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SIMPLIFIED PRESENTATION OF "TRANSPORTATION-PROBLEM PROCEDURE" IN LINEAR PROGRAMMING [1/]

Milton M. Snodgrass and Charles E. French [2/]
Purdue University

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PROCEDURE" IN LINEAR PROGRAMMING [1]

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To date, most agricultural economists using linear programming have employed the "general procedure" or "simplex method." Certain agricultural economics problems seem better adapted and more easily solved by the "transportation-problem procedure" or "MODI method." [3]

The "transportation-problem procedure" was conceived to give a minimum transport cost in satisfying a given set of needs from a given set of sources. The need of each location and the capacity of each source were predetermined. Total needs equalled total capacity. Thus, all coefficients of the matrix could be converted to one or zero.

This would seem unduly restricting, but it is not difficult to generalize the procedure to cover a large group of problems where the objective is to give a <u>minimum cost, or maximum profit</u>, in satisfying <u>any set of outputs from</u> a <u>given set of inputs</u>. Any problem meeting the following formal characteristics can qualify:

"(1) One unit of <u>any</u> input can be used to produce one unit of <u>any</u> output.
(2) The cost or margin which will result from conversion of one unit of a particular input into one unit of a particular output can be expressed by a single figure regardless of the number of units converted.
(3) The quantity of each individual input and output is fixed in advance, and the total of the inputs equals the total of the outputs." [4]

Thus, the transportation problem differs explicitly from the general problem in at least two ways. Any input can satisfy any output in the transportation problem, while this is usually not true in the general problem. Also, total inputs must equal total outputs in the transportation problem, but this need not be true in the general problem. [5]

Koopmans [6] illustrated early use of the "transportation problem procedure" and Samuelson [7] later showed its relationship to the broader spatial equilibrium problem. Judge [8] has solved with this procedure an interregional competition problem in eggs. The authors [9] have recently completed a somewhat similar study in dairy products. Waugh [10] used a modification of it to award contracts in connection with the school lunch program.

This procedure makes for a much more simple formulation of a given problem than the "general procedure." Often hand calculation will yield answers for sizeable problems. [11] Also, recent improved machine programs developed by A. Charnes at Purdue have made digital computer solution quite simple for this type of problem.

The "general procedure" has been well illustrated in this journal. [12] The "transportation-problem procedure" has not been so illustrated in sources readily available to agricultural economists. [13] Thus, this article will illustrate this procedure in detail. In the same vein as the articles cited above by Heady and Boles, no complicated mathematics will be used.

This procedure can best be illustrated by example. Let us assume an interregional problem of determining the lowest transport cost in satisfying fixed consumption needs of given regions from fixed production levels within the same regions.

Let us suppose that the United States is composed of four regions which are contiguous. Let us choose one point in each region (preferably near the center) which will serve as a point of production for the region and also as a point of consumption for the region. The next logical step is to specify a given production and consumption requirement for each region. Suppose that 100 units of the product in question are produced in the whole country. If we eliminate the possibility of storing the commodity, then 100 units will also be consumed. Let us assume the following:

Region	Production		Consumption	Surplus		Deficit
1	10		30	—		20
2	15		25	—		10
3	30		25	5		—
4	45		20	25		—
Check	100	=	100	30	=	30

With regard to transportation; let us assume the following structure of rates which are independent of volume or direction:

	Region 1	Region 2	Region 3	Region 4
	(dollars per unit)			
Region 1	0	2	4	6
2	2	0	6	2
3	4	6	0	4
4	6	2	4	0

It should be noted that there is no cost involved in shipping from any one region to the same region because we assumed the same point in the region for production and consumption. Hence, diagonal rates from upper left to lower right are all zero.

Given these data, the problem becomes one of moving 100 units of production into consumption at a minimum total cost. By inspection of the data, one would guess that Region 1 will consume all 10 of the units it produces since no cost is involved. Similarly, it would be expected that Region 2 will consume all 15 of the units it produces. Since Region 3 and 4 are surplus regions, it would seem likely that they would meet all of their consumption needs from their own production. As a result, then, a guess that only 30 units of the product will move is quite logical. It will be seen later how this process of inspection can cut down considerably the time involved in actually solving the problem.

