Empirical Specification Considerations for Two-Constraint Models of Recreation Demand

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Introduction

The literature on recreation demand is gradually becoming more sophisticated as researchers respond to the myriad of conceptual and empirical challenges that are associated with this particular area of demand analysis.[1] These challenges arise from several distinguishing features of recreation as a commodity: its time-intensiveness, the lack of markets to signal relevant own and substitute prices, and the important role that space plays in consumption. The time-intensiveness of recreation means that, unlike consumption of most other commodites, one cannot ignore the time "price" of consumption and, perhaps more importantly (but less well addressed), the constraints that time places on consumption opportunities.[2] The lack of markets means, among other things, that the researcher must construct prices of recreation and substitute activitities since they are not observed.[3] The role of space in outdoor recreation consumption leads to several important complications of the usual demand analysis. Since outdoor recreation is usually consumed at a location other than one's home, there are fixed and variable cost elements to the choice which, unlike in demand analysis for most other commodities, cannot be ignored. This gives rise to two separate margins of choice: how often to incur fixed costs (trips to the site), and how much of the commodity to consume (days or hours per year). It also means that choice sets of consumers will vary depending on their location, which causes problems in defining relevant substitutes and their prices. The presence of a fixed cost element also raises cost allocation questions when trips are taken for multiple purposes.

Progress has been made in each of these areas. McConnell has articulated the correct correspondence between the different margins of quantity choice (trips and days) and their respective own- and cross-prices (travel cost and onsite cost); for measurement of unobserved travel cost, Englin and Shonkwiler have recently advanced a model in which the travel cost is a latent variable but is correlated with a set of observed proxy variables. A rich and rapidly-growing literature based on the random utility modelling framework focuses better attention on the role of substitute sites in recreation demand analysis (e.g., Hausman *et al.*; Morey *et al.*). Mendelsohn et. al have implemented a model for the cost allocation problem that treats combination trips as a separate choice from sole-purpose trips to individual sites. Several papers have presented frameworks that explain the joint trips-days consumption choice in two-constraint models that include time prices and budgets (Bockstael *et al*; McConnell; Larson). Empirical estimates of the value of leisure time have been made from sample data on recreational choices by McConnell and Strand, Smith *et al.*, and Bockstael *et al.*

This focus of this paper is on further refinement of the two-constraint recreation demand model, both theoretically and empirically. We develop the theoretical restrictions on the two-constraint recreation demand model that follow from the fact that there are two versions of Roy's Identity when consumption is made subject to two constraints. This implies coefficient restrictions on the relationships between money and time prices and money and time budget coefficients, whether or not individuals are presumed to be making marginal labor-leisure choices. In either case, the two-constraint model can be written as a single-constraint problem, with the marginal value of leisure time serving as the conversion factor between time and money in full prices and full budgets.

The results we develop encompass the standard case, assumed by most of the literature, where the marginal value of leisure time is assumed to be an exogenous parameter. This occurs in two ways: either recreation choice is analyzed subject to two (time and money) constraints, with marginal labor supply choice by some individuals leading to the conclusion that the appropriate marginal value of leisure time is the discretionary wage (e.g., Bockstael *et al.*); or an arbitrary assumption is made, usually a fraction of one-third to one-half the wage rate, based on the suggestions of Cesario. In either case, the presence of optimization with respect to an exogenous parameter leads to that parameter serving as the relevant "terms of trade" of time for money.

Our results also pertain to the case where the marginal value of leisure time, which is the ratio of shadow values on the time and money budget constraints, is endogenous. In this case, the full prices and full budget contain the endogenous value of time. Because the structure of how it must appear in the demand equation(s) is clear from theory, it can be estimated as part of the demand structure.

Our empirical application is based on the Almost Ideal Demand System (AIDS) modelling framework of Deaton and Muellbauer, which has proved to be an exceptionally useful empirical specification for a wide variety of consumer demand applications. When consumer choice is subject to two binding constraints, there are two dual minimum expenditure functions (Smith) and, in the AIDS modelling context, two systems of Marshallian and Hicksian share systems with cross-system, as well as the usual cross-equation, restrictions. We implement a version of the AIDS model which is consistent with the theoretical restrictions of the two constraint framework, estimating incomplete share systems that explain the share of time budget and of money budget devoted to whalewatching trips off California.

Two-Constraint Recreation Choice Models

A good starting point for model development is the work on two-constraint choice models is the work by Smith and by Bockstael *et al.*[4] Let **x** be a vector of consumption goods with corresponding money prices **p** and time prices **t**, and choices are made subject to a money budget constraint $M \geq \mathbf{px}$ and a strictly binding time constraint $T = \mathbf{tx}$. Intuitively, the reason the time constraint always binds is that time must always be "spent" in some activity, whereas it is possible (though unlikely) that the income constraint will not bind, indicating satiation. As special cases, some of the elements of **t** or **p** could be zero, indicating activities that require time but no money (such as walks on the beach) or money but no–or little–time, such as making charitable contributions. The individual's utility is also influenced by an exogenous nonpriced quality variable z.

