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A NEW APPROACH TO MODEL GNP FUNCTIONS: AN APPLICATION OF NON-SEPARABLE TWO-STAGE TECHNOLOGIES *

by

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Abstract

This paper shows that two-stage technologies can provide a general procedure for combining profit and value-added functions to obtain new specifications of import demand and output supply systems. In such technologies, we assume that imports interact with other exogenous variables to produce intermediate inputs, which are in turn used to produce final outputs. To show the utility of this new approach, we use it to specify and estimate the Australian GNP function. As will be seen, our proposed framework has an attractive property: the capability of incorporating exogenous effects such as labour and capital endowments within a strong theoretical underpinning. We investigate a new GNP function for which the demand and supply systems are effectively globally regular. Our results demonstrate that the new approach is feasible and promising while the estimated elasticities are not significantly different from those of the traditional models.

JEL Classification: D24; F12.

Keywords : Profit Functions; GNP Functions; Nominal Value-added; Two-Stage Technologies; Regularity Conditions.

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1. Introduction

Estimates of export-supply and import-demand elasticities are of great interest to policy makers for a variety of reasons. They may wish to know what effects the realignments of exchange rates, tariffs, domestic indirect taxes, and relative price level may have on the trade balance. Certainly, each of these changes will affect a country's trade flows and level of income and employment, depending on the magnitude of the elasticities.

In view of the large Australian current account deficit in recent years, modelling of import demand and output supply relations becomes an interesting and important field for investigation. Unfortunately, relatively little is known about the responsiveness of Australian foreign trade to changes in relative prices and in factor endowments. Although there are some estimates of aggregate import price elasticities available for Australia, the vast majority of studies have specified a single-equation functional form. Despite the simplicity of such an approach and hence its wide-spread popularity, this traditional form suffers from at least three deficiencies:

1. Imports are treated as final goods or intermediate goods which are assumed to be grouped in large aggregates, thereby implying that they are separable from all other commodities in the utility function, or from other primary inputs in the production function. Elasticity estimates that refer to large aggregates of commodities or inputs are rarely of interest for policy analysis, as long as government policies refer to specific import items.

2. The functional forms used by previous studies are either *ad hoc* or rather restrictive, overlooking much of the information available on the industrial structure of the economy.

3. The use of the ordinary least squares method has ignored much of the theoretical knowledge available on a complete demand system.

These deficiencies have motivated Kohli (1978, 1982a, 1982b, 1990 & 1993) to use the GNP function approach to estimate the import demand and output supply systems. Kohli's approach has been implemented with various flexible functional forms and has been applied to a number of countries. An attractive feature of this approach is that it is based on duality theory and thus, it is consistent with solid theoretical foundations. However, it is not necessarily flawless. It has the drawback of assuming technology satisfies constant returns to scale, which in many cases may not be a very good approximation of reality. Furthermore, the method of introducing factor endowment variation rests largely on *ad hoc* considerations. This rules out complicated interactions of those exogenous effects with input and output prices, as well as eliminating many potential models that are attractive in their simplicity.[1]

In response to these arguments, this paper is primarily concerned with the specification and estimation of technology. In particular, we apply the concept of a two-stage technology to obtain empirically tractable specifications of an overall GNP function and of the import demand and output supply systems. We first show that Lewbel's framework (1985) as well as two-stage technologies can provide a general procedure for combining profit and value added functions to obtain new specifications of GNP functions. Then, we demonstrate the usefulness of this procedure by estimating a new GNP function. Unlike previous work based on two-stage technologies, we need not impose restrictive assumptions on the structure of production process; that is, our underlying technology needs not separable, homothetic and linear homogeneous. The advantage gained is that the extended approach, to be referred to as the EGNP function model, is reasonably general in allowing input factors and other exogenous variables to interact with input and output prices in the demand system in an almost unlimited variety of ways. Also, in doing econometric analysis, our new specification of GNP function has a tempting feature: the capability of satisfying global regularity conditions in an unbounded region, and if those conditions are not satisfied, they can be easily imposed. Last but not least, this is the first time Australian data has been used to estimate disaggregated import demand elasticities. We believe such a new approach will make our estimates more useful for policy analysis application as our demand systems are specified in terms of disaggregated import demands.

Throughout this paper, we assume one output and imports are disaggregated into six components according to the types of commodities and services. The remainder of this paper is structured as follows. In Section 2, the theoretical background will be briefly elaborated. The GNP and the EGNP function models will be described in Section 3. The specification of the GNP function is provided in Section 4, and Section 5 details our data set, estimation method, and presents the empirical results. Conclusions will be drawn in the last section of this paper.

2. Theoretical Background

2.1 Transformation and Production Functions

Let R^N denote the N-tuples of real numbers, let Ω^N be the non-negative orthant, and let Ω^N_+ be the strictly positive orthant. Assume that the technology set is represented by Γ which is the firm's production possibilities set or the set of all feasible input and output combinations. Symbolically, Γ is defined as:

(2.1)[2] $$\Gamma = \{(y, m) : y \in \Omega^{N1}, m \in \Omega^{N2}, m \text{ can produce } y\}$$

[1] For example, it is hard to introduce the factor endowment variables into models as simple as Cobb-Douglas or CES form.

where $y \in \Omega^{N1}$ is the N1-dimensional vector of non-negative outputs, and $\mathbf{m} \in \Omega^{N2}$ is the N2-dimensional vector of non-negative inputs. Suppose that Γ satisfies the following regularity conditions (**RΓ**):

RΓ1: $\Gamma \in \Omega^{N1+N2}$
RΓ2: Γ is a non-empty set
RΓ3: Γ is a closed set
RΓ4: Γ is a convex set
RΓ5: Γ is a bounded set.

Instead of representing the technology by a set Γ, many authors prefer to use the concept of a transformation function $T(y, \mathbf{m})=0$ to describe the set of efficient input-output combinations. When the firm's possibilities set Γ satisfies Conditions **RΓ**, we could explicitly define the firm's transformation function as the maximum amount of the first output y_1 that the firm can produce given that it has amounts of $\hat{y} = \{y_2,......, y_{N1}\}$ and the input vector **m**. In notation,

(2.2)[3] $$Y1(\hat{y}, \mathbf{m}) = \text{Max}_{y_1} \{y_1: (y_1, \hat{y}, \mathbf{m}) \in \Gamma\}$$

which is the asymmetric form of the transformation function T. It can be shown that the asymmetric transformation function defined by (2.2) satisfies Conditions **RY1**:

RY11: Y1: $\Omega^{(N1-1)+N2} \rightarrow \Omega^1$
RY12: Y1 is continuous and twice differentiable
RY13: Y1 is concave in $\hat{y}$ and **m**
RY14: Y1 is non-increasing in $\hat{y}$
RY15: Y1 is non-decreasing in **m**.

For a given Γ and the assumption that $N1=1$, Y1 becomes the direct production function which is defined as the maximum obtainable output that can be produced for a given input vector **m**:

(2.3) $$Y(\mathbf{m}) = \text{Max}_y \{y: T(y, \mathbf{m})=0\}.$$

The direct production function $Y(\mathbf{m})$ is said to be regular if it satisfies the following conditions (**RY**):

RY1: Y: $\Omega^{N2} \rightarrow \Omega^1$
RY2: Y is continuous and twice differentiable

[2] Vectors of values or functions are denoted here by dropping the subscripts and using the bold letters.
[3] The notation $y1=Y1(\mathbf{m}, \hat{y})$ is indicative of that used in the rest of this paper. Upper case letters denote functions. Lower case letters denote the scalar values of those functions.

RY3: Y is non-decreasing in **m**
RY4: Y is concave in **m**.

2.2 The Profit Function

Another way in which the technology can be described is with the help of the profit function, defined as follows:

$$\Pi(\mathbf{p}, \mathbf{w}) = \text{Max}_{\mathbf{m}, \mathbf{y}}\{\mathbf{p}'\mathbf{y} - \mathbf{w}'\mathbf{m} : T(\mathbf{m}, \mathbf{y}) = 0\} \tag{2.4}$$

where Π is the maximum attainable profit subject to the production technology, $\mathbf{p} \in \Omega_+^{N1}$ is the N1x1 output price vector corresponding to $\mathbf{y}$, $\mathbf{p}'$ is the row-vector transpose of $\mathbf{p}$, and $\mathbf{w} \in \Omega_+^{N2}$ is the N2x1 input price vector corresponding to $\mathbf{m}$. Duality theory states that the profit function Π corresponding to the technology set Γ possesses the following properties (**RΠ**):

RΠ1: $\Pi : \Omega_+^{N1+N2} \rightarrow R$
RΠ2: Π is continuous and twice differentiable
RΠ3: Π is non-increasing in **w**
RΠ4: Π is non-decreasing in **p**
RΠ5: Π is homogeneous of degree one (HD1) in (**p**, **w**)
RΠ6: Π is convex in (**p**, **w**).

We can use Hotelling's lemma to derive the profit-maximising output supply (**Y**) and input demand (**M**) functions; that is,

$$\mathbf{Y}(\mathbf{p}, \mathbf{w}) = \nabla_{\mathbf{p}} \Pi$$
$$\mathbf{M}(\mathbf{p}, \mathbf{w}) = -\nabla_{\mathbf{w}} \Pi$$

where ∇ is the gradient vector for a particular functional system.