Next, the formulation of the problem matrix is necessary.[14] To designate the squares in the matrix, the notation x_{ij} is used where i refers to the region of production or origin and j refers to the region of consumption or destination. Thus the sum of the x_{ij}'s would equal 100 as all 100 that are produced are moved into consumption. The following table illustrates the situation where x_{ij} equals the amount (if any) moving from state i to state j.

(A)

x_{11}	x_{12}	x_{13}	x_{14}
x_{21}	x_{22}	x_{23}	x_{24}
x_{31}	x_{32}	x_{33}	x_{34}
x_{41}	x_{42}	x_{43}	x_{44}

Example: The amount in this cell (x_{24}) will represent the amount produced and shipped from Region 2 for consumption in Region 4.

The total number of cells in table matrix (A) totals 16 as would be expected; four rows and four columns each with four cells. If we let production be indicated by rows and consumption by columns, and at the same time let P's (production) represent rows and U's, columns (consumption), the following relationships are established:

(B)

$$P_1 = x_{11} + x_{12} + x_{13} + x_{14} = 10$$

$$P_2 = x_{21} + x_{22} + x_{23} + x_{24} = 15$$

$$P_3 = x_{31} + x_{32} + x_{33} + x_{34} = 30$$

$$P_4 = x_{41} + x_{42} + x_{43} + x_{44} = 45$$

$$P_1 + P_2 + P_3 + P_4 = 100 = \sum x_{ij} = \text{Total production}$$

(C)

$$U_1 = x_{11} + x_{21} + x_{31} + x_{41} = 30$$

$$U_2 = x_{12} + x_{22} + x_{32} + x_{42} = 25$$

$$U_3 = x_{13} + x_{23} + x_{33} + x_{43} = 25$$

$$U_4 = x_{14} + x_{24} + x_{34} + x_{44} = 20$$

$$U_1 + U_2 + U_3 + U_4 = 100 = \sum x_{ij} = \text{Total consumption}$$

Finally:

$$\sum_{i=1}^{4} P_i = \sum_{i=1}^{4} U_j = 100$$

The next step is the formulation of a basic solution or, in other words, to formulate a type of solution to the problem which will satisfy the above requirements. It may also be called a first approximation and it will no doubt yield a total cost figure greater than the optimum. The solving for the optimum therefore is nothing more than a series of approximations each of which gives a lesser total cost than the previous.

To formulate a base, or first approximation, it is necessary to fill seven of the 16 cells or x_{ij}'s. The number of cells that must be filled to constitute a basic solution is always obtained by adding the number of rows to the number of columns and subtracting one. In this case, four plus four minus one, equals seven.

The next question is, which seven are chosen for the first approximation? It is here that inspection of the data "pays off" -- in guiding the problem solver in making the best first approximation possible. To the extent the researcher has either empirical or theoretical knowledge of his problem, he can hypothesize logical interregional flows. These flows can be built into the basic solution within certain limits (explained later). A good first approximation saves considerable time in reaching the optimum.

The combination of seven cells chosen must meet certain requirements to be valid. The first requirement is that it must satisfy all row and column requirements designated in (B) and (C) above.

Following are two examples of basic solutions. Included in the upper right of each cell is the appropriate transportation rate.

In example (D), the seven x_{ij}'s or cells in the solution and their amounts chosen were x_{14} (10), x_{24} (10), x_{23} (5), x_{33} (20), x_{32} (10), x_{42} (15), and x_{41} (30). The total cost of this approximation would be the sum of each of the amounts times its particular rate. If we designate the transportation rates by "t_{ij}", then the total cost of this solution would be $\sum x_{ij}t_{ij}$ or $(10 \times 6) + (10 \times 2) + (5 \times 6) + (20 \times 0) + (10 \times 6) + (15 \times 2) + (30 \times 6)$ or $60 + 20 + 30 + 0 + 60 + 30 + 180 = \$380$. A quick check shows that this solution meets the requirements of (B) and (C) above.