The primal version of the problem leads to the indirect utility function V(**p**,**t**,z,M,T), defined as

$$V(\mathbf{p},\mathbf{t},z,M,T) \equiv \max_{\mathbf{x}} u(\mathbf{x},z) + \lambda\{M - \mathbf{px}\} + \mu\{T - \mathbf{tx}\} \tag{1}$$

where the ratio of the Lagrange multipliers on the time and money constraints, $\mu/\lambda = V_T(\cdot)/V_M(\cdot)$[5], is the money value of time. To simplify notation, we let this function be represented by $\rho \equiv \rho(\mathbf{p},\mathbf{t},z,M,T)$, since Lagrange multipliers from constrained optimization problems are, in general, functions of all parameters in the problem. Note that in the case where the money budget is slack, ρ goes infinite. Because the time constraint holds as an identity, ρ can have any sign and the marginal value of discretionary time can be positive or negative. When time is abundant relative to money, ρ approaches zero.

The dual money expenditure function e(**p**,**t**,z,T,u) is defined as

$$e(\mathbf{p},\mathbf{t},z,T,u) \equiv \min_{\mathbf{x}} \mathbf{px} \text{ s.t. } T = \mathbf{tx},\ u=u(\mathbf{x},z),$$

and the dual time expenditure function is defined as

$$\xi(\mathbf{p},\mathbf{t},z,M,u) \equiv \min_{\mathbf{x}} \mathbf{tx} \text{ s.t. } M \geq \mathbf{px},\ u=u(\mathbf{x},z).$$

The marginal value of leisure time can be expressed in terms of these dual functions as $\rho = -\partial e(\cdot)/\partial T$ and $\rho = -[\partial \xi(\cdot)/\partial M]^{-1}$.

Much of the literature on recreation demand based on utility-theoretic foundations for the value of time (e.g., McConnell; McConnell and Strand; Bockstael *et al.*) notes that individuals observed at "interior" solutions with respect to labor supply effectively reveal their marginal value of time through their observed trades of time for money at the marginal or discretionary wage rate. This parameter can be used to collapse the two-constraint choice problem into a single-constraint problem of maximizing utility subject to full prices and full budgets, with the wage acting as the terms of trade between time and money (e.g., Becker). On the other hand, individuals at "corner solutions" in the labor market work fixed hours and do not (or are not observed to) trade time for money at a parametric marginal wage do not reveal their value of leisure time, and one cannot infer their marginal values of time from an exogenous parameter.

Bockstael *et al.* took account of this distinction explicitly in estimating the demand for sportfishing in Southern California. They argue that individuals making marginal labor supply choices have demand functions of the form $x_i = h^I(\mathbf{p}+w^D\mathbf{t},z,M+w^D T)$ where w^D is the observed marginal wage; and demands for individuals not making such choices are $x_i = h^C(\mathbf{p},\mathbf{t},z,M,T)$. They estimated both types of demand functions using a specification of the direct utility function that solves for linear demands in each case.

The presence of an additional (time) constraint implies additional structure for the relationships between demand coefficients for money price and time price, and for money budget and time budget. These relationships, which hold whether or not an individual is making a marginal labor supply choice, arise from the two versions of Roy's Identity relating the slopes of the indirect utility function with respect to time arguments to the slopes with respect to money arguments.

The relationships between money and time prices and budgets, which are developed in the next section, have not to our knowledge appeared in the recreation demand literature. The result is a unification of the demand estimation strategy for both types of individuals, because in both cases demand arguments appear as full prices and full budgets. This difference is that for individuals who make marginal labor supply choices, the "terms of trade" between time and money arguments is an exogenous parameter (the marginal wage), whereas for others it is a parameter (or function) to be estimated.

Parameter Restrictions on Demands

As noted by Smith (p. 81), there are two versions of Roy's Identity for the two-constraint choice problem. From (1), we can see that $V_{p_i} = -\lambda x_i$, $V_{t_i} = -\mu x_i$, $V_M = \lambda$, and $V_T = \mu$, so that. Thus the two versions of Roy's Identity operating on the two-constraint system are

$$x_i(\mathbf{p},\mathbf{t},M,T) \equiv -V_{p_i}/V_M \equiv -V_{t_i}/V_T. \qquad (2)$$

The implications of equation (2) for parameter restrictions in the demand system appear not to have been developed, and prove useful for specification and estimation of the marginal value of leisure time. Rewriting (2) slightly, we have

$$V_{p_i}V_T \equiv V_{t_i}V_M \qquad (3)$$

which holds as an identity if the two share systems are representing expenditure on the same good(s). Differentiating (3) in turn with respect to p_j, t_j, M, and T, one obtains

$$V_{p_ip_j}V_T + V_{p_i}V_{Tp_j} = V_{t_ip_j}V_M + V_{t_i}V_{Mp_j} \qquad (4)$$

$$V_{p_it_j}V_T + V_{p_i}V_{Tt_j} = V_{t_it_j}V_M + V_{t_i}V_{Mt_j} \qquad (5)$$

$$V_{p_iM}V_T + V_{p_i}V_{TM} = V_{t_iM}V_M + V_{t_i}V_{MM} \qquad (6)$$

$$V_{p_iT}V_T + V_{p_i}V_{TT} = V_{t_iT}V_M + V_{t_i}V_{MT}. \qquad (7)$$

Dividing each equation by $V_M V_T$ and using Roy's Identities from (2), (4)-(7) can be rewritten as

