little basis to choose one method over the other. Where possible, it would seem that exact aggregation ought to be considered since it allows for differences in the functional forms as well as attributes of micro-agents. It is the non parametric approach to aggregation.

IV. Risk Aggregation

The question now arises as to what any of this has to do with aggregation under risk. To illustrate, consider first the case where all variables are identical across firms with the exception of risk aversion. In such case, p represents means, variances, or probabilities and y_i is the unobserved risk aversion. An index of risk aversion independent of p implies micro-demands of the form:

$$(13) \quad x_i = \alpha_i(p) + \beta(p)g_i(y_i) \qquad i = 1,\dots,N.$$

where g_i is any function of y_i. Monotonic transformations of $\sum_{i=1}^{N} g_i(y_i)$ could serve as the aggregate index when micro decisions must sum up [see 3].

If (3) is adopted and average decisions are linearly related to average risk aversion, the distribution of risk aversion does not alter average decisions in the sense that a mean preserving spread of risk aversion would not alter aggregate behavior. Incidentally, one could let $y_i = \bar{y} + \epsilon_i$ ($i = 1,\dots,N.$) and obtain the random parameter case commonly used to yield firm effects. The marginal effect of aggregate risk aversion on aggregate decisions unbiasedly (and/or consistently) estimates the corresponding micro response. The linear case is indeed attractive.

However, it may also be the case that distributional considerations matter. Indeed the whole motivation for Gorman's, Lau's and Muellbaurer's work was in this direction.

Using Lau's result, let

$$(14)\quad x_i = \alpha(p) + \beta^1(p)\bar{y}_i + \beta^2(p)\sigma_i^{\,2} \qquad i = 1,\ldots,N.$$

where β^1 and β^2 might be related to indices of risk aversion, then aggregate decisions corresponding to (3) are:

$$(15)\quad \bar{x} = \alpha(p) + \beta^1(p)\bar{y} + \beta^2(p)\bar{\sigma}^2.$$

and decisions add up regardless of the values of y_i. Again linearity yields very simple indices which are used to compute and identify the aggregate marginal effect of the mean and variance of price (revenue). Note however, that the researcher need not estimate marginal responses for each micro unit but must collect means and variances at the micro level and average them to use in (15). This is not equivalent to calculating the mean of aggregate price (revenue) which uses formulas like (7) to calculate the national average ex post price (revenue).

This point could perhaps be made more convincing by explicitly introducing attributes into the analysis for multiple indices. Let A_i represent attributes, then a nonsymmetrical aggregate index can be obtained from the following micro equations

$$(16)\quad x_i = \alpha_i(p) + \beta^1(p)g_i^1(A_i,y_i) + \beta^2(p)g_i^2(A_i,y_i), \quad i=1,\ldots,N.$$

Letting A_i be risk aversion and employing separability yields:

$$(17)\quad x_i = \alpha_i(p) + \beta^1(p)g_i^1(A_i)h_i^1(y_i) + \beta^2(p)g_i^2(A_i)h_i^2(y_i), \quad i=1,\ldots, N.$$

Letting h^1 and h^2 be the functionals for means and variances, respectively, yields the aggregate model

$$\bar{x} = \alpha(p) + \beta^1(p)\,\frac{1}{N}\sum_{i=1}^{N} g_i^1(A_i)h_i^1(y_i) + \beta^2(p)\,\frac{1}{N}\sum_{i=1}^{N} g_i^2(A_i)h_i^2(y_i).$$

where α, β^1, β^2, $g_i^{\,1}$ and $g_i^{\,2}$ would be estimated econometrically.

Note that distributional considerations can be entered with respect to any variable by entering nonlinearities as in Muellbauer. For example, let $h_i^{\,2}(y_i)$ be $\sigma_i^{\,2\lambda_i}$ and the distribution of the variances $\sigma_i^{\,2}$ will disturb aggregate response. Thus, testing whether $\lambda_i = 1$ for all $i = 1,\ldots,N$ could be useful in understanding aggregate issues.

In any of the above models one could introduce random economic parameters, $\theta_i = \theta + e$, $E(e) = 0$ and test for heterogeneity by $\sigma_e^{\,2} = E(e^2) = 0$. Finally, how would Stoker's method be employed? The distribution of exogenous variables (e.g., moments of prices or revenues) could be specified in the population. Returning to the earlier example, a consistent estimate of $\partial\bar{x}/\partial\sigma^2$ can be obtained by regressing x_i on $\sigma_i^{\,2}$ $(i = 1,\ldots,N)$ and using $\ln \sigma_i^{\,2}$ as the instrument in an instrumental variable technique.