2.3 Structure of the Profit Function

In empirical applications, any specification involving a reasonable number of inputs and outputs will require us to impose some structure on the demand and supply systems. The most common assumption used is the separability hypothesis which permits us to keep the estimation process manageable by merely dealing with certain aspects of a model without spelling out all details of the rest of the model. Following Chambers (1988), let the set of the indices for **w** be:

$${}_{o}\mathbf{I} = \{1, 2, \ldots\ldots, N2\}.$$

We next order the input prices in N3 separable groups defined by the mutually exclusive and exhaustive partition:

$$\hat{\mathbf{I}} = \{I^1, I^2, \ldots\ldots, I^{N3}\} \qquad N3 \leq N2.$$

Corresponding to Partition $\hat{\mathbf{I}}$ is a Cartesian decomposition of Ω_+^{N3} such that:

$$\Omega_+^{N2} = \Omega_+^{(1)} \times \Omega_+^{(2)} \times \cdots\cdots \times \Omega_+^{(N3)} .$$

Then, the input price vector **w** can be analogously partitioned as:

$$\mathbf{w} = \{\mathbf{w}^1, \mathbf{w}^2, \ldots\ldots, \mathbf{w}^{N3}\}$$

where $\mathbf{w}^r \in \Omega_+^{(r)}$. If $\Pi(\mathbf{p}, \mathbf{w})$ is weakly separable in Partition $\hat{\mathbf{I}}$, this implies that the function will have the following structure:

(2.5) $$\Pi(\mathbf{p}, \mathbf{w}) = \overline{\Pi}[\mathbf{p}, \Pi^1(\mathbf{p}, \mathbf{w}^1), \ldots\ldots, \Pi^{N3}(\mathbf{p}, \mathbf{w}^{N3})] .$$

Browning (1982) refers to this structure as **p** - decentralisability. With this form, a producer can allocate cost optimally within sector r knowing only the sectoral prices ($\mathbf{w}^r$), aggregate sectoral cost and the output prices. Any weakly separable profit function is legitimate when each Π^r $\forall$ r satisfies **RΠ**, and $\overline{\Pi}$ is HD1 and convex in (Π^r, **p**), and non-decreasing and non-increasing in **p** and Π^r respectively.

Consider another structural form of a profit function:

(2.6) $$\Pi(\mathbf{p}, \mathbf{w}) = \overline{\overline{\Pi}}[\mathbf{p}, G^1(\mathbf{w}^1), \ldots\ldots, G^{N3}(\mathbf{w}^{N3})]$$

which is equivalent to a similar structure for the cost function:

(2.7) $$\begin{aligned} C(\mathbf{w}, \mathbf{y}) &= \mathrm{Max}_\mathbf{p}\{\mathbf{p}'\mathbf{y} - \Pi(\mathbf{p}, \mathbf{w})\} \\ &= \mathrm{Max}_\mathbf{p}\left\{\mathbf{p}'\mathbf{y} - \overline{\overline{\Pi}}\left[\mathbf{p}, G^1(\mathbf{w}^1), \ldots\ldots, G^{N3}(\mathbf{w}^{N3})\right]\right\} \\ &= \tilde{C}\left[\mathbf{y}, G^1(\mathbf{w}^1), \ldots\ldots, G^{N3}(\mathbf{w}^{N3})\right]. \end{aligned}$$

This is, in turn, equivalent to homothetic separability wherein the cost minimisation problem can be decomposed into two stages.

2.4 Intermediate Inputs and Nominal Value-Added Function

Economists normally treat the production technology as a "black-box". In this sense, empirical work of production analysis only focuses on purchases of primary inputs and sales of final outputs. A natural extension of this work is to allow for the existence of intermediate inputs. Consider a firm producing a single gross output y and its technology is described by the following direct production function:

$$y = Y(\mathbf{q}, \mathbf{z})$$

where $\mathbf{q} \in \Omega^{J}$ is a Jx1 vector of intermediate inputs whose prices **h** are assumed to be exogenous, and $\mathbf{z} \in \Omega^{N2\text{-}J}$ is a (N2-J) $\times$ 1 vector of primary input.

Define the nominal value-added as:

$$\tilde{\pi} = py - \mathbf{h}'\mathbf{q}.$$

Attempts to measure value-added are central to national income accounting and the attribution of the income generated from the sale of final outputs among primary product. Formally, the nominal value-added function is defined as:

(2.8) $$\tilde{\Pi}(p, \mathbf{h}, \mathbf{z}) = \text{Max}_{y, \mathbf{q}} \{p \cdot y - \mathbf{h}'\mathbf{q} \text{ s.t. } y = Y(\mathbf{q}, \mathbf{z})\}.$$

Using Hotelling's lemma, we can derive the following demand and supply systems:

(2.9) $$Q_i(p, \mathbf{h}, \mathbf{z}) = -\frac{\partial \tilde{\Pi}}{\partial h_i}$$
$$Y(p, \mathbf{h}, \mathbf{z}) = \frac{\partial \tilde{\Pi}}{\partial p}.$$

Under the restrictions that the production function satisfies Conditions **RY**, we obtain the following properties of the value-added function $\tilde{\Pi}$ ($\mathbf{R}\tilde{\Pi}$):

R$\tilde{\Pi}$1: $\tilde{\Pi} : \Omega_{+}^{N2+1} \rightarrow R$
R$\tilde{\Pi}$2: $\tilde{\Pi}$ is continuous and twice differentiable
R$\tilde{\Pi}$3: $\tilde{\Pi}$ is HD1 in (p, **h**)
R$\tilde{\Pi}$4: $\tilde{\Pi}$ is convex in (p, **h**)
R$\tilde{\Pi}$5: $\tilde{\Pi}$ is concave in **z**.

3 The Standard and Extended GNP Function Models

3.1 The Standard GNP Function Model

Over the years, duality theory has extended the determination of imports in a number of directions. Kohli treated imports (m_o) as the variable inputs to the technology, together with fixed inputs capital (*k*) and labour (*l*), to produce exports (*x*), consumption goods (*c*), investment goods (*iv*) and government purchases (*g*). This leads to the standard GNP function model which was first implemented in 1978 by using the Translog form to model the Canadian GNP function. Technology was represented by a short run variable profit function, with the endowments of capital and labour fixed, and the prices of imports, exports, investment goods, consumption goods and government purchases exogenous. Furthermore, the domestic and international demand and supply decisions were made by profit maximising firms operating under perfect competition in all commodity and factor markets.

Let $T^L(\mathbf{y}, \mathbf{m})=0$ be a long run transformation function. Suppose that input vector $\mathbf{m}$ can be partitioned into two components ($\mathbf{m}=\{\mathbf{m}_o, \delta\}$), with $\mathbf{m}_o$ containing perfectly variable inputs and $\delta=\{k, l\}$ containing labour and capital endowments that are fixed at the level of $\hat{\delta}=\{\hat{k}, \hat{l}\}$. Then, we define the short run production possibility set for the economy as:

$$\Gamma^S=\{\mathbf{m} \in \Gamma^L : k = \hat{k} \text{ and } l = \hat{l}\}$$

which implies that the short run transformation corresponding to Γ^S is written as:

$$T^S(\mathbf{m}_o, \hat{\delta}, \mathbf{y})=0.$$

Here, we assume the technology described by Γ^S satisfies the following conditions: constant returns to scale on $\hat{\delta}$, it is a non-empty, closed and convex cone, and it allows free disposal and is bounded.

Let the short run variable profit function (hereafter we call it the standard GNP function) be:

$$\text{(3.1)} \qquad \Pi^S(\mathbf{p}, \mathbf{w}_o, \hat{\delta}) = \text{Max}_{\mathbf{m}_o, \mathbf{y}}\left\{\mathbf{p}'\mathbf{y} - \mathbf{w}_o'\mathbf{m}_o : T^S(\mathbf{m}_o, \hat{\delta}, \mathbf{y}) = 0\right\}$$

where $\mathbf{w}_o$ is the price vector corresponding to imports $\mathbf{m}_o$, and Π^S is a well-defined function for all positive price vectors. In the standard GNP function model, $\mathbf{m}_o=\{\hat{m}\} \in \Omega^1$ denotes the aggregate import quantity which is the only variable input, $\mathbf{y}=\{c, inv, g, x\} \in \Omega^4$ denotes the 4 x 1 vector of outputs, and $\delta=\{k, l\} \in \Omega^2$ denotes a 2x1 vector of capital and labour endowments (or fixed inputs) utilised by our economy. Under the assumptions made on Γ^S, the standard GNP function Π^S will inherit a more restrictive set of regularity conditions ($\mathbf{R\Pi^S}$):

$R\Pi^S 1$: $\Pi^S : \Omega^2 \times \Omega_+^{4+1} \to R$
$R\Pi^S 2$: Π^S is continuous and twice differentiable
$R\Pi^S 3$: Π^S is non-increasing in $\mathbf{w}_o$
$R\Pi^S 4$: Π^S is non-decreasing in $\mathbf{p}$
$R\Pi^S 5$: Π^S is non-decreasing in $\hat{\delta}$
$R\Pi^S 6$: Π^S is HD1 in $\{\mathbf{p}, \mathbf{w}_o\}$

$R\Pi^S 7$: Π^S is HD1 in $\hat{\delta}$

$R\Pi^S 8$: Π^S is convex in $\{\mathbf{p}, \mathbf{w}_o\}$

$R\Pi^S 9$: Π^S is concave in $\hat{\delta}$.

Using Hotelling's lemma, the economy's observed net output vector and net import function are equal to:

$$\mathbf{Y}(\mathbf{p}, \mathbf{w}_o, \hat{\delta}) = \nabla_p \Pi^S$$

$$\mathbf{M}_o(\mathbf{p}, \mathbf{w}_o, \hat{\delta}) = -\nabla_{w_o} \Pi^S.$$

3.2 The EGNP Function Model

By considering the structural form of a GNP function, and the impact of factor endowment variables on import demands and output supply, we extend the standard approach to modelling import demand into another direction. This extended approach modifies the standard model on at least three fronts: a) it advocates a totally new procedure for specifying GNP functions, b) it describes how the fixed input factors and the other exogenous variables interact with demand and supply systems with a strong theoretical underpinning, and c) unlike previous work based on two-stage technologies, it does not require separability as well as homotheticity, and it relaxes some restrictive constraints imposed on the technology.

We believe that such modifications are empirically important in at least two respects. For instances, import demand and output supply systems will be very useful for trade policy analysis if those systems are specified in terms of disaggregated commodities based on relevant conditioning variables, i.e., we usually need to specify two large complete import demand and output supply systems. Nonetheless, due to data constraints and to a degree of freedom problem, it is a common practice to impose *a priori* restrictions on the structure of the underlying technology. Another important aspect is that the method of introducing input factor endowments into any demand and supply systems should be non-restrictive and applicable to any general technology within a tight theoretical framework. A systematic way of eschewing the foregoing problems is to postulate that the process of output production can be decomposed into two stages. This assumption leads to our Extended GNP function model which is based on the idea of a two-stage technology. [Pollak & Wales (1987)].

A two-stage technology, as its name suggests, is a sequential production process that first uses primary inputs to produce intermediate inputs and then uses the intermediate inputs to produce final output. [Pollak & Wales (1987)]. Now, we propose another application of this theory. It should be emphasised that we have no intention to recover the underlying intermediate factor technologies whereas we want to show how this idea can be utilised to model the GNP function, and examine the effect of factor endowment and other exogenous variables on the demand and supply systems.

