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Production Economics Paper 6213
Purdue University
July 27, 1962

DISTRIBUTED LAGS AS A TECHNIQUE FOR OBTAINING LONG-AND SHORT-RUN ELASTICITIES

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LONG-AND SHORT-RUN ELASTICITIES

Economists from the time of Marshal have made a distinction between long-run and short-run relationships. This distinction is made in evaluating the effect of a change in an economic variable or cause on its subsequent consequences. In the theory of the firm the concept is specifically related to the proportion of inputs that are permitted to change in response to a change in product price.

The exact division with regard to what will be called short-run and what will be called long-run responses is quite arbitrary, but long-run has usually referred to a length of time sufficient to permit complete adjustment to the changed situation. In the theory of the firm this is considered to be the length of run in which all inputs are permitted to vary. On the demand side the concept is less precise, but does suggest that economic units do not make a complete adjustment or change in response to a changed condition in an arbitrary period of time, but rather spread their reaction over multiple periods of time.

Distributed Lags as a Means of Estimating Long-Run Elasticities

Historically, alternative approaches have been made to the problem of obtaining empirical estimates of long-run elasticities. Koych, 1/Nerlove, 2/

1/L. M. Koyck, Distributed Lags and Investment Analysis (Amsterdam: North Holland Publishing Company, 1954).

2/Marc Nerlove, and William Addison, "Statistical Estimation of Long-run Elasticities of Supply and Demand," Journal of Farm Economics, XL (November, 1958), 861-880. See also the following works by Nerlove: "Distributed Lags and Estimation of Long-Run Supply and Demand Elasticities: Theoretical Considerations," Journal of Farm Economics, XL (May, 1958), 301-311; Distributed Lags and Demand Analysis for Agricultural and Other Commodities, Agriculture Handbook No. 141, Agricultural Marketing Service, 1958; and The Dynamics of Supply: Estimation of Farmers' Response to Price (Baltimore: The Johns-Hopkins Press, 1958).

and Hildreth and Jarrett[3] have proposed one method, while Working[4] has used an alternative. The approach discussed here is that originally proposed by Koyck, but recently developed by Nerlove and others. The method is based fundamentally on the concept of a distributed lag.

Distributed lags arise in theory when any economic cause produces its affect only after some lag in time, so that this effect is not felt all at once, at a single point of time, but is distributed over a period of time. So stated this is the basic idea behind the distinction between short- and long-run elasticities and has led Nerlove to argue that the formulation of economic relationships containing distributed lags is related to the problem of formulating meaningful relationships among observable variables. He goes on to point out that the problem of estimating distributions of lag is really the problem of estimating long-run elasticities.

His approach involves the development of a dynamic model of producer or consumer behavior which implies a distributed lag only incidentally. These models may be used directly in an analysis designed to estimate the long-run elasticity of demand or supply. It leads to a direct interpretation of the distribution of lag in terms of producer or consumer behavior and, therefore, in terms of the difference between short- and long-run elasticities of supply or demand. Such models may be used without reference to the distributions of lag which are implied.

3/ C. Hildreth and F. G. Jarrett, A Statistical Study of Livestock Production and Marketing (New York: John Wiley and Sons, 1955), pp. 112-113.

4/ E. J. Working, "Appraising the Demand for American Agricultural Output during Rearmament," Journal of Farm Economics, XXXII (May, 1952), 206-224; Demand for Meat (Chicago: University of Chicago Press, 1954); "How Much Progress Has Been Made in the Study of the Demand for Farm Products?" Journal of Farm Economics, XXXV (December, 1955), 968-974.

The model can be illustrated by considering the demand equation suggested by the theory of demand for a productive input. A long-run labor demand function is postulated which may be written:

$$\overline{X}_{8t} = aX_{7t} + bX_{2t} + cX_{6t} + d$$

where: $\overline{X}_{8t}$ = the long-run or equilibrium quantity demanded,

X_{7t} = the agricultural wage rate,

X_{2t} = the "real" price of farm products, and

X_{6t} = an index of technology.

The long-run or equilibrium quantity demanded cannot be observed because the other variables are continually changing. Therefore, this equation cannot be estimated directly.

Let X_{8t} be the current quantity demanded. In the absence of changes in the independent variables upon which demand depends, it is assumed that the current quantity demanded would change in proportion to the difference between the long-run equilibrium quantity and the current quantity.[5] This assumption may be expressed by the following difference equation:

$$X_{8t} - X_{8t-1} = \gamma \left[\overline{X}_{8t} - X_{8t-1}\right] \qquad 0 < \gamma < 1$$

where the variables are identified as earlier and γ is a coefficient of adjustment, showing what proportion of the disequilibria is removed in one time period.