(D)

					Production ↓
	[0]	[2]	[4]	[6] (10)	$P_1 = 10$
	[2]	[0]	[6] (5)	[2] (10)	$P_2 = 15$
	[4]	[6] (10)	[0] (20)	[4]	$P_3 = 30$
	[6] (30)	[2] (15)	[4]	[0]	$P_4 = 45$
Consumption →	$U_1 = 30$	$U_2 = 25$	$U_3 = 25$	$U_4 = 20$	$\sum_{i=1}^{4} P_i = \sum_{j=1}^{4} U_j = 100$

A second requirement that must be met concerns the cells which are <u>not</u> chosen as a part of the basic solution. This requirement is best explained in nonmathematical terms by use of an analogy. Imagine the matrix above (D) as a pond of water and each x_{ij} or cell which is one of the seven in the solution as a stepping stone on which one could stand without getting wet. Suppose also the pond was of a sufficiently small size so that one could easily walk from one stepping stone to another. At once it is evident that it would be possible to walk from x_{14} in the upper right or northeast corner of the matrix to x_{41} in the lower left or southwest corner of the matrix without getting wet. It is also pertinent to require that in making this journey one would never walk diagonally (northwest, southwest, northeast, or southeast) in direction but always either north, south, east, or west. In this particular case, one would walk only south and west.

The combination of seven cells in matrix (D) constitutes a valid base if and only if it is possible to accomplish a check for each x_{ij} or cell that is not a stepping stone. Since seven of the 16 cells are stepping stones, it is necessary to check the remaining nine which are not; namely, x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{31}, x_{34}, x_{43}, and x_{44}. To check a cell, move directly east or west in the same row as the cell that is being checked until a stepping-stone cell is found. Then, if it is possible to move alternately north or south and then east or west to stepping-stone cells until a stepping stone is reached that is in the same column as the cell originally being checked, the test for that particular cell is completed. For example to check cell x_{11} in the matrix (D), the first move is to x_{14} (which is in the same row), thence south to x_{24}, west to x_{23}, south to x_{33}, west to x_{32}, south to x_{42}, and west to x_{41} where the check is complete because x_{41} is in the same column as the cell being checked, x_{11}. It is important to note that it is not necessary to move to an adjacent stepping-stone cell with each move. It is only necessary that a stepping-stone cell be used at the corners or when one is moving from a row to a column or vice versa. It is "legal" to skip over cells to reach a stepping stone at a corner. These skipped cells can be either other stepping-stone cells or empty cells.

Finally, then, the following paths of movement would constitute a final check as to the validity of the base in matrix (D):

Cell to be checked		Movement path of check
x_{11}	-	x_{14} to x_{24} to x_{23} to x_{33} to x_{32} to x_{42} to x_{41}
x_{12}	-	x_{14} to x_{24} to x_{23} to x_{33} to x_{32}
x_{13}	-	x_{14} to x_{24} to x_{23}
x_{21}	-	x_{23} to x_{33} to x_{32} to x_{42} to x_{41}
x_{22}	-	x_{23} to x_{33} to x_{32}
x_{31}	-	x_{32} to x_{42} to x_{41}
x_{34}	-	x_{33} to x_{23} to x_{24}
x_{43}	-	x_{42} to x_{32} to x_{33}
x_{44}	-	x_{42} to x_{32} to x_{33} to x_{23} to x_{24}

(E)

In each case, the first stepping-stone cell listed is in the same row as the one being checked and the last stepping-stone cell listed in the movement path is in the same column as the cell being checked. There is only one possible movement path which will give these results. In a large matrix, the path is not always immediately evident as in this simple case.

