2-38

Production Economics Paper No. 6207
Purdue University
June 15, 1962

SELECTION OF HOG MANURE HANDLING SYSTEMS BY MATHEMATICAL PROGRAMMING

by

Anders Nygaard

SELECTION OF HOG MANURE HANDLING SYSTEMS BY MATHEMATICAL PROGRAMMING

by

Anders Nygaard

1. The Problem

Automatic methods or "push button" systems for the feeding and watering of fattening hogs have been developed recently, while methods of manure handling have received relatively little attention. The problems of manure handling have become important by changing over from the traditional pasture system to intensified confined systems. The major reasons for this may be found in the increasing value of land used for pasture and the possibility of larger one-man operations in confinement.

It is assumed that the manure, from health and sanitation point of view, must be removed from the pens. The accumulation of manure and waste feed on the floor also produces a characteristic odor and a suitable breeding ground for the common fly (Musca domestica).

The manure is also considered to be of value as plant nutrition. However, manure is not a stable mixture, so the value of the manure is highly dependent upon its handling from dunging time until it is injected into the soil. The most common way of evaluating the manure is to equate its content of N, P_2O_5 and K_2O injected into the soil to the same amount of N, P_2O_5 and K_2O in commercial fertilizer.

A flow chart for the different systems to be considered is given in Figure 1. The process of manure handling is divided into seven "steps". The nodes are placed at the changing over points from one step to the next. This