Assume that $\mathbf{q} \in \Omega^{N3}$ denote a non-negative N3x1 vector of intermediate inputs. We assume imports ($\mathbf{m}_o$) interact with the exogenous variables ($\mathbf{d}$) via the transformation function to produce $\mathbf{q}$ which can directly generate aggregate gross output (y) for exports and domestic absorption.[4] In this sense, imports do not enter directly into the production process of y but they only serve as inputs for producing output-bearing intermediate inputs via a transformation function. This treatment of imports seems appropriate to the case of Australia, since a substantial share (about 70%) of Australian imports consists of non-finished and semi-finished products, and they are typically subject to domestic handling, transportation, banking and retailing changes before they actually reach final demand. As a result, these goods flow through the domestic production sector, wherein they are combined with other exogenous variables like labour and capital inputs.

Three additional assumptions made here should be noted. First, all commodity markets in the economy are perfectly competitive. On the other hand, input factors are not freely mobile between firms, revealing that their markets are not competitive and their market prices should be different from their shadow prices. Moreover, capital and labour inputs are not treated as fixed inputs in the EGNP function model. In contrast to the standard approach, they are treated as the exogenous or conditional variables in the production technology which are similar to the demographic factors in the consumer case.

To show the aforesaid approach in a technical framework, let us define the first stage technology by a transformation function T^1:

$$T^1(\mathbf{m}_o, \mathbf{q}, \mathbf{d}) = 0$$

wherein $\mathbf{m}_o$ interacts with $\mathbf{d}$ to produce $\mathbf{q}$. In the second stage, q_j are treated as intermediate inputs to produce the maximum amount of single output y via another transformation function T^2, specified as:

$$T^2(\mathbf{q}, y) = 0.$$

Pulling all pieces of information together, the technology in the EGNP function model can be best described by an implicit representation of y:

$$T^1(\mathbf{m}_o, \mathbf{q}, \mathbf{d}) = T^2(\mathbf{q}, y). \tag{3.2}$$

By duality theory, the first stage technology T^1 can be equivalently represented by a cost function:

[4] For simplicity, our models will consider only 2 intermediate inputs (N3=2) and one output (N1=1).

(3.3) $$C(\mathbf{w}_o, \mathbf{q}, \mathbf{d}) = \text{Min}_{\mathbf{m}_o} \mathbf{w}_o'\mathbf{m}_o \text{ s.t. } T^1(\mathbf{m}_o, \mathbf{q}, \mathbf{d}) = 0$$

where the function $C(\mathbf{w}_o, \mathbf{q}, \mathbf{d})$ gives the least expensive collection of imported goods capable of producing the characteristic vector **q** conditional on **d** and import prices $\mathbf{w}_o$. We also assume that T^1 satisfies Conditions **R**Γ, but it is *not necessarily constant returns to scale in d*. Suppose that the cost function C has the following structure:

(3.4) $$C(\mathbf{w}_o, \mathbf{q}, \mathbf{d}) = \bar{C}\left[\tilde{C}(\mathbf{w}_o, \mathbf{q}, \mathbf{d}), \mathbf{w}_o, \mathbf{d}\right]$$

and $\tilde{C}$ is written as:

(3.5) $$\tilde{C}(\mathbf{w}_o, \mathbf{q}, \mathbf{d}) = \mathbf{H}(\mathbf{w}_o, \mathbf{d})'\mathbf{q}$$

where H_j are non-negative functions of imported commodity prices and the exogenous variables **d**.

Next we try to provide a concrete interpretation of the functions $\bar{C}$ and $\tilde{C}$. Let C* be a legitimate cost function defined by the following optimisation problem:

(3.6) $$C^*[\mathbf{H}(\mathbf{w}_o, \mathbf{d}), y] = \text{Min}_{\mathbf{q}} \mathbf{H}'\mathbf{q} \text{ s.t } T^2(\mathbf{q}, y) = 0.$$

As can be seen, **H** becomes the implicit price vector corresponding to **q**. It should also be stressed that c* represents the minimum scalar value of $\tilde{C}$, and its function C* can sufficiently describe the second stage technology in certain circumstances.

With the help of (3.6), the cost function C corresponding to a two-stage technology is given by:

(3.7) $$\begin{aligned} C(\mathbf{w}_o, y, \mathbf{d}) &= \text{Min}_{\mathbf{m}_o, \mathbf{q}} \left\{ \mathbf{w}_o'\mathbf{m}_o \text{ s.t. } T^1(\mathbf{m}_o, \mathbf{q}, \mathbf{d}) = 0, T^2(\mathbf{q}, y) = 0 \right\} \\ &= \bar{C}\left\{ \left\langle \text{Min}_{\mathbf{q}}\left[\mathbf{H}'\mathbf{q} \text{ s.t. } T^2(\mathbf{q}, y) = 0\right]\right\rangle, \mathbf{w}_o, \mathbf{d} \right\} \\ &= \bar{C}\left\{ C^*\left[\mathbf{H}(\mathbf{w}_o, \mathbf{d}), y\right], \mathbf{w}_o, \mathbf{d} \right\} \end{aligned}$$

or

$$c = G(c^*, \mathbf{w}_o, \mathbf{d}).$$

Indeed (3.7) is similar to that in Lewbel's (1985) modifying functions approach, with T^1 playing the role of the household utility function, and T^2 the role of commodity production functions.[5] According to Lewbel's definitions, C* is the kernel cost function which inherits all properties of a legitimate cost function. Besides, G and **H** can be thought of as the modifying functions to transform the kernel cost function C* to the modified cost function C.

[5] See also Pollak & Wachter (1975).

Modifying functions, G and **H**, could also be interpreted as the indirect specification of the relationship between **q** and $\mathbf{m}_o$. To illustrate this argument, let us apply Shephard's lemma to (3.5) to obtain the import demand system:

$$
\begin{aligned}
(3.8) \qquad M_{oi}(\mathbf{w}_o, \mathbf{q}, \mathbf{d}) &= \frac{\partial C}{\partial w_{oi}} \\
&= \left(\frac{\partial \overline{C}}{\partial \tilde{C}}\right)\left(\sum_j \frac{\partial \tilde{C}}{\partial H_j}\frac{\partial H_j}{\partial w_{oi}}\right) + \frac{\partial \overline{C}}{\partial w_{oi}} \\
&= \left(\frac{\partial \overline{C}}{\partial \tilde{C}}\right)\left(\sum_j q_j \frac{\partial H_j}{\partial w_{oi}}\right) + \frac{\partial \overline{C}}{\partial w_{oi}} .
\end{aligned}
$$

Evidently, $\mathbf{m}_o$ and **q** are closely related by the above system. By virtue of (3.8), we can rewrite the cost function as:

$$
\begin{aligned}
(3.9) \qquad C(\mathbf{w}_o, \mathbf{q}, \mathbf{d}) &= \sum_i w_{oi} M_{oi} \\
&= \left(\sum_i \frac{\partial \overline{C}}{\partial w_{oi}} w_{oi}\right) + \left(\frac{\partial \overline{C}}{\partial \tilde{C}}\right)\left(\sum_i \sum_j q_j \frac{\partial H_j}{\partial w_{oi}} w_{oi}\right) \\
&= \sum_i \frac{\partial \overline{C}}{\partial w_{oi}} w_{oi} + \left[\sum_j q_j \left(\sum_i \frac{\partial H_j}{\partial w_{oi}} w_{oi}\right)\frac{\partial \overline{C}}{\partial \tilde{C}}\right] .
\end{aligned}
$$

The cost function is seem to have two components. Lewbel refers to the first term as the overhead cost, or the minimum amount of cost that must be obtained before production begins.[6] Notice moreover that if the H_j functions are homogeneous of degree φ, the second term of (3.9) has an appealing form:

$$
(3.10) \qquad \sum_j q_j \varphi H_j \frac{\partial \overline{C}}{\partial \tilde{C}} = \sum_j q_j Z_j
$$

where $Z_j = \varphi H_j \frac{\partial \overline{C}}{\partial \tilde{C}}$, and note that $\frac{\partial C}{\partial q_j} = H_j \frac{\partial \overline{C}}{\partial \tilde{C}}$. These forms reveal an interesting property that the shadow price functions H_j are proportional to the marginal costs of producing intermediate goods **q**. Consequently, the second term in (3.9) can be interpreted as the total variable cost in the production of intermediate inputs.

To keep matters simple, we will specialise Lewbel's approach in the sense that φ is set to be one, and the functions H_j are restricted to be of the form:

[6] Note that the term $\sum_i (\partial \overline{C} / \partial w_{oi}) w_{oi}$ can be viewed as the overhead cost if and only if it is independent of the quantity of intermediate inputs.

$$h_j = H_j(\mathbf{w}_o^j, \mathbf{d})$$

where $\mathbf{w}_o$ is partitioned as $\{\mathbf{w}_o^r\}$ (r=1,......, N3) according to the partition $\hat{\mathbf{I}}$ defined in Section 2.3. After the simplification, H_j now become the price indices depending on the sectoral input prices and the exogenous input factor **d**. Recall also that each input price w_{ok} appears in one and only one of the price functions H_j, thereby indicating the kernel cost function $\tilde{C}$ (but not $\overline{C}$) is additive and strongly separable in Partition $\hat{\mathbf{I}}$.

