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ESTIMATION OF CROP WATER RESPONSE AND ECONOMIC OPTIMIZATION FOR CONTINUOUSLY GROWING CROPS*

by

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ABSTRACT

Further insight is provided regarding an appropriate model of crop-water response for continuously growing crops. A dated input production function was estimated for pineapple oranges in Florida. Generalized least squares regression techniques were used to ameliorate the effects of contemporaneously and serially correlated errors. The continuously growing citrus crops also lead to factorially determined production relations over time. This causes the economic optimum level of water to apply in any given time period to be a function of water applied in the past and expectations for the future.

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Models of biological systems are often a prerequisite to sound economic analyses. The economic researcher must often develop these models, but is sometimes chastised for failure to adequately model the biological system [3, p. 826; 5, p. 660]. Despite this, agricultural economists have been leaders in the modeling effort, especially with respect to crop and livestock production systems. Much of this literature has been reviewed by Woodworth [18], with crop-water response relations being of particular concern in this paper.

The overall purpose of this paper is to provide further insight into appropriate models of crop-water response, in particular as this response varies with growth stages in a continuously growing crop. The data analysis was performed using generalized least-squares regression techniques to correct for the effects of contemporaneously and serially correlated errors. The optimization procedure is also illustrated for this factorially determined (with respect to time) case, using the theoretical model suggested by Frisch [4].

Recent Crop Water Response Analysis

Major journals and other articles in the crop, soil, agronomic, engineering, and economic sciences for the period 1960-78 were reviewed. Several approaches have been proposed and used in this recent literature for modeling plant-soil-water relations. There appear to be two general categories. One group of researchers advocates relating yield to the available soil water (usually some measure of soil water tension), while another group generally takes the approach of relating yield to the evapotranspiration (ET) or relative ET of the plant. These two groups are referred to herein as the soil water (WS) group and the plant water (WP) group, respectively.

In the WS approach, the plant is thought to respond directly to soil-water conditions, with articles by Moore and Beringer as excellent examples[1] of this approach. More recently, however, the approach of the WP group has dominated the literature (see for example, [8], [17], and [5]).[2] In general, the production function estimation approach taken by many economists (see for example, [10], [6], and [19]) also fits the WP category. In this respect, the economic literature is replete with attempts to relate yield to "water applied" (from irrigation and rainfall), where water applied is used as a proxy for ET.

A discussion of the relative advantages and disadvantages of the WS versus the WP approaches is beyond the scope of this paper. It appears, however, that the WP approach is becoming more generally accepted across the several disciplines concerned with crop-water research. Suffice it to say, the WP approach is further examined herein. In particular, insight is provided regarding an appropriate dated input model.

The three most frequently referenced dated input models (where ET is estimated for smaller time periods within a single cropping season) are the Hiler and Clark model [7], the Minhas model [12] and the Jensen model [9]. Howell and Hiler found these models to yield essentially the same results [8, p. 873]. Accepting this as a working hypothesis, the Jensen model (with some modification) is used in this paper. This model has the form

$$\frac{q_t}{q_o} = \prod_{i=1}^{n} \left(\frac{ET_{it}}{ET_{oit}}\right)^{\lambda_i} \qquad (1)$$

where q_t and ET_{it} represent the actual yield in time t and actual ET in growth stage i, respectively, and λ_i is the "sensitivity coefficient" [8]. The variables q_o and ET_{oit} represent the maximum potential yield and ET. As noted in [14, p. 556], the Jensen model is analagous to the Cobb-Douglas form often used in economic models.

Intra-seasonal Crop Growth and Economic Relations

The Jensen model represents a "short-run" production relation where, with the exception of the water variable, ceterus paribus conditions prevail. Hence, the model is best suited for estimation over limited geographic areas. This model also derives its primary usefulness from the need for firm-level decision making within a season (as only the short run demand or marginal value product for water can be determined). The resulting model may have limited usefulness for area-wide water management, however, unless the ceterus paribus conditions describe a truely representative firm in the area.

