MONASH UNIVERSITY

AUSTRALIA

A COMPARATIVE ANALYSIS OF DIFFERENT MONTE CARLO METHODS

László Mátyás and Mark Harris

Working Paper No. 10/93

September 1993

DEPARTMENT OF ECONOMETRICS

ISSN 1032-3813

ISBN 0 7326 0378 1

A COMPARATIVE ANALYSIS OF DIFFERENT MONTE CARLO METHODS

László Mátyás and Mark Harris

Working Paper No. 10/93

September 1993

DEPARTMENT OF ECONOMETRICS

MONASH UNIVERSITY, CLAYTON, VICTORIA 3168, AUSTRALIA.

A Comparative Analysis of Different Monte Carlo Methods

László Mátyás* and Mark Harris**

* Monash University, Australia and Budapest University of Economics
** Monash University, Australia

Abstract: The purpose of this paper is to investigate how different ways to generate data in a Monte Carlo experiment can influence the outcome of an analysis.

Key words: Monte Carlo experiment, Simulation, Estimation, Durbin–Watson test.

1. Introduction

Monte Carlo experiments have become essential in econometrics to analyse the behaviour of procedures that analytically cannot (at least easily) be treated. Because we rely heavily on these simulation methods we must fully understand what we do. Sometimes, unfortunately, we do not realize that there may be important differences in the way the data is (artificially) generated and this may have a strong effect on the outcome of an experiment. The purpose of this paper is to compare three very similar and widely used approaches of Monte Carlo data generation and show how the dissimilarities can influence the results.

We focus on the question how the exogenous variables of a model in a Monte Carlo experiment are generated. In practice three different approaches are used:[1]

1. *Pure Monte Carlo Data Generation:*
   The exogenous variables are considered as random and are generated using an autoregressive process. In its basic form: the variable x_t is generated in the experiment as $x_t = \gamma x_{t-1} + \varepsilon_t$, where γ is a known parameter, ε_t is a random variable with known distribution and the starting point of the process x_0 is a

[1] A quick review for the period 1985–1993 of the Journal of Econometrics, the International Economic Review, and Economics Letters shows that about 40% of the papers using Monte Carlo methods (with exogenous variables) applied Pure, 25% Mixed and 25% Randomized data generation, while for about 10% there is not enough information for the identification of the method.

simple random variable (say, for example, with $N(0,1)$ distribution). Then for each Monte Carlo run the x_t observations are generated again and again. This approach takes into account the real life fact that in practice these variables are likely to be observed with measurement error(s) and cannot be considered as fixed. (Good examples of this type of experiments can be found in *Verbeek and Nijman* [1993] and *Inder* [1993].)

2. *Randomized Monte Carlo Data Generation:*
   The exogenous variables are real variables, they are not generated. In the hypothesis testing literature (see, for example, *Giles and Scott* [1992] or *Silvapulle and King* [1993]) the *Durbin and Watson* [1951] data on spirit consumption, spirit prices, etc. in the U.K. is frequently used. We call this randomization because the random disturbances are the only variables of the model generated artificially in the experiment. The big advantage of this approach is that the results of different studies can easily be compared. The disadvantage is that the results are conditioned on the real data, therefore the researcher does not control all parameters of the experiment, which can be considered as scientifically peculiar.

3. *Mixed Data Generation:*
   The exogenous variables are generated as in the case of a pure data generation (1.) but only once, then they are considered as fixed, so the same observations are used again and again in the Monte Carlo runs as in the case of the randomized data generation (2.). In this approach the experiment is fully under control, but the outcome of the analysis depends on (is conditioned on) a single realization vector of a (pseudo) random number generator. (See, for example, *Veall and Zimmermann* [1992] for this type of Monte Carlo study.) Moreover, both in cases 2. and 3. the elements of the observation vector of the exogenous variables will change as the sample size in the experiment changes. Therefore it can be very difficult to separate the effects due to the change in the sample size from those due to the change in the elements of that observation vector.

In this paper, using the three different ways of generating the exogenous variables, we compare the results obtained by Monte Carlo analysis for two well known problems:
a) The small sample bias of the OLS estimator in a linear dynamic regression model and the small and large sample bias of the OLS and GLS estimators in the same model but with autocorrelated residuals;
b) The empirical critical values of the Durbin–Watson test (for a given size) for the above two models.