V. A Simple Monte Carlo Experiment

Let $N=3$ and let there be $T=41$ time periods. Prices are assumed to follow a tri-variate normal distribution with means $\{\mu_1=2.5, \mu_2=3.0, \mu_3=3.5\}$. The respective standard deviations are $\{\sigma_1=0.5, \sigma_2=0.6, \sigma_3=0.7\}$. The correlation matrix is:[2/]

$$\begin{bmatrix} 1.00 & 0.62 & 0.91 \\ 0.62 & 1.00 & 0.67 \\ 0.91 & 0.67 & 1.00 \end{bmatrix}$$

Using Cholesky factorization the trivariate normal price distribution was simulated. Yields for all three units and time periods were generated as $33 + 4.5z$, where z is a standard normal random variable. Supply is yield times acreage. Each unit had acreage micro-responses equal to:

$$(18)\quad X_{it} = 50. + 300P_{it}^{e} - 50V_{it}^{e} + e_{it}$$

where e_{it} is an independent standard normal deviate, P_{it}^{e} is the expected price, and V_{it}^{e} is the subjective variance. Exact linear aggregation would allow the constants 300 and -50 to vary as functions of time or input prices in a common way and the intercept could be firm or unit specific. However (18) is sufficient for this preliminary analysis.

Subjective means and variances of prices are calculated with Fisher Weights for the lag structure. That is

$$(19)\quad P_{it}^{e} = 0.5P_{it-1} + 0.33334P_{it-2} + .16667P_{it-3}$$

$$(20)\quad V_{it}^{e} = 0.5(P_{it-1} - P_{it}^{e})^2 + 0.33334(P_{it-2} - P_{it-1}^{e})^2 + 0.16667(P_{it-3} - P_{it-2}^{e})^2$$

$$i=1,\ 2,\ 3;\ t=1,\ \ldots,41.$$

This can be interpreted as a Bayesian learning model. The above expectations procedure was used so that 41 usable observations resulted for each unit.

Exact aggregation would allow $\sum_i P_{it}^{e}$ or $\sum_i V_{it}^{e}$ or any monotonic transformation of each as the aggregator functions. Only the linear case is considered using the sums as indices. Thus, the aggregate regression consists of regressing aggregate acreage on the sum of expected prices and variance of prices (equivalently average acreage, expected prices, or variance could be used).

Allowing for 500 repetitions of the 3 x 41 design and using ordinary least squares to estimate each aggregate regression led to the following average estimates and standard deviations:

	Means	Std.Dev.
Constant	150.18	2.85
P^e	299.98	0.31
V^e	-50.06	1.05

The correlations of coefficient estimates of P^e and V^e is small at 0.005.

As is apparent, the procedure led to very little actual bias in the estimated coefficients. This of course is the prediction of consistent linear aggregation.

A contrasting procedure used by most who do aggregate analysis is to consider the aggregate price index to be (7). That is, the share weighted index is used as price. Using the random draws above for prices the aggregate price realizations are

$$(21)\quad P_t = \sum_i P_{it} Y_{it} / \sum_i Y_{it} \quad t=1, \ldots, T$$

where Y is output. Expected prices are calculated using the sam Fisher weight procedure as in (19) and (20) to calculate P_t^e and V_t^e. Then, the aggregate acreage $\sum_i X_{it}$ is regressed on P_t^e and V_t^e using ordinary least squares.

The means and standard deviations of the estimated coefficients for 500 replications gave the results.

	Means	Std.Dev.
Intercept	173.97	45.90
P^e	875.74	15.03
V^e	-146.49	60.06

The bias in the aggregate intercept is substantial at almost 24. Note that the slope coefficients are no where near their actual values. This leads to question the meaning of P^e and V^e using aggregate data. Suppose that there is a small increase in P^e and we interpret this increase as occurring in each of the three units. Then the increase in acreage should be 3 x 300 or 900. In such case, the coefficient of P^e is biased downwards. In contrast, if 1/3 of the change in P^e comes from each unit, then the coefficient of P^e should be 300 and the bias substantially is upward. Similarly, if the coefficient of V^e is interpreted to mean a one unit use in all three supplying units then, the aggregate marginal response should be -150 with a bias of -3.6.

Note, also, that the standard deviation of the slope on V^e is both high relative to that to P^e and for the coefficient estimate under exact linear aggregation.