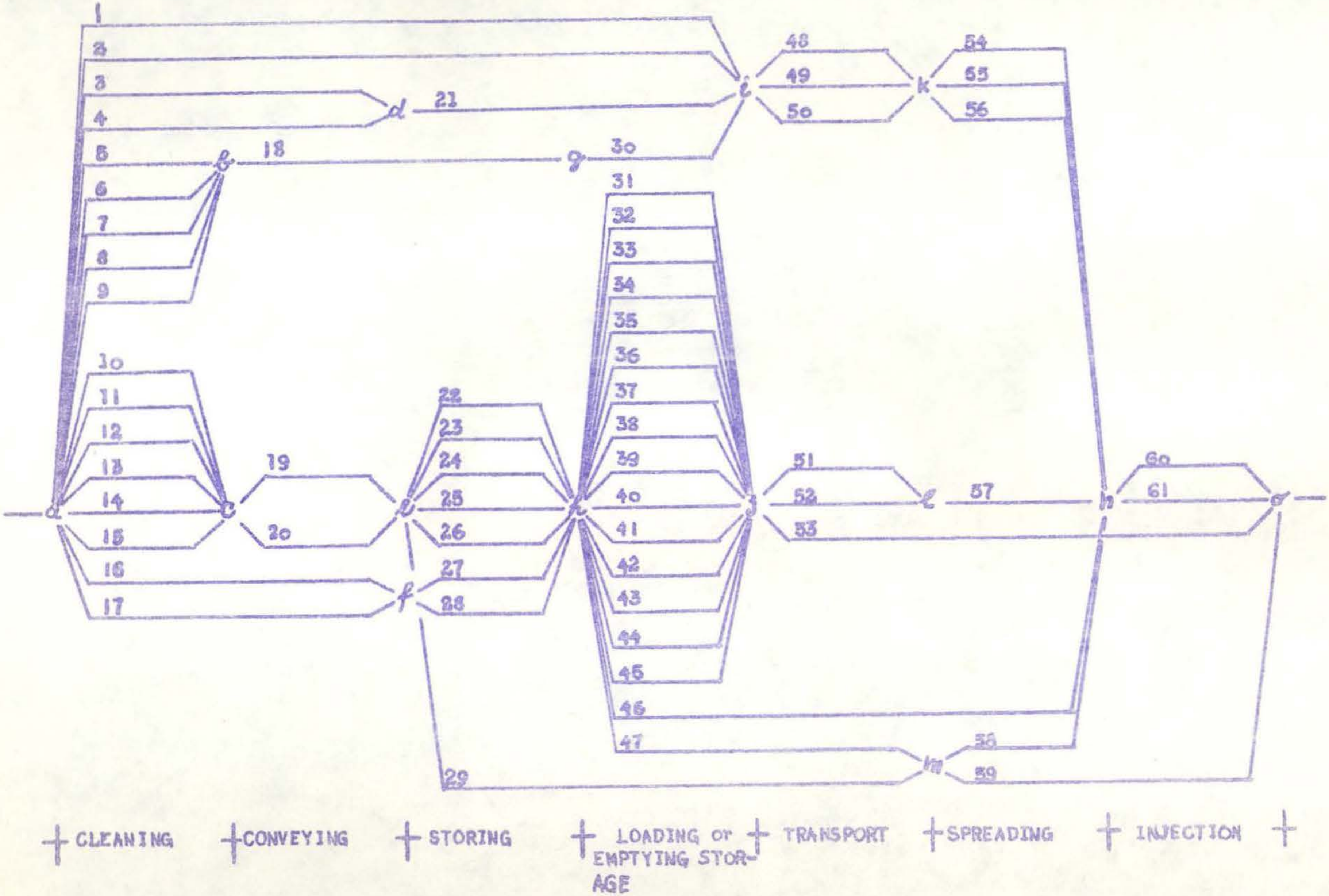

CLEANING
CONVEYING
STORING
LOADING or EMPTYING STORAGE
TRANSPORT
SPREADING
INJECTION

FIG. 1.

means that the amount coming into a node equals the amount of manure leaving the node. The links represent the total cost of each handling method.

The amount of manure may change between the nodes because some of the liquid drains off. However, the unit on which the method (link) cost is based, is the same for all the links in the flow chart. This unit may be called the chart constant.

Finding the least cost system (or path) through the flow chart may be done by ordinary budgeting methods. However, in this paper, two different mathematical programming methods will be applied to select the least cost path:

1. Simplex method
2. PERT (Program Evaluation Review Technique).

(The solution will be computed for only a part of the chart.)

2. The unit flow model solved by the simplex method.

2.1 The mathematical model.

The objective function to be minimized is:

$$\text{Min } Z = \sum_{j=1}^{m_1} c_j x_j + \sum_{k=1}^{m_2} c_k x_k + \sum_{\ell=1}^{m_3} c_\ell x_\ell$$

$$\left.\begin{matrix} x_j \\ x_k \\ x_l \end{matrix}\right\} \geq 0$$

Subject to: (1) Network equations

(2) Purchase inequalities.

Where:

X_j = Methods (links) with no multiple-use machines.

X_k = Methods (links) with multiple-use machines.

X_1 = Multiple-use machines (purchase variable)

C_j = Total cost (fixed + variable) for methods with no multiple-use machines

C_k = Total cost minus fixed cost for multiple use machines for methods with multiple-use machines

C_1 = Fixed cost for multiple use machines

n_1 = Number of methods (links) without multiple-use machines

n_2 = Number of methods with multiple use machines

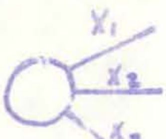
n_3 = Number of multiple-use machines.

And:

The number of network equations to be written equals the number of nodes minus one. The one node to be left out could be either the first or the last one. For the other nodes, the amount of product coming into a node is equal to the amount leaving the node.

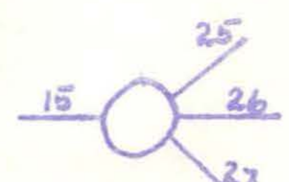
Examples:

First node:



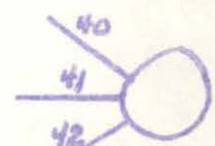
$X_1 + X_2 + X_3 = 1$

Other nodes:



$X_{15} - X_{25} - X_{26} - X_{27} = 0$

Last node:



$X_{40} + X_{40} + X_{41} = 1$

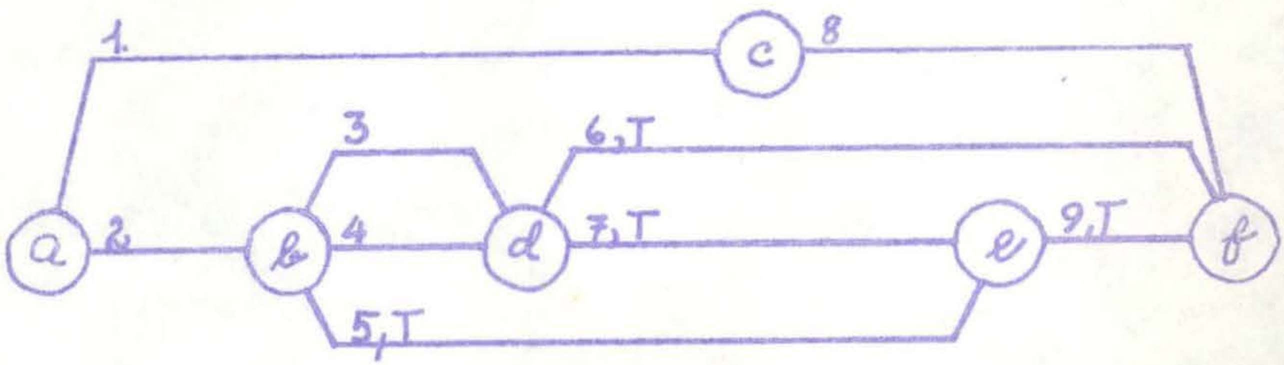
The purchase inequalities for the multiple-use machines are written so that the value of the purchase variable takes on at least the value equal to the fixed cost of the link associated with it. If X_T is the fixed cost for a multiple-use machine, the purchase variable X_{PT} must take on a value which satisfies the inequality:

$$X_T - X_{PT} \leq 0$$

2.2 A simplified flow chart

It is assumed that the flow chart for the whole problem presented in Figure 1 will be too big to solve by hand in a simplex tableau.

As an example, only a part of the chart is taken for solution, Figure 2.



Link number:
1. Subfloor storage drained once per month into a lagoon.
2. Subfloor lagoon with one year capacity.
3. Loading manure with auger.
4. Loading manure with centrifugal pump.
5. Loading manure into vacuum tank wagon, drive to field and spread manure.
6. Drive to field with tank wagon with injection facilities and inject manure into the soil.
7. Drive to field with tank wagon and spread manure.
8. Dispose of manure in lagoon.
9. Mix manure into soil by harrowing.

T = Multiple-use tractor.

FIGURE 2.

2.3 Classification of machines and equipment.

The machines and equipment used on the flow chart have to be classified in

1. Single-use machines (use optional).

2. Must-use machines (use essential to the process).

3. Multiple ordered-use machines (used between different pairs of nodes).

The fixed cost of these three types of machines is added to the operation costs of each method in which they are specified.

4. Multiple-use machines.

A multiple-use machine has two or more series uses that may each be in a system without the other.

There is no correct place on the flow chart to assign the fixed cost of a multiple-use item. The solution of this problem lies in constructing separate variables for the multiple-use machines and assuring an integer solution for these variables.

On the flow chart in Figure 2, there is only one multiple-use machine. This is a tractor denoted by a capital letter T on the appropriate links.

2.4 The methods (links) costs

For computing the methods costs a chart constant (unit) has been used:

Floor capacity	400 hogs
or Annual output $2\frac{1}{2}\cdot 400$	= 1000 hogs
Animal manure production:	
1.25 gallon/hog·1000·365	= 456250 gallons
Unused N, P_2O_5 and K_2O in lagoon	
1.15 \$/hog·1000 hogs	= \$1150

The fixed cost for machines and storages is calculated by using an average annual use figure:

10.6% of first cost for buildings and

20.0% of first cost for machines.

The final cost is given in Table 1.

Table 1. Methods costs for the flow chart.

