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COMPARISONS OF ESTIMATORS AND TESTS BASED ON MODIFIED LIKELIHOOD AND MESSAGE LENGTH FUNCTIONS

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Abstract

The presence of nuisance parameters causes unexpected complications in econometric inference procedures. A number of modified likelihood and message length functions have been developed for better handling of nuisance parameters but all of them are not equally efficient. In this paper, we empirically compare different modified likelihood and message length functions in the context of estimation and testing of parameters from linear regression disturbances that follow either a first-order moving average or first-order autoregressive error processes. The results show that estimators based on the conditional profile likelihood and tests based on the marginal likelihood are best. If there is a minor identification problem, the sizes of the likelihood ratio and Wald tests based on simple message length functions are best. The true sizes of the Lagrange multiplier tests based on message length functions are rather poor because the score functions of message length functions are biased.

1. Introduction

Satisfactory statistical analysis of non-experimental data, is an important problem in econometrics. Econometric models usually involve a large number of influences, most of which are not of immediate interest. This means that such models contain two kinds of parameters, those of interest and those not of immediate interest that are known as nuisance parameters. Their presence causes unexpected complications in econometric inference. A fairly standard procedure in likelihood based statistical inference is to concentrate the likelihood function by replacing nuisance parameters by their respective maximum likelihood (ML) estimators conditional on the parameters of interest. In such situations, estimators and tests can perform poorly in small samples (Bewley 1986, Cox and Reid 1987, King 1987, King and McAleer 1987, Moulton and Randolph 1989, Chesher and Austin 1991). Earlier, Neyman and Scott (1948) warned that nuisance parameters can seriously compromise likelihood based inference. In relation to this, King (1996) observed that when nuisance parameters are present, statistical theory is generally less helpful in suggesting reliable diagnostic tests. Also, Cordus (1986) noted that the presence of nuisance parameters causes a shift in the estimated mean of the null distribution of the likelihood ratio test.

The question which then arises is which methods should be used to tackle the problem of nuisance parameters in order to improve estimators and tests. The marginal likelihood is one such method for handling nuisance parameters. Estimators and tests based on this likelihood have better small sample properties compared to those based on the classical likelihood function (Ara 1995, Cordus 1986, Rahman and King 1998). In the context of estimating variance components in the linear regression model, a related approach known as residual (or restricted) maximum likelihood (REML) (Patterson and Thompson 1971) has gained considerable importance. The marginal likelihoods cannot be constructed in all situations and REML applies only to the disturbances parameters in the linear model. As an alternative, Barndorff-Nielsen (1983) proposed the modified profile likelihood (MPL) and Cox and Reid (1987) initiated the idea of the conditional profile likelihood (CPL) which requires that the parameter(s) of interest and nuisance parameters are orthogonal. Also, using the combination of REML and CPL, Laskar and King (1998) derived the conditional profile restricted log-likelihood function (CPRL) for better handling of nuisance parameters. They investigated the small sample properties of estimators and tests based on this likelihood function and three other modified likelihood functions and compared with those based on the profile likelihood function.

An alternative approach, known as minimum message length (MML), is a information theoretic criteria for parameter estimation and model selection. The MML principle needs a prior distribution of the parameters, the square root of the determinant of the information matrix for the parameters and a likelihood function. In this context, Wallace and Dowe (1993) mentioned that the inclusion of the first two factors helps reduce the measure of uncertainty, their ratio is dimension free and invariant to reparameterization. Extending their research, Laskar and King (1996) derived six different message length functions using different prior distributions of the parameters and combinations of CPL and message length functions. They investigated the small sample properties of estimators based on these message length functions. Moreover, Laskar and King (1997) investigated the small sample properties of different tests based on these message length functions. There are many different modified likelihood and message length functions for handling nuisance parameters but for econometric problems where estimation and diagnostic testing are of main interest, all of them are not equally efficient. Thus, it is important to investigate and find out the best approaches for handling nuisance parameters.

The aim of this paper is to empirically compare all the likelihood and message length functions in the context of estimation and testing of parameters involved in the variance-covariance matrix of linear regression disturbances. We extend and compare the Monte Carlo results of Laskar and King (1996, 1997a, 1997b, 1998). This will enable us to recommend the best functions in estimation and testing problems. In section 2, different likelihood and message length functions are presented. A Monte Carlo experiment, conducted to compare the estimators and tests based on all the likelihood and message length functions are reported in section 3. Some concluding remarks are made in section 4.

2. Theory

Consider the linear regression model with non-spherical disturbances

$$y = X\beta + u\,;\; u \sim N(0,\sigma^2\Omega(\theta)) \qquad (1)$$

where y is $n\times 1$, X is $n\times k$, nonstochastic and of rank $k<n$, β is a $k\times 1$ vector, $\Omega(\theta)$ is a symmetric matrix and θ is a $p\times 1$ vector. This model generalizes a wide range of disturbance processes of the linear regression model of particular interest to statisticians and econometricians. These include all parametric forms of autocorrelated disturbances, all parametric forms of heteroscedasticity (in which case $\Omega(\theta)$ is a diagonal matrix), and error components models including those that result from random regression coefficients. The likelihood and log-likelihood for this model (excluding constants) are respectively

$$L(y;\theta,\sigma^2,\beta)\propto\sigma^{-n}|\Omega(\theta)|^{-\frac{1}{2}}\exp\{-\frac{1}{2\sigma^2}(y-X\beta)'\Omega(\theta)^{-1}(y-X\beta)\}, \tag{2}$$

$$l(y;\theta,\sigma^2,\beta)\propto-\frac{n}{2}\log\sigma^2-\frac{1}{2}\log|\Omega(\theta)|-\frac{1}{2\sigma^2}(y-X\beta)'\Omega(\theta)^{-1}(y-X\beta) \tag{3}$$

and the log profile (or concentrated) likelihood is

$$l_p(y;\theta)\propto-\frac{n}{2}\log\hat{\sigma}_\theta^2-\frac{1}{2}\log|\Omega(\theta)| \tag{4}$$

where $\hat{\sigma}_\theta^2=(y-X\hat{\beta}_\theta)'\Omega(\theta)^{-1}(y-X\hat{\beta}_\theta)/n$ and $\hat{\beta}_\theta=(X'\Omega(\theta)^{-1}X)^{-1}X'\Omega(\theta)^{-1}y$.

2.1. Modified Likelihood Functions

Tunnicliffe and Wilson (1989) derived the marginal likelihood for θ in (1) as

$$\ell_m(y;\theta)=-\frac{1}{2}\log|\Omega(\theta)|-\frac{1}{2}\log|X'\Omega(\theta)^{-1}X|-\frac{m}{2}\log(\hat{u}'\Omega(\theta)^{-1}\hat{u}) \tag{5}$$

where $m=n-k$. Using the combination of REML and CPL, Laskar and King (1998) derived the CPRL function of θ for model (1) as

$$\bar{\ell}^*_{cpe}(y;\theta)=-\frac{(m-2)}{2m}\left[\log|\Omega(\theta)|-\log|X'\Omega(\theta)^{-1}X|-m\log(\hat{u}'\Omega(\theta)^{-1}\hat{u})\right] \tag{6}$$

Using the idea of Cox and Reid (1987), Laskar (1998) derived the CPL for θ in (1) as

$$l_{cp}(y;\theta)=-\frac{1}{2}\log|X'\Omega(\theta)^{-1}X|-\frac{(n-2)}{2n}\log|\Omega(\theta)|-\frac{(m-2)}{2}\log(\hat{u}'\Omega(\theta)^{-1}\hat{u}). \tag{7}$$

Based on the idea of Cox and Reid (1993), Laskar (1998) also derived an approximate conditional profile likelihood (ACPL) for θ in (1) as

$$l_{acp}(y;\theta)=-\frac{m-2}{2}\log(\hat{u}'\Omega(\theta)^{-1}\hat{u})-\frac{1}{2}\log|\Omega(\theta)|-\frac{1}{2}\log|X'\Omega(\theta)^{-1}X|$$

$$+\frac{1}{n}tr\left[\Omega(\theta)^{-1}\frac{\partial\Omega(\theta)}{\partial\theta}\right]_{\theta=\hat{\theta}}(\theta-\hat{\theta}). \qquad (8)$$

From (5) and (6)

$$\bar{l}^{*}_{cpr}(y;\theta)=\frac{(m-2)}{m}l_m(y;\theta)$$

so that for the purposes of estimating θ, the marginal likelihood function and the CPRL are equivalent. This is not necessarily true for likelihood based tests of θ, because scores, Hessians and maximized likelihood will be different, although any differences will obviously disappear as n increases.

2.2. Message Length Functions

Minimum message length is a Bayesian method which chooses estimators to minimize the length of an encoded form of the data made up of a model and the deviations from that model (residuals). Wallace and Dowe (1993) state that the MML principle is that the best possible conclusion to draw from the data is the theory which maximizes the product of the probability of the data occurring in the light of the theory with the prior probability of that theory.

For model (1), an approximate message length function given by Wallace and Freeman (1987) and accurate to $\delta=1/\sqrt{K_s^s F(\theta,\sigma^2,\beta)}$ is

$$-\log\left[\frac{\pi(\theta,\sigma^2,\beta)L(\theta,\sigma^2,\beta)}{\sqrt{F(\theta,\sigma^2\beta)}}\right]+\frac{s}{2}(1+\log K_s) \qquad (9)$$

where $\pi(\theta,\sigma^2,\beta)$ is a prior density for $\gamma=(\theta',\sigma^2,\beta')'$, $F(\theta,\sigma^2,\beta)$ is the determinant of the information matrix, $s=k+p+1$, K_s is the s dimensional lattice constant which is independent of parameters, as given by Conway and Sloan (1988, p. 59-61). For example $K_1=\frac{1}{12}$, $K_2=\frac{5}{36\sqrt{3}}$ and $K_3=\frac{19}{36.\sqrt[3]{2}}$. Wallace and Dowe (1994) mentioned, maximizing (9) is equivalent to maximizing the average of the log-likelihood function over region of size proportional to $1/\sqrt{F(\theta,\sigma^2,\beta)}$ while the ML estimator maximizes the likelihood function at a single point. The value of θ which minimizes (9) is the MML estimate of θ with accuracy $\delta=1/\sqrt{K_s^s F(\theta,\sigma^2,\beta)})$.