Before we proceed, we should pause a moment to address another issue. That is how could we specify a GNP (profit) function by applying the modifying function approach. Let's start with a GNP function corresponding to a two-stage technology described in (3.2):

$$\begin{aligned}(3.11)\quad \hat{\Pi}(p, \mathbf{w}_o, \mathbf{d}) &= \text{Max}_{\mathbf{q}, \mathbf{m}_o}\{py - \mathbf{w}_o'\mathbf{m}_o \text{ s.t. } T^1(\mathbf{m}_o, \mathbf{d}, \mathbf{q}) = 0,\ T^2(\mathbf{q}, y) = 0\} \\ &= \text{Max}_{\mathbf{q}}\{py - [\text{Min}_{\mathbf{m}_o} \mathbf{w}_o'\mathbf{m}_o \text{ s.t. } T^1(\mathbf{m}_o, \mathbf{d}, \mathbf{q}) = 0] \text{ s.t. } T^2(\mathbf{q}, y) = 0\} \\ &= \text{Max}_{\mathbf{q}}\{p \cdot y - \overline{C}[\tilde{C}(\mathbf{w}_0, \mathbf{q}, \mathbf{d}), \mathbf{w}_o, \mathbf{d}] \text{ s.t. } T^2(\mathbf{q}, y) = 0\} \\ &= \text{Max}_{\mathbf{q}}\ \{p \cdot y - \overline{C}[\mathbf{H}'\mathbf{q}, \mathbf{w}_o, \mathbf{d}] \text{ s.t. } T^2(\mathbf{q}, y) = 0\} \\ &= \overline{\Pi}[p, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d}), \mathbf{w}_o, \mathbf{d}].\end{aligned}$$

Denote the function P as the inverse of $\hat{\Pi}$. Thus,

$$(3.12)\qquad p = P[\hat{\pi}, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d}), \mathbf{w}_o, \mathbf{d}].$$

Providing $\hat{\Pi}$ is strictly increasing in the output price, then the output price or the inverse function P is well defined. In addition, it will inherit the standard properties of a distance function which defines the maximum radial expansion of input prices with the level of GNP remaining no less that $\hat{\pi}$.[7]

We next define the kernel GNP function (or the kernel profit function) as:

$$(3.13)\qquad \hat{\Pi}^*[p, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d})] = \text{Max}_{\mathbf{q}}\ \{py - \mathbf{H}'\mathbf{q} \text{ s.t. } T^2(\mathbf{q}, y) = 0\}.$$

A straightforward way of interpreting (3.13) is to treat it as a nominal value-added function corresponding to a two-stage technology. As shown in Section 2.4, (3.13) is a special case of a proper nominal value-added function in which the gross output y is solely produced by the intermediate inputs **q** rather than the primary and fixed inputs. If T^2 is well-behaved, the kernel GNP function will inherit the regularity conditions ($\mathbf{R}\hat{\Pi}^*$):

[7] See Cornes (1992, pp.79-85), for a concise introduction to the distance function, and see Cooper (1994) for a formal proof of his argument.

$R\hat{\Pi}*1$: $\hat{\Pi}*: \Omega^3 \times \Omega_+^{1+N3} \rightarrow R$
$R\hat{\Pi}*2$: $\hat{\Pi}*$ is continuous and twice differentiable
$R\hat{\Pi}*3$: $\hat{\Pi}*$ is non-decreasing in p
$R\hat{\Pi}*4$: $\hat{\Pi}*$ is non-increasing in **H**
$R\hat{\Pi}*5$: $\hat{\Pi}*$ is HD1 in (p, **H**)
$R\hat{\Pi}*6$: $\hat{\Pi}*$ is convex in (p, **H**).

When the associated output price function (P) for $\hat{\Pi}*$ is well defined, we could write it as:

$$(3.14) \qquad P(\hat{\pi}, \mathbf{w}_o, \mathbf{d}) = \overline{P}*\left[\hat{\pi}*, \ \mathbf{H}(\mathbf{w}_o^r, \mathbf{d})\right].$$

It is intuitively clear that (3.12) and (3.14) are specifying the same scalar value. By equating both equations, we can easily derive the implicit relationship between the explicit GNP function ($\hat{\Pi}$) and the kernel GNP function ($\hat{\Pi}*$):

$$(3.15) \qquad P[\hat{\pi}, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d}), \mathbf{w}, \mathbf{d}] = \overline{P}*\left[\hat{\pi}*, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d})\right].$$

Suppose that it is feasible to solve explicitly for $\hat{\Pi}$ as a function of $\hat{\Pi}*$, **H**, **w** and **d** and thus, we can generate a function F by rearranging (3.15) as:

$$(3.16) \qquad \begin{aligned} \hat{\Pi} &= F[\hat{\pi}*, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d}), \mathbf{w}_o, \mathbf{d}] \\ &= F\{\hat{\Pi}*[p, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d})], \mathbf{H}(\mathbf{w}_o^r, \mathbf{d}), \mathbf{w}_o, \mathbf{d}\}. \end{aligned}$$

Then the resulting GNP function will have the following structural form:

$$(3.17) \qquad \hat{\pi} = \hat{\Pi}[p, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d}), \mathbf{w}_o, \mathbf{d}]$$

Technically, the derivation of (3.16) tells us how the F function starts its life. Based upon our intuition, however, we might think of F as a transformation function to convert the nominal value-added $\hat{\pi}*$ into the GNP amount $\hat{\pi}$. We also realise that $\hat{\pi}$ is not necessarily equal to $\hat{\pi}*$. Of course the discrepancy depends heavily on the first stage production technology described by T^1.

Another point worth noting concerns the structural form of the F function. Such a complicated structure like (3.17) will make our extended approach unpopular and inapplicable for econometric work. To remedy this problem, we make this approach more empirically operational by simplifying (3.15) as:

$$(3.18) \qquad \hat{\Pi} = F[\hat{\pi}*, \mathbf{w}_o]$$

or

$$\hat{\Pi}[p, \mathbf{w}, \mathbf{d}] = F\{ \hat{\Pi}^*[p, \mathbf{H}(\mathbf{w}_o^r, \mathbf{d})], \mathbf{w}_o \} \tag{3.19}$$

which nests the structure of a standard GNP function (3.1).

Once the kernel profit function is regulated by Conditions **R**$\hat{\Pi}$* and F is simplified by (3.18), two sets of restrictions on F and **H** have to be derived which are sufficient to guarantee that $\hat{\Pi}$ (our explicit GNP function) is a legitimate profit function. These restrictions are listed as follows:

<u>**RH**</u> (For the H_j functions)
RH1: H_j are non-decreasing in $\mathbf{w}_o$
RH2: H_j are continuous and twice differentiable
RH3: H_j are HD1 in $\mathbf{w}_o$
RH4: H_j are concave in $\mathbf{w}_o$,

and

<u>**RF**</u> (For the F Function)
RF1: F: $R \times \Omega^{N2}_{+} \rightarrow R$
RF2: F is non-degreasing in $\hat{\Pi}^*$
RF3: F is non-increasing in **H**
RF4: F is convex in $\hat{\Pi}^*$ and $\mathbf{w}_o$
RF5: F is HD1 in $\hat{\Pi}^*$ and $\mathbf{w}_o$.

Any pair of functions F[$\hat{\pi}^*$, **H**] and $\mathbf{H}(\mathbf{w}_o^r, \mathbf{d})$ satisfying Conditions **RF** and **R**$\hat{\Pi}$* respectively are referred to as modifying functions, and the resulting $\hat{\Pi}$ is simply called the modified GNP function as opposed to the kernel GNP function $\hat{\Pi}$*.

From Hotelling's lemma, the import demand (M_{oi}) and output supply (Y) equations are obtained by differentiating $\hat{\Pi}$ with respect to (the negative of) import prices and output prices respectively. Thus,

$$M_{oi}(p, \mathbf{w}_o, \mathbf{d}) = -\frac{\partial \hat{\Pi}}{\partial w_{oi}} = -\hat{\Pi}_i \tag{3.20}$$

$$= -\left(\frac{\partial F}{\partial \hat{\Pi}^*}\right)\left(\frac{\partial \hat{\Pi}^*}{\partial H_j}\ \frac{\partial H_j}{\partial w_{oi}}\right) - \frac{\partial F}{\partial w_{oi}}$$

$$= \left(\frac{\partial F}{\partial \hat{\Pi}^*}\right)\left(\frac{\partial H_j}{\partial w_{oi}}\right) Q_j(p, \mathbf{w}_o; \mathbf{d}) - \left(\frac{\partial F}{\partial w_{oi}}\right)$$
$$= \hat{M}_{oi}\left[p, \mathbf{H}\left(\mathbf{w}_o^r, \mathbf{d}\right), \mathbf{w}_o\right]$$

(3.21)
$$Y(p, \mathbf{w}_o, \mathbf{d}) = \frac{\partial \hat{\Pi}}{\partial p} = \hat{\Pi}_p$$
$$= \left(\frac{\partial F}{\partial \hat{\Pi}^*}\right)\left(\frac{\partial \hat{\Pi}^*}{\partial p}\right)$$
$$= \hat{Y}\left[p, H\left(\mathbf{w}_o^r, \mathbf{d}\right), \mathbf{w}_o\right].$$

(3.20) and (3.21) reveal the structure of our demand and supply systems associated with a two-stage technology. As we can see, the factor endowment variables (*k*, *l*) which are the elements of **d** are incorporated into the systems solely through the H_j functions. Therefore, the changes in factor input endowments influence the quantity of output supply and import demand only indirectly through the changes in the shadow prices of intermediate inputs. On the other hand, the overall impact of **d** on M_{oi} and Y is still ambiguous since we cannot predetermine the effect of **d** on H_j.

3.3 Interpreting the F function

Empirical analysis of GNP function often proceeds by specifying a regular profit function, using Hotelling's lemma to derive the implied demand and supply systems and estimating the systems. This approach, however, lacks the general applicability of modifying functions as the incorporation of factor endowment variables depends completely on the exact functional form of the chosen GNP function. In addition, this approach lacks any general interpretation such as that of the shadow price function and overhead cost function as elaborated previously. Regarding these, it seems advantageous to explore other approaches to the specification of our demand and supply systems.

Following McFadden (1978, p. 49), a convenient method of forming a GNP function with a structural form like (3.19) is to build it up from simple functions for which the duality mappings are known. Recall that $\hat{\Pi}$ is a legitimate profit function providing the functions F, H_j and $\hat{\Pi}^*$ satisfy Conditions **RF**, **RH** and **R**$\hat{\Pi}^*$ respectively. Obviously, this property could offer a convenient starting point for empirical analysis, even when the allocation of imports to intermediate inputs are unobservable. Unlike directly specifying variations of known GNP functions, our new approach constructs the new demand and supply systems as variants of know systems. More specifically, we will first specify the kernel profit function $\hat{\Pi}^*$ satisfying **R**$\hat{\Pi}^*$ and the H_j functions satisfying **RH**. Notice that $\hat{\Pi}^*$, known as the value-added function, only corresponds to the second stage technology $T^2(y, \mathbf{q})=0$. That means $\hat{\Pi}^*$ is not a "complete GNP function" and so, it is not equivalent to the implicit production function in the representation of production technology. To finish our task, it is necessary to do the second step wherein we transform the kernel GNP function into the explicit GNP function by specifying the F function satisfying **RF**.