Link no.	Description	Operating Cost: Annual oper. Hours 2	Operating Cost: Cost per hour $ 3	Operating Cost: Annual oper. cost $ 4	Fixed Cost: First cost $ 5	Fixed Cost: Annual ownership % 6	Fixed Cost: Annual fixed cost $ 7	Total cost $ 8
1.	Labor	6	2.00	12				
	Storage				1,208	10.60	128	140
	Sum				1,208		128	140
2.	Subfloor lagoon Sum				1,680	10.60	178	178
3.	Labor	160	2.00	320				
	Auger				90	20.00	18	
	Power, electric	152	.06	9	120	20.00	24	
	Sum			329			42	371
4.	Labor	46	2.00	92				
	Centrifugal pump	-	-	-	350	20.00	70	
	Power, electric	38	.06	2	120	20.00	24	
	Sum			94			104	198
5.	Labor	433	2.00	866				
	Vacuum tank and pump				650	20.00	130	
	Power, tractor	433	0.50	216				
	Sum			1,082			130	1,212
6.	Labor	335	2.00	670				
	Tank wagon and injectors				550	20.00	110	
	Power, tractor	335	0.50	167				
	Sum			837			110	947
7.	Labor	335	2.00	670				
	Tank wagon				400	20.00	80	
	Power, tractor	335	0.50	167				
	Sum			837			80	917
8.	Lagoon				408	10.60	43	43
	Losses of N, P_2O_5 and K_2O							1,150
	Sum							1,193
9.	Labor	13	2.00	26				
	Harrow				148	20.00	30	
	Power, tractor	30	0.50	15				
	Sum			41			30	71
	Fixed cost of tractor							220

2.5 The objective function and restrictions

The objective function to be minimized is:

(The subscripts of the variables correspond to the link number in the flow chart, Figure 2, plus purchase variable number 10.)

$$\text{Min } Z = 140x_1 + 178x_2 + 371x_3 + 198x_4 + 1212x_5 + 947x_6 + 917x_7 + 1193x_8 + 71x_9 + 220x_{10}$$

Subject to:

$$
\begin{aligned}
x_1 + x_2 &= 1 \\
x_2 - x_3 - x_4 - x_5 &= 0 \\
x_1 - x_8 &= 0 \\
x_3 + x_4 - x_6 - x_7 &= 0 \\
x_5 + x_7 - x_9 &= 0 \\
x_5 - x_{10} &\leq 0 \\
x_6 - x_{10} &\leq 0 \\
x_7 - x_{10} &\leq 0 \\
x_9 - x_{10} &\leq 0
\end{aligned}
$$

2.6 The simplex tabeaux and solution

The first and last simplex tableaux are shown in Tables 2 and 3 respectively. The matrix is 19 times 9 and the optimal solution is reached in the seventh iteration. The optimal solution consists of $X_1 = 140$, and $X_8 = 1193$, which gives Min. Z = \$1333 per 1000 hogs per year.

I.

TABLE 2.

λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	c_i
C_j	140	178	371	198	1212	947	917	1193	71	220	M	M	M	M	M	0	0	0	0		
11	1	1									1									1	M
12		1	−1	−1	−1							1								0	M
13	(1)							−1					1							0	M
14			1	1		−1	−1							1						0	M
15					1		1		−1						1					0	M
16					1					−1						1				0	0
17						1				−1							1			0	0
18							1			−1								1		0	0
19									1	−1									1	0	0
$Z_j - C_j$: M	2	2	0	0	0	−1	0	−1	−1	0	0	0	0	0	0	0	0	0	0	1	
$Z_j - C_j$: $	−140	−178	−371	−198	−1212	−947	−917	−1193	−71	−220	0	0	0	0	0	0	0	0	0	0	

VII. TABLE 3.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	λo	Ci
11					1			1		1	1	−1	−1	−1			−1			1	1193
12		1			−1					−1		1		1			1			0	178
13	1				1					1	1	−1		−1			−1			1	140
14			1	1						−1				1			1			0	198
15					1		1		−1						1					0	917
16					1					−1						1				0	0
17					−1	1			1	−1					−1		1			0	947
18										−1								1		0	0
19									1	−1									1	0	0
$Z_j - C_j$: M	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	0	0	0	0	0	
[illegible]	0	0	−173	0	−87	0	0	0	−41	−210	1333	1155	−1193	−957	−30	0	−10	0	0	1333	

Conclusions: The least cost manure handling system of the systems compared is obtained by using slatted floor with subfloor storage and disposal of the manure in a lagoon.

Appendix A gives the complete seven iterations of the simplex method.

3. Program Evaluation and Review Technique (PERT)

The Program Evaluation and Review Technique might be applied in this problem to compute the least cost path through the flow chart.