2. Framework of the experiment

We used the models:

$$y_t = \alpha y_{t-1} + \beta^{(1)} x_t^{(1)} + \beta^{(2)} x_t^{(2)} + u_t \tag{1}$$

and

$$y_t = \alpha y_{t-1} + \beta^{(1)} x_t^{(1)} + \beta^{(2)} x_t^{(2)} + \varepsilon_t \tag{2}$$

where $\varepsilon_t = \rho \varepsilon_{t-1} + v_t$ and the disturbance terms u_t and ε_t are assumed to be white noise in a first round and then in a second round u_t and ε_t are considered to be lognormal $(1,1)$. Model (1) is estimated by OLS and model (2) by OLS and FGLS and the bias of these estimators is analysed. In addition the Durbin–Watson test statistic is calculated for both models using the OLS residuals and the empirical critical values are determined for a test size of 5%. (About the legitimacy of using the DW test in dynamic models, see *Inder* [1984], [1986].)

The exogenous variables were generated in the following way:

— Pure Monte Carlo experiment:

$$x_0^{(i)} = \varepsilon_0^{*(i)} \quad x_t^{(i)} = \gamma x_{t-1}^{(i)} + \varepsilon_t^{*(i)} \quad i = 1,2.$$

Scenarios:
- $\varepsilon_t^{*(i)} \sim N(0,1), \quad \gamma = 0.5 \quad t = 0 \ldots T, \ i = 1,2.$
- $\varepsilon_t^{*(i)}$ exponential distribution, with parameter $= 1$,
$\gamma = 0.5 \quad t = 0 \ldots T, \ i = 1,2.$

— Randomized Monte Carlo experiment:
The exogenous variables $x^{(1)}$ and $x^{(2)}$ are not generated, they are the annual consumption of spirit (1870–1938) and the log relative price of spirits in the U.K. drawn from *Durbin and Watson* [1951].

— Mixed Monte Carlo experiment:
The exogenous variables $x^{(1)}$ and $x^{(2)}$ are generated as in the Pure case but only once and then they are kept fixed.

3. Results of the Monte Carlo analysis

The simulation results are summarized in Tables 1–4. The first striking result is that the use of a given data set (here the Durbin–Watson data) leads to biases (and partly DW critical values) completely different from those obtained by the Pure and Mixed methods for both models (1) and (2) regardless the distribution of the residual terms. How to interpret this contradictory outcome? Does this mean that the randomized experiment is worthless because the biases are unrealistically large? Or just the other way round: does this mean that the Pure and Mixed experiments are worthless because the biases are unrealistically small? Or should we conclude that the Monte Carlo experiment, as it is, is completely useless to estimate the magnitude of the bias for the analysed estimators? The obvious answer to all these questions would be that the results are conditioned on the set up of the experiment and therefore their validity is limited. Fine, but is all this relevant from a practical point of view? When one estimates a model, in general, the distribution (the DGP) of the exogenous variables is unknown. Is it fair to say that it is likely that the bias is going to be around the values shown by the Pure and Mixed methods but in some cases it may be much larger? (When?) Are these results due to the distribution of the DGP for the Durbin–Watson data, or just to the given (atypical) realization of the process? (Graphs 1–2 show that it is not very likely that the DGP in the Durbin–Watson data can be well represented by any usual uni-modal distribution.) Unfortunately we do not know the answer(s) to most of these questions, but at least we must be aware of them.

It seems (in our framework) that there is not too much difference between the Pure and Mixed methods, at least in large samples when the residuals are white noise. In small samples, however, important differences can occur. When the residuals have an asymmetric (lognormal) distribution it can be a non negligible difference between these two methods. We must emphasize that according to our results the Mixed method is not a midway approach between the Pure and the Randomized data generation but it seems to be closer to the Pure generation process.

The introduction of lognormal residuals has increased substantially the bias of the estimators in model (1) while this seems to have little effect for model (2).