To illustrate that there is another combination of seven cells which constitute a basic solution, the following matrix is shown:

(F)

[0] (10)	[2]	[4]	[6]	10
[2]	[0] (15)	[6]	[2]	15
[4]	[6]	[0] (25)	[4] (5)	30
[6] (20)	[2] (10)	[4]	[0] (15)	45
30	25	25	20	100

First, compare this matrix with (D) as regards total cost. In (F) above, total cost is (10 x 0) + (15 x 0) + (25 x 0) + (5 x 4) + (20 x 6) + (10 x 2) + (15 x 0) which totals $160. This is $220 less than the cost of (D). Therefore, if this solution is a valid base, it is a much better one to start with.

Check:

	Cell to be checked		Movement path of check
	x_{12}	–	x_{11} to x_{41} to x_{42}
	x_{13}	–	x_{11} to x_{41} to x_{44} to x_{34} to x_{33}
	x_{14}	–	x_{11} to x_{41} to x_{44}
	x_{21}	–	x_{22} to x_{42} to x_{41}
(G)	x_{23}	–	x_{22} to x_{42} to x_{44} to x_{34} to x_{33}
	x_{24}	–	x_{22} to x_{42} to x_{44}
	x_{31}	–	x_{34} to x_{44} to x_{41}
	x_{32}	–	x_{34} to x_{44} to x_{42}
	x_{43}	–	x_{44} to x_{34} to x_{33}

This completes the check and establishes the basic solution as valid. Thus, it is a "legal" starting point from which to work towards the optimal. It should be noted that the value of inspection mentioned previously would lead one to use matrix (F) as a basic solution instead of matrix (D).

The next process involves the computation of the marginal <u>row</u> and column values which will be designated by a lower case letters "c" or "r". These fit into the matrix in the following manner:

(H)

$r \downarrow$ \ $c \rightarrow$	c_1	c_2	c_3	c_4	$\downarrow P_i$
r_1	0; x_{11} (circled)	2; x_{12}	4; x_{13}	6; x_{14}	P_1
r_2	2; x_{21}	0; x_{22} (circled)	6; x_{23}	2; x_{24}	P_2
r_3	4; x_{31}	6; x_{32}	0; x_{33} (circled)	4; x_{34} (circled)	P_3
r_4	6; x_{41} (circled)	2; x_{42} (circled)	4; x_{43}	0; x_{44} (circled)	P_4
$U_j \rightarrow$	U_1	U_2	U_3	U_4	$\sum x_{ij}$

To compute these marginal values (r and c), only stepping-stone cells are used. In (H) above, those x_{ij}'s which were stepping stones in (F) are circled. One value can be assigned and can be either an "r" or a "c" value. The value assigned can be of any magnitude, however to ease computations, it is most advantageous to assign a zero. While the zero might be assigned to any r or c value, here it will be assigned to r_1. To compute the remaining marginal values, for each x_{ij} that is a stepping-stone cell, $r \neq c$ is made equal to t_{ij} (transportation rate). Since the t_{ij} values are known and we have already assigned the value zero to r_1, the rest can be computed in the following order:

1. (c_1) If $r_1 + c_1$ must equal t_{11} then $c_1 = t_{11} - r_1$, or

$c_1 = 0 - 0 = \underline{0}$

2. (r_4) If $r_4 + c_1 = t_{41}$, then $r_4 = t_{41} - c_1$, or

$r_4 = 6 - 0 = \underline{6}$

3. (c_2) If $r_4 + c_2 = t_{42}$, then $c_2 = t_{42} - r_4$, or

$c_2 = 2 - 6 = \underline{-4}$

4. (r_2) If $r_2 + c_2 = t_{22}$, then $r_2 = t_{22} - c_2$, or

$r_2 = 0 - (-4) = \underline{4}$

5. (c_4) If $r_4 + c_4 = t_{44}$, then $c_4 = t_{44} - r_4$, or

$c_4 = 0 - 6 = \underline{6}$

6. (r_3) If $r_3 + c_4 = t_{34}$, then $r_3 = t_{34} - c_4$, or

$r_3 = r - (-6) = \underline{10}$

7. (c_3) If $r_3 + c_3 = t_{33}$, then $c_3 = t_{33} - r_3$, or

$c_3 = 0 - 10 = \underline{-10}$

The completed matrix including these marginal values is shown below (I).