The flow chart in Figure 2 is reworked in Figure 3. The link number 3, which is a parallel to 4, is taken out. Node number a is "opened" in a_1 and a_2. The fixed cost of multiple-use machines might cause some difficulties. In this case the fixed cost of the tractor is added to link numbers 5, 6 and 7.

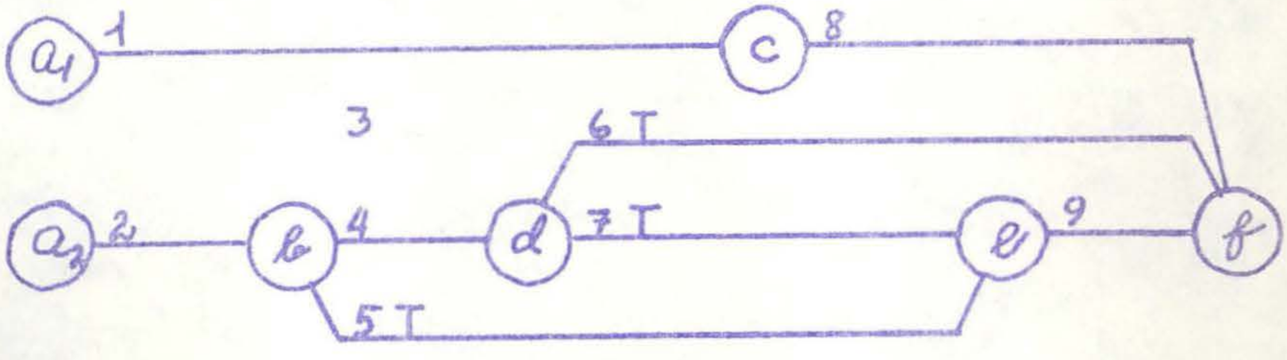


FIG. 3.

The next step is to form the P-matrix, Figure 4.

	a_1	a_2	b	c	d	e	f
a_1	0	∞	∞	140	∞	∞	∞
a_2	∞	0	178	∞	∞	∞	∞
b	∞	∞	0	∞	198	1432	∞
c	∞	∞	∞	0	∞	∞	1193
d	∞	∞	∞	∞	0	1137	1167
e	∞	∞	∞	∞	∞	0	71
f	∞	∞	∞	∞	∞	∞	0

Figure 4. The P-matrix for the flow chart in Figure 3.

The M-matrix is computed by the formula:

$$m_{rs} = \underset{i}{Min} \left\{ m_{ri} + p_{is} \right\}$$

Except: (1) $m_{ij} = 0$

(2) $m_{ij} + \infty = \infty$

(3) $\infty + p_{ij} = \infty$

(4) $0 + p_{ij} = \infty$

(5) $m_{ij} + 0 = \infty$

$m_{a_1a_2}$	m_{a_1b}	m_{a_2b}	m_{a_1c}	m_{a_2c}
$0+\infty=\infty$	$0+\infty=\infty$	$\infty+\infty=\infty$	$0+140=140$	$\infty+140=\infty$
$?+0=\infty$	$\infty+178=\infty$	$0+178=178$	$\infty+\infty=\infty$	$0+\infty=\infty$
$?+\infty=\infty$	$?+0=\infty$	$178+0=\infty$	$\infty+\infty=\infty$	$178+\infty=\infty$
$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$\infty+\infty=\infty$	$?+0=\infty$
$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$\infty+\infty=\infty$	$?+\infty=\infty$
$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$\infty+\infty=\infty$	$?+\infty=\infty$
$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$\infty+\infty=\infty$	$?+\infty=\infty$
∞	∞	178	140	∞

m_{bc}	m_{a_1d}	m_{a_2d}	m_{bd}	m_{cd}
$\infty+140=\infty$	$0+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$
$\infty+\infty=\infty$	$\infty+\infty=\infty$	$0+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$
$0+\infty=\infty$	$\infty+198=\infty$	$178+198=376$	$0+198=198$	$\infty+198=\infty$
$?+0=\infty$	$140+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$0+\infty=\infty$
$?+\infty=\infty$	$?+0=\infty$	$?+0=\infty$	$?+0=\infty$	$?+0=\infty$
$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$
$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$
∞	∞	376	198	∞