The DW test critical values turned out to be less sensitive to the changes in the design of the experiment (as far as the normality of the residual terms is maintained), which confirms the well known robust behaviour of this test, especially when the exogenous variables are artificially generated. However, a small change in these critical values (or the size of the test) can make the difference between a correct and an incorrect decision, so the seemingly minor changes can be very important.

4. A further method

As far as we are aware, the above three methods cover econometric practice of how to generate the exogenous variables of a model in a Monte Carlo study. However, there is at least one other approach worth considering in this context. It is possible to combine the Pure and the Mixed procedures by introducing an extra loop into the data generation process. First a Mixed Monte Carlo experiment is carried out. Then in the next step the exogenous variables are re-generated again, but only once, and the Mixed experiment is re-run. We proceed like this as many times as we wish. This procedure is similar to the Pure data generation, but unlike there, the exogenous variables here are generated in an additional external loop. This method is able to combine some the advantages of the Pure and the Mixed approach: while the exogenous variables are kept fixed we do not have to rely on a single realization vector of a random process.

From the point of view of our analysis the Combined simulation is able to answer two questions: a) whether the fact that only a single realization of a random vector was used has important effect on the Mixed method; and b) is the Combined method an improvement vis-a-vis the Mixed method. It can be seen from Tables 5 and 6 that the average biases are nearly the same as in Tables 1–4, therefore it seems that this is not performing better than the Mixed method. However the magnitudes of the maximum biases are a warning to us that in the Mixed approach the range of the biases obtained is quite large. This means that the use of a single realization of the exogenous variables may lead to quite unstable results. Therefore the Combined should be preferred to the Mixed method, because the range of the results gives us valuable information about the reliability of the simulation (of course within the framework of the setup).

5. Conclusion

In this paper we investigated how different ways to generate data in a Monte Carlo experiment can influence the outcome of an analysis. The results teach us to be very careful not only with the interpretation of Monte Carlo simulations, but also with their use as benchmarks for empirical studies. The results obtained are very sensitive to the setup of the experiment(s) and the way the exogenous variables are generated so the extrapolation of the results to real life problems seems to be less than obvious.

SIMTABLMD1

Table 1. Simulation results for Model (1)
(normal residuals)

Normal exogenous Variables	OLS* bias			DW** Critical Value
"Pure" MC	α	β_1	β_2	
T = 10	0.254	0.343	0.333	2.869
T = 25	0.118	0.165	0.162	2.466
T = 60	0.071	0.096	0.097	2.305
"Mixed" MC				
T = 10	0.267	0.249	0.289	2.599
T = 25	0.114	0.157	0.130	2.490
T = 60	0.065	0.068	0.073	2.322
Exponential exogenous variables	----------------------			--------------------
"Pure" MC				
T = 10	0.194	0.342	0.356	2.947
T = 25	0.096	0.160	0.156	2.584
T = 60	0.059	0.091	0.088	2.369
"Mixed" MC				
T = 10	0.154	0.227	0.244	2.943
T = 25	0.081	0.179	0.131	2.576
T = 60	0.057	0.095	0.080	2.334
Fixed exogenous variables	----------------------			--------------------
T = 10	0.184	14.48	13.58	3.114
T = 25	0.137	7.30	6.87	2.47
T = 60	0.092	0.96	0.89	2.19

* $\sum_{i=1}^{n} \frac{|\hat{\beta}_i - \beta|}{n}$, where n = number of replications

** Critical value of the DW test for a size of 5%

Table 2. Simulation results for Model (2)
(normal residuals)

Normal exogenous variables:	OLS* bias			FGLS* bias			DW** Critical Value
"Pure" MC	α	β_1	β_2	α	β_1	β_2	
T = 10	0.250	0.361	0.384	0.285	0.345	0.345	2.512
T = 25	0.191	0.196	0.197	0.182	0.190	0.183	1.966
T = 60	0.200	0.125	0.127	0.168	0.108	0.114	1.777
"Mixed" MC							
T = 10	0.298	0.260	0.337	0.274	0.245	0.320	2.324
T = 25	0.181	0.166	0.175	0.173	0.156	0.164	1.977
T = 60	0.187	0.113	0.143	0.179	0.100	0.111	1.814
Exponential exogenous variables							
"Pure" MC							
T = 10	0.186	0.364	0.393	0.240	0.398	0.439	2.744
T = 25	0.130	0.198	0.198	0.220	0.188	0.194	2.00
T = 60	0.144	0.147	0.153	0.239	0.105	0.114	1.746
"Mixed" MC							
T = 10	0.134	0.641	0.293	0.141	0.733	0.290	2.778
T = 25	0.100	0.205	0.164	0.164	0.207	0.160	1.998
T = 60	0.126	0.148	0.118	0.216	0.108	0.096	1.737
Fixed exogenous variables							
T = 10	0.248	20.90	19.53	0.486	24.60	22.16	2.921
T = 25	0.147	10.19	9.601	0.333	13.47	12.90	2.054
T = 60	0.212	1.096	1.027	0.354	3.162	2.677	1.825