$$V_{p_i p_j}/V_M - x_i(V_{Tp_j}/V_T) = V_{t_i p_j}/V_T - x_i(V_{Mp_j}/V_M) \tag{8}$$

$$V_{p_i t_j}/V_M - x_i(V_{Tt_j}/V_T) = V_{t_i t_j}/V_T - x_i(V_{Mt_j}/V_M) \tag{9}$$

$$V_{p_i M}/V_M - x_i(V_{TM}/V_T) = V_{t_i M}/V_T - x_i(V_{MM}/V_M) \tag{10}$$

$$V_{p_i T}/V_M - x_i(V_{TT}/V_T) = V_{t_i T}/V_T - x_i(V_{MT}/V_M). \tag{11}$$

Since each of (8)-(11) must hold for any nonnegative values of x_i, it must be true that the terms independent of x_i in each equation must be equal, giving

$$V_{p_i p_j}/V_M = V_{t_i p_j}/V_T, \tag{12}$$

$$V_{p_i t_j}/V_M = V_{t_i t_j}/V_T,$$

$$V_{p_i M}/V_M = V_{t_i M}/V_T,$$

and

$$V_{p_i T}/V_M = V_{t_i T}/V_T .$$

Similarly, the coefficients on x_i in each equation must also be equal, so that

$$V_{Tp_i}/V_T = V_{Mp_i}/V_M, \tag{13}$$

$$V_{Tt_i}/V_T = V_{Mt_i}/V_M,$$

$$V_{TM}/V_T = V_{MM}/V_M,$$

and

$$V_{TT}/V_T = V_{MT}/V_M.$$

Relating Price Slopes of the Demand Functions

Now consider the implications for comparative statics from the demand systems derived from the 2-constraint model. Restrictions arise relating the time and money price slopes within and across the demand systems. Using the usual (money) version of Roy's Identity ($x_i = -V_{p_i}/V_M$), the effect of a change in any money price p_j is

$$\partial x_i/\partial p_j = -[V_M V_{p_i p_j} - V_{p_i} V_{Mp_j}]/V_M^2$$
$$= -V_{p_i p_j}/V_M - x_i(V_{Mp_j}/V_M) \quad (14)$$

while

$$\partial x_i/\partial t_j = -[V_M V_{p_i t_j} - V_{p_i} V_{Mt_j}]/V_M^2$$
$$= -V_{p_i t_j}/V_M - x_i(V_{Mt_j}/V_M). \quad (15)$$

From (12), $V_{p_i t_j} = [V_T/V_M]V_{p_i p_j}$ and $V_{Mt_j} = [V_T/V_M]V_{Mp_j}$, so using (14), (15) can be written as

$$\partial x_i/\partial t_j = -[V_T/V_M]V_{p_i p_j}/V_M - [V_T/V_M]x_i(V_{Mp_j}/V_M).$$

$$= \rho \cdot \partial x_i/\partial p_j, \quad (16)$$

recalling that $[V_T/V_M] \equiv \rho$. Equation (16) is the key result regarding the relationship of time and money price slopes in all equations of the demand system. It says that in all two-constraint demand systems, the ratio of all time price slopes to corresponding money price slopes of Marshallian demand must be equal, and the factor of proportionality is the marginal value of leisure time.

Relating Budget Slopes of the Demand Functions

One can follow the same procedure to derive the relationship among budget slopes in the demand system. Differentiating the money version of Roy's Identity with respect to M and T, one obtains

$$\partial x_i/\partial M = -[V_M V_{p_iM} - V_{p_i} V_{MM}]/V_M^2$$
$$= -V_{p_iM}/V_M - x_i(V_{MM}/V_M) \qquad (17)$$

and
$$\partial x_i/\partial T = -[V_M V_{p_iT} - V_{p_i} V_{MT}]/V_M^2$$
$$= -V_{p_iT}/V_M - x_i(V_{MT}/V_M). \qquad (18)$$

From (13), $V_{p_iT} = [V_T/V_M]V_{p_iM}$ and $V_{MT} = [V_T/V_M]V_{MM}$, so using (17), (18) can be written as

$$\partial x_i/\partial T = -[V_T/V_M]V_{p_iM}/V_M - [V_T/V_M]x_i(V_{MM}/V_M).$$
$$= \rho \cdot \partial x_i/\partial M. \qquad (19)$$

Equation (19) is the key equation relating the time budget and money budget comparative statics. Taken with (16), it implies that demands can be expressed equivalently well as functions of full prices ($p_i+\rho t_i$) and full budgets ($M+\rho T$), with ρ as the terms of trade of time for money.

Note that this result is general, pertaining to all recreationists whether or not they are observed making a marginal labor-leisure choice. The usual motivation for recreation choice is that it is nested within a longer-run labor supply choice and that work time is not a source of (dis)utility (see, e.g., Bockstael *et al.*). Thus the exogenous money and time budgets M and T can be thought of as resulting from a prior labor supply decision concerning the individual's "primary" job.