Inclusion of $\pi(\theta,\sigma^2,\beta)$ and $\sqrt{F(\theta,\sigma^2,\beta)}$ help reduce the measure of uncertainty, their ratio is dimension free and invariant to reparameterization (Wallace and Dowe 1993). Since MML is a Bayesian method and depends on the choice of prior density of the parameters, there is scope in selecting the prior. Using different prior densities and combinations of CPL and message length functions, Laskar and King (1996) derived six different message length functions which are

$$ML_1 = \frac{m-1}{2}\log\sigma^2 + \frac{1}{2}\log|\Omega(\theta)| + \frac{1}{2\sigma^2}u'\Omega(\theta)^{-1}u + \frac{1}{2}\log|X'\Omega(\theta)^{-1}X|$$
$$+\frac{1}{2}\log\left(n\times tr\left[-\frac{\partial\Omega(\theta)^{-1}}{\partial\theta}\frac{\partial\Omega(\theta)}{\partial\theta}\right]-\left\{tr\left[\Omega(\theta)^{-1}\frac{\partial\Omega(\theta)}{\partial\theta}\right]\right\}^2\right)$$
$$+\frac{s}{2}(1+\log K_s)-\log 2, \tag{10}$$

$$ML_2 = \frac{m}{2}\log\sigma^2 + \frac{1}{2}\log|\Omega(\theta)| + \frac{1}{2\sigma^2}u'\Omega(\theta)^{-1}u + \frac{1}{2}\log|X'\Omega(\theta)^{-1}X|$$
$$+\frac{1}{2}\log\left(n\times tr\left[-\frac{\partial\Omega(\theta)^{-1}}{\partial\theta}\frac{\partial\Omega(\theta)}{\partial\theta}\right]-\left\{tr\left[\Omega(\theta)^{-1}\frac{\partial\Omega(\theta)}{\partial\theta}\right]\right\}^2\right)$$
$$+\frac{s}{2}(1+\log K_s)-\log 2, \tag{11}$$

$$CPML_1 = \frac{m-k-3}{2}\log\hat{\delta}_1 + \frac{k+1}{n+k+1}\log|\Omega(\theta)| + \log\left|X_\theta^{\dagger\prime}X_\theta^\dagger\right|$$
$$+\frac{1}{2}\log\left(n\times tr\left[-\frac{\partial\Omega(\theta)^{-1}}{\partial\theta}\frac{\partial\Omega(\theta)}{\partial\theta}\right]-\left\{tr\left[\Omega(\theta)^{-1}\frac{\partial\Omega(\theta)}{\partial\theta}\right]\right\}^2\right) \tag{12}$$

where $u_\theta^\dagger = y_\theta^\dagger - X_\theta^\dagger\beta$, $X_\theta^\dagger = D(\theta)^{-\frac{1}{2}}X$, $D(\theta)=\Omega(\theta)/|\Omega(\theta)|^{\frac{1}{(n+k+1)}}$, $y_\theta^\dagger = D(\theta)^{-\frac{1}{2}}y$, $\hat{\delta}_1 = \hat{u}_\theta^{\dagger\prime}\hat{u}_\theta^\dagger/(n-k-1)$, $\hat{u}_\theta^\dagger = y_\theta^\dagger - X_\theta^\dagger\hat{\beta}_\theta^\dagger$ and $\hat{\beta}_\theta^\dagger = (X_\theta^{\dagger\prime}X_\theta^\dagger)^{-1}X_\theta^{\dagger\prime}y_\theta^\dagger$.

$$CPML_2 = \frac{m-k-2}{2}\log\hat{\delta}_2 + \frac{k}{n+k}\log|\Omega(\theta)| + \log\left|X_\theta^{*\prime}X_\theta^*\right|$$
$$+\frac{1}{2}\log\left(tr\left[-\frac{\partial\Omega(\theta)^{-1}}{\partial\theta}\frac{\partial\Omega(\theta)}{\partial\theta}\right]-\left\{tr\left[\Omega(\theta)^{-1}\frac{\partial\Omega(\theta)}{\partial\theta}\right]\right\}^2\right) \tag{13}$$

where $\hat{\delta}_2 = \hat{u}_\theta^{*\prime}\hat{u}_\theta^*/m$, $\hat{u}_\theta^* = y_\theta^* - X_\theta^*\hat{\beta}_\theta^*$, $\hat{\beta}_\theta^* = (X_\theta^{*\prime}X_\theta^*)^{-1}X_\theta^{*\prime}y_\theta^*$, $X_\theta^* = G_1(\theta)^{-\frac{1}{2}}X$, $y_\theta^* = G_1(\theta)^{-\frac{1}{2}}y$ and $G_1(\theta) = \Omega(\theta)/|\Omega(\theta)|^{\frac{1}{(n+k)}}$.

$$AML_1 = \frac{m-1}{2}\log\delta + \frac{1}{2\delta}u_\theta' u_\theta + \frac{1}{2}\log|X_\theta' X_\theta| + \frac{1}{2}\log|C(\theta)|, \quad (14)$$

$$AML_2 = \frac{m}{2}\log\delta + \frac{1}{2\delta}u_\theta' u_\theta + \frac{1}{2}\log|X_\theta' X_\theta| + \frac{1}{2}\log|C(\theta)| \quad (15)$$

where $\sigma^2 = \delta / |\Omega(\theta)|^{\frac{1}{n}}$, $G(\theta)$ is an $n \times n$ matrix comprised of $\Omega(\theta)$ with each element divided by $|\Omega(\theta)|^{\frac{1}{n}}$ and the $(i,j)^{th}$ element of the $p \times p$ matrix $C(\theta)$ is $\frac{1}{2}tr\left[\frac{\partial^2 G(\theta)^{-1}}{\partial\theta_i \partial\theta_j} G(\theta)\right]$.

Details of the LR, LM, Wald, AW and NW tests based on all the likelihood and message length functions in the context of testing $H_0: \theta = \theta_0$ against $H_a: \theta \neq \theta_0$ in (1) are given in Laskar and King (1998), Laskar and King (1997a) and Laskar and King (1997b). Laskar and King (1998) estimated the MA(1) disturbances parameter constrained between -1 to 1, because of the identification problem for MA(1) disturbances. It is well known that there is a non-zero probability of getting ML estimators of -1 or 1 for MA(1) disturbances parameter (Shephard 1993). The score with respect to the MA(1) parameter is discontinuous and the information matrix is not well defined at those two points. As a result, Laskar and King (1998) faced the problem of nonmonotonicity of the power curve of the Wald test. They initially tackled this problem by rejecting the null hypothesis whenever the estimate of the MA(1) disturbance parameter is ±1 and called this the AW test. Unfortunately the AW test cannot totally solve this problem because it takes into account boundary values of the parameter estimates only. The power curve may be nonmonotonic at some other points of the parameter space. Laskar and King (1997a) fully overcame this problem by replacing the unknown parameter values in the variance component of the Wald test with their null hypothesis values rather than their estimated values and denoted it as the NW test.

3. Monte Carlo Experiment

Laskar and King (1998) investigated the small sample properties of estimators and LR, LM, Wald and AW tests based on different modified likelihood functions in the context of MA(1) and AR(1) regression disturbances. Also, Laskar and King (1997a) investigated the small sample properties of NW tests based on different modified likelihood functions in the context of MA(1) regression disturbances. When message length functions based estimation and testing are concern, Laskar and King (1996) investigated the small sample properties of estimators in the context of MA(1) regression disturbances and Laskar and King (1997b) investigated the small sample properties of tests in the context of MA(1) regression disturbances.

In order to compare the small sample properties of estimators and small sample size and power properties of the LR, LM, Wald, AW and NW tests for testing $H_0: \gamma = 0$ for MA(1) regression disturbances or $H_0: \rho = 0$ for AR(1) regression disturbances i.e. $H_0: \theta = 0$ based on different modified likelihoods, classical (profile) likelihood and message length functions, we considered results from above papers and further a Monte Carlo experiment was conducted for computing the estimators and small sample sizes and powers based on message length functions with the disturbances of (1) generated by the AR(1) process

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{16}$$

in which $\varepsilon_t \sim IN(0, \sigma^2)$, $t = 0,1,\ldots,n$. Under (16), $u \sim N(0, \sigma^2 \Omega(\rho))$, where $u_0 \sim N(0, \sigma^2/(1-\rho^2))$, $\Omega(\rho)$ is the $n \times n$ symmetric matrix whose $(i,j)^{th}$ element is $\rho^{|i-j|}/(1-\rho^2)$. For the model (16), all the message length functions are not defined at $\rho = \pm 1$. So, the best way of tackling this problem is to restrict ρ to the interval

$$-0.9999 \le \rho \le 0.9999 . \tag{17}$$

For our purposes, the need to impose the restrictions (17), has a positive implication. Often when estimators are being investigated, there is uncertainty about which moments of the estimator's distribution exist. If, for example, the second-order moment does not exist, then any estimate of it obtained from a Monte Carlo experiment will be finite but meaningless. In our case, while we do not know the distributions of our estimators, the restrictions (17) implies that all moments will exist.

3.1. Experimental Design

The first part of the study covered a comparison of the different MML estimators for the AR(1) parameter. The estimates based on (i) ML_1, (ii) ML_2, (iii) $CPML_1$, (iv)

$CPML_2$, (v) AML_1 and (v) AML_2 when ρ = -0.8, -0.4, 0, 0.4, 0.8 were used for the first comparison. The second part involved a comparison of sizes of different tests using asymptotic critical values at the five percent level. The third part of the experiment was divided into two parts. In first part, the Monte Carlo method was used to estimate appropriate critical values of each of the tests in order to compare the powers of all tests at approximately the same level. These critical values were calculated using 2000 replications. In second part, powers of all the tests were calculated using these (simulated) critical values. The tests involved LR, LM, Wald and NW tests.

All the calculations were repeated 2000 times using the GAUSS (1996) constrained optimization routine but with particular care taken in choosing starting valus (see Laskar, 1998). The following X matrices were used with $n = 30$ and $n = 60$:

$X1$: (k = 5). A constant, quarterly Australian private capital movements, Government capital movements commencing 1968(1) and these two variables lagged one quarter as two additional regressors.

$X2$: (k = 3). A constant, quarterly seasonally adjusted Australian household disposable income and private final consumption expenditure commencing 1959(4).

$X3$: (k = 3). The regressors are the eigenvectors corresponding to the three smallest eigenvalues of the $n \times n$ tridiagonal matrix whose main diagonal elements are 2, except for the top left and bottom right elements which are both 1 and whose elements in the leading off-diagonals are all -1.

$X4$: ($k = 2$). A constant and a linear trend.

These matrices reflect a variety of behaviour. The capital movements regressors in $X1$ are rapidly changing with a high degree of seasonality. This is in contrast to the relatively smooth regressors $X2$ (seasonally adjusted quarterly data). The regressors in $X3$ are smoothly evolving and include an intercept. They cause the Durbin-Watson statistic, which is an approximately locally best one-sided test against both MA(1) and AR(1) disturbances (King and Evans 1988), to attain its upper bound. Also Laskar and King (1998), Laskar and King (1997a), Laskar and King (1997b) and Laskar and King (1996) considered the same set of X matrices.