* $\sum_{i=1}^{n} \frac{|\hat{\beta}_i - \beta|}{n}$, where n = number of replications

** Critical value of the DW test for a size of 5%.

Table 3. Simulation results for Model (1)
(lognormal residuals)

Normal exogenous Variables		OLS* bias		DW** Critical Value
"Pure" MC	α	β_1	β_2	
T = 10	0.306	0.405	0.406	2.879
T = 25	0.284	0.206	0.204	2.460
T = 60	0.303	0.136	0.139	2.341
"Mixed" MC				
T = 10	0.553	0.349	0.254	2.383
T = 25	0.325	0.178	0.208	2.239
T = 60	0.304	0.093	0.195	2.624
Exponential exogenous variables		----------------------		--------------------
"Pure" MC				
T = 10	0.037	0.231	0.244	2.796
T = 25	0.101	0.209	0.146	2.593
T = 60	0.092	0.140	0.103	2.400
"Mixed" MC				
T = 10	0.144	0.402	0.406	2.985
T = 25	0.094	0.199	0.209	2.601
T = 60	0.102	0.122	0.121	2.431
Fixed exogenous variables		----------------------		--------------------
T = 10	0.134	12.42	11.50	3.202
T = 25	0.097	6.724	6.336	2.582
T = 60	0.094	1.073	0.850	2.179

* $\sum_{i=1}^{n} \frac{|\hat{\beta}_i - \beta|}{n}$, where n = number of replications

** Critical value of the DW test for a size of 5%

Table 4. Simulation results for Model (2) (lognormal residuals)

Normal exogenous variables:	OLS* bias			FGLS* bias			DW** Critical Value
"Pure" MC	α	β_1	β_2	α	β_1	β_2	
T = 10	0.243	0.391	0.383	0.274	0.387	0.376	2.509
T = 25	0.193	0.199	0.195	0.181	0.193	0.178	2.001
T = 60	0.199	0.127	0.129	0.178	0.110	0.109	1.788
"Mixed" MC							
T = 10	0.304	0.280	0.342	0.267	0.248	0.324	2.346
T = 25	0.181	0.169	0.175	0.170	0.156	0.165	1.950
T = 60	0.187	0.110	0.144	0.178	0.100	0.111	1.813
Exponential exogenous variables							
"Pure" MC							
T = 10	0.188	0.398	0.407	0.245	0.413	0.409	2.748
T = 25	0.127	0.197	0.207	0.226	0.181	0.191	1.986
T = 60	0.138	0.149	0.152	0.241	0.111	0.114	1.705
"Mixed" MC							
T = 10	0.138	0.671	0.299	0.148	0.720	0.286	2.759
T = 25	0.097	0.200	0.165	0.171	0.208	0.160	1.970
T = 60	0.124	0.144	0.120	0.221	0.108	0.097	1.724
Fixed exogenous variables							
T = 10	0.298	23.08	21.45	0.501	23.82	21.96	2.779
T = 25	0.161	9.890	9.304	0.382	13.48	13.12	2.023
T = 60	0.181	1.219	1.137	0.337	3.217	2.775	1.769

* $\sum_{i=1}^{n} \frac{|\hat{\beta}_i - \beta|}{n}$, where n = number of replications