4. The Specification Of An Empirical Model

In the previous sections, we have introduced a new approach for specifying an overall GNP function corresponding to a non-homothetic and non-separable two-stage technology. To demonstrate the usefulness of this procedure, we will provide a specific example for our empirical analysis. Define the normalised input price vector as $\tilde{w}_o = (w_{o1}/p, w_{o2}/p, \ldots\ldots, w_{oN2}/p)$ and $\tilde{w}_o \in \Omega_+^{N2}$. Since the kernel and explicit GNP functions are both HD1 in (p, $\mathbf{w}_o$), and the function F is HD1 in ($\hat{\pi}^*$, $\mathbf{w}_o$), without loss of generality we can represent the explicit GNP function as:

$$
\begin{aligned}
(4.1)\qquad \hat{\Pi}(p, \mathbf{w}_o, \mathbf{d}) &= p\hat{\Pi}(\mathbf{w}_o/p, \mathbf{d}) \\
&= pF\left[\hat{\Pi}^*(p, \mathbf{H})/p, \mathbf{w}_o/p\right] \\
&= pF\left\{\hat{\Pi}^*\left[\mathbf{H}(\mathbf{w}_o^r/p, \mathbf{d}), \mathbf{w}_o/p\right]\right\} \\
&= pF\left\{\hat{\Pi}^*\left[\mathbf{H}(\tilde{\mathbf{w}}_o^r, \mathbf{d})\right], \tilde{\mathbf{w}}_o\right\}
\end{aligned}
$$

where $\tilde{\mathbf{w}}_o^r$ is a vector of normalised prices in Partition $\hat{\mathrm{I}}$. Suppose the general form of F function in (4.1) is characterised by:

$$
(4.2)\qquad F\left\{\hat{\Pi}^*\left[\mathbf{H}(\tilde{\mathbf{w}}_o^r, \mathbf{d})\right], \tilde{\mathbf{w}}_o\right\} = \frac{\hat{\Pi}^*\left[\mathbf{H}(\tilde{\mathbf{w}}_o^r, \mathbf{d})\right]}{\left[\tilde{W}_3(\tilde{\mathbf{w}}_o)\right]^{\eta}}
$$

where $\tilde{W}_3(\tilde{\mathbf{w}}_o) = W_3(\mathbf{w}_o)/p$, and W_3 is the import price index satisfying the following conditions (**RW**):

RW1: W_3 is non-negative
RW2: W_3 is HD1 in $\mathbf{w}_o$
RW3: W_3 is non-decreasing in $\mathbf{w}_o$
RW4: W_3 is concave in $\mathbf{w}_o$.

Suppose further that the kernel profit function is specified as:

$$
(4.3)\qquad \hat{\Pi}^*\left[\mathbf{H}(\tilde{\mathbf{w}}_o^r, \mathbf{d})\right] = \frac{\left[\tilde{H}_4(\tilde{\mathbf{w}}_o)\right]^{-\alpha} - 1}{\alpha},
$$

where $\tilde{H}(\tilde{\mathbf{w}}_o) = \overline{H}/p$, and $\overline{H}$ is a function of $\mathbf{H}$ written as:

$$\overline{H}(\mathbf{w}_o, \mathbf{d}) = \left(H_1^K + H_2^K\right)^{1/K}. \tag{4.4}$$

By substituting (4.3) and (4.4) into (4.2) after rearrangement, we obtain the general form of our GNP function or the Fractional Profit Function System (FPFS):[8]

$$\hat{\Pi}(p, \mathbf{w}_o, \mathbf{d}) = p\left\{\frac{[\tilde{H}(\tilde{\mathbf{w}}_o)]^{-\alpha} - 1}{\alpha}\right\}[\tilde{W}_3(\tilde{\mathbf{w}}_o)]^{-\eta} \tag{4.5}$$

$$= \left[\frac{(p/\overline{H})^{\alpha} - 1}{\alpha}\right]\left(\frac{p^{1+\eta}}{W_3^{\eta}}\right).$$

In the form (4.5), we see the direct connection between the FPFS and Normalised Translog Profit Function System; setting η and α equal to zero and $\tilde{H}(\tilde{\mathbf{w}}_o)$ is replaced by a Translog form of $\mathbf{w}_o$. Another attractive property of FPFS is that $\hat{\Pi}$ is general enough to leave functional separability as hypothesis to be tested rather than maintained. As can be seen, imposing homothetic separability of in Partition $\hat{I}$ is equivalent to η being zero. Henceforth, (4.5) will collapse to the following form:

$$\hat{\Pi}(p, \mathbf{w}_o, \mathbf{d}) = p\left[\frac{(p/\overline{H})^{\alpha} - 1}{\alpha}\right]$$

$$= \hat{\Pi}(p, \mathbf{H})$$

which is a function of the output price and N3 aggregate input indices $H_r(\mathbf{w}_o^r, \mathbf{d})$.

Using the intuition stemming from Chung's (1996) model, we choose the functional form of Hj (for j=1,2) as follows:

$$H_1(\mathbf{w}_o^1, \mathbf{d}) = \tilde{D}(\mathbf{d})\left(\prod_i w_{oi}^{\beta_i}\right) \text{ with } 0 \le \beta_i \le 1 \text{ and } \Sigma_i\beta_i = 1 \ \forall\ i \tag{4.6}$$

$$H_2(\mathbf{w}_o^2, \mathbf{d}) = \tilde{D}(\mathbf{d})\left(\prod_i w_{oi}^{\gamma_i}\right) \text{ with } 0 \le \gamma_i \le 1 \text{ and } \Sigma_i\gamma_i = 1 \ \forall\ i$$

where $\tilde{D}(\mathbf{d}) = \prod_k d_k^{\phi_k}$, $\mathbf{w}_o^r$ are the sectoral input price vectors corresponding to Partition $\hat{I}$, and β_i, γ_i and ϕ_k are the parameters to be estimated.

Our specified kernel profit and shadow price functions have several intuitive interpretations. First of all, the form (4.6) tells us that the economy assembles only

[8] See Wong (1997).

two intermediate inputs, q_1 and q_2, from the imported goods **m**. By analogy with Gorman's (1976) terminology, the term $\tilde{D}(\mathbf{d})$ can be viewed as the scaling parameter which means the quantity of each intermediate input is normalised by $\tilde{D}(\mathbf{d})$ or the corresponding shadow price is inflated by $\tilde{D}(\mathbf{d})$. Also, this term bears resemblance to Barten's demographic scaling parameters because both parameters depend on the exogenous factors rather than prices or expenditure. Another interpretation of $\tilde{D}(\mathbf{d})$ comes from Ng's (1997) explanation for trends in a general demand system. Based upon her idea, this term is added to create an avenue for business cycle influences on the demand systems.

As indicated by Hotelling's lemma, differentiation of (4.5) after some manipulation will give the supply (S_p) and demand systems (S_i) in terms of profit share form:

(4.7)
$$S_p = \frac{p\hat{\Pi}_p}{\hat{\Pi}} = 1 + \alpha + \eta + 1/\tilde{R}$$
$$S_i = \frac{-w_{oi}\hat{\Pi}_i}{\hat{\Pi}} = \eta EW_{3i} + E\overline{H}_i\left(\alpha + 1/\tilde{R}\right)$$

where

$$\tilde{R} = \frac{\left(p/\overline{H}\right)^{\alpha} - 1}{\alpha}$$
$$EW_{3i} = \frac{\partial \log(W_3)}{\partial \log(w_{oi})}$$
$$E\overline{H}_i = \frac{\partial \log(\overline{H})}{\partial \log(w_{oi})} = \left[\frac{\partial \log(\overline{H})}{\partial \log(H_i)}\right]\left[\frac{\partial \log(H_i)}{\partial \log(w_{oi})}\right]$$
$$= E\overline{H}_{h1}EH_{1i} \text{ for } i \in \hat{I}^1$$
$$= E\overline{H}_{h2}EH_{2i} \text{ for } i \in \hat{I}^2,$$
$$E\overline{H}_{hi} = \frac{\partial \log(\overline{H})}{\partial \log(H_i)} = \frac{H_i^{\kappa}}{\sum_j H_j^{\kappa}}, \; i=1,\, 2 \text{ and}$$
$$EH_{ji} = \frac{\partial \log(H_j)}{\partial \log(w_{oi})}, \; j=1,\, 2.$$

Let E_{ypp} denote the partial own price elasticity of output, E_{ypi} the elasticity of output with respect to w_{oi}, E_{mip} the elasticity of imported good i with respect to output price, E_{mij} the price elasticity of imported good i with respect to w_{oj}, E_{ypn} the Rybczynski elasticities of output with respect to input endowments [n=capital stock (*k*) and labour endowment (*l*)], and E_{min} the Rybczynski elasticities of imports with respect to input endowments. The specifications of import and output price elasticity equations associated with the systems (4.6) are expressed as follows:[9]

$$E_{ypp} = \frac{p\hat{\Pi}_{pp}}{\hat{\Pi}_p} = S_p - 1 - (1+\alpha\tilde{R})/S_p$$

$$E_{ypi} = \frac{w_{oi}\hat{\Pi}_{pi}}{\hat{\Pi}_p} = (1/S_p)E\overline{H}_i(1+\alpha\tilde{R})/\tilde{R}^2 - S_i$$

$$E_{mip} = \frac{p\hat{\Pi}_{ip}}{\hat{\Pi}_i} = S_p - \frac{E\overline{H}_i(1+\alpha\tilde{R})}{S_i\tilde{R}^2}$$

$$E_{mij} = \frac{w_{oj}\hat{\Pi}_{ij}}{\hat{\Pi}_i} = -S_j - \delta_{ij} + \left[\eta EW_{3ij} + \left(\alpha + \frac{1}{\tilde{R}}\right)E\overline{H}_{ij} + \frac{E\overline{H}_i E\overline{H}_j}{\tilde{R}^2}(1+\alpha\tilde{R})\right]$$

$$E_{ypn} = \frac{\hat{\Pi}_{pn}N}{\hat{\Pi}_p} = E\overline{H}_n(\alpha + 1/\tilde{R})\left(\frac{1}{S_p\tilde{R}} - 1\right)$$

$$E_{min} = \frac{\hat{\Pi}_{in}N}{\hat{\Pi}_p} = -E\overline{H}_n(\alpha + 1/\tilde{R})(1 + E\overline{H}_i/S_i)$$

where $EW_{3ij}=\partial EW_{3i}/\partial\log(w_{oj})$, $E\overline{H}_{ij}=\partial E\overline{H}_i/\partial\log(w_{oj})$ and $E\overline{H}_n=\partial\log(\overline{H})/\partial\log(n)$ for n=*k* and *l*.