(I)

r ↓ \ c →	0	-4	-10	-6	P_i ↓
0	[0] (10)	[2]	[4]	[6]	10
4	[2]	[0] (15)	[6]	[2]	15
10	[4]	[6]	[0] (25)	[4] (5)	30
6	[6] (20)	[2] (10)	[4]	[0] (15)	45
U_j →	30	25	25	20	100

Now we are ready to make the next approximation and determine how much it will reduce cost. The marginal values were computed by use of the transportation rates in stepping-stone cells by making the two corresponding marginal values equal to the rate. However, if a check is made for the nonstepping-stone cells, various answers result. For example, if for x_{12} we add $r_1 + c_2$, the answer is a -4 which is less than the transportation rate, plus two (t_{12}). For x_{43}, $r_4 + c_3$ is $6 + (-10)$ or -4 which is less than a plus four (t_{43}). For the cells which are not stepping stones the results are:

	For: x_{12}	$-$	$r_1 + c_2 = 0 + (-4) = -4$	which is	$\leq 2\ (t_{12})$
	x_{13}	$-$	$r_1 + c_3 = 0 + (-10) = -10$	" "	$\leq 4\ (t_{13})$
	x_{14}	$-$	$r_1 + c_4 = 0 + (-6) = -6$	" "	$< 6\ (t_{14})$
	x_{21}	$-$	$r_2 + c_1 = 4 + 0 = 4$	" "	$> 2\ (t_{21})$
(J)	x_{23}	$-$	$r_2 + c_3 = 4 + (-10) = -6$	" "	$\leq 6\ (t_{23})$
	x_{24}	$-$	$r_2 + c_4 = 4 + (-6) = -2$	" "	$\leq 2\ (t_{24})$
	x_{31}	$-$	$r_3 + c_1 = 10 + 0 = 10$	" "	$> 4\ (t_{31})$
	x_{32}	$-$	$r_3 + c_2 = 10 + (-4) = 6$	" "	$= 6\ (t_{32})$
	x_{43}	$-$	$r_4 + c_3 = 6 + (-10) = -4$	" "	$\leq 4\ (t_{43})$

In (J) above, it is important to note that for all cells except x_{21} and x_{31}, the $r + c$ value is always less than or equal to the t_{ij} value of the cell being checked. If all of the cells in (J) had checked out to be less than or equal to the t_{ij} value of the cell being checked, then our first approximation would have been optimum and no further approximation would be necessary. However, since x_{21} and x_{31} did not check, further reductions in cost can be made. The introduction of either of the cells as stepping stones and the elimination of an existing one would decrease cost.

Since only one cell can be changed at a time, a choice must be made as to which will be brought in as a stepping stone. To choose the one that will bring us the greatest reduction in cost, a check is made to see for which cell the value is greater when the calculation $r + c - t_{ij}$ is made. For x_{21}, the value is $(4 + 0 - 2)$ or 2; for x_{31} it is $(10 + 0 - 4)$ or 6. Therefore the choice is the latter, x_{31}.

The next step is to determine how x_{31} was checked previously in (G). Repeating the movement path for the check on x_{31}, it shows: x_{34} to x_{44} to x_{41}. The stepping-stone cells in this movement path are next assigned plus and minus alternately starting with plus. In this particular case, x_{34} is assigned plus, x_{44}, minus, and x_{41}, plus. Since the values in these cells are 5, 15, and 20 respectively, they are thought of as $+5$ -15, and $+20$. Now, the value that is moved to the new stepping-stone cell (x_{31}) is the smallest plus value, or in this instance the 5 from cell x_{34}. Therefore, x_{31} becomes a new stepping-stone cell and x_{34} is eliminated and now becomes a cell that is not a stepping stone. Then, in order that the new matrix will meet the requirements of (B) and (C), each of the other quantities in cells x_{44} and x_{41} must be adjusted. A rule to follow is that for each square that was assigned a plus value, the quantity moved (in this case 5) should be subtracted and for each cell assigned a minus, the quantity moved should be added. In this case, then, the quantity in x_{44} becomes 20 and the quantity in x_{41}, 15.