Substitution of this adjustment equation into the long-run or equilibrium demand equation leads to the following estimating equation:

5/ This model assumes, rather arbitrarily, that prices adjust immediately to changed conditions, while the quantity variable is adjusted with a lag. The reasonableness of this assumption must be evaluated in the individual instance.

$$X_{8t} = a\gamma X_{7t} + b\gamma X_{2t} + c\gamma X_{6t} + (1-\gamma)\, X_{8t-1} + d\gamma$$

estimated in the form:

$$X_{8t} = \pi_1 X_{7t} + \pi_2 X_{2t} + \pi_3 X_{6t} + \pi_4 X_{8t-1} + \pi_5$$

This equation is not any sort of a demand function but merely a relationship among observable variables. It is useful because it is possible to derive estimates of the parameters in the long-run equation from its parameter estimates.

More specifically, γ can be obtained by subtracting π_4 from one. Divinding the other parameter estimates of the estimating equation in turn by γ leads to estimates of the parameters of the long-run or equilibrium equation. These can then be used to compute the long-run elasticities. Short-run elasticities are obtained from the coefficients of the estimating equation. The coefficient of adjustment determines the relation among the short-run elasticities and the long-run elasticities.

Similar considerations apply when dealing with the supply equation. Operationally, the essence of the model when quantity demanded or supplied is taken as the dependent variable is to add the dependent variable lagged one period as an additional independent variable in the original demand or supply equation. The coefficient of the lagged variable is then given a special interpretation. If the price is taken as the dependent variable, the appropriate procedure when quantity is predetermined, the model is much more complicated.[6/] Problems of collinearity are likely to arise, in addition to having several parameters from which to choose the coefficient of adjustment.

6/ Marc Nerlove and William Addison, "Statistical Estimation of Long-Run Elasticities of Supply and Demand", Journal of Farm Economics, XL (November, 1958), 877.

If price and quantity are assumed to be mutually determined, Nerlove[7/] suggests no change in the form of the model, but only that limited information or some other appropriate estimation procedure be followed rather than ordinary least squares. It is important to normalize the equations on quantity rather than price, however, in order to avoid the complications of making price the dependent variable.

Problem of Estimation and Interpretation

The introduction of a lagged dependent variable as an independent variable gives rise to problems in obtaining perameter estimates with desirable properties. If the independent variables and the error terms satisfy the usual least squares assumptions, the parameter estimates will be consistent, though in small samples the estimate of γ will be biased. However, if the error terms are serially correlated, the least squares estimate of γ will be biased even in large samples. This suggests that in statistically evaluating the model, it is important to test for serial correlation in the calculated residuals.

In past applications of the model this has not been a problem, however. Nerlove[8/] feels that perhaps the most significant finding in early applications of the model is that this dynamic approach tends to eliminate evidence of positive serial correlation found in the calculated residuals of regressions based on static models. And further, that is provides an economic rationale for the presence of serial correlation in economic time series, one of the sources of serial correlation in the residuals.

On the assumption that producers and consumers do not react instantaneously to the fullest extent to changed conditions, current quantities, both

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7/ Ibid., p. 876.
8/ Ibid.

supplied and demanded, are positively related to lagged quantity. The failure to include lagged quantity as an explicit variable, then, leads to positive serial correlation in the calculated residuals.[9] Including lagged quantity eliminates the serial correlation in the residuals, and the important point in this context is that Nerlove's model provides an economic rationale for the presence of serial correlation in times series data. The presence of serial correlation in the residuals indicates that something systematic is left to be explained. This model provides a meaningful way of accounting for it.

This leads to one of the major criticisms of the model. There are strong reasons for suspecting that the coefficients of adjustment are subject to a greater extent than other parameters to specification bias, or the omission of relevant variables. This was pointed out by Brandow[10], Halvorson[11], and Griliches[12] and recognized by Nerlove.[13] The introduction of the lagged dependent variable into the regression is a very useful device to take into account the empirical fact that economic variables are serially correlated. As indicated above its use as an addition independent variable introduces the fact of serial correlation explicitly into the model. But it

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9/ This is because the error or residual terms in the economic relation estimated usually arise from the omission of relevant variables. Because these variables are themselves serially correlated, they give rise to serial correlation in the calculated residuals.