3.2. Empirical Comparisons of Estimators Based on Likelihood and Message Length Functions

Estimated bias, standard deviation, skewness and kurtosis of all the estimators were computed and summarized using a loss function, $|\text{bias}| + \frac{1}{\lambda}(\text{standard deviation}) + \frac{1}{\lambda^2}|\text{skewness}| + \frac{1}{\lambda^3}|\text{kurtosis} - 3|$ where $\lambda = 3$ (Laskar and King, 1998). Then we compare all the estimators by ranking their absolute value of bias, standard deviation, the absolute value of skewness, the absolute value of (kurtosis - 3) and loss. These statistics of the estimators based on profile likelihood, marginal likelihood, CPL, ACPL, ML_1, $CPML_1$, ML_2, $CPML_2$, AML_1 and AML_2 were combined from this Monte Carlo experiment, Laskar and King (1998) and Laskar and King (1996). They were ranked from 1 to 10 in ascending order, from the smallest to the largest values, for each X matrix and each value of n, γ and ρ. The average ranks with their standard error in parenthesis and the rank of this average rank for MA(1) processes, AR(1) processes and combined MA(1) and AR(1) processes are presented in Table 1. The average ranks of rankings of all statistics with their standard error in parenthesis and the ranks of this average ranks for different processes and values of γ and ρ are presented in Table 2.

The ranking of average ranks based on all statistics for combined MA(1) and AR(1) processes with n = 30 and 60 from Table 2 indicates that the CPL based estimators are best, but not for all processes and values of n. The same table reflects that for MA(1) processes with $n = 30$ and $n = 60$, they are the second and the third best respectively, but for combined $n =$ 30 and 60, they are best. For AR(1) processes with $n = 30$ they are the fifth best, but for $n = 60$, they are best and for combined $n =$ 30 and 60, they are the second best. They are best for combined MA(1) and AR(1) processes with all sample sizes. The separate average rank for losses and all individual statistics sheds more light on the performance of the CPL based estimators. The skewness of the CPL based estimators are smallest for combined MA(1) and AR(1) processes and MA(1) processes with n = 30 and 60. The absolute value of estimated kurtosis minus three, of the CPL based estimators are smallest for all processes and all sample sizes, except for AR(1) processes with $n = 30$. This pattern is not clear for losses, biases and standard deviations. The losses of the CPL based estimators are smallest for AR(1) processes with $n = 60$, MA(1) processes with $n = 30$ and 60 and combined MA(1) and AR(1) processes with $n = 30$ and 60. The performance of the CPL based estimators are relatively poor in average ranks based on biases and standard deviations, in particular, the average ranks based on standard deviations are relatively large and typically eighth and ninth in most of the cases. The CPL based estimators reduce bias, but not very much, their average ranks vary from two to five, but they reduce estimators' skewness and kurtosis compared to all other likelihood and message length functions based estimators.

The average ranks of all rankings from Table 2 reflect that the marginal likelihood based estimators are the second best, but they are more uniform in terms of their ranking with the smallest standard error of 0.103, compared to 0.113 for the CPL based estimators. For AR(1) processes, they perform relatively better compared to their performance for MA(1) processes. For AR(1) processes with $n = 30$ and combined $n = 30$ and 60, they are best and when $n = 60$, they are the second best. In contrast, for MA(1) processes with $n = 30$ and $n = 60$, they are the fourth best and for combined $n = 30$ and 60, they are the fifth best. The average ranks based on losses for the marginal likelihood based estimators are the smallest for combined MA(1) and AR(1) processes with $n = 60$, MA(1) processes with $n = 60$ and all cases of AR(1) processes. For combined MA(1) and AR(1) processes with $n = 30$ and 60, they are the second best, but for $n = 30$, they are the third best. The marginal likelihood based estimator reduces estimators' bias compared to all other likelihood and message length functions. This is the reason why Ara and King (1993) and Rahman and King (1998) have found better small sample properties of the marginal likelihood based tests compared to the classical likelihood based tests.

The estimators based on AML_2 are the third best overall, but the standard error of their average rank namely, 0.150, is relatively large, being seventh in ranking; which suggests a lack of uniformity in terms of their ranking. Their ranking is third due to the smaller ranking based on standard deviations and small values of γ and ρ. The average rank based on standard deviations from Table 1 suggests that for AR(1) processes, it is best, for MA(1) processes and combined MA(1) and AR(1) processes, it is the second best except for MA(1) processes with $n = 30$, where is the third best. All other cases their average ranks are higher.

Average ranks of all ranking from Table 2 suggest that the estimators based on ML_l are the fourth best. The ranking of ML_2, AML_l, ACPL, profile likelihood, $CPML_2$ and $CPML_l$ based estimators are fifth, sixth, seventh, eighth, ninth and tenth respectively. The areas in which the ML_l based estimators do well are MA(1) processes with $n = 60$, where they are best and for MA(1) processes with combined $n = 30$ and 60 and combined MA(1) and AR(1) processes with $n = 30$ they are the second best. The average ranks based on losses from Table 1 exhibit that the ML_l based estimators have smallest losses for combined MA(1) and AR(1) processes with $n = 30$ and MA(1) processes with $n = 30$. For MA(1) processes with $n = 30$, skewness of the ML_l based estimators are smallest and for $n = 60$ and combined $n = 30$ and 60, they are the second best. The average rank based on kurtosis of the ML_l based estimators is relatively larger and only for AR(1) processes with $n = 30$, it is the smallest and for $n = 60$ and combined $n = 30$ and 60, it is the second smallest. For combined MA(1) and AR(1) processes with $n = 30$, it is also the second smallest. There is a tendency for the ML_l based estimators to do slightly better when $n = 30$. In contrast, the marginal likelihood based estimators do slightly better for $n = 60$.

The estimator based on the ML_2 is the fifth best with a relatively low standard error of 0.112. The average ranks based on losses reveal that for MA(1) processes with $n = 30$, losses of the ML_2 based estimators are the second smallest and in all other cases they are typically larger. Biases of the ML_2 based estimator are smallest for MA(1) processes with $n = 30$ and for combined $n = 30$ and 60 but for $n = 60$ they are the second smallest. Average ranks of the ML_2 estimator based on standard deviations, skewness and kurtosis are relatively larger.

The estimators based on the AML_l are the sixth best with relatively large standard error of 0.160, ranking ninth. The average rank based on standard deviations are the smallest for all processes and all sample sizes, except for AR(1) processes with $n = 60$ and with combined $n = 30$ and 60, in which it is the second smallest. The average ranks based on skewness for AR(1) processes with $n = 30$ and for combined $n = 30$ and 60 are also smallest. Table 2 exhibits that for $\gamma = \rho = 0$, the average ranks based on the AML_l are smallest. These are the factors responsible for the smallest average rank of the AML_l based estimator.

The average ranks based on all ranking from Table 2 display that for $\gamma = \rho =$ -0.4, the estimators based on the CPL are best and those based on the AML_1 are the second best. The CPL based estimators are also less discrepant with smallest standard error of 0.198 compared to 0.317 for the AML_2 based estimator. For $\gamma = \rho = 0$, on the basis of average ranks of all rankings, the estimators based on AML_2 are best and those based on AML_1 are the second best. The ranking of marginal likelihood and CPL based estimators are fifth and sixth respectively. For $\gamma = \rho = 0.4$, the AML_1 based estimators are best and the CPL based estimators are the second best. The results for different values of γ and ρ suggest that for γ and ρ close to zero, AML_2 and AML_1 based estimators perform better. In contrast for γ and ρ away from zero, marginal and CPL based estimators perform better. There is a case for $\gamma = \rho = -0.8$, where the ML_2 based estimators are best with a relatively large standard error.

Overall, this analysis displays that the estimators based on the CPL are best with the marginal likelihood based estimators being second best. Although the marginal likelihood based estimators are the second best, they are more uniform in terms of ranking on a range of criteria with the smallest standard error of 0.103 compared to 0.113 for the CPL based estimator. The biases of the marginal likelihood based estimators are smallest compared to those of the CPL based estimator. On the other hand, skewness and kurtosis of the CPL based estimators are smaller compared to those of the marginal likelihood based estimators. Bias reduction is considered more important in the literature but, the latter two findings are not. The CPL is also capable of reducing estimators' skewness and kurtosis. When the values of γ and ρ are closer to zero, both marginal likelihood and CPL based estimators perform relatively poorly and we are able to favour AML_1 and AML_2 based estimators, but still they are nonuniform in terms of their ranking. It is not difficult to conclude that marginal likelihood and CPL based estimators are close competitors and it is very difficult to state which one is best. The estimators based on ML_1 and ML_2 may be ranked third and fourth best respectively, considering the standard error of their average ranks. Although the AML_2 based estimators are fourth in average rank, the standard error of their average rank is 0.150, which is much larger compared to 0.112 for ML_2 based estimators. Typically, $CPML_1$, $CPML_2$ and profile likelihood based estimators are the worst of all other estimators considered.

3.3. Empirical Comparisons Among Likelihood and Message Length Based Tests

The sizes and powers of all tests based on profile likelihood, marginal likelihood, CPL, CPRL, ACPL, ML_1, $CPML_1$, ML_2, $CPML_2$, AML_1 and AML_2 were combined from this Monte Carlo experiment, Laskar and King (1998) and Laskar and King (1997b). The absolute value of (size - 0.05) for all eleven tests are ranked from the smallest to the largest values, and powers are ranked from the largest to the smallest values for each X matrix and values of n, γ and ρ. The average rank of size of different tests with its standard error in parenthesis and the ranking of the average rank for MA(1) processes, AR(1) processes and combined MA(1) and AR(1) processes are presented in Table 3. The average rank of power with its standard error in parenthesis and the ranking of the average rank for all tests based on eleven likelihood and message length functions for different values of γ and ρ are presented in Table 4 and those of all the tests based on marginal likelihood, CPL, ACPL, ML_1, ML_2, AML_1 and AML_2 for MA(1) processes, AR(1) processes and combined MA(1) and AR(1) processes are presented in Table 5. The marginal likelihood and CPRL are equivalent for the purpose of estimating γ and ρ, but they have scores and information matrices that differ by a multiplicative constant. As a result, the small sample sizes of asymptotic tests based on these two likelihood functions will differ, although the tests are identical if simulated (or exact) critical values are used. Some minor variations may be observed due to rounding errors. As a result, the CPRL was not considered for the second power comparisons. Also the $CPML_1$, $CPML_2$ and profile likelihood were dropped from the second power comparison because they produce highly biased power curves.