m_{a_1e}	m_{a_2e}	m_{be}	m_{ce}	m_{de}
$0+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$
$\infty+\infty=\infty$	$0+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$
$\infty+1432=\infty$	$178+1432=1610$	$0+1432=1432$	$\infty+1432=\infty$	$\infty+1432=\infty$
$140+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$0+\infty=\infty$	$\infty+\infty=\infty$
$\infty+1137=\infty$	$376+1137=1513$	$198+1137=1335$	$\infty+1137=\infty$	$0+1137=1137$
$?+0=\infty$	$?+0=\infty$	$?+0=\infty$	$?+0=\infty$	$?+0=\infty$
$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$	$?+\infty=\infty$
∞	1513	1335	∞	1137

m_{a_1f}	m_{a_2f}	m_{bf}	m_{cf}	m_{df}	m_{ef}
$0+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$
$\infty+\infty=\infty$	$0+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$
$\infty+\infty=\infty$	$178+\infty=\infty$	$0+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$	$\infty+\infty=\infty$
$140+1193=1333$	$\infty+1193=\infty$	$\infty+1193=\infty$	$0+1193=1193$	$\infty+1193=\infty$	$\infty+1193=\infty$
$\infty+1167=\infty$	$376+1167=1543$	$198+1167=1365$	$\infty+1167=\infty$	$0+1167=1167$	$\infty+1167=\infty$
$\infty+71=\infty$	$1513+71=1584$	$1335+71=1406$	$\infty+71=\infty$	$1137+71=1208$	$0+71=71$
$?+\infty=\infty$	$?+0=\infty$	$?+0=\infty$	$?+0=\infty$	$?+0=\infty$	$\infty+\infty=\infty$
1333	1543	1365	1193	1167	71

	a_1	a_2	b	c	d	e	f
a_1	0	∞	∞	140	∞	∞	1333 / c
a_2		0	178	∞	376	1513	1543 / bd
b			0	∞	198	1335	1365
c				0	∞	∞	1193
d					0	1137	1167
e						0	71
f							0

Figure 5. M-matrix.

The least cost path could be read off from the M-matrix. This is found to be a_1 to f through c with $1333 per 1000 hogs per year.

By opening the node a on the graph, the next best path could likewise be found. The next best path is a_2-b-d-f with $1543 per 1000 hogs per year.

This splitting of the first node expands the matrix, but many times we are interested not only in the least cost path, but also the relation to other paths. The PERT allows for doing this. However, precaution should be taken in ranking the computed paths, then some links or part of a path might have been "knocked out" during the computation.

I.

Apendix A.

λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	c_i
C_j	140	178	371	198	1212	947	917	1193	71	220	M	M	M	M	M	0	0	0	0		
11	1	1									1									1	M
12		1	-1	-1	-1							1								0	M
13	(1)							-1					1							0	M
14			1	1		-1	-1							1						0	M
15					1		1		-1						1					0	M
16					1					-1						1				0	0
17						1				-1							1			0	0
18							1			-1								1		0	0
19									1	-1									1	0	0
$z_j - c_j$: M	2	2	0	0	0	-1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	1	
$	-140	-178	-371	-198	-1212	-947	-917	-1193	-71	-220	0	0	0	0	0	0	0	0	0	0	

II.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	Cc
11		1						1			1		-1							1	M
12		(1)	-1	-1	-1							1								0	M ←
13	1							-1					1							0	140
14			1	1		-1	-1							1						0	M
15					1		1		-1						1					0	M
16					1					-1						1				0	0
17						1				-1							1			0	0
18							1			-1								1		0	0
19									1	-1									1	0	0
$Z_j - C_j$: M	0	2	0	0	0	-1	0	1	-1	0	0	0	-2	0	0	0	0	0	0	1	
$	0	-178	-571	-198	-1212	-947	-917	-1333	-71	-220	0	0	140	0	0	0	0	0	0	0	
	↓																				

17.