Conditions (16) and (19) state that the correctly formulated two-constraint model is $\mathbf{x}(\mathbf{p}+\rho\mathbf{t}, M+\rho T)$ for all recreationists. A subset of these recreationists will be "moonlighting," making marginal labor leisure choices beyond their primary labor supply decision. These individuals and their choices are encompassed as a special case of the model. For this special case, let x_1 represent the consumption of time spent at a second job, with time price 1 (an hour worked costs an hour) and money price $-w_D$ (an hour of work "costs" the negative of the discretionary wage rate w_D). Since work is not a source of utility, x_1 does not enter the utility function and the first order condition from (1) is $-\lambda(-w_D) - \mu = 0$, so as is well-known the first order condition for how much discretionary labor to offer reveals the marginal value of the discretionary time which is offered, as $\rho = \mu/\lambda = w_D$.

The point is that these individuals provide more information about their values of leisure time that obviate the need to estimate ρ. If, however, the correct marginal wage is not collected, either because it is not asked or because of difficulties in collecting such information accurately, the marginal value of time can be estimated for these individuals like for any other individuals. Presumably, given a correct model specification and a sufficiently flexible specification for ρ, the estimated value of time would approximate, even approach, the individual's discretionary wage rate

It is interesting to note that the demand functions estimated by Bockstael *et al.* satisfy the coefficient restrictions between time and money price arguments in (16) and time and money budget arguments in (19). This is not surprising as their empirical demand specification was derived explicitly from an underlying utility function. Their demand coefficients of $\partial x_i/\partial T = 2.982$ and $\partial x_i/\partial M = .024$ imply a marginal value of time ρ = (2.982 units x/hour)/(.024 units x/\$) $\approx$ \$124/hour. This is approximately double the estimate of \$60/hour that they infer from consumer's surplus estimates of the welfare loss from eliminating the resource, denominated in dollar and time units. This difference arises because one is a marginal estimate (\$124) and the other is an average estimate (\$60) of the money-time tradeoff for a discrete change in resource availability conditions.

Parameter Restrictions on Share Systems

In a setting with two constaints on choice, consumer demand models constructed to explain expenditure shares must explain two budget shares: in the recreation demand context, they are time and money shares. Marshallian shares for commodity i are, by definition, $s_i \equiv p_i x_i(p,t,M,T)/M$ and $s_i^T \equiv t_i x_i(p,t,M,T)/T$. Since they share in common the Marshallian quantity $x_i(p,t,M,T)$, we can expect some cross share-system restrictions to result. Additionally, the second constraint on choice imposes some restrictions on relationships between coefficients on time and money prices and budgets within share systems.

Within-System Restrictions

In addition to the usual homogeneity, symmetry, and adding up restrictions on each share system implied by theory (Deaton and Muellbauer), there are also restrictions on the relationship of time and money slopes within each share equation.

Cross-price Restrictions

To convert (15) to a restriction on share equations, note that for $i \neq j$, the cross-money price slopes in the share equation for good i in expenditure can be written

$$\begin{aligned} \partial s_i/\partial p_j &= \partial(p_i x_i/M)/\partial p_j \\ &= (p_i/M)\partial x_i/\partial p_j \end{aligned} \tag{20}$$

while the time price slope is

$$\begin{aligned} \partial s_i/\partial t_j &= \partial(p_i x_i/M)/\partial t_j \\ &= (p_i/M)\partial x_i/\partial t_j. \end{aligned} \tag{21}$$

Combining (20) and (21), the cross-price share slopes can be related as

$$\begin{aligned} \partial s_i/\partial t_j &= (p_i/M)[(V_T/V_M)\partial x_i/\partial p_j]. \\ &= (p_i/M) \cdot \rho \cdot [(\partial s_i/\partial p_j)(M/p_i)]. \end{aligned}$$

$$= (p_i/M) \cdot \rho \cdot [(\partial s_i/\partial p_j)(M/p_i)].$$
$$= \rho \cdot (\partial s_i/\partial p_j). \qquad (22)$$

Converting to logarithmic derivatives by noting that $\partial s_i/\partial t_j = (1/t_j)\partial s_i/\partial \log(t_j)$ and $\partial s_i/\partial t_j = (1/t_j)\partial s_i/\partial \log(t_j)$, one can write (22) as

$$\partial s_i/\partial \log(t_j) = \rho \cdot (t_j/p_j) \cdot \partial s_i/\partial \log(p_j). \qquad (23)$$

Own-price Restrictions

Own-price restrictions are asymmetric due to the fact that own-money prices appear twice in the money share (while own-time prices do not), and own-time prices appear twice in the time share (while own-money prices do not). For $i = j$, the own-money and own-time price slopes in the share of good i are

$$\partial s_i/\partial p_i = (p_i/M)\partial x_i/\partial p_j + s_i/p_i$$

and

$$\partial s_i/\partial t_i = (p_i/M)\partial x_i/\partial t_i$$

where $s_i/p_i = x_i/M$. Using the same logic as before to relate the logarithmic own-money and own-time price slopes of the share equation, one gets

$$\partial s_i/\partial \log(t_i) = t_i(p_i/M)[\rho \cdot (1/p_i)(\partial s_i/\partial \log(p_i) - s_i/p_i)(M/p_i)].$$
$$= \rho \cdot (t_i/p_i)[\partial s_i/\partial \log(p_i) - s_i]. \qquad (24)$$