3.3.1. Comparisons of Sizes

The ranking of the average ranks for sizes of the LR tests reflect that for MA(1) processes with $n = 30$, sizes of the ML_1 based tests are best overall, being closest to the nominal size of 0.05 and those of CPRL and AML_1 are jointly the second best. The fourth best are the sizes of the AML_2 based LR tests. However, for $n = 60$, those of the CPRL based tests are closest to the nominal size and those of ML_1 and ACPL are the second and third best respectively. The sizes of the profile likelihood based

LR tests have the worst performance, being always away from the nominal size in both the cases. It appears that for MA(1) processes, sizes of the marginal likelihood based LR tests are away from the nominal size. This may be because of the identification problem associated with MA(1) processes. The sizes of the marginal likelihood based LR tests for AR(1) processes are closest to the nominal size while those for MA(1) processes are the ninth best, so clearly they depend on data generating processes. For AR(1) processes, sizes of the ML_1 based LR tests are the second best and very similar both for MA(1) and AR(1) processes.

The sizes of the marginal likelihood based LM tests are the most promising, of the eleven tests, with sizes ranked closest to the nominal size for each of MA(1) and AR(1) processes; those based on ACPL, CPL and CPRL are typically the second, third and fourth best respectively. In contrast, sizes of the message length based LM tests are relatively poor, being away from the nominal size. These results are not surprising as Mahmood and King (1997) observed that the LM test based on an unbiased score function has best small sample properties and reported that the score functions based on marginal likelihood and CPRL are unbiased.

The Wald test was not constructed for MA(1) processes for all the likelihood and message length functions because of the identification problem mentioned in section 3.1. Sizes of the Wald test based on ML_2 are best for AR(1) processes with both n = 30 and 60 and those based AML_1 with n = 30 and the marginal likelihood with n = 60 are the second best and they are the same in terms of rankings. AW and NW tests were constructed only for MA(1) processes and in terms of rankings, their sizes seem to be more accurate when modified likelihood functions are replaced by message length functions. The sizes based on ML_1 are best for both n = 30 and 60, those of the NW tests based on ML_2 are best and those of the ML_1 are the second best. These results are tied up with the identification problem outlined in section 3.1. When we get a problem of lack of identification, the information matrix reacts and solves this problem. In this regard, Martin (1997) used the Bayesian method for inference about fractional cointegration using autoregressive fractionally integrated moving average processes and reported that the use of Jeffreys prior helped to offset an identification problem in the likelihood function. Jeffreys prior is proportional to the determinant of the information matrix. The message length function contains the square root of this determinant. Inclusion of this factor may help to tackle the identification problem. Consequently all versions of message length based Wald tests have better small sample sizes compared to those based on modified likelihood functions.

Overall, the results show that sizes of the marginal likelihood based LM tests for both MA(1) and AR(1) processes and those of LR tests for AR(1) processes are most accurate and closest to the nominal size. The LM test based on all the message length functions perform poorly and show strong size distortion. On the other hand, the LR test based on ML_1 is quite impressive, having best sizes for MA(1) processes with $n = 30$ and second best sizes for MA(1) processes with $n = 60$ and AR(1) processes. Wald, AW and NW tests perform relatively better for message length functions and, particularly those tests based on ML_1, ML_2 and AML_1, have desirable sizes. Clearly, sizes of profile likelihood, $CPML_1$ and $CPML_2$ based tests are away from the nominal size, indicating their poor performance in testing problems. There is no clear pattern of sizes for other tests, but a ranking of the average ranks of sizes for all tests (second bottom column of Table 3) indicates that sizes of the marginal likelihood based tests are best with those based on ML_1, CPL, CPRL, ACPL, AML_2 being second, third, fourth, fifth and sixth best respectively, and those based on AML_1 and CPL are the same in average ranks. However, average ranks based on classical (LR, LM and Wald) tests show an interesting picture, where the sizes of all the modified likelihood based are better compared to those of message length based tests. Those of marginal likelihood based tests are again best and sizes of the ACPL based tests are the second best. The third and fourth best sizes are those of the CPRL and CPL based tests. Clearly, modified likelihood based classical tests have better overall small sample sizes.

3.3.2. Comparisons of Powers

Table 4 shows that powers of the profile likelihood based tests are largest for positive values of γ and ρ and smallest for negative values of γ and ρ. In contrast, powers of the $CPML_1$ and $CPML_2$ based tests are smallest for positive values of γ and ρ and largest for negative values of γ and ρ. All of them produce highly biased power curves. As a result their average ranks will be similar to those of the tests which produce unbiased power curves. Consequently, we may arrive at wrong conclusions. So we have decided it is not safe to include them in the power comparison.

The ranking of the average rank from Table 5 reflects that for MA(1) processes, power curves of marginal likelihood and AML_1 based LR tests are best centred and the second and fourth best, respectively. In contrast, power curves of the CPL based LR test show greater bias, being best powers for positive values of γ and ρ and worst powers for negative values of γ and ρ. This pattern of powers is completely the opposite for AR(1) processes, being highly biased power curves for marginal likelihood and AML_1 and relatively better centred power curves for CPL, ACPL and ML_1. Finally, for MA(1) processes, powers of the marginal likelihood based LR test are largest overall and those based on ACPL and CPL are the second and the third largest respectively. For AR(1) processes, powers of the CPL based test are largest and those based on the marginal likelihood are the second largest. In contrast, for combined MA(1) and AR(1) processes, powers of the marginal likelihood based LR tests are largest and those based on CPL and ACPL are the second largest.

When powers of the LM tests are concerned, there is no clear pattern, but it appears from the average ranks that power curves of the CPL based tests are poorly centred compared to those of the marginal likelihood. The power curves of AML_1 and AML_2 based LM tests are best centred for AR(1) processes and those of CPL and ACPL are the best and second best respectively for combined MA(1) and AR(1) processes, while those based on CPL and ACPL have the same ranks and are best for AR(1) processes. It seems that power curves of the ACPL based LM test are bettered centred, the second best for combined MA(1) and AR(1) processes and best for MA(1) processes. The power curves based on the ML_2 are also quite impressive, being the third best for combined MA(1) and AR(1) processes. Those based on ML_1 are close competitors, being the fourth best power having very consistent rankings with the smallest standard error of 0.207.

With respect to powers of Wald tests, when based on marginal likelihood and ML_1, they are highly biased and those based on AML_2 are best centred. However power curves of AW tests based on ML_2 are best centred and powers are relatively low, being the sixth best, while those of AML_2 and AML_1 are best and the second best respectively. On the other hand, power curves of the NW tests based on AML_1 and

AML_2 are best centred with relatively low power and they are the sixth and seventh best respectively. There is no other clear pattern, but it seems that powers of the ML_2 based tests are uniformly best and those based on the ML_1 are the second best.

When we consider the average ranks based on all tests (last seven column of Table 5), a slightly different pattern in powers is observed. Powers of the marginal likelihood based tests are consistently best, followed by the powers of the ACPL based tests which are the second best for both MA(1) and AR(1) processes. Those based on CPL, AML_2, AML_1, ML_2 and ML_1 are the third, fourth, fifth, sixth and seventh best respectively. This ranking pattern is slightly different for different processes and different values of γ and ρ. For MA(1) processes, power curves based on ACPL and AML_1 are best centred and those based on CPL are highly biased. In contrast, for AR(1) processes, only the power curves based on CPL are best centred and those based on marginal likelihood and ML_1 are highly biased.

Overall, these results reflect that based on overall power in small samples, the marginal likelihood is uniformly best and powers of LR tests are best centred for MA(1) processes, while being highly biased for AR(1) processes. Also, powers of all versions of the marginal likelihood based Wald tests are highly biased. The powers of the ACPL based tests are the second best with smallest standard error and those for LR and LM tests are also the second best for both the MA(1) and AR(1) processes but their performance is relatively poor for all versions of Wald tests and is the fifth best in all cases. Those of the CPL based tests are the third best. The powers of AML_2, AML_1, ML_2 and ML_1 based tests are the fourth, fifth, sixth and seventh best respectively. It is clear that on the basis of average ranks, powers of all tests based on modified likelihood functions are larger compared to those of the message length based tests. There are some cases where message length based tests produce better centred power curves with relatively lower powers and modified likelihood based tests dominate in terms of power. Above all it is easy to conclude in favour of marginal likelihood based tests when power is the criteria.

4. Conclusions

In this paper we empirically compare the estimators and tests of the parameters involved in the variance-covariance matrix of the linear regression disturbances based on eleven different likelihood and message length functions, most of which are designed for proper handling of nuisance parameters. The estimation results show that estimators based on the CPL are the best out of the eleven estimation methods. The CPL reduces skewness and brings kurtosis close to three. The second best are marginal likelihood based estimators which show the best uniformity in terms of their ranking over the various situations considered. The average sizes of all likelihood and message length based tests show that overall, marginal likelihood based tests have the most accurate sizes, particularly for the LM test; these are most impressive. All message length functions perform poorly based on power compared to the likelihood functions. Powers of the marginal likelihood based LR tests and the CPL based LM tests are best for both MA(1) and AR(1) processes. Overall, powers of the marginal likelihood based tests are best and those of the ACPL based tests are the second best and the CPL based tests are the third best.

In conclusion, it can be said that at least in the context of MA(1) and AR(1) linear regression disturbances, the marginal likelihood may be the best likelihood for handling nuisance parameters in estimation and testing problems. But, for more general inference problems there are some situations where marginal likelihood cannot be applied. In such situations, the CPL and ACPL are the preferred alternatives. The former has the problem of non-uniqueness of reparameterization and orthogonality and cannot always be found. The latter is applicable for scalar parameters and depends on the profile likelihood based estimators. In addition to these, MML is a Bayesian method and depends on the prior distribution of the parameters. Our empirical results exhibit that, in some situations, ML_1 and ML_2 perform very well and have the ability to deal with a minor identification problem. However, there is flexibility in choosing a different prior. Also, message length based LM tests have poor small sample properties due to their biased score functions. All these clarify that none of the methods is superior in all situations, but their performance may depend on the nature of the problem at hand. However, for the problems considered in this paper, we conclude in favour of the marginal likelihood.

References

Ara, I. (1995), Marginal likelihood based tests of regression disturbances, unpublished Ph.D. thesis, Monash University.

Ara, I. and M.L. King (1993), Marginal likelihood based tests of regression disturbances, mimeo (Monash University).

Barndorff-Nielsen, O.E. (1983), On a formula for the distribution of the maximum likelihood estimator, *Biometrika* 70, 343-365.

Bewley, R. (1986), *Allocation Models: Specification, Estimation and Applications*, Ballinger, Boston.

Chesher, A. and G. Austin (1991), The finite sample distributions of heteroskedasticity robust Wald statistics, *Journal of Econometrics* 47, 153-173.

Conway, J.H. and N.J.A. Sloan (1988), *Sphere Packings, Lattices and Groups*, Springer-Verlag, London.

Cordus, M. (1986), The use of the marginal likelihood in testing for serial correlation in time series regression, unpublished M. Phil. thesis, University of Lancaster.

Cox, D.R. and N. Reid (1987), Parameter orthogonality and approximate conditional inference (with discussion), *Journal of the Royal Statistical Society* B 49, 1-39.