10/ George E. Brandow, "A Note on the Nerlove Estimate of Supply Elasticity," Journal of Farm Economics, XL (August, 1958), 719-722.

11/ Harlow W. Halvorson, "The Response of Milk Production to Price," Journal of Farm Economics, XL (December, 1958), 1101-1113.

12/ Zvi Griliches, "Distributed Lags, Disaggregation, and Regional Demand Functions for Fertilizer," Journal of Farm Economics, XLI (February, 1959), 90-102.

13/ Marc Nerlove, "On the Nerlove Estimate of Supply Elasticity: A Reply," Journal of Farm Economics, XL (August, 1958), 719-728; and "On the Estimation of Long-Run Elasticities: A Reply," Journal of Farm Economics, XLI (August, 1959), 632-640.

may be quite wrong to attribute all of the serial correlation to the adjustment mechanism. Some of it may be due to the serial correlation in other variables that are left out. If this is the case, the adjustment coefficient will be underestimated, and some of the sluggishness in the omitted variables will be attributed to people's slowness to react to the included variables.

Griliches[14/] has shown that the coefficient of adjustment will be consistently underestimated. But he and Nerlove[15/] have shown that this need not necessarily imply an overestimation of the long-run elasticities. It can be shown that in absolute terms the short-run elasticity may be underestimated also.

Griliches[16/] has also shown that a low coefficient of adjustment may be due to multicollinearity. If quantity lagged is highly correlated with other independent variables, it may be the "stronger" variable and the estimation procedure will attribute some of the effects of the other variables to it. He derives maximum estimates of the adjustment coefficients that are consistent with the data by use of the covariance of the respective parameters. In his example the maximum adjustment coefficient can be as high as three or four times the estimated coefficient and still be consistent with the data. But in this case, too, the estimates of the long-run elasticities are not altered very much. The possibilities of evaluating the extent to which the coefficient of the lagged variable is reflecting the effects of other variables is more difficult in larger models than Griliches has used, however,

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14/ Zvi Griliches, "Distributed Lags, Disaggregation, and Regional Demand Functions for Fertilizer," Journal of Farm Economics, XLI (February, 1959), 90-102.

15/ Ibid.

16/ Ibid.

Ironmonger,[17] in another examination of the model, raised two criticisms. He argued that the lagged quantity variable should be used to represent changing tastes rather than the distributed lag model. His hypothesis was that in the demand studies reported by Nerlove there had been changing tastes, and that this accounted for the significance of the lagged variable. Nerlove acknowledged that taste might be an omitted variable and that, as in other cases pointed out by Brandow and Griliches, this could bias the estimate of the coefficient of adjustment. This does not mean, however, that the distributed lag model does not apply. Rather it means that care should be exercised in obtaining a properly specified model.

The second criticism Ironmonger raised was the relevance of the time period selected for the data. To the extent that actual adjustment is made in a period of less than a year in length, the coefficient of adjustment estimated from annual data will "understate" the "true" coefficient of adjustment. This suggests that analyses should be based on data from as short a time period as is meaningfully possible.

These criticisms give rise to positive suggestions for using the distributed lag model. In the first place it is important to have a properly specified model. Specification bias is more of a problem than in more conventional models. Secondly, low coefficients of adjustment should be viewed with suspicion. Though there are no objective criteria for what is "low," intuitive judgment can be exercised in terms of the adjustment characteristics of the factor or commodity being analyzed. And thirdly, the interpretation of the coefficient of adjustment should be done with care, especially with regard to what it means in terms of the measurement time period.

17/ D. S. Ironmonger, "A Note on the Estimation of Long-Run Elasticities," Journal of Farm Economics, XL (August, 1959), 626-632.

In general it is a good idea to introduce a trend variable into any equation in which the distributed lag model is used. In this way the effects of omitted variables that are lightly correlated with time will be isolated and not attributed to the adjustment mechanism. This improves the specification of the model and provides a means of reducing the potential specification bias. The trend variable should be introduced both in addition to the lagged dependent vaiable and as an alternative hypothesis.

An Alternative Interpretation

Introduction of the lagged dependent variable into a model also provides a very crude test of the reliability of the over-all statistical results. If the apparent relationship between a set of variables was due entirely to the presence of trend in each of them, or more generally to high autocorrelation in each of the, the regression coefficient of the lagged dependent variable would tend to be nearly one, and its presence in the regression would tend to make the other regression coefficients (and partial correlations) zero. Significance of other variables in the presence of the lagged variable lends them greater credibility.