Budget Coefficient Restrictions

The relationship between money and time budget coefficients is analogous to that for own prices, because the same general form of asymmetry arises. The money and time budget slopes of share i are

$$\partial s_i/\partial M = (p_i/M)\partial x_i/\partial M \; - \; s_i/M$$

and

$$\partial s_i/\partial T = (p_i/M)\partial x_i/\partial T$$

where here $s_i/M = p_i x_i/M^2$. Once again relating the logarithmic own-money and own-time price slopes of the share equation, one gets

$$\begin{aligned} \partial s_i/\partial \log(T) &= T(p_i/M)(\rho)[(1/M)(\partial s_i/\partial \log(M) - s_i/M)(M/p_i)]. \\ &= \rho \cdot (T/M) \cdot [\partial s_i/\partial \log(M) - s_i]. \end{aligned} \tag{25}$$

Across-system restrictions

With the Marshallian money and time shares as defined above, it must be true in specifying an internally consistent pair of share systems that

$$\begin{aligned} s_i^T &\equiv [(t_i/T)/(p_i/M)]s_i. \\ &= F_i s_i, \end{aligned} \tag{26}$$

where $F_i \equiv [(t_i/T)/(p_i/M)]$ is the relative time-intensity of consumption of good x_i. It measures the relative resource requirements of consuming the good, expressed as percent of time budget to percent of money budget. Immediately it follows that for cross- money and time prices,

$$\partial s_i^T/\partial p_j = F_i \, \partial s_i/\partial p_j \tag{27}$$

$$\partial s_i^T/\partial t_j = F_i \, \partial s_i/\partial t_j \quad \text{for } j \neq i, \tag{28}$$

since F_i is independent of p_j and t_j. Own-price and budget effects are more complicated, owing to the fact that F_i depends on all these terms. Applying the chain rule to (23), own price effects are

$$\partial s_i^T/\partial \mathrm{p}_i = \mathrm{F}_i\,(\partial s_i/\partial \mathrm{p}_i \;-\; s_i/\mathrm{p}_i) \tag{29}$$

$$\partial s_i^T/\partial \mathrm{t}_i = \mathrm{F}_i\,(\partial s_i/\partial \mathrm{t}_j \;+\; s_i/\mathrm{t}_i), \tag{30}$$

while the budget effects are

$$\partial s_i^T/\partial \mathrm{M} = \mathrm{F}_i\,(\partial s_i/\partial \mathrm{M} \;+\; s_i/\mathrm{M}) \tag{31}$$

$$\partial s_i^T/\partial \mathrm{T} = \mathrm{F}_i\,(\partial s_i/\partial \mathrm{T} \;-\; s_i/\mathrm{T}). \tag{32}$$

An Empirical Two-Constraint Shares Model

Beginning with any one of the optimized choice functions $\mathrm{v}(\cdot)$, $\mathrm{e}(\cdot)$, or $\xi(\cdot)$, one can derive the others from the dual structure of the optimization problem. The AIDS model of Deaton and Muellbauer is an attractive candidate because of its ease of use and consistency with theory in the single-constraint case. In the two-constraint case, where estimated share systems must satisfy (23) through (32), the standard AIDS cost function with additional time price and budget terms does not work. A model which does satisfy (23) through (32) is

$$\mathrm{e}(\mathrm{p,t,z,T,u}) = \alpha_0 + \sum_i \alpha_i^z \cdot [\mathrm{p}_i + \rho \mathrm{t}_i] + \tfrac{1}{2}\Big\{\sum_i\sum_j \gamma_{ij}^z \cdot [\mathrm{p}_i + \rho \mathrm{t}_i][\mathrm{p}_j + \rho \mathrm{t}_j]\Big\}$$
$$- \rho \mathrm{T} + \mathrm{u}\beta_0 \prod_i \mathrm{p}_i^{\beta_i} \mathrm{t}_i^{\rho\beta_i}. \tag{33}$$

where $\alpha_i^z \equiv \alpha_i^z + \gamma_i \log(\mathrm{z})$ and $\gamma_{ij}^z \equiv \gamma_{ij} + \epsilon_{ij} \log(\mathrm{z})$ are intercept and slope coefficients of the share equations, respectively, that may shift with a quality variable z. This is essentially the linear expenditure system cost function with two differences: the presence of a second constraint, and a quadratic in prices term that allows for flexible substitution between goods in consumption.

The utility dual of the money expenditure function, obtained by inverting e(**p**,**t**,z,T,u) with respect to the utility argument, is the indirect utility function

$$V(\mathbf{p},\mathbf{t},z,M,T) = \Big\{[M+\rho T] - \alpha_0 - \sum_i \alpha_i^z \cdot [p_i+\rho t_i] - \tfrac{1}{2}\Big\{\sum_i\sum_j \gamma_{ij}^z \cdot [p_i+\rho t_i][p_j+\rho t_j]\Big\}\Big\} \cdot \beta_0^{-1}\prod_i p_i^{-\beta_i}\, t_i^{-\rho\beta_i}. \tag{34}$$

The time expenditure function, obtained by inverting v(**p**,**t**,z,M,T) with respect to T, is

$$\xi(\mathbf{p},\mathbf{t},z,M,u) = \tfrac{1}{\rho}\Big\{\alpha_0 + \sum_i \alpha_i^z \cdot [p_i+\rho t_i] + \tfrac{1}{2}\Big\{\sum_i\sum_j \gamma_{ij}^z \cdot [p_i+\rho t_i][p_j+\rho t_j]\Big\} - M + u\beta_0\prod_i p_i^{\beta_i}\, t_i^{\rho\beta_i}.\Big\}. \tag{35}$$