Given CES specification for the input price index W_3:

$$W_3(w_o) = \left[\sum_i a_i w_{oi}^{\rho}\right]^{1/\rho}, \quad \sum_i a_i = 1,$$

the elasticity term (E_{4i}) in (4.6) takes the form:

$$EW_{3i} = \frac{a_i w_{oi}^{\rho}}{\sum_j a_j w_{oj}^{\rho}}. \tag{4.8}$$

With this specification, it can be shown that when $0 \le a_i \le 1$, $\rho \le 1$, $0 \le \eta \le 1$, $\kappa \le 1$ and $\alpha \ge -1$, the regularity conditions **RH** and **RW** are sufficient to ensure (4.5) to be a legitimate GNP function over the region $\{(p, w_o): p > W_4\}$.[10]

---

[9] $\hat{\Pi}_{pp}$ denotes the second order partial differentiation of $\hat{\Pi}$ with respect to p. $\hat{\Pi}_{pi}$ is the cross partial derivatives of $\hat{\Pi}$ with respect to p and w_{oi}, and $\hat{\Pi}_{ij}$ is the second order partial differentiation of $\hat{\Pi}$ with respect to w_{oi} and w_{oj}.

[10] See Cooper & McLaren (1993) for a formal proof of the regularity conditions of (4.4).

5 Data, Empirical Estimation and Results

5.1 Brief Remarks on the Data Base

The empirical investigation of the present work will be carried out on annual data for Australia spanning the period 1968-69 to 1995-96 (1966-67 and 1967-68 are used to create lags). Most of these data are obtained directly from sources of the RBA (Reserve Bank of Australia) data base; while the updated series for net stock of capital are taken from Australian National Account (Australian Bureau of Statistics). Total imports are disaggregated into six components and specifically, the six import components are:

1. m_{o1}=Food, Beverages & Tobacco;
2. m_{o2}=Crude Material;
3. m_{o3}=Mineral Fuel & lubricants (or petroleum products);
4. m_{o4}=Other Products[11]
5. m_{o5}=Motor Vehicles & Transport Equipment; and
6. m_{o6}=Machinery.

Also, we can further divide the six items into two broad categories. We postulate that series on import components for food, beverages and tobacco (m_{o1}), crude material (m_{o2}), fuel (m_{o3}) and other products (m_{o4}) are classified as the imports for the group "non-capital items". Time series data for m_{o5} (motor vehicles) and m_{o6} (machinery) are grouped as "capital items". Therefore, the shadow price functions H_j are specified as:

$$(5.1) \qquad \begin{aligned} H_1(\mathbf{w}_o^1, \mathbf{d}) &= \tilde{D}(\mathbf{d})\left(w_{o1}^{\beta_1} \cdot w_{o2}^{\beta_2} \cdot w_{o3}^{\beta_3} \cdot w_{o4}^{\beta_4}\right) \\ H_2(\mathbf{w}_o^2, \mathbf{d}) &= \tilde{D}(\mathbf{d})\left(w_{o5}^{\gamma_5} \cdot w_{o6}^{\gamma_6}\right). \end{aligned}$$

The reason for restricting our form to the case of two intermediate inputs is to minimise the number of parameters. Admittedly, the number of parameters increases substantially with the number of intermediate factors which creates estimation convergence problems.

The exogenous variables are: d1= net stock of capital lagged by one period, d2=total labour force lagged by one period, d3=household saving rate, d4=female participation rate, d5=unemployment rate, d6=inflation rate, d7=the index of terms of trade (setting 1966-67=100), and d8=the time trend. Gross output (y) is obtained by subtracting total imports from gross national product; while the data for output price is the implicit price index obtained by dividing the current price series

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[11] Imports for other products ($w_{04}m_{04}$) are calculated as total imports net of imports of other components. Besides, the data of price of component 4 (w_{o4}) is its implicit price index obtained by dividing the current price series by the corresponding constant price series.

by the corresponding constant price series. All prices are normalised to one in 1966-67, and the data of the net stock of capital are measured on a per capita basis.

5.2 Estimation

To implement our empirical analysis, we postulate that all our demand and supply systems are the exact representation of the technology, but our observed profit share systems are random due to errors of optimisation. We thus specify the disturbances in the ith equation at time t as $\tilde{u}_t^i$, and the column vector of disturbances at time t as $\tilde{\mathbf{u}}_t$. The adding up property of profit share equation produces an additive error structure i.e. $\iota'\tilde{\mathbf{u}}_t = 0 \ \forall$ t (where ι is 7x1 vector with all elements equal to unity). This would produce a singular covariance matrix of errors which are contemporaneously correlated across equations. To circumvent this problem, all estimation will be carried out using the LSQ option of TSP version 4.3 computer package, which mainly accounts for across equation error correlations. Since the covariance matrix is singular, we should estimate only six import demand equations of the systems and the result of the seventh equation (output supply) can be obtained by using the theoretical restrictions in conjunction with the estimated coefficients of the other 6 import demand equations. As usual, the estimation should be independent of which equations are excluded.

Serial correlation of the error terms $\tilde{u}_t^i$ must be handled separately. Here we assume that the error vector $\tilde{\mathbf{u}}_t$ follows a first order autoregressive process:

(5.2) $$\tilde{\mathbf{u}}_t = \tilde{\tilde{\mathbf{R}}}\tilde{\mathbf{u}}_{t-1} + \tilde{\mathbf{v}}_t \qquad \text{for } t=2, 3, \ldots, 29$$

where $\tilde{\tilde{\mathbf{R}}}$ is the 7x7 matrix of autocorrelation coefficients, and the $\tilde{\mathbf{v}}_t$ are serially uncorrelated error terms that are characterised by a multivariate normal distribution with zero mean and constant contemporaneous covariance matrix. Once again the presence of adding-up property implies that $\iota'\tilde{\mathbf{u}}_t = 0$ which also entails that all the columns $\tilde{\tilde{\mathbf{R}}}$ add to the same constant; that is:

$$\iota'\tilde{\tilde{\mathbf{R}}}_t = \tilde{k}\iota'$$

where $\tilde{k}$ is a scalar constant. Denote the 6x6 transformed matrix of $\tilde{\tilde{\mathbf{R}}}$ as $\overline{\mathbf{R}}$ which is obtained by first subtracting the last column of $\tilde{\tilde{\mathbf{R}}}$ from each of other columns of $\tilde{\tilde{\mathbf{R}}}$, and then deleting the last row and column. Therefore, a typical element in the matrix $\overline{\mathbf{R}}$ is $\tilde{\tilde{R}}_{ij} - \tilde{\tilde{R}}_{i7}$ (for i, j=1 to 6). Berndt and Savin (1975) note that the vector autocorrelation of interest can be reduced to:

(5.3) $$\mathbf{u}_t = \overline{\mathbf{R}}\mathbf{u}_{t-1} + \mathbf{v}_t$$

where $\mathbf{u}_t$ and $\mathbf{v}_t$ are both 6x1 vectors, and $\mathbf{v}_t$ are independently normally distributed with zero mean and the constant covariance matrix $\hat{\Omega}$. Suppose further that the process is stationary, so that $\mathbf{u}_t$ will have a multivariate normal distribution with mean zero and a contemporaneous covariance matrix $\hat{\Sigma}$, satisfying the following relationship:

$$\hat{\Sigma} = \overline{\mathbf{R}}\hat{\Sigma}\overline{\mathbf{R}} + \hat{\Omega}. \tag{5.4}$$

The normal remedial action to cope with the autoregressive error terms is to reparameterise the matrix $\tilde{\tilde{\mathbf{R}}}$ in a number of ways. In this paper, we adopt the procedure based on a scalar autocorrelation correction calculated using the FIML algorithm of Beach & MacKinnon (1979); that is we replace $\tilde{\tilde{\mathbf{R}}}$ by the following scalar matrix:

$$\tilde{\tilde{\mathbf{R}}} = \theta \mathbf{I}_7,\ 1 \leq \theta \leq 0, \tag{5.5}$$

where $\mathbf{I}_7$ is the 7x7 identity matrix, and θ is the autocorrelation coefficient.

5.3 Empirical Results and Their Interpretation

Table 1 presents some of the data. Some figures to note from this table are: the share of motor vehicles as a percentage of total imports has increased by 35.9%; while the food, crude material and fuel shares have fallen by 14.7%, 70.16% and 31.7% respectively over the 30 years of the sample. It is generally believed that those figures are mirrored in the substantial structural changes of import since 1966. At the same time, there have also been large changes in relative prices. As shown in columns 3 and 4, all price levels have increased in absolute value and the range has been quite wide. The log of price of fuel has ballooned by 2.6; while the prices (log) of food, others, motor vehicles, machinery and output have increased by 1.63, 1.44, 2.3, 2.08 and 2.01 respectively. Those figures translate into about an annual 3% relative price change between fuel and food.

The empirical findings of the unconstrained version of FPFS are summarised in Table 2a. Estimates of asymptotic t-ratios are shown in the parentheses; these values, however, must be interpreted with care because standard asymptotic theory is inapplicable when parameters are subject to inequality constraints.[12] The algorithm takes 52 iterations to converge. The log-likelihood function after 52 iterations is 912.511. Our findings also indicate that the estimated Durbin-Watson (D-W) statistics are reasonably high, and the estimate of θ is highly significant indicating possibly the appropriateness of the correction for autoregressive errors.[13]

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[12] See Diewert & Morrison (1988) and Kohli (1993).

[13] As D-W statistics are not well-defined in highly non-linear systems of equations, the use of this diagnostic statistics in the context of demand systems are just viewed as an aid to data analysis and interpretation, rather than as a conclusive attempt for formal hypothesis testing.