An examination of the data shows that the aggregate expected price is higher for the Laspeyres type (share weighted) index than for the exact index divided by three. This accounts for the lower estimated coefficient on P^e compared to that for $\sum_i P_{it}^e$. Similarly, the variance is almost always lower for the non-exact index of variance resulting in a larger coefficient on V^e than on $\sum_i V_{it}^e$.

Consider why $3P^e$ is consistently less than $\sum_i P_{it}^e$. The probability limit is complicated but it is apparent that large random draws of P_{it} are associated with large P_{it}^e's and larger acreages and hence supplies. The probability limit in the numerator of P^e contains second order moments of P while the denominator contains only first order moments.

The essential and concluding point of this section is that exact aggregation procedures identify correctly both micro and aggregate responses. Since there is not a large cost in pursuing exact aggregation, it should be considered whenever heterogeneity is substantial.

VI. Concluding Remarks

There is a tendency many times for agricultural economists to ignore developments in general economics about aggregation (including index numbers). For those who do only farm level work this is perhaps understandable. Presumably one merely classifies farms and some one else does the aggregation and little thought is given to consistent aggregate plans directly (e.g. by solving an aggregate L.P.).

For those interested in policy, inevitably an aggregate model of some form will be directly estimated or solved. Heterogeneity is seldom discussed, and yet "we" run regressions of acreage on the mean and variance of revenue (Just) where heterogeneity of yields is apparent for even the most homogeneous of products. Further, as argued earlier, price distributions are objectively heterogeneous across spatial dimensions. These are dimensions where data on counties and states at a minimum are available on price and yield. Is it any wonder that a regression of aggregate acreage on the variance of the aggregate price often yields "insignificant" or unbelievable coefficient estimates? I believe we can do better but we need to do more work in order to fully understand the significance of aggregation issues. In conclusion, it should be noted that nothing here constrains the entities being aggregated to have the same name. Many product aggregations use either the Tornqist or Laspeyres type index numbers.

1/ Measured attributes can be introduced into $\Sigma_i g^k(y_i)$ as a symmetric function.

2/ The data here resembles in many respects the estimated distributional parameters for wheat for the states of Washington, North Dakota, and Oklahoma. Yield parameters resemble those for the average of the three states for the current decade.

References

Antle, J. "Econometric Estimation of Procedures' Risk Attitudes," American Journal of Agricultural Economics 69(1987): 509-522.

Chambers, R. and R. Pope, "What do Aggregate Agricultural Supply and Demand Curves Mean?," unpublished paper, University of Maryland, 1989.

Gorman, W.M., "Community Preference Fields," Econometrica 21(1953): 63-80.

Haltiwanger, J. and M. Waldman, "Rational Expectations in the Aggregate," Economic Inquiry 1989.

Hoch, I. and Y. Mundlak, "Consequences of Alternative Specifications in Estimation of Cobb-Douglas Production Functions," Econometrica 33(1985): 814-28.

Holt, M., "Uncertainty and the Micro Foundations of Supply Responses: Some Theoretical Considerations and an Empirical Application in the U.S. Corn Market," Ph.D. Dissertation, University of Missouri, 1987.

Just, R. "An Investigation of the Importance of Risk in Farmers' Decisions," American Journal of Agricultural Economics, 56(1974): 14-25.

Lau, L.J., "A Note on the Fundamental Theorem of Exact Aggregation," Economics Letters, 9(1982): 119-126.

McDonald, J. and R.J. Butler, "Some Generalized Mixture Distributions with an Application to Unemployment Duration," Review of Economics and Statistics, 64(1987): 232-400.

Muellbauer, J., "Aggregation, Income Distribution, and Consumer Demand," Review of Economic Studies, 42(1975): 525-43.

Onal, H. and B. McCarl, "Exact Aggregation in Mathematical Programming Sector Models," Unpublished paper, Texas A&M University, 1989.

Paris, Q. "Perfect Aggregation and Disaggregation of Complementarity Problems," American Journal of Agricultural Economics, 62(1980): 681-88.

Pope, R. and R. Chambers, "Price Aggregation When Competitive Firms' Prices Vary," Review of Economic Studies, 56(1989): 297-309.

Swamy, P.A. V.B.; Conway, R. and M.R. Leblanc, "The Coefficients Approach to Econometric Modeling: Parts 1, 2, and 3," The Journal of Agricultural Economics, 1988.

Stoker, T.M. "Aggregation, Efficiency, and Cross-Section Regressions," Econometrica, 1(1986): 171-88.

Theil, H. Linear Aggregation of Economic Relations, Amsterdam: North Holland, 1954.