It proves convenient to estimate share equations for time and money requirements of consumption. These Hicksian (utility constant) money share equations come from differentiating (33) with respect to money prices; substituting the utility index from (34) into the Hicksian money share equations yields the Marshallian money share equations, which are of the form

$$s_i(\mathbf{p},\mathbf{t},z,M,T) = \alpha_i^z(p_i/M) + \sum_j \gamma_{ij}^{z*}[p_j+\rho t_j](p_i/M) + \beta_i[(M\text{-}MI) + \rho(T\text{-}TI)](p_i/M) \tag{36}$$

where $\gamma_{ij}^{z*} = ½(\gamma_{ij}^z + \gamma_{ji}^z)$ under symmetry. The terms MI and TI are money income and time budget deflators, respectively.[6]

Time share equations are derived analogously, noting that by the envelope theorem Hicksian time share equations come from differentiating (35) with respect to t_i. Again substituting the utility index, the Marshallian time share equations are

$$s_i^T(\mathbf{p},\mathbf{t},z,M,T) = \alpha_i^z(t_i/T) + \sum_j \gamma_{ij}^{z*}\,[p_j+\rho t_j](t_i/T) + \beta_i[(M-MI)+\rho(T-TI)](t_i/T). \qquad (37)$$

Equations (36) and (37) define two share systems or blocks of share equations, one for money expenditure and the other for time expenditure. Each activity x_i that has two prices t_i and p_i has two share equations, one explaining the share of time budget the activity consumes and the other explaining the share of money budget it consumes. Each of these share equations is a function of own time price and an own money price, as well as cross-money and time prices and time and money budgets. Those activities for which either t_i=0 or p_i=0 are represented by only a single share equation; thus there may be asymmetries in the number of equations in each share system.

An Application to California Whalewatching

The data used to illustrate the model are from on-site intercepts of whalewatchers at four sites in California during the winter of 1991-92. The survey instrument was pretested using individuals who had gone whale watching in the previous year. It collected information on trips taken so far that season, expected future trips, travel time, travel costs, whether the trip was their primary destination, etc., were asked. Also collected was information including actual contributions to marine mammal groups, time spent reading, watching, or thinking about wildlife and whales, as well as purchases of whale-related merchandise. Lastly, demographic information including work status, wage rates, and income was asked. The survey was presented in booklet form.

In total, 1,402 visitor surveys were handed out, and 1,003 were returned, for an overall response rate of 71.3%. The response rate was reasonably similar across the four locations, varying from a low of 65.2% for intercepts at Point Loma (San Diego) to a high of 80.3% for intercepts at Point Reyes. On-site refusals were not a problem. For example, at Point Reyes, only 10 people of roughly 600 contacted (about 1.6%) refused to take a survey packet.

Four goods were used to define the time and money share systems from the whalewatching data set: whalewatching trips; monetary donations to whale- and marine mammal-related organizations; time volunteered for such organizations; and consumption of all other goods. Recreation trips (x_1) involve money costs, both in travel and onsite, and time costs in the form of travel time required to gain access to the site.[7] Volunteering of time (x_2) appears only in the time share system as it involves primarily time costs, which were not well measured in the survey. Monetary donations (x_3) appears only in the money share system as it has primarily money costs (the "tax price" varies across households depending on income bracket) but little time costs.

The numeraire good is x_4, the residual expenditures of time and money from their respective budgets after accounting for trips and the two donations activities. Smith has shown (p. 81) that the two-constraint model is homogeneous of degree zero in all prices and budgets. The model is normalized on the time price of x_4, so that t_4 is unity and does not appear as an argument in the share systems. This normalization defines x_4 as “all other activities,” and the money price of x_4 is then $p_4 = (M-p_1x_1-p_3x_3)/(T-t_1x_1-x_2)$, the money expenditure per unit of residual time.

In addition to the time and money prices, it is expected that the individual's whalewatching success will influence both trips demand and, potentially, the willingness to make donations of time and money. The success variable (z) is the individual's *ex ante* expectation of whale sightings for the whalewatching trip when they were contacted. Money budget (M) is the household income before taxes, and the time budget (T) is amount of nonworking time in the number of weekend and paid vacation days. The unconditional budgets are used because incomplete, as opposed to partial, demand systems are estimated.

A feature of the model is an estimate of the marginal value of leisure time, which is ρ. In the utility-expenditure model of (33)-(35), the marginal value of leisure time is a constant. This is an undesirable feature of the model, because as noted earlier since the marginal value of time is the ratio of multipliers on the budget constraints, one would expect more generally that it would vary with at least some prices and budgets. The difficulty is that more general formulations of $\rho(\mathbf{p},\mathbf{t},M,T)$ in the model (33)-(35) are not consistent with the share system parameter restrictions required from (23)-(32).

While generalizing the model to allow ρ to vary systematically would be a useful extension, this treatment of the marginal value of time is similar to those of Bockstael *et al.* and Hausman *et al.* Hausman *et al.* estimated a travel mode choice model to infer the marginal value of travel time, which they inferred was a constant $5.35/hour for everyone regardless of income or other characteristics. Bockstael *et al.*, as noted earlier, also estimated a model which implies a constant marginal value of time for everyone.