Cox, D.R. and N. Reid (1993), A note on the calculation of adjusted profile likelihood, *Journal of the Royal Statistical Society* B 55, 467-471.

GAUSS (1996), *GAUSS Constrained Optimization*: *Application Module*, Aptech System, Inc., Washington.

King, M.L. (1987), Testing for autocorrelation in linear regression models: A survey, *Specification Analysis in the Linear Model*, M.L. King and D.E.A. Giles (eds.), Routledge and Kegan Paul, London, 19-73.

King, M.L. (1996), Hypothesis testing in the presence of nuisance parameters, *Journal of Statistical Planning and Inference* 50, 103-120.

King, M.L. and M.A. Evans (1988), Locally optimal properties of the Durbin-Watson test, *Econometric Theory* 4, 509-516.

King, M.L. and M. McAleer (1987), Further results on testing AR(1) against MA(1) disturbances in the linear regression model, *Review of Economic Studies* 54, 649-663.

Laskar M.R. (1998), Estimation and testing of linear regression disturbances based on modified likelihood and message length functions, Unpublished Ph.D. thesis, Monash University.

Laskar, M.R. and M.L. King (1996), Estimation of regression disturbances based on minimum message length, *Proceedings of the Conference ISIS'96: Information, Statistics and Induction in Science,* D.L. Dowe, K.B. Korb and J.J. Oliver (eds.), World Scientific, Singapore, 92-101.

Laskar, M.R. and M.L. King (1997a), Modified Wald test for regression disturbances, *Economics Letters* 56, 5-11.

Laskar, M.R. and M.L. King (1997b), Testing of regression disturbances based on minimum message length, *Proceedings of the Econometric Society Australasian Meeting*, P. Bardsley and V.L. Martin (eds.), University of Melbourne 2, 179-202.

Laskar, M.R. and M.L. King (1998), Estimation and testing of regression disturbances based on modified likelihood functions, *Journal of Statistical Planning and Inference*, forthcoming.

Mahmood, M. and M.L. King (1997), Dealing with nuisance parameters in likelihood based estimating equations, mimeo (Monash University).

Martin, G.M. (1997), Fractional cointegration: Bayesian inference using Jeffreys prior, mimeo (Monash University).

Moulton, B.R. and W.C. Randolph, (1989), Alternative tests of the error components model, *Econometrica* 57, 685-693.

Neyman, J. and E.L. Scott (1948), Consistent estimates based on partially consistent observations, *Econometrica* 16, 1-32.

Patterson, H.D. and R. Thompson (1971), Recovery of interblock information when block sizes are unequal, *Biometrika* 58, 545-554.

Rahman, S. and M.L. King (1998), Marginal likelihood score based tests of regression disturbances in the presence of nuisance parameters, *Journal of Econometrics* 82, 81-106.

Shephard, N. (1993), Distribution of the ML estimator of an MA(1) and a local level model, *Econometric Theory* 9, 377-401.

Tunnicliffe Wilson, G. (1989), On the use of marginal likelihood in time series model estimation, *Journal of the Royal Statistical Society* B 51, 15-27.

Wallace, C.S. and D.L. Dowe (1993), MML estimation of the von Mises concentration parameter, Technical Report No.93/193, Monash University.

Wallace, C.S. and D.L. Dowe (1994), Intrinsic classification by MML - the Snob program, *Proceedings of Seventh Australian Joint Conference on Artificial Intelligence*, 37-44.

Wallace, C.S. and P.R. Freeman (1987), Estimation and inference by compact coding, *Journal of the Royal Statistical Society* B 49, 240-265.

Table 1. Average ranks, with standard errors in the parenthesis and ranking of the average ranks of estimated bias, standard deviation, skewness, kurtosis and loss of the estimators based on different message length and likelihood functions for MA(1) and AR(1) processes.

ML and MML estimators									
Profile	Marg.	CPL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank based on estimated losses for MA(1) and AR(1) processes, $n = 30$ and 60									
7.05	3.513	3.413	4.563	3.9	8.613	4.438	7.825	5.95	5.6
(.369)	(.1649	(.201)	(.221)	(.199)	(.223)	(.210)	(.259)	(.325)	(.328)
8	2	1	5	3	10	4	9	7	6
Average rank based on estimated losses for MA(1) and AR(1) processes, $n = 30$									
7.5	3.7	3.35	4.875	3.25	8.425	4.050	7.375	6.30	6.025
(.533)	(.217)	(.290)	(.291)	(.240)	(.338)	(.286)	(.372)	(.425)	(.442)
9	3	2	5	1	10	4	8	7	6
Average rank based on estimated losses for MA(1) and AR(1) processes, $n = 60$									
6.6	3.325	3.475	4.25	4.475	8.8	4.825	8.275	5.6	5.175
(.537)	(.173)	(.284)	(.288)	(.300)	(.218)	(.310)	(.286)	(.524)	(.492)
8	1	2	3	4	10	5	9	7	6
Average rank based on estimated losses for MA(1) processes, $n = 30$ and 60									
8.325	4.125	3.7	5.6	3.775	7.85	3.85	6.975	5.6	5.175
(.409)	(.203)	(.271)	(.248)	(.361)	(.382)	(.331)	(.35)	(.544)	(.533)
10	4	1	6	2	9	3	8	6	5
Average rank based on estimated losses for MA(1) processes, $n = 30$									
9.3	4.45	3.3	6.15	2.8	7.1	3	6.2	6.5	6.15
(.385)	(.336)	(.391)	(.319)	(.345)	(.657)	(.348)	(.569)	(.587)	(.662)
10	4	3	5	1	9	2	7	8	5
Average rank based on estimated losses for MA(1) processes, $n = 60$									
7.35	3.8	4.1	5.05	4.75	8.6	4.7	7.75	4.7	4.2
(.662)	(.213)	(.362)	(.344)	(.561)	(.328)	(.503)	(.339)	(.886)	(.793)
8	1	2	7	6	10	4	9	4	3
Average rank based on estimated losses for AR(1) processes, $n = 30$ and 60									
5.775	2.9	3.125	3.525	4.025	9.375	5.025	8.675	6.3	6.025
(.548)	(.171)	(.293)	(.284)	(.174)	(.163)	(.225)	(.335)	(.355)	(.378)
6	1	2	3	4	10	5	9	8	7
Average rank based on estimated losses for AR(1) processes, $n = 30$									
5.804	5.019	5.313	5.636	6.925	8.861	6.313	8.25	6.25	5.9
(.757)	(.254)	(.421)	(.409)	(.273)	(.165)	(.258)	(.507)	(.514)	(.561)
4	1	2	3	6	10	8	9	7	5

Table 1. (continued)

ML and MML estimators									
Profile	Marg.	CPL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank based on estimated losses for AR(1) processes, $n = 60$									
5.85 (.828) 6	2.85 (.233) 1	2.85 (.399) 1	3.45 (.394) 3	4.2 (.213) 4	9 (.290) 10	4.95 (.373) 5	8.8 (.439) 9	6.5 (.505) 8	6.15 (.514) 7
Average rank based on estimated biases for MA(1) and AR(1) processes, $n = 30$ and 60									
6.375 (.392) 8	3.088 (.182) 1	3.725 (.234) 3	4.05 (.221) 5	3.588 (.153) 2	8.825 (.186) 10	3.95 (.228) 4	8.2 (.225) 9	6.088 (.301) 7	5.325 (.318) 6
Average rank based on estimated biases for MA(1) and AR(1) processes, $n = 30$									
6.55 (.560) 7	3.275 (.237) 1	4 (.338) 4	4.225 (.344) 5	3.575 (.214) 2	8.775 (.290) 10	3.7 (.289) 3	8 (.356) 9	6.575 (.363) 8	5.675 (.406) 6
Average rank based on estimated biases for MA(1) and AR(1) processes, $n = 60$									
6.2 (.554) 8	2.9 (.277) 1	3.45 (.322) 2	3.875 (.280) 4	3.6 (.220) 3	8.875 (.235) 10	4.2 (.351) 5	8.4 (.275) 9	5.6 (.473) 7	4.975 (.488) 6
Average rank based on estimated biases for MA(1) processes, $n = 30$ and 60									
7.25 (.475) 8	3.525 (.322) 3	4.252 (.328) 5	5 (.300) 6	3.425 (.258) 2	8.6 (.318) 10	3.05 (.305) 1	7.775 (.303) 9	5.55 (.492) 7	4.475 (.465) 4
Average rank based on estimated biases for MA(1) processes, $n = 30$									
7.6 (.665) 9	3.7 (.385) 3	4.8 (.499) 4	5.1 (.512) 5	3.1 (.340) 2	8.4 (.450) 10	2.45 (.276) 1	7.5 (.459) 8	6.45 (.689) 7	5.25 (.636) 6
Average rank based on estimated biases for MA(1) processes, $n = 60$									
6.9 (.692) 8	3.35 (.494) 1	4.25 (.458) 5	4.9 (.289) 7	3.75 (.383) 4	8.8 (.367) 10	3.65 (.525) 2	8.05 (.320) 9	4.65 (.765) 6	3.7 (.726) 3
Average rank based on estimated biases for AR(1) processes, $n = 30$ and 60									
5.5 (.598) 6	2.65 (.146) 1	2.925 (.285) 2	3.1 (.250) 3	3.75 (.163) 4	9.05 (.189) 10	4.85 (.274) 5	8.625 (.322) 9	6.625 (.333) 8	6.175 (.394) 7

Table 1. (continued)

ML and MML estimators									
Profile	Marg.	CPL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank based on estimated biases for AR(1) processes, $n = 30$									
5.5 (.860) 6	2.85 (.182) 1	3.2 (.421) 2	3.35 (.350) 3	4.05 (.223) 4	9.15 (.233) 10	4.95 (.328) 5	8.5 (.478) 9	6.7 (.465) 8	6.1 (.598) 7
Average rank based on estimated biases for AR(1) processes, $n = 60$									
5.5 (.854) 6	2.45 (.223) 1	2.65 (.386) 2	2.85 (.357) 3	3.45 (.223) 4	8.95 (.303) 10	4.75 (.446) 5	8.75 (.441) 9	6.55 (.489) 8	6.25 (.528) 7
Average rank based estimated S.D. for MA(1) and AR(1) processes, $n = 30$ and 60									
5.275 (.416) 3	6.213 (.196) 8	6.313 (.215) 9	6.613 (.200) 10	6.038 (.171) 6	6.188 (412) 7	5.588 (.255) 4	5.863 (.332) 5	1.789 (.109) 1	1.988 (.113) 2
Average rank based estimated S.D. for MA(1) and AR(1) processes, $n = 30$									
5.65 (.623) 5	6.9 (.217) 8	7.075 (.230) 9	7.475 (.224) 10	6.05 (.223) 7	5.275 (.615) 4	5.775 (.355) 6	5.075 (.473) 3	2.05 (.152) 1	2.425 (.171) 2
Average rank based estimated S.D. for MA(1) and AR(1) processes, $n = 60$									
4.9 (.553) 3	5.525 (.291) 5	5.55 (.322) 6	5.75 (.272) 7	6.025 (.262) 8	7.1 (.517) 10	5.4 (.368) 4	6.65 (.438) 9	1.525 (.148) 1	1.55 (.113) 2
Average rank based on estimated standard deviations for MA(1) processes, $n = 30$ and 60									
7.8 (.536) 10	6.75 (.335) 7	5.725 (.302) 5	6.825 (.336) 8	6.7 (.266) 6	3.85 (.404) 3	7.5 (.240) 9	4.125 (.346) 4	1.8 (.157) 1	2.3 (.183) 2
Average rank based on estimated standard deviations for MA(1) processes, $n = 30$									
7.5 (.854) 10	7.7 (.242) 8	6.35 (.244) 5	7.75 (.376) 9	6.6 (.373) 6	2.55 (.336) 2	7.65 (.350) 7	3.15 (.274) 4	2.15 (.233) 1	2.9 (.261) 3
Average rank based on estimated standard deviations for MA(1) processes, $n = 60$									
7.4 (.666) 10	5.8 (.555) 6	5.1 (.523) 3	5.9 (.481) 7	6.8 (.388) 8	5.15 (.617) 5	7.35 (.335) 9	5.1 (.561) 3	1.45 (.185) 1	1.7 (.179) 2