The reduction in total cost is computed by the formula, $r_3 + c_1 - t_{31}$ times amount moved (5). Thus $10 + 0 - 4$ times $5 = \$30$. Total cost of the first approximation was \$160. Since the cost was reduced \$30, the total cost has been reduced to \$130.

The new matrix is as follows:

(K)

r ↓ \ c →	0	-4	-4	-6	P_i ↓
0	[0] (10)	[2]	[4]	[6]	10
4	[2]	[0] (15)	[6]	[2]	15
4	[4] (5)	[6]	[0] (25)	[4]	30
6	[6] (15)	[2] (10)	[4]	[0] (20)	45
U_j →	30	25	25	20	100

Total cost check: (10 x 0) ≠ (15 x 0) ≠ (25 x 0) ≠ (5 x 4) ≠ (15 x 6) ≠ (10 x 2) ≠ (20 x 0) = 0 ≠ 0 ≠ 0 ≠ 20 ≠ 90 ≠ 20 ≠ 0 = <u>$130</u>.

From this point, operations already performed are merely repeated. First, new marginal row and column values are computed. This is necessary because they are computed with use of stepping-stone cells and in the new approximation, one stepping stone has been eliminated and a new one added. New marginal values are shown in (K) above.

If the process of (J) above is repeated, it is found that only for x_{21} is the r ≠ c value greater than the transportation rate (4 ≠ 0 ≫ 2). Again, assigning alternate plus and minus values to the movement path (G): x_{22} (15) if plus; x_{42} (10) is minus; and x_{41} (15) is plus. In this case, there are two cells in the movement path that tie for being the smallest plus value (x_{22} and x_{41}, both are ≠ 15). Either one can be eliminated. In this case, x_{41} is the better choice. This is because x_{22} will be able to remain in the matrix and t_{22} equals zero. [15/]

After adjusting the movement path cells by 15, x_{21} has a value of 15, x_{22} has a value of zero, x_{42} has a value of 25, and x_{41} is eliminated. It should be noted that the value zero in x_{22} is a part of the matrix or constitutes one of the seven cells in the solution. Although it means the same with regard to movement of product as an empty cell, it is different in that it is a part of the matrix and a stepping stone.

At all times, the number of original stepping-stone cells in the basic solution (in this case, seven) must be retained. It is possible to remove only one chosen cell (in this case, x_{41}) with each new approximation. Thus, any other cells that become zero after the adjustments on the movement path are made, must remain as a stepping-stone cell and part of the basic matrix.

To compute further reduction in costs: $r_2 + c_1$ minus t_{21} times $15 = 4 + 0 - 2 \times 15 = \$30$. Cost of second approximation was \$130; less \$30 = \$100, cost of third approximation. The new matrix is in (L) below.

(L)

r ↓ \ c →	0	-2	-4	-4	↓ P_i
0	[0] (10)	[2]	[4]	[6]	10
2	[2] (15)	[0] (0)	[6]	[2]	15
4	[4] (5)	[6]	[0] (25)	[4]	30
4	[6]	[2] (25)	[4]	[0] (20)	45
U_j →	30	25	25	20	100

Total cost check: $(10 \times 0) + (15 \times 2) + (5 \times 4) + (0 \times 6) + (25 \times 2) + (25 \times 0) + (20 \times 0) = 0 + 30 + 20 + 0 + 50 + 0 + 0 =$ <u>\$100</u>.

New marginal values are shown in (L) above. In checking the empty cells, it is found that in each case $r + c$ is equal to or less than the transportation rate. Thus, the optimum solution has been reached.