** Critical value of the DW test for a size of 5%.

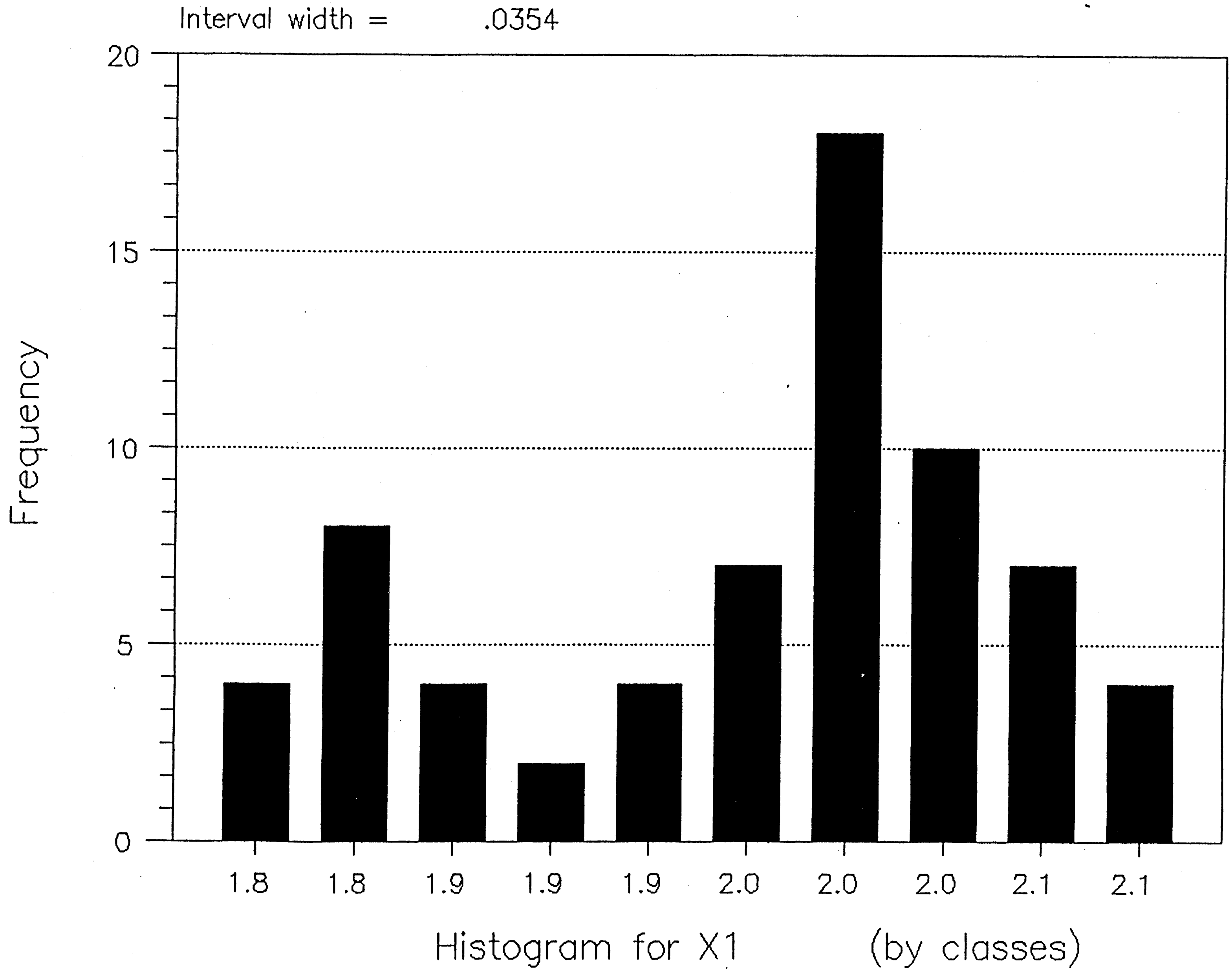
Table 5. Combined MC results for model (1)