Table 1. (continued)

ML and MML estimators									
Profile	Marg.	CPL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank based on estimated standard deviations for AR(1) processes, $n = 30$ and 60									
2.95 (.371) 3	5.675 (.169) 6	6.9 (.279) 8	6.4 (.217) 7	5.375 (.159) 5	8.525 (.495) 10	3.675 (.236) 4	7.6 (.456) 9	1.775 (.154) 2	1.675 (.115) 1
Average rank based on estimated standard deviations for AR(1) processes, $n = 30$									
3.5 (.613) 3	6.1 (.261) 6	7.8 (.321) 9	7.2 (.236) 8	5.5 (.185) 5	8 (.811) 10	3.9 (.161) 4	7 (.673) 7	1.95 (.198) 1	1.95 (.170) 1
Average rank based on estimated standard deviations for AR(1) processes, $n = 60$									
2.4 (.393) 3	5.25 (.176) 5	6 (.363) 8	5.6 (.266) 7	5.25 (.260) 5	9.05 (.564) 10	3.45 (.211) 4	8.2 (.468) 9	1.6 (.234) 2	1.4 (.134) 1
Average rank based on estimated skewness for MA(1) and AR(1) processes, $n = 30$ and 60									
6.713 (.349) 9	4.688 (.211) 4	3.738 (.227) 1	4.938 (.229) 5	4.575 (.280) 3	7.388 (.350) 10	4.388 (.262) 2	5.675 (.352) 8	5.013 (.402) 7	5 (.379) 6
Average rank based on estimated skewness for MA(1) and AR(1) processes, $n = 30$									
7.45 (.427) 10	4.85 (.303) 4	3.775 (.355) 1	5.325 (.339) 5	4.275 (.370) 2	7.25 (.461) 9	4.325 (.364) 3	5.6 (.454) 8	5.35 (.564) 6	5.575 (.531) 7
Average rank based on estimated skewness for MA(1) and AR(1) processes, $n = 60$									
6.119 (.531) 9	4.595 (.292) 3	3.714 (.282) 1	4.667 (.306) 5	4.738 (.422) 6	7.286 (.549) 10	4.381 (.376) 2	5.643 (.539) 8	4.881 (.581) 7	4.643 (.542) 4
Average rank based on estimated skewness for MA(1) processes, $n = 30$ and 60									
7.975 (.444) 10	4.875 (.261) 4	3.5 (.256) 1	5.7 (.310) 5	3.675 (.389) 2	6.225 (.462) 9	4.2 (.364) 3	5.775 (.436) 6	6.15 (.577) 8	6.075 (.556) 7
Average rank based on estimated skewness for MA(1) processes, $n = 30$									
9.2 (.236) 10	4.95 (.407) 4	2.8 (.287) 2	5.85 (.519) 7	2.75 (.331) 1	5.6 (.494) 6	3.65 (.460) 3	5.15 (.488) 5	7.3 (.689) 8	7.7 (.607) 9

Table 1. (continued)

ML and MML estimators									
Profile	Marg.	CPL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank based on estimated skewness for MA(1) processes, $n = 60$									
6.75 (.771) 10	4.8 (.337) 4	4.2 (.367) 2	5.55 (.352) 7	4.6 (.651) 1	6.85 (.769) 6	4.75 (.547) 3	6.4 (.709) 5	5 (.868) 8	4.45 (.790) 9
Average rank based on estimated skewness for AR(1) processes, $n = 30$ and 60									
5.45 (.464) 7	4.5 (.332) 5	3.975 (.375) 3	4.175 (.295) 4	5.475 (.353) 8	8.55 (.463) 10	4.575 (.379) 6	5.575 (.558) 9	3.875 (.507) 1	3.925 (.462) 2
Average rank based on estimated skewness for AR(1) processes, $n = 30$									
5.7 (.607) 7	4.75 (.458) 3	4.75 (.580) 3	4.8 (.414) 5	5.8 (.457) 8	8.9 (.584) 10	5 (.533) 6	6.05 (.766) 9	3.4 (.659) 1	3.45 (.559) 2
Average rank based on estimated skewness for AR(1) processes, $n = 60$									
5.2 (.713) 10	4.25 (.486) 4	3.2 (.421) 2	3.55 (.380) 7	5.15 (.539) 5	8.2 (.724) 6	4.15 (.534) 3	5.1 (.817) 5	4.35 (.772) 8	4.4 (.734) 9
Average rank based on estimated kurtosis for MA(1) and AR(1) processes, $n = 30$ and 60									
6.187 (.350) 9	4.625 (.236) 2	3.962 (.243) 1	4.725 (.284) 3	5.112 (.294) 5	7.112 (.395) 10	5.425 (.247) 7	6.037 (.370) 8	5.1 (.347) 4	5.25 (.305) 6
Average rank based on estimated kurtosis for MA(1) and AR(1) processes, $n = 30$									
6.675 (.433) 9	4.85 (.383) 2	4.35 (.378) 1	4.95 (.431) 5	4.85 (.400) 2	7.45 (.521) 10	5.15 (.357) 7	6.425 (.522) 8	4.925 (.463) 4	5.125 (.436) 6
Average rank based on estimated kurtosis for MA(1) and AR(1) processes, $n = 60$									
5.7 (.546) 8	4.4 (.277) 2	3.575 (.299) 1	4.5 (.373) 3	5.375 (.434) 5	6.775 (.596) 10	5.7 (.342) 8	5.65 (.526) 7	5.275 (.523) 4	5.375 (.434) 5
Average rank based on estimated kurtosis for MA(1) processes, $n = 30$ and 60									
6.875 (.500) 10	4.8 (.365) 2	4.125 (.266) 1	5.65 (.412) 7	5.125 (.461) 4	6.65 (.549) 9	5.25 (.359) 5	5.875 (.453) 8	5.15 (.522) 3	5.375 (.465) 6

Table 1. (continued)

ML and MML estimators									
Profile	Marg.	CPL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank based on estimated kurtosis for MA(1) processes, $n = 30$									
7.15 (.670) 10	4.85 (.617) 3	4.45 (.336) 1	6.15 (.642) 9	4.65 (.646) 2	6.1 (.774) 8	4.85 (.534) 3	5.25 (.664) 5	5.6 (.659) 6	5.9 (.684) 7
Average rank based on estimated kurtosis for MA(1) processes, $n = 60$									
6.6 (.755) 9	4.75 (.410) 3	3.8 (.408) 1	5.15 (.509) 5	5.6 (.659) 6	7.2 (.780) 10	5.65 (.477) 7	6.5 (.600) 6	4.7 (.815) 2	4.85 (.625) 4
Average rank based on estimated kurtosis for AR(1) processes, $n = 30$ and 60									
5.5 (.472) 7	4.45 (.304) 3	3.8 (.410) 1	3.8 (.338) 2	5.1 (.371) 5	7.575 (.566) 10	5.6 (.343) 8	6.2 (.592) 9	5.05 (.465) 4	5.125 (.402) 6
Average rank based on estimated kurtosis for AR(1) processes, $n = 30$									
6.2 (.546) 8	4.85 (.472) 5	4.25 (.688) 2	3.75 (.446) 1	5.05 (.484) 6	8.8 (.569) 10	5.45 (.478) 7	7.6 (.731) 9	4.25 (.632) 2	4.35 (.499) 4
Average rank based on estimated kurtosis for AR(1) processes, $n = 60$									
4.8 (.753) 4	4.05 (.366) 3	3.35 (.443) 1	3.85 (.519) 2	5.15 (.577) 6	6.35 (.913) 10	5.75 (.502) 7	4.8 (.835) 4	5.85 (.650) 8	5.9 (.593) 9

Table 2. Average ranks of all statistics with standard errors in the parenthesis and ranking of the average ranks of estimators based on different message length and likelihood functions for MA(1) and AR(1) processes.

ML and MML estimators									
Profile	Marg.	CPL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank of MA(1) processes, $n = 30$									
8.081 (.270) 10	4.929 (.226) 4	4.182 (.197) 2	5.97 (.237) 8	4.091 (.255) 1	6.141 (.308) 9	4.394 (.261) 3	5.606 (.266) 7	5.556 (.315) 6	5.495 (.309) 5
Average rank of MA(1) processes, $n = 60$									
7.11 (.325) 10	4.73 (.212) 4	4.43 (.194) 3	5.55 (.185) 7	4.99 (.249) 5	7.1 (.317) 9	5.18 (.241) 6	6.58 (.265) 8	4.14 (.353) 2	3.89 (.306) 1
Average rank of MA(1) processes, $n = 30$ and 60									
7.605 (.213) 10	4.815 (.155) 5	4.315 (.138) 1	5.755 (.150) 7	4.54 (.180) 2	6.635 (.223) 9	4.77 (.180) 4	6.105 (.190) 8	4.85 (.241) 6	4.68 (.224) 3
Average rank of AR(1) processes, $n = 30$									
5.21 (.315) 8	4.18 (.183) 1	4.5 (.257) 5	4.4 (.204) 3	4.85 (.171) 7	9.12 (.203) 10	4.84 (.176) 6	7.72 (.285) 9	4.43 (.291) 4	4.27 (.277) 2
Average rank of AR(1) processes, $n = 60$									
4.86 (.342) 6	3.89 (.189) 2	3.79 (.245) 1	4 (.215) 3	4.64 (.187) 4	8.11 (.306) 10	4.65 (.202) 5	6.95 (.336) 9	5.02 (.304) 8	4.9 (.292) 7
Average rank of AR(1) processes, $n = 30$ and 60									
5.035 (.232) 8	4.035 (.132) 1	4.145 (.179) 2	4.2 (.149) 3	4.745 (.127) 6	8.615 (.187) 10	4.745 (.134) 6	7.335 (.222) 9	4.725 (.211) 5	4.585 (.202) 4
Average rank of combined MA(1) and AR(1) processes, $n = 30$									
6.655 (.231) 8	4.54 (.147) 3	4.35 (.162) 1	5.18 (.165) 7	4.47 (.155) 2	7.645 (.212) 10	4.6 (.159) 4	6.675 (.208) 9	4.995 (.217) 6	4.87 (.211) 5
Average rank of combined MA(1) and AR(1) processes, $n = 60$									
5.985 (.247) 8	4.31 (.140) 2	4.11 (.166) 1	4.775 (.150) 5	4.815 (.145) 6	7.605 (.206) 10	4.915 (.147) 7	6.765 (.212) 9	4.58 (.225) 4	4.395 (.208) 3