To allow for some variation in the value of time elasticity, ρ, we estimated different constants for different subsets of the data. Two sets of dummy variables were created to reflect low, medium, and high ranges of household income per wage earner (M) and leisure budget (T). D_1 took the value 1 for the medium income group (and zero otherwise); D_2 was 1 for the high M group, D_3 was 1 for the medium T group, and D_4 was 1 for the high T group. Nine groups resulted from this classification: low M–low T ($D_1 = D_2 = D_3 = D_4 = 0$), medium M–low T ($D_1 = 1$, $D_2 = D_3 = D_4 = 0$), and so on. The rationale is that differences in the absolute and relative levels of the two resources required for recreation trips and other activities may influence their shadow values and, hence, the value of time ρ. The model was also specified with a value ρ_0 that applied to all individuals, so that ρ_0 is interpreted as the value of time for the low M–low T group, and the coefficients on the dummy variables (ρ_1, etc.) are deviations from ρ_0 for the other groups.

Another consideration is the distinction between those who work fixed hours and those with flexible hours. As noted above, both types of individuals are addressed by the model, though the discretionary wage will reveal the marginal value of leisure time for the latter group. Though the survey did not collect information on marginal or discretionary wage, because of concerns about accurately capturing this variable, individuals did indicate whether they were on salary or worked for an hourly wage.

As Bockstael *et al.* note, one would expect a discrepancy between the marginal value of time and the marginal wage rate if the hours constraint is binding on salaried workers; in principle the difference can have any sign, though if individuals are working more than what they would freely choose at a salaried job, one would expect the marginal value of leisure time to be higher than the marginal wage. They found this relationship in their empirical application.

To reflect premia or discounting associated with fixity in work hours, the dummy variable ρ_F was created. It takes the value ρ_F=1 for salaried individuals, and 0 otherwise.

Results

The linear approximate version of the trips money share and trips time share equations in (36) and (37) were estimated as incomplete demand systems, with Stone's price index for the money and time deflators, using the nonlinear systems estimator in SHAZAM 8.0. The coefficient restrictions implied by (16), (19), and (23)-(32) were maintained across share systems.

The estimation results for the trips money and time share equations are given in Table 1. The first model includes all the dummy variables for value of time estimates. The money income dummies were not highly significant, so a second model was run using only the time budget dummies ρ_3 and ρ_4. A Wald test of the hypothesis H$_0$: $\rho_1 = \rho_2 = 0$

yielded a χ^2 statistic of -2(3706.4-3709.0) = 5.21, less than the critical $\chi^2_{.05,2\,df}$ value of 5.99. Given this failure to reject the null hypothesis, the more parsimonious model results are also presented as Model 2 in Table 1.

The coefficients of both models are both mostly significant and have the expected signs and magnitudes. Trips demand is full-price inelastic and inferior at the means of the data, though normal for a fraction of the data points. The cross-price effects for money donations and the other activities variable both enter with significance, as do the quality slope shifters. The coefficient on ρ_0 is an estimate of the marginal dollar value of leisure time for those with low money and time budgets; it is nearly \$17/hour in Model 1 and roughly \$18.25 in Model 2.

The estimated deviations for those with larger time budgets, in Model 2, are small in magnitude though statistically different from zero. As one would intuit, those with higher time budgets have (slightly) lower marginal money values of time. The coefficient on ρ_F is positive, suggesting a relatively small (\$0.25/hour) but statistically significant premium on the value of leisure time associated with salaried workers. Other specifications with ρ_F interacting with the other value of time dummies were also explored, but did not yield significantly better fits.

Table 2 compares the Model 2 estimates of the marginal values of leisure time to the sample average hourly wages reported by each group. Mean wages are \$23/hour, \$24/hour, and \$31 per hour, so the direction of change in mean wages with increases in discretionary time budget is opposite to the predictions of the marginal value of time. The reason this occurs is that mean income increases more rapidly than mean time budget as time budget increases. While these results are preliminary, they suggest that values of time that exceed 50% of the wage rate may be appropriate.

Conclusions

We have developed the structural implications of the two-constraint recreation demand model for coefficients on time and money prices and time and money budgets, in both demand and share systems. The implication is that two-constraint models should be formulated and estimated as functions of full prices and budgets, with the marginal value of leisure time serving as the "terms of trade" between time and money prices and time and money budgets. This actually simplifies demand estimation for these models, because this structure applies for individuals working variable hours as well as those on fixed salaries. The marginal value of time can be estimated as a parameter or function for both groups, and for individuals making a marginal labor supply choice one could validate the model by comparing the estimated marginal value of time to the discretionary wage they report.

A two-constraint model consistent with the theoretical restrictions was introduced and estimated using a sample of whalewatchers in California. The model fit well, with significant own- and cross-full prices and full-budgets, and indicated price inelasticity and income inferiority, the latter a not-uncommon finding in recreation demand analyses. One of the parameters estimated in the model is the marginal value of leisure time, which is a constant that we allowed to vary by subgroups within the sample based on magnitudes of the time and money budget. The estimated values of time were of plausible magnitude and statistically significant, generally ranging from somewhat over 50% to somewhat over 75% of the reported wage for the different groups of whalewatchers we analyzed.