	OLS					
	bias			max bias		
	α	β_1	β_2	α	β_1	β_2
Normal residuals						
- Normal exogenous variables						
T = 10	0.242	0.359	0.319	0.338	0.986	0.700
T = 25	0.120	0.162	0.163	0.148	0.405	0.248
T = 60	0.069	0.098	0.095	0.079	0.134	0.130
- Exponential exogenous variables						
T = 10	0.136	0.356	0.331	0.376	1.096	0.907
T = 25	0.097	0.158	0.157	0.142	0.267	0.284
T = 60	0.058	0.090	0.092	0.069	0.138	0.133
Lognormal residuals						
- Normal exogenous variables						
T = 10	0.303	0.417	0.410	0.536	1.209	1.047
T = 25	0.289	0.224	0.233	0.403	0.586	0.615
T = 60	0.298	0.137	0.141	0.348	0.300	0.369
- Exponential exogenous variables						
T = 10	0.144	0.420	0.409	0.327	1.151	1.051
T = 25	0.097	0.185	0.195	0.163	0.322	0.530
T = 60	0.101	0.122	0.115	0.132	0.131	0.211

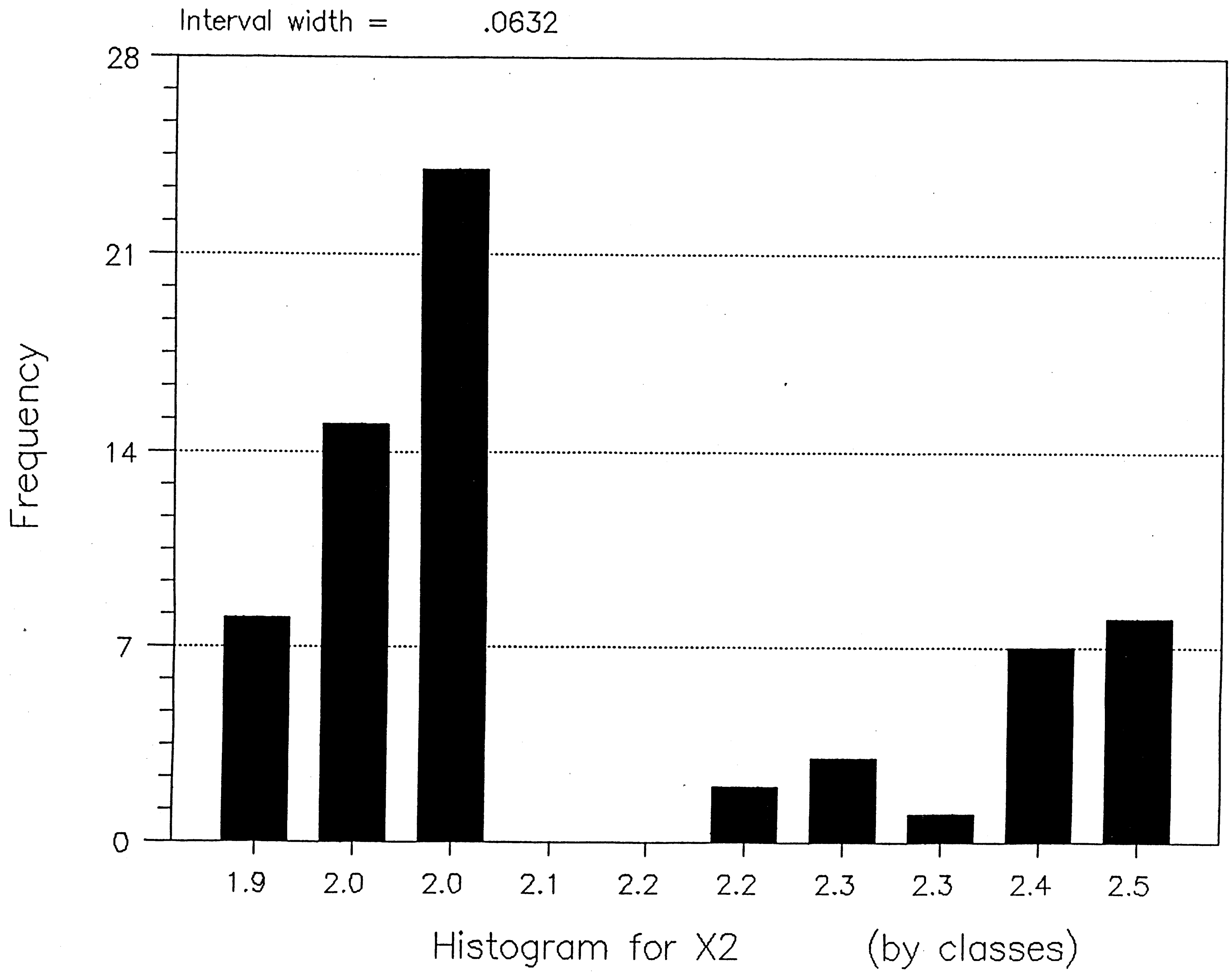
Table 6. Combined MC results for model (2)