Table 2. (continued)

ML and MML estimators									
Profile	Marg.	CPL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank of combined MA(1) and AR(1) processes, n = 30 and 60									
6.32 (.170) 8	4.425 (.103) 2	4.23 (.113) 1	4.978 (.112) 7	4.643 (.110) 4	7.625 (.153) 10	4.758 (.112) 5	6.72 (.149) 9	4.788 (.160) 6	4.633 (.150) 3
Average rank for $\gamma = \rho$ = -0.8, n = 30 and 60									
5.975 (.419) 8	4.388 (.245) 2	4.5 (.226) 3	4.95 (.192) 5	4.588 (.315) 4	7.413 (.289) 10	3.813 (.256) 1	6.738 (.317) 9	5.75 (.370) 7	5.525 (.324) 6
Average rank for $\gamma = \rho$ = -0.4, n = 30 and 60									
5.325 (.422) 8	4.463 (.243) 3	3.988 (.198) 1	4.647 (.189) 5	4.963 (.268) 6	8.188 (.316) 10	5.313 (.209) 7	7.488 (.318) 9	4.463 (.326) 3	4.025 (.317) 2
Average rank for $\gamma = \rho$ = 0, n = 30 and 60									
7.338 (.339) 9	4.888 (.274) 5	5.388 (.219) 6	6.35 (.278) 7	4.4 (.159) 4	7.463 (.386) 10	4.013 (.257) 3	6.563 (.336) 8	2.863 (.244) 2	2.538 (.239) 1
Average rank for $\gamma = \rho$ = 0.4, n = 30 and 60									
6.925 (.328) 9	4.5 (.210) 3	4.313 (.306) 2	4.963 (.300) 5	5.063 (.247) 6	7.55 (.391) 10	5.4 (.232) 7	6.6 (.345) 8	4.05 (.300) 1	4.5 (.300) 3
Average rank for $\gamma = \rho$ = 0.8, n = 30 and 60									
6.038 (.347) 6	3.888 (.155) 2	2.963 (.227) 1	3.95 (.202) 3	4.2 (.205) 4	7.513 (.322) 10	5.25 (.237) 5	6.213 (.342) 7	6.813 (.365) 9	6.575 (.318) 8

Table 3. Average ranks, with standard errors in parentheses and ranking of the average ranks of sizes of the LR, LM, Wald, AW and NW tests for MA(1) and AR(1) disturbances based on different likelihood and message length functions.

ML and MML estimators										
Profi.	Marg.	CPL	CPRL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank based on sizes of LR tests for MA(1) with $n = 30$										
10.75 (.250) 11	8 (.913) 9	7.25 (1.32) 7	2.25 (.750) 2	8.25 (.629) 10	2 (.408) 1	7 (1.29) 6	6.25 (.479) 5	7.25 (1.75) 7	2.25 (0.75) 2	3.75 (.479) 4
Average rank based on sizes of LR tests for MA(1) with $n = 60$										
10.75 (.250) 11	7.5 (.866) 9	4.5 (.866) 5	1.5 (.500) 1	4 (2.38) 3	2.5 (1.50) 2	7 (2.04) 8	5 (1.08) 6	7.5 (1.50) 9	4.25 (1.37) 4	6 (1.08) 4
Average rank based on sizes of LR tests for AR(1) with $n = 30$										
9.25 (1.18) 9	1.5 (.500) 1	5.25 (1.11) 6	7.75 (.629) 8	6.5 (.645) 7	2.75 (.479) 2	9.75 (.750) 10	3.75 (.629) 4	10 (000) 11	3.75 (1.55) 4	3.25 (1.25) 3
Average rank based on sizes of LR tests for AR(1) with $n = 60$										
5.5 (1.55) 6	2 (.577) 1	5 (.577) 4	8 (.707) 9	5.25 (.479) 5	2.5 (.645) 2	10.25 (.479) 11	2.75 (.629) 3	10 (.408) 10	5.75 (1.79) 7	6.25 (1.79) 8
Average rank based on sizes of LM tests for MA(1) with $n = 30$										
8.5 (.500) 8	1.25 (.250) 1	3.5 (.289) 4	3.25 (.250) 3	1.75 (.250) 2	8 (.707) 7	9.75 (1.25) 11	9 (.707) 9	9.25 (.750) 10	5 (000) 5	6.25 (.629) 6
Average rank based on sizes of LM tests for MA(1) with $n = 60$										
8 (1.29) 9	1.25 (.250) 1	3.5 (.645) 3	4 (1.41) 4	3 (.707) 2	7.25 (.854) 7	7 (2.04) 6	8.25 (.854) 10	8.5 (1.89) 11	6.25 (1.70) 5	7.5 (.866) 8
Average rank based on sizes of LM tests for AR(1) with $n = 30$										
7 (2.12) 6	1 (000) 1	3.25 (.479) 3	4 (.707) 4	3 (.707) 2	8 (.816) 9	8.25 (2.13) 10	9.25 (.250) 11	7.25 (1.70) 7	6.25 (.479) 5	7.25 (.250) 7
Average rank based on sizes of LM tests for AR(1) with $n = 60$										
5.75 (1.49) 5	1.5 (.500) 1	3.25 (.750) 3	4 (.913) 4	2.75 (.479) 2	5.75 (.854) 5	7.75 (2.35) 9	7.5 (.645) 8	6.75 (2.13) 7	8.25 (.854) 10	9.5 (.957) 11

Table 3. (continud)

ML and MML estimators										
Profi.	Marg.	CPL	CPRL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank based on sizes of Wald tests for AR(1) with $n = 30$										
8.75 (1.65) 5	3.75 (1.11) 3	5.25 (.250) 5	5.75 (.479) 6	3.75 (.629) 3	7.5 (.289) 7	10 (.577) 11	8.5 (.289) 9	8.25 (1.75) 8	2 (.408) 2	1.75 (.750) 1
Average rank based on sizes of Wald tests for AR(1) with $n = 60$										
4.5 (1.32) 4	2 (.408) 2	5.75 (.479) 6	5.75 (.750) 6	5.25 (.629) 5	10 (.707) 11	7.75 (1.43) 8	9.5 (.500) 9	8.5 (.645) 9	2.75 (.629) 3	1.5 (.289) 1
Average rank based on sizes of AW tests for MA(1) with $n = 30$										
11 (000) 11	8.25 (.479) 9	7.25 (.479) 8	6.75 (.629) 7	9.25 (.479) 10	1.25 (.250) 1	5.5 (1.19) 5	1.25 (.250) 1	6.25 (1.43) 6	3.5 (.500) 3	4.5 (.500) 4
Average rank based on sizes of AW tests for MA(1) with $n = 60$										
11 (000) 11	9.25 (.479) 10	5.5 (.645) 6	6.25 (.750) 8	6 (.707) 7	1.75 (.750) 1	5.25 (1.93) 5	3.75 (.854) 3	6.5 (2.06) 9	2.5 (.645) 2	4.75 (1.18) 4
Average rank based on sizes of NW tests for MA(1) with $n = 30$										
11 (000) 11	8.75 (.629) 9	5.5 (.500) 5	6 (.816) 7	9.75 (.250) 10	2.75 (.750) 2	5.25 (1.11) 4	1 (000) 1	3.75 (1.11) 3	6.5 (.957) 8	5.5 (1.19) 5
Average rank based on sizes of NW tests for MA(1) with $n = 60$										
11 (000) 11	7.5 (.500) 9	3.5 (1.04) 2	4.5 (.645) 5	3.75 (1.25) 4	3.5 (.645) 2	6.25 (1.25) 7	1 (000) 1	6 (1.47) 6	9.25 (.750) 10	7.25 (1.75) 8
Average rank of all tests for MA(1)and AR(1) with $n = 30$ and 60										
8.768 (.396) 11	4.563 (.462) 1	4.875 (.247) 3	4.982 (.308) 5	5.161 (.386) 6	4.679 (.420) 2	7.625 (.425) 10	5.482 (.443) 8	7.554 (.408) 9	4.875 (.379) 3	5.357 (.382) 7
Average rank of LR, LM and Wald tests for MA(1)and AR(1) with $n = 30$ and 60										
7.857 (.489) 9	2.95 (.436) 1	4.5 (.279) 4	4.475 (.386) 3	4.225 (.391) 2	5.575 (.507) 7	8.35 (.486) 11	6.325 (.402) 8	8.325 (.441) 10	4.675 (.445) 5	5.275 (.471) 6

Table 4. Average ranks, with standard errors in parentheses and ranking of the average ranks of powers of the LR, LM, Wald, AW and NW tests for MA(1) and AR(1) disturbances based on different likelihood and message length functions.