III.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	λo	Ci
11			1	1	1			1			1	-1	-1							1	M
12		1	-1	-1	-1							1								0	178
13	1							-1					1							0	140
14			1	(1)		-1	-1							1						0	M ←
15					1		1		-1						1					0	M
16					1					-1						1				0	0
17						1				-1							1			0	0
18							1			-1								1		0	0
19									1	-1									1	0	0
$Z_j - C_j$ M	0	0	2	2	2	-1	0	1	-1	0	0	-2	-2	0	0	0	0	0	0	1	
$	0	0	-549	-376	-1390	-947	-917	-1333	-71	-220	0	178	140	0	0	0	0	0	0	0	
				↓																	

$\overline{IV}$.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	Ci
	11					1	1	1	1			1	-1	-1	-1						1	M
	12		·1			-1	-1	-1					1		1						0	178
	13	1							-1					1							0	140
	14			1	1		-1	-1							1						0	198
	15					1		(1)		-1						1					0	M ←
	16					1					-1						1				0	0
	17						1	1			-1							1			0	0
	18										-1								1		0	0
	19									1	-1									1	0	0
$Z_i - C_j$	M	0	0	0	0	2	1	2	1	-1	0	0	-2	-2	-2	0	0	0	0	0	1	
	[illegible]	0	0	-173	0	-1390	-1323	-1293	-1333	-71	-220	0	178	140	376	0	0	0	0	0	0	
								↓														

V.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	λ_0	C_i
11						1		1	1		1	-1	-1	-1	-1					1	M
12		1				-1			-1			1		1	1					0	178
13	1							-1					1							0	140
14			1	1	1	-1			-1					1	1					0	198
15					1		1		-1						1					0	917
16					1					-1						1				0	0
17					-1	(1)			1	-1					-1		1			0	0
18										-1								1		0	0
19									1	-1									1	0	0
$Z_j - C_j$: M	0	0	0	0	0	1	0	1	1	0	0	-2	-2	-2	-2	0	0	0	0	1	
[illegible]	0	0	-175	0	-97	-1323	0	-1653	-1964	-220	0	178	140	376	1293	0	0	0	0	0	

20.

$\overline{VI}$.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	λ_0	c_i	
11					1			(1)		1	1	−1	−1	−1			−1			1	M	
12		1			−1					−1		1		1			1			0	178	
13	1							−1					1							0	140	←
14			1	1						−1				1			1			0	198	
15					1		1		−1						1					0	917	
16					1					−1						1				0	0	
17					−1	1			1	−1					−1		1			0	947	
18										−1								1		0	0	
19									1	−1									1	0	0	
$z_j - c_j$: M	0	0	0	0	1	0	0	1	0	1	0	−2	−2	−2	−1	0	−1	0	0	1		
$	0	0	−173	0	−1420	0	0	−1553	−41	−1543	0	178	140	376	−30	0	1323	0	0	0		
								↓														

VII.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	λ_0	C_i
11					1			1		1	1	-1	-1	-1			-1			1	1193
12		1			-1					-1		1		1			1			0	178
13	1				1					1	1	-1		-1			-1			1	140
14			1	1						-1				1			1			0	198
15					1		1		-1						1					0	917
16					1					-1						1				0	0
17					-1	1			1	-1					-1		1			0	947
18										-1								1		0	0
19									1	-1									1	0	0
$z_j - c_j$: M	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	
B	0	0	-173	0	-87	0	0	0	-41	-210	1333	1155	-1193	-957	-30	0	-10	0	0	1333	

Literature used:

1. Class notes from Course IE 535.

2. Morris, W. H. M., The Economics of Disposal of Hog Manure in Confinement Systems, Dept. of Agr. Econ., Purdue University.

3. Peart, Robert M, Gerald W. Isaacs and Charles E. French, Optimizing Materials-Handling Systems by Mathematical Programming, Purdue Agr. Exp. Sta., 1960.

4. Robinson, Thomas W., The Handling of Hog Manure from Confinement Systems of Management, unpublished M.S. Thesis, Purdue University, 1961.

5. Suter, Robert C., Farm Planning Props, Dept. of Agr. Econ., Purdue University, 1960.

6. Unpublished data from Dept. of Agr. Econ., Purdue University.