<u>Comments</u>

In solving a problem, care should be taken to avoid mistakes that can be costly with regard to time. A mistake has been made if any of the following occurs: (1) more than one movement path can be found in checking any cell; (2) more than one value can be computed for any of the marginal values; (3) the formula ($r + c - t_{ij}$ times amount moved) gives an amount for reduction in cost such that when subtracted from the total cost of the previous approximation it does not equal $\sum x_{ij} t_{ij}$; (4) when $r + c - t_{ij}$ does not equal zero for a stepping-stone cell; or (5) at any time when the following relationships are violated: (a) the sum of x_{ij}'s in any row do not equal P_i or the sum of x_{ij}'s in any column do not equal U_j, (b) $\sum_{i=1}^{n} P_i = \sum_{j=1}^{m} U_j = \sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij}$.

Errors of types (1), (2), and (5) above are serious. The other types of errors will tend to wash out as succeeding approximations are made. However, for the final or optimal solution, all these checks should be made.

The maximum transportation cost or a profit maximizing scheme can be solved with this model. To solve for a maximum necessitates eliminating cells for the opposite reason. For example, in (J) above, cells should be eliminated until the $r + c$ value is always <u>greater</u> than or equal to the t_{ij} value of the cell being checked. Although it seems absurd to compute a maximum cost of transportation, in some instances it might be valuable to know the range between the minimum and maximum cost.

Footnotes

1. Purdue Journal Paper No. 1013, approved for publication August 8, 1956. **In press for Journal of Farm Economics.**
2. The authors are especially indebted to their colleague at Purdue University, A. Charnes, who has been instrumental in developing the method illustrated in this article. His helpful suggestions on the manuscript are also appreciated. Acknowledgment is also due Earl W. Kehrberg of the Dept. of Agricultural Economics, Purdue University for his review of the manuscript and worthwhile suggestions.
3. This distinction between the "general" and "transportation-problem" procedures is made explicitly by Alexander Henderson and Robert Schlaifer, "Mathematical Programming-Better Information for Better Decision Making", Harvard Business Review, May-June, 1954, pp 94-100.
4. Ibid., p. 98.
5. Henderson and Schlaifer, loc. cit., the authors illustrate some of these working differences with actual problems.
6. T.C. Koopmans, "Optimum Utilization of the Transportation System", Econometrica, Vol. 17, Suppl. (July 1949) pp. 136-46.
7. P.A. Samuelson, "Spatial Price Equilibrium and Linear Programming", The American Economic Review, Vol. 42, No. 3, June 1952, pp. 283-303.
8. George G. Judge, Competitive Position of the Connecticut Poultry Industry - A Spatial Equilibrium Model for Eggs, Storrs Agr. Expt. Station Bulletin 318, University of Connecticut, January 1956.
9. M.M. Snodgrass, "Linear Programming Approach to Optimum Resource Use in Dairying" unpublished Ph.D. Thesis, Purdue University, August 1956.
10. Reported to the authors in letter of June 27, 1955.
11. The authors found that a matrix 24 by 24 could be handled by hand without too much difficulty. However, if the researcher has little idea

as to the general nature of the optimum solution, a 24 x 24 matrix would be extremely time consuming to solve by hand.

12. E.O. Heady, "Simplified Presentation and Logical Aspects of Linear Programming Technique," Journal of Farm Economics, Dec., 1954 pp. 1035-1048 and J.M. Boles "Linear Programming and Farm Management Analysis", Journal of Farm Economics, Feb., 1955 pp. 1-24.

13. General discussions of this procedure are outlined in Henderson and Schlaifer op. cit., pp 73-100 and Robert O. Ferguson "Linear Programming", American Machinist, April 11, 1955 pp. 121-136.

14. For purposes of illustration, a matrix of 4 x 4 will be formulated including all four states as both producing and consuming states. However, since two are surplus and two are deficit states, a matrix of 2 x 2 is all that would be necessary to solve the problem.

15. In larger matrices, the right choice made here may save considerable time. It pays to scan the row and column which pass through the cell in question to see if a larger saving can be made by eliminating another cell. With the selection of the best choice, some of the other possibilities may be eliminated.