ML and MML estimators										
Profi.	Marg.	CPL	CPRL	ACPL	ML_1	$CPML_1$	ML_2	$CPML_2$	AML_1	AML_2
Average rank of powers of LR tests for $\gamma = \rho$ = 0.4 and 0.8 with n = 30 and 60										
9.031 (.696) 11	5.219 (.491) 7	3.813 (.363) 3	5.656 (.581) 8	4.88 (.530) 6	6.38 (.540) 9	2.25 (.486) 1	7.125 (.588) 10	2.594 (.442) 2	4.375 (.457) 4	4.688 (.478) 5
Average rank of powers of LR tests for $\gamma = \rho$ = -0.4 and -0.8 with n = 30 and 60										
1.375 (.317) 1	2.813 (.226) 2	4.75 (.426) 5	3.281 (.281) 3	3.66 (.360) 4	5.63 (.490) 8	9.844 (.520) 11	5.063 (.460) 6	9 (.473) 10	6.25 (.614) 9	5.219 (.555) 7
Average rank of powers of LM tests for $\gamma = \rho$ = 0.4 and 0.8 with n = 30 and 60										
9.406 (.514) 11	5.719 (.412) 6	3.813 (.282) 3	5.719 (.412) 6	4.375 (.380) 4	5.188 (.334) 5	1.781 (.314) 1	6.719 (.490) 9	2.219 (.300) 2	6.813 (.616) 10	6.656 (.726) 8
Average rank of powers of LM tests for $\gamma = \rho$ = -0.4 and -0.8 with n = 30 and 60										
1.5 (.127) 1	3.75 (.254) 3	6.563 (.481) 9	3.75 (.254) 3	4.344 (.453) 5	6.156 (.419) 7	9.563 (.449) 11	2.907 (.306) 2	8.719 (.471) 10	6.156 (.613) 7	5.688 (.734) 6
Average rank of powers of Wald tests for ρ = 0.4 and 0.8 with n = 30 and 60										
8.5 (.979) 11	6.813 (.802) 10	6.625 (.865) 9	6.5 (.775) 7	6.5 (.830) 7	2.313 (.285) 3	1.75 (.403) 1	2.88 (.360) 4	2.188 (.400) 2	4.938 (.755) 5	5.063 (.692) 6
Average rank of powers of Wald tests for ρ = -0.4 and -0.8 with n = 30 and 60										
1 (000) 1	1.875 (.155) 2	3.563 (.540) 4	1.938 (.170) 3	3.69 (.540) 5	6.563 (.837) 9	9.313 (.820) 11	6.5 (.840) 8	9.063 (.793) 10	4.563 (.570) 7	3.938 (.566) 6
Average rank of powers of AW tests for γ = 0.4 and 0.8 with n = 30 and 60										
9.313 (.865) 11	3.563 (.516) 4	3.5 (.796) 1	3.5 (.532) 1	3.688 (.650) 5	8.313 (.514) 9	6.75 (.981) 8	8.375 (.605) 10	5.875 (.903) 7	3.5 (.532) 1	3.813 (.627) 6
Average rank of powers of AW tests for γ = -0.4 and -0.8 with n = 30 and 60										
4.25 (.588) 5	3.375 (.272) 3	4.875 (.562) 6	3.5 (.258) 4	6.75 (.403) 7	9.438 (.223) 10	9.688 (.445) 11	9 (.342) 9	8.813 (.390) 8	2.813 (.493) 2	1.813 (.332) 1

Table 5. Average ranks, with standard errors in parentheses and ranking of the average ranks of powers of the LR, LM, Wald, AW and NW tests for MA(1) and AR(1) disturbances based on different likelihood and message length functions.

			ML and MML estimators						
γ	ρ		Marg	CPL	ACPL	ML_1	ML_2	AML_1	AML_2
LR test									
0.4 and 0.8		Average rank S.D. of ranks Rank	2.875 (.328) 2	1.688 (.299) 1	3.313 (.669) 3	4.131 (.435) 6	5.313 (.435) 7	3.875 (.417) 4	4.063 (.478) 5
-0.4 and -0.8		Average rank S.D. of ranks Rank	2.375 (.315) 2	4.125 (.515) 5	2.063 (.359) 1	5.5 (.387) 7	4.625 (.375) 6	3.938 (.636) 4	3.063 (.551) 3
	0.4 and 0.8	Average rank S.D. of ranks Rank	4.625 (.605) 7	3.125 (.375) 3	3.25 (.413) 4	4 (.555) 5	4.5 (.645) 6	1.25 (.112) 1	1.688 (.151) 2
	-0.4 and -0.8	Average rank S.D. of ranks Rank	1.25 (.112) 1	2.5 (.303) 2	2.813 (.368) 5	2.688 (.416) 4	2.5 (.474) 2	5.188 (.660) 7	4.25 (.581) 6
±0.4 and ±0.8		Average rank S.D. of ranks Rank	2.625 (.228) 1	2.906 (.366) 3	2.688 (.390) 2	4.906 (.306) 6	4.969 (.286) 7	3.906 (.374) 5	3.563 (.370) 4
	±0.4 and ±0.8	Average rank S.D. of ranks Rank	2.938 (.428) 2	2.813 (.244) 1	3.031 (.275) 4	3.344 (.361) 6	3.5 (.433) 7	3.219 (.483) 5	2.960 (.374) 3
±0.4 and ±0.8	±0.4 and ±0.8	Average rank S.D. of ranks Rank	2.781 (.241) 1	2.859 (.218) 2	2.859 (.238) 2	4.125 (.254) 6	4.234 (.273) 7	3.563 (.306) 5	3.266 (.264) 4
LM test									
0.4 and 0.8		Average rank S.D. of ranks Rank	4.875 (.352) 7	2.625 (.272) 1	3.375 (.364) 4	3.813 (.319) 5	4.5 (.398) 6	3 (.532) 3	2.813 (.634) 2
-0.4 and -0.8		Average rank S.D. of ranks Rank	3.25 (.214) 3	5.938 (.359) 7	2.563 (.376) 2	5.125 (.301) 6	3.25 (.214) 3	3.438 (.570) 5	2.438 (.577) 1
	0.4 and 0.8	Average rank S.D. of ranks Rank	3.188 (.493) 4	1.625 (.352) 1	1.875 (.446) 2	2.813 (.306) 3	4.5 (.585) 5	5.438 (.329) 6	5.625 (.473) 7
	-0.4 and -0.8	Average rank S.D. of ranks Rank	2.5 (.354) 2	3.813 (.534) 4	3.563 (.524) 3	3.875 (.507) 5	1.188 (.188) 1	4.813 (.400) 6	5 (.585) 7

Table 5. (continued)

γ	ρ		Marg	CPL	ACPL	ML_1	ML_2	AML_1	AML_2
ML and MML estimators									
±0.4		Average rank	4.063	4.281	2.969	4.469	3.875	3.219	2.625
and		S.D. of ranks	(.250)	(.371)	(.267)	(.246)	(.249)	(.386)	(.423)
±0.8		Rank	5	6	2	7	4	3	1
	±0.4	Average rank	2.844	2.719	2.719	3.344	2.844	5.125	5.313
	and	S.D. of ranks	(.305)	(.371)	(.371)	(.307)	(.424)	(.261)	(.374)
	±0.8	Rank	3	1	1	5	3	6	7
±0.4	±0.4	Average rank	3.946	3.135	3.311	3.473	3.378	4.041	3.973
and	and	S.D. of ranks	(.210)	(.278)	(.227)	(.207)	(.252)	(.260)	(.3270
±0.8	±0.8	Rank	5	1	2	4	3	7	6
Wald test									
	0.4	Average rank	4.625	4.438	4.25	1.188	1.75	3.25	3.5
	and	S.D. of ranks	(.562)	(.547)	(.504)	(.101	(.214)	(.461)	(.447)
	0.8	Rank	7	6	5	1	2	3	4
	-0.4	Average rank	1.125	2.25	2.375	5.063	5	3.063	2.563
	and	S.D. of ranks	(.085)	(.359)	(.340)	(.616)	(.626)	(.370)	(.376)
	-0.8	Rank	1	2	3	7	6	5	4
	±0.4	Average rank	2.875	3.344	3.313	3.125	3.375	3.156	3.031
	and	S.D. of ranks	(.421)	(.377)	(.343)	(.464)	(.437)	(.291)	(.299)
	±0.8	Rank	1	6	5	3	7	4	2
AW test									
0.4		Average rank	2.75	2.5	2.563	6.063	6.063	2.438	2.688
and		S.D. of ranks	(.359)	(.563)	(.365)	(.309)	(.370)	(.387)	(.435)
0.8		Rank	5	2	3	6	6	1	4
-0.4		Average rank	2.813	3.5	4.563	6.625	6.25	2.25	1.5
and		S.D. of ranks	(.245)	(.365)	(.203)	(.125)	(.112)	(.250)	(.204)
-0.8		Rank	3	4	5	7	6	2	1
±0.4		Average rank	2.781	3	3.563	6.344	6.156	2.344	2.094
and		S.D. of ranks	(.259)	(.380)	(.264)	(.212)	(.246)	(.279)	(.296)
±0.8		Rank	3	4	5	7	6	2	1
NW test									
0.4		Average rank	3.5	1.813	3.813	1.313	2.125	5.813	6
and		S.D. of ranks	(.387)	(.228)	(.534)	(.151	(.301)	(.262)	(.465)
0.8		Rank	4	2	5	1	3	6	7
-0.4		Average rank	2.188	4.063	3.188	2.938	1.688	4.875	6.625
and		S.D. of ranks	(.387)	(.520)	(.510)	(.370)	(.198)	(.554)	(.125)
-0.8		Rank	2	5	4	3	1	6	7
±0.4		Average rank	2.844	2.938	3.5	2.125	1.906	5.344	6.313
and		S.D. of ranks	(.397)	(.487)	(.520)	(.346)	(.257)	(.443)	(.345)
±0.8		Rank	3	4	5	2	1	6	7

Table 5. (continued)

ML and MML estimators									
γ	ρ		Marg	CPL	ACPL	ML_1	ML_2	AML_1	AML_2
LR, LM, Wald, AW and NW tests									
0.4 and 0.8		Average rank S.D. of ranks Rank	3.5 (.204) 3	2.156 (.185) 1	3.266 (.250) 2	3.875 (.265) 5	4.5 (.262) 7	3.781 (.257) 4	4.047 (.299) 6
-0.4 and -0.8		Average rank S.D. of ranks Rank	2.656 (.148) 1	4.406 (.247) 6	3.094 (.219) 2	5.047 (.228) 7	3.953 (.243) 5	3.625 (.282) 4	3.25 (.313) 3
	0.4 and 0.8	Average rank S.D. of ranks Rank	4.082 (.325) 7	3.082 (.288) 2	3.143 (.286) 3	2.755 (.275) 1	3.633 (.341) 6	3.306 (.303) 4	3.551 (.315) 5
	-0.4 and -0.8	Average rank S.D. of ranks Rank	1.625 (.151) 1	2.854 (.248) 2	2.917 (.242) 4	3.875 (.319) 5	2.896 (.343) 3	4.354 (.303) 7	3.938 (.324) 6
±0.4 and ±0.8		Average rank S.D. of ranks Rank	2.925 (.131) 1	3.381 (.183) 3	3.131 (.166) 2	4.575 (.182) 7	4.063 (.180) 6	4.05 (.190) 5	3.756 (.219) 4
	±0.4 and ±0.8	Average rank S.D. of ranks Rank	2.866 (.217) 1	2.969 (.190) 2	3.031 (.187) 3	3.309 (.217) 5	3.268 (.243) 4	3.825 (.220) 7	3.742 (.226) 6
±0.4 and ±0.8	±0.4 and ±0.8	Average rank S.D. of ranks Rank	2.996 (.121) 1	3.143 (.125) 3	3.112 (.125) 2	3.951 (.145) 7	3.804 (.151) 6	3.759 (.145) 5	3.701 (.159) 4