To assess the estimated coefficients of determinant (R^2 values), the unconstrained FPFS is not satisfactory. R^2 values range from 27.12% for machinery to 87.77% for vehicles. These low R^2 values may be caused by our division of the quantities on the left-hand sides of (3.14) by GNP to eliminate trends resulting from the cyclical changes in the economy. Moreover, the low R^2 value for the machine equation in the GNP function systems might be due to the failure to allow for imperfect adjustment to price changes as the actual profit share of machine has reasonable variation.

Another important point to highlight from Table 2a is that most of the parameter estimates satisfy Conditions **RF**, **RH**, **R$\hat{\Pi}$*** and **RW** without the need to impose constraints. Unfortunately, α is significantly above its limiting value of -1 thereby defying the curvature conditions of **RW**. In fact, an inspection of the eigenvalues of the Hessian matrix $\{\hat{\Pi}_{ij}\}$ of FPFS reveals some violations of the convexity requirements for 19 observations in the sample period. This outcome is quite usual in a model of small sample size. Therefore, global convexity in (p, w_o) has to be imposed; this can be done by setting α equal to -1.

The detailed parameter estimates for the constrained FPFS in which α is set to -1 are reported in Table 2b. Comparisons of the results with those in Table 2a show that in most cases the estimates of constrained model are of the same sign and order of magnitude as the corresponding unconstrained case. One crucial point should be acknowledged; that is the freeing-up of α is of little statistical value, and there is no gain of fit. As shown in the table, the log of likelihood function after 93 iterations is 911.7. Our estimated R^2s are ranged from 30.6% (machinery) to 87.9% (petroleum product). According to our findings, the null hypothesis $H_o{:}\alpha = -1$ or the restriction required to preserve the global convexity is not rejected by our data on the bases of both likelihood ratio and Wald tests.

Our next task is to formally test the homothetic separability restriction of the FPFS using the results of the constrained model. The testing has been done by using a standard Wald test. Reference to this testing result summarised in Table 2b shows that our null hypothesis H_o: $\eta=0$ is rejected by our data at 99% of confidence. It could be seen that the computed χ^2 value (13.1402) is obviously greater than the critical value for χ^2_1 (6.635) at the 1% level, thereby concluding that FPFS specification represents a substantial improvement over the homothetic separable profit function for our data set.

Once our constrained model is not statistically inferior to its unconstrained version, in the following sections we will use the constrained estimates to compute the fitted profit shares and the point estimates of elasticities. A plot of the fitted systems is shown in Figure 1. Inspection of these diagrams highlights some specification problems with the model. The profit share equations of fuel (S_3) and machinery (S_5) perform poorly and their actual fits are highly volatile through time. Especially, the profit shares of fuel and machines are almost unpredictable starting from the mid seventies. These are precisely when S_3 and S_5 were poorly fitted, and they coincided with a period when the economy was subjected to two OPEC shocks. Note also that some import items are wide composites and some might include items that are physically durable. For example, most items in "machinery" should be durable goods; "mineral fuel & lubricants" may include coal and other storable fuels that might not be well explained by a static model.

The elasticity estimates for constrained FPFS for selected years are reported in Table 3. Own price and cross price elasticities of imports and output are all of correct signs and sensible magnitudes over the period. Of considerable interest is the own price elasticity of output (ε_{ypp}). ε_{ypp} on average (.075) is somehow similar to the earlier estimates by Kohli (1982) (ranging from .017 to .09) but apparently smaller than the most recent estimates by Appelbaum & Kohli (1997).

The own-price elasticities of the demand for food (ε_{m11}) and the demand for crude material (ε_{m22}) are of interest as well. ε_{m11} decreases from about -.33 in 1968 to -.57 in 1995; ε_{m22} increases substantially from -.64 in 1968 to -.32 in 1995. Therefore, the imported raw material becomes less price elastic whereas the imported food becomes more price elastic by the march of time. It can also be seen that the disaggregate import own price elasticities (on average) are quite small ranging from -0.35 for petroleum products to -.66 for other products. In particular, 1% increase in the price of imported petroleum products (w_{o3}) in 1980 due to, say, by a tariff would reduce its own demand by only 0.46%. This elasticity response of petroleum products is considerably lower than the results (ε_{m33}=-1.14 in 1980) found in and Appelbaum & Kohli (1997). Notice however that comparisons should be made with care since the explanatory variables vary greatly between different studies.

Analysis of the cross-price elasticities indicates that the effects of cross price changes on imports and output are rather weak over the period. It is interesting to note that an increase in the output price actually stimulates the demand for all imports (see the positive signs of ε_{mip}). Conversely, the increases in import prices would reduce the domestic output supply for all of the year (see the negative signs of ε_{ypi}). Looking at the other cases, ε_{mij} (for i, j=1 to 6) are all negative except ε_{m14}, ε_{m46}, ε_{m56}, ε_{m64} and ε_{m65}.

Trade economists are primarily concerned with Rybczynski elasticities. Judging from our estimates in Table 3, they show that the changes in the capital and labour input endowments do not significantly affect the demand for imports and the domestic output production, though the magnitudes of impact have substantially increased since the early years of the sample. It happens that an increase in capital endowment slightly stimulates the domestic output production and the demand for all types of imports. An increase of labour force, on the other hand, results in actual decreases in all types of import demand and output supply.

6 Concluding Remarks

In this paper, an attempt has been made to introduce a modification (EGNP function approach) for specifying any legitimate GNP function corresponding to non-homothetic and non-separable two-stage technologies. As illustrated in the previous sections, this extended approach has several appealing features like allowing the incorporation of input factor endowments with a strong theoretical underpinning, and allowing expansion of the conventional approaches to import demand determination into a new direction. It should be emphasised that we are not primarily concerned with providing improved import demand elasticity estimates for input to any computational general equilibrium model. Instead, we intend to advocate a straightforward but promising procedure for combining profit functions and nominal value-added functions to obtain empirically tractable import demand and output supply systems.

In an empirical illustration we have specified a new GNP function which is not necessarily globally regular, whilst the function can be restricted to be regular over an unbounded region. Some of the results which are of special interest to trade economists are the relatively small own price and cross price effects of the demand for imports and the supply of domestic output. In particular, our results indicate that the demand for petroleum products in Australia is less price elastic than in United States. We also find that the input factor endowments did not play an influential role in domestic production. On the other hand, a rise in total labour force would reduce import demand and output supply.

The failure of some observations to satisfy the global convexity conditions might reflect the quality of the data. More likely, this finding might cast doubt on the reliability of FPFS or the level of aggregation of our data series. A valid suggestion on this line is to approach the specification procedure using dynamic optimisation theory as advocated by McLaren & Cooper (1980), who exploited the duality relationships between production and profit functions in the context of an intertemporal economic model. However, no such attempt is made here as it is out of the scope of this paper.

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Figure 1. FPFS fitted in the levels to Australian data on profit shares, 1968-69 through 1995-96

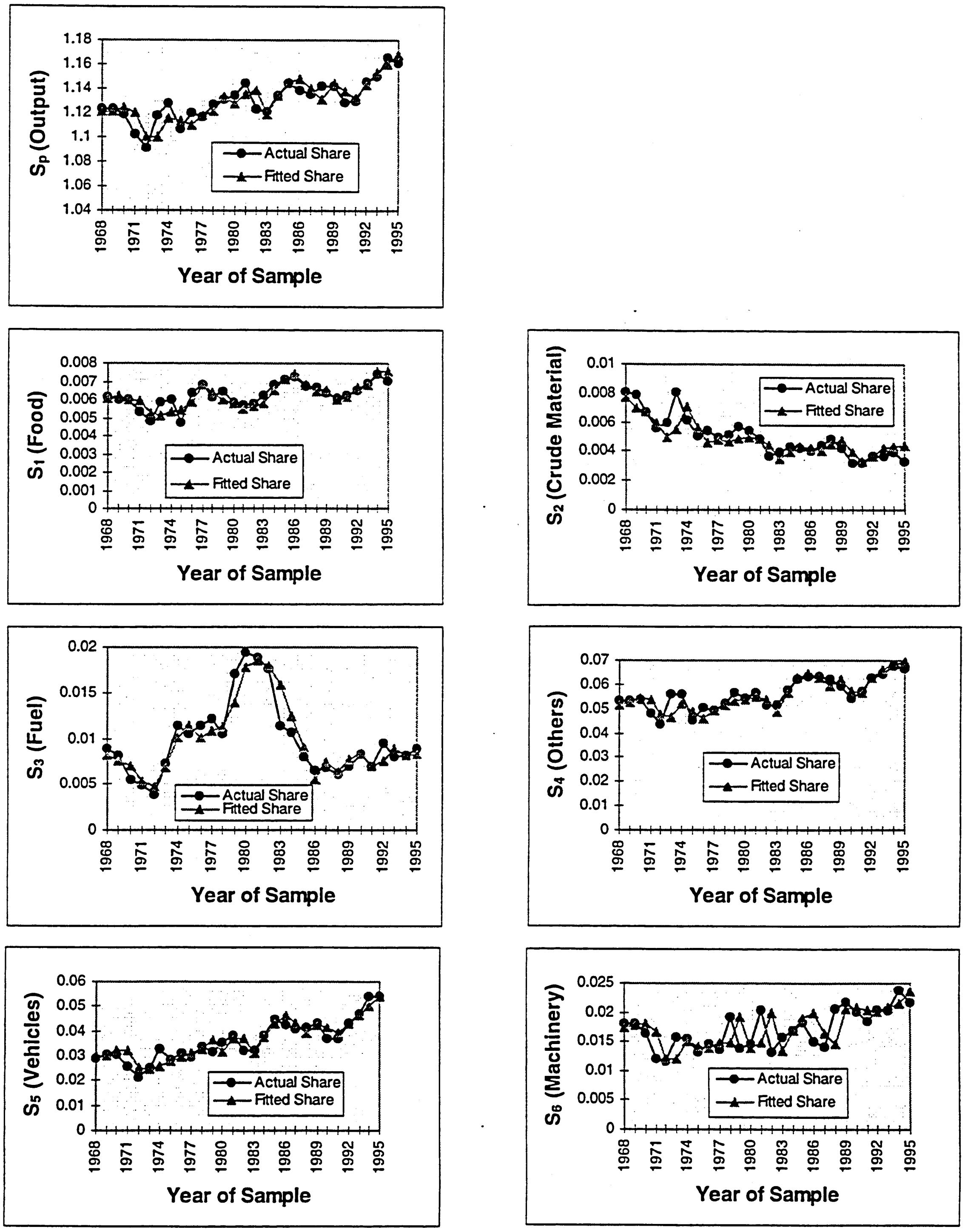

Table 1: Some Data

Imports/Output	Import Share (%)		Log Price (1966-67=0)	
	1966-67	1995-96	1966-67	1995-96
	(1)	(2)	(3)	(4)
1) Food	5.123	4.370	0	1.630
2) Crude Material	6.770	2.020	0	1.984
3) Mineral Fuel	8.110	5.540	0	2.620
4) Others	42.43	41.20	0	1.438
5) Motor Vehicles	24.57	33.39	0	2.267
6) Machinery	13.00	13.48	0	2.084
7) Output			0	2.010