Some caveats are in order. First, while the two-constraint model estimated is consistent with the requirements of theory, it is extremely simple with respect the marginal value of leisure time, which is estimated as a parameter from the consumer's optimization problem. While it is possible to stratify the sample and estimate a series of values of time for each subgroup, this model appears insufficiently flexible with respect to the marginal value of time. It is not obvious how best to "slice" the sample with respect to defining groups with homogeneous marginal values of leisure time. As a result, the model is probably best interpreted as predicting conditional mean values of time rather than individual values of leisure time. Developing a more flexible utililty-theoretic model is a challenge that remains to be addressed. The two-constraint approach patterned after the LES and AIDS consumer demand models seems a promising approach with respect to estimation of recreation demand.

Footnotes

1. The list of issues not usually confronted in market demand analysis, but commonly encountered in nonmarket demand analysis, extends to nearly every variable relevant to the explanation of recreation choice, since by definition markets to signal marginal value are largely absent. A not-necessarily exhaustive list would include definition of the own-quantity variable (trips, days per trip, or days per season); identification of, and inclusion of prices for, relevant substitute goods and activities; the measurement of own price (the money cost of travel, which is constructed, not observed); incorporation of time costs and constraints on choice; how to value leisure time spent in recreation; how to allocate costs of trips taken for multiple purposes.

2. A long literature going back to the earliest applications recognizes the importance of measuring time costs, particularly for its effect on the money price coefficient used to infer changes in consumer's surplus (e.g., Knetsch; Clawson and Knetsch).

3. See Randall for a discussion of this issue.

4. Smith (pp. 78-83), in particular, provides a thorough treatment of the primal and dual properties of the two-constraint problem.

5. Parameters appearing as subscripts refer to partial derivatives; e.g., $V_T \equiv \partial V(\mathbf{p},\mathbf{t},z,M,T)/\partial T$.

6. The full deflators in the two-constraint model are $MI \equiv \alpha_{0m} + \sum_i \alpha_i^z p_i + \frac{1}{2}\left\{\sum_i \sum_j \gamma_{ij}^z [p_j + \rho t_j] p_i\right\}$ and $TI \equiv \alpha_{0t} + \sum_i \alpha_i^z t_i + \frac{1}{2}\left\{\sum_i \sum_j \gamma_{ij}^z [p_i + \rho t_i] t_j\right\}$, respectively. We estimated the linear approximate version of the model, substituting Stone's price indices $MI \approx \prod_i p_i^{s_i}$ and $TI \approx \prod_i t_i^{s_i^T}$ for the money and time deflators.

7. We take the onsite time to be exogenous, because all whalewatching trips covered in this analysis are day trips and roughly half of all whalewatching trips represented

are boat trips of fixed duration. Other variations in time spent onsite, for example for shoreline whalewatchers, are small enough to raise questions about how precisely they can be measured.

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Table 1. Estimated Coefficients of the Trips Money and Time Share Equations

Variable	Coefficient	Model 1[a]	Model 2
Intercept	γ_1	33.139 (3.02)	30.253 (3.10)
Intercept Shift	ϵ_1	-0.25639 (-1.23)	-0.26700 (-1.31)
Own-Full Price	γ_{11}	-0.79100E-03 (-4.35)	-0.84903E-03 (-4.99)
Own-Price Shift	ϵ_{11}	-0.63893E-04 (-1.89)	-0.39440E-04 (-1.29)
Tax Price	γ_{13}	-2.7706 (-0.97)	-3.2873 (-1.24)
Tax Price Shift	ϵ_{13}	0.30098 (1.23)	0.30886 (1.30)
Other Activities	γ_{14}	1.6391 (3.44)	1.3426 (3.18)
Other Act. Shift	ϵ_{14}	0.78805E-02 (1.44)	0.73325E-02 (1.40)
Full Income	β_1	-0.25434E-03 (-3.63)	-0.20822E-03 (-3.40)
Value of Time Estimates:			
	ρ_0	16.878 (2.91)	18.247 (2.93)
Shift (Medium M)	ρ_1	-0.21888 (-2.07)	
Shift (High M)	ρ_2	0.50734E-01 (0.12)	
Shift (Medium T)	ρ_3	-0.24430 (-2.72)	-0.26222 (-2.72)
Shift (High T)	ρ_4	-0.44425 (-2.66)	-0.44459 (-2.51)
Fixed Hours	ρ_f	0.25289 (2.56)	0.29987 (2.39)
Log-L		3709.0	3706.6
N		362	362

[a] Student's-t statistics in parentheses.

Table 2. Imputed Marginal Values of Leisure Time and Reported Wages by Whalewatchers

Variable	Symbol	Units	Group		
			Low T	Medium T	High T
Time Budget	T	hours	6680	6770	6862
			(1.74)	(2.99)	(6.35)
Money Budget	M	dollars	34970	39295	51392
			(3518)	(1889)	(3214)
Wage	w	\$/hr.	23.06	24.18	31.95
			(2.216)	(1.729)	(2.416)
Marginal Value of Leisure Time	ρ	\$/hr.	18.25	17.99	17.80
			(6.23)	(6.15)	(6.08)
Count			97	136	97