	OLS bias (max bias)			FGLS bias (max bias)		
	α	β_1	β_2	α	β_1	β_2
Normal residuals						
- Normal exogenous variables						
T = 10	0.258 (0.328)	0.373 (0.741)	0.374 (0.735)	0.268 (0.347)	0.383 (0.881)	0.370 (0.696)
T = 25	0.189 (0.238)	0.196 (0.277)	0.204 (0.297)	0.181 (0.223)	0.184 (0.269)	0.195 (0.301)
T = 60	0.159 (0.236)	0.125 (0.160)	0.124 (0.162)	0.172 (0.204)	0.109 (0.135)	0.110 (0.138)
- Exponential variable						
T = 10	0.194 (0.307)	0.347 (0.785)	0.394 (1.666)	0.239 (0.356)	0.407 (1.207)	0.424 (1.831)
T = 25	0.130 (0.192)	0.200 (0.348)	0.199 (0.397)	0.213 (0.279)	0.189 (0.410)	0.186 (0.353)
T = 60	0.144 (0.179)	0.152 (0.221)	0.151 (0.253)	0.236 (0.290)	0.113 (0.171)	0.110 (0.168)
Lognormal residuals						
- Normal exogenous variable						
T = 10	0.260 (0.342)	0.404 (0.988)	0.399 (1.130)	0.275 (0.395)	0.403 (1.044)	0.408 (1.033)
T = 25	0.190 (0.252)	0.199 (0.343)	0.196 (0.285)	0.180 (0.213)	0.186 (0.328)	0.186 (0.268)
T = 60	0.197 (0.239)	0.126 (0.165)	0.126 (0.172)	0.174 (0.210)	0.109 (0.140)	0.109 (0.150)
- Exponential exogenous variables						
T = 10	0.195 (0.306)	0.412 (1.384)	0.425 (1.107)	0.240 (0.390)	0.409 (1.392)	0.436 (1.179)
T = 25	0.127 (0.185)	0.202 (0.385)	0.207 (0.331)	0.219 (0.290)	0.187 (0.327)	0.198 (0.458)
T = 60	0.139 (0.183)	0.150 (0.237)	0.149 (0.234)	0.239 (0.277)	0.115 (0.195)	0.114 (0.182)

Graph 1. Histogram for the Durbin-Watson data



Graph 2. Histogram for the Durbin-Watson data



References

Durbin, J. and G. Watson [1951]: Testing for Serial Correlation in Least Squares Regression – II; *Biometrika*, 38, pp. 159–178.

Giles, D. and M. Scott [1992]: Some Consequences of Using the Chow test in the Context of Autocorrelated Disturbances; *Economics Letters*, 38, pp. 145–150.

Inder, B. [1984]: Finite-sample Power of Tests for Autocorrelation in Models Containing Lagged Dependent Variables; *Economics Letters*, 14, pp. 179–185.

Inder, B. [1986]: An Approximation to the Null Distribution of the Durbin–Watson Test Statistic in Models Containing Lagged Dependent Variables; *Econometric Theory*, 2, pp. 413–428.

Inder, B. [1993]: Estimating Long Run Relationships in Economics; *Journal of Econometrics*, 57, pp. 53–68.

Silvapulle, P. and M. L. King [1993]: Non Nested Testing for Autocorrelation in the Linear Regression Model; *Journal of Econometrics*, 58, pp. 295–314.

Veal, M. R. and K. F. Zimmermann [1993]: Performance Measures from Prediction–Realization Tables; *Economics Letters*, 39, pp. 129–134.

Verbeek, M. and T. Nijman [1992]: Testing for Selectivity Bias in Panel Data Models; *International Economic Review*, 33, pp. 681–703.