Table 2a: Empirical Results (Unconstrained Model)

(t-ratios in parentheses)

Parameters					
β_1	.0609(7.7713)	ϕ_4	-.4137(-.8475)	a_1	.0835(5.2052)
β_2	.0370(1.8003)	ϕ_5	-.1818(-2.6371)	a_2	.0437(2.3766)
β_3.	.0456(1.5824)	ϕ_6	-.0175(-1.1390)	a_3	.0517(3.9821)
β_4	.8565(26.2620)	ϕ_7	.4248(2.8052)	a_4	.4525(2.0373)
γ_5	.8332(9.6379)	ϕ_8	.8107(2.7706)	a_5	.1608(2.2713)
γ_6	.1668(1.9230)	κ	-.4122(-1.7967)	a_6	.2078(3.5704)
ϕ_1	-.1886(-1.1961)	α	-1.9011(-3.4916)	ρ	.9702(5.7390)
ϕ_2	-.0137(-.0522)	η	.0562(3.8866)	θ	.7993(18.8808)
ϕ_3	-.0981(-2.7298)				

Log-Likelihood	912.511
R^2	
1) Food, Beverages & Tobacco	.7405
2) Crude Material & Lubricants	.7386
3) Mineral Fuel	.8777
4) Other Products	.7775
5) Motor Vehicles	.8588
6) Machinery	.2712
7) Output Supply	.7498
Durbin-Watson Statistics	
1) Food, Beverages & Tobacco	2.0209
2) Crude Material & Lubricants	1.7504
3) Mineral Fuel	1.6178
4) Other Products	2.1421
5) Motor Vehicles	1.9153
6) Machinery	2.5267
7) Output Supply	2.0279

Table 2b: Empirical Results (Constrained Model)

(t-ratios in parentheses)

Functional Form Restriction: α=-1							
Parameters							
β_1	.0646(8.7961)	ϕ_1	-.4551(-1.6238)	ϕ_7	.0179(.0657)	a_1	.0822(4.9846)
β_2	.0290(1.4108)	ϕ_2	.6913(1.4502)	ϕ_8	1.6562(3.3170)	a_2	.0539(2.7909)
β_3.	.0380(1.2524)	ϕ_3	-.0919(-1.4527)	κ	-.4099(-1.9178)	a_3	.0536(4.0939)
β_4	.8684(25.2936)	ϕ_4	-1.6978(-1.7531)	η	.0548(3.6249)	a_4	.4417(12.5929)
γ_5	.7688(10.1637)	ϕ_5	-.3982(-2.9304)	ρ	.9843(5.4222)	a_5	.1973(3.1549)
γ_6	.2312(3.0559)	ϕ_6	-.0384(-1.3100)	θ	.7671(16.1059)	a_6	.1713(3.2181)
Log-Likelihood				911.702			

R^2		**Durbin-Watson Statistics**	
1) Food	.7526	1) Food	1.9641
2) Crude Material	.7326	2) Crude Material	1.6453
3) Mineral Fuel	.8791	3) Mineral Fuel	1.5788
4) Other Products	.7929	4) Other Products	1.9676
5) Motor Vehicles	.8625	5) Motor Vehicles	1.9247
6) Machinery	.3055	6) Machinery	2.5493
7) Output Supply	.7680	7) Output Supply	1.9938

FPFS Wald Test Result for Homothetic Separability Hypothesis (η=0):

Wald Test Statistics = 13.1402
1% Critical Value: χ_1^2 =6.63.

Table 3: Estimates of Elasticities For Selected Years

(Using the Estimates of Constrained FPFS)

	1968	1980	1990	1995	Mean
Price Elasticities of Import Demand/Output Supply					
e_{ypp}	0.1058	0.0731	0.0707	0.0807	0.0749
e_{yp1}	-0.0056	-0.0038	-0.0033	-0.0036	-0.0041
e_{yp2}	-0.0075	-0.0041	-0.0027	-0.0026	-0.0039
e_{yp3}	-0.0078	-0.0166	-0.0068	-0.0059	-0.0085
e_{yp4}	-0.0447	-0.0271	-0.0211	-0.0161	-0.0270
e_{yp5}	-0.0242	-0.0075	-0.0128	-0.0190	-0.0126
e_{yp6}	-0.0156	-0.0066	-0.0123	-0.0130	-0.0100
e_{m1p}	1.0281	0.7460	0.6302	0.5533	0.7473
e_{m11}	-0.3328	-0.4625	-0.5187	-0.5697	-0.4503
e_{m12}	-0.0475	-0.0265	-0.0222	-0.0233	-0.0293
e_{m13}	-0.0442	-0.1521	-0.0779	-0.0478	-0.0854
e_{m14}	-0.3538	-0.1594	-0.0793	0.0259	-0.1772
e_{m15}	-0.1196	0.0873	0.1057	0.0946	0.0504
e_{m16}	-0.1301	-0.0328	-0.0379	-0.0329	-0.0555
e_{m2p}	1.0882	0.9270	0.7879	0.6917	0.8826
e_{m21}	-0.0373	-0.0310	-0.0340	-0.0403	-0.0368
e_{m22}	-0.6411	-0.5504	-0.3904	-0.3167	-0.4834
e_{m23}	-0.0274	-0.1305	-0.1004	-0.0800	-0.0837
e_{m24}	-0.2160	-0.1710	-0.1744	-0.1010	-0.1878
e_{m25}	-0.0866	0.0043	-0.0019	-0.0454	-0.0185
e_{m26}	-0.0799	-0.0484	-0.0867	-0.1081	-0.0724
e_{m3p}	1.0800	1.0542	0.9248	0.8385	0.9620
e_{m31}	-0.0331	-0.0497	-0.0555	-0.0435	-0.0533
e_{m32}	-0.0262	-0.0364	-0.0466	-0.0422	-0.0397
e_{m33}	-0.6860	-0.4577	-0.1841	-0.3685	-0.3496
e_{m34}	-0.1920	-0.2983	-0.3304	-0.1507	-0.2979
e_{m35}	-0.0728	-0.1053	-0.1400	-0.1081	-0.1001
e_{m36}	-0.0700	-0.1067	-0.1680	-0.1256	-0.1214
e_{m4p}	0.9738	0.5717	0.4204	0.2706	0.5610
e_{m41}	-0.0417	-0.0173	-0.0083	0.0028	-0.0202
e_{m42}	-0.0324	-0.0158	-0.0119	-0.0064	-0.0176
e_{m43}	-0.0302	-0.0988	-0.0487	-0.0180	-0.0549
e_{m44}	-0.7545	-0.6796	-0.6253	-0.5893	-0.6628
e_{m45}	-0.0398	0.2142	0.2478	0.2874	0.1863
e_{m46}	-0.0752	0.0256	0.0260	0.0529	0.0082

Table 3 (Continued)

	1968	1980	1990	1995	Mean
e_{m5p}	0.8973	0.2668	0.3529	0.4085	0.3973
e_{m51}	-0.0133	-0.0314	-0.0109	0.0000	-0.0111
e_{m52}	-0.0221	0.0007	-0.0002	-0.0037	-0.0037
e_{m53}	-0.0195	-0.0589	-0.0285	-0.0165	-0.0283
e_{m54}	-0.0676	0.3618	0.3423	0.3680	0.2810
e_{m55}	-0.7030	-0.5990	-0.6592	-0.7260	-0.6385
e_{m56}	-0.0612	0.0127	-0.0227	-0.0435	-0.0155
e_{m6p}	1.0038	0.5361	0.6708	0.6427	0.6559
e_{m61}	-0.0453	-0.0137	-0.0109	-0.0105	-0.0207
e_{m62}	-0.0355	-0.0173	-0.0162	-0.0200	-0.0208
e_{m63}	-0.0326	-0.1365	-0.0677	-0.0441	-0.0702
e_{m64}	-0.2225	0.0989	0.0712	0.1558	0.0274
e_{m65}	-0.1066	0.0290	-0.0449	-0.1001	-0.0316
e_{m66}	-0.5613	-0.4965	-0.6023	-0.6238	-0.5401
Rybczynski Elasticities					
e_{ypK}	0.0007	0.0016	0.0018	0.0024	0.0016
e_{ypL}	-0.0011	-0.0025	-0.0028	-0.0037	-0.0024
e_{m1K}	0.0494	0.1928	0.2510	0.3024	0.1930
e_{m1L}	-0.0750	-0.2928	-0.3813	-0.4594	-0.2931
e_{m2K}	0.0225	0.1154	0.1847	0.2458	0.1353
e_{m2L}	-0.0341	-0.1754	-0.2806	-0.3734	-0.2056
e_{m3K}	0.0261	0.0611	0.1272	0.1857	0.1020
e_{m3L}	-0.0397	-0.0929	-0.1931	-0.2821	-0.1549
e_{m4K}	0.0736	0.2672	0.3392	0.4181	0.2719
e_{m4L}	-0.1118	-0.4059	-0.5152	-0.6351	-0.4130
e_{m5K}	0.1079	0.3975	0.3675	0.3616	0.3424
e_{m5L}	-0.1639	-0.6037	-0.5583	-0.5493	-0.5201
e_{m6K}	0.0602	0.2824	0.2339	0.2658	0.2323
e_{m6L}	-0.0915	-0.4290	-0.3553	-0.4038	-0.3529

Note: The elasticities in the last column are estimated at the mean values of exogenous variables.
The Index Set for output, imports and input factor endowments = {P=Output, 1=Food, Beverages & Tobacco, 2=Crude Material & Lubricants, 3=Mineral Fuel, 4=Others, 5=Motor Vehicles and 6=Machinery, K=Stock of Net Capital, L=Labour Endowment}.