Model Formulation and Data Sources

A total of four citrus crop response functions were generated in this study. The results for one crop, pineapple oranges, are presented here. Similar results were obtained for all crops. The particular functional form used to represent the process for pineapple oranges was

$$q_{jt} = AW_{1jt-1}^{\alpha_1} W_{2jt-1}^{\alpha_2} W_{3jt-1}^{\alpha_3} W_{4jt-1}^{\alpha_4} W_{5jt}^{\alpha_5} D_1^{\gamma_1} D_2^{\gamma_2} \varepsilon_{jt} \qquad (2)$$

where q_{jt} = yield of oranges measured in pounds solids. In each case the W_{ijt} variable defines the amount of irrigation water applied plus the effective rainfall received during growth stage period i on plot j during year t.[3] The growth stage periods and "dummy" variables were defined as:[4]

W_{1jt-1} = January, February, and March,

W_{2jt-1} = April and May,

W_{3jt-1} = June, July, August and September,

W_{4jt-1} = October, November and December,

W_{5jt} = January of the harvest year, the last month before harvest

D_1 = 2.716 ... before the major freeze which occurred during the 1964-65 crop year; D_1 = 1 otherwise, and

D_2 = 2.716 ... in years of early harvest; D_2 = 1 otherwise.

The D_2 variable was necessary because the harvest date varied as much as two months.

Absolute yield and water levels are included in Equation (2) as compared to the relative magnitudes recommended by Jensen. This was due to the difficulty in establishing ET_{io} and q_o. The level of q_o could be taken to be the maximum yield for the experiment; however, there is no easily defensible manner for arriving at ET_{oi}, short of empirical measurement. Use of absolute levels poses no particular problems except in generalization of results to other areas.

The data used in this study was derived from experimental trials conducted at Lake Alfred, Florida over the period 1961-68 [11]. Actual precipitation and temperature records were used to calculate effective rainfall using procedures outlined by the Soil Conservation Service [16].

It is assumed that little or no runoff or deep percolation occurred in the test plots, giving rise to the contention that W_i is a reasonable measure of ET_i. To the extent that these assumptions are unfulfilled, ET_i will be measured by W_i with error. The soil is a deep, highly permeable sand, however, which would tend to result in negligible runoff. Irrigation was set at 2 acre inches [11, p. 1]. The soil was found to be capable of holding 3.1 inches in the top 5 feet [11, p. 1]. Thus, it is further expected that irrigations did not contribute significantly to deep percolation.

It was expected the data would exhibit both contemporaneously correlated (cross-sectional) and serially correlated errors. With respect to cross-sectional correlation, the data were derived from 12 plots divided among 4 treatments, with each plot subjected to ceterus paribus conditions. Despite this, it was recognized that plots would differ with respect to location, slope, soil content, etc. in a predictable fashion. Thus, no two plots were expected to be independent (within a time period). Serial correlation of the error was also expected for each plot. Because citrus trees continue to develop over time, output at any point in time is a function of the trees relative maturity and previous output. There is a "biological carry-over" affecting output over time. This time period was assumed to be one year, implying first-order auto-regressive error structure within plots.

The hypothesis of correlated errors was tested by comparison of the results obtained by ordinary least-squares regression (OLS) with those results obtained using the Park's method and generalized least-squares (GLS) [15]. Briefly, the Park's method first corrects for first-order serial correlation of the errors within each plot and then corrects for contemporaneous correlation of the errors among plots within any given year.

Empirical Results

The estimated coefficients and t-test statistics for both the OLS and the GLS are presented in Table 1. The $R^2 = 0.61$ for the OLS equation. This is a significant divergence from 1.0 and probably indicates ceterus paribus conditions did not hold over the study years. Generally, the t-statistics indicate significant levels in acceptable ranges, lending further credance to the Jensen model form. Corrections for contemporaneous and serial correlation in the errors reduced variances of the estimates, resulting in improved t-statistics and more efficient estimates of the regression coefficients. The most dramatic change was in variable $\ln W_{4t-1}$ where the t-statistic

Table 1. Pineapple orange water response estimation, Lake Alfred, Florida 1961-68

Variable[a]	OLS		GLS	
	Estimate	t	Estimate	t
Constant	5.350	9.740	5.334	22.320
$\ln W_{1t-1}$	-0.938	-3.629	-0.988	-7.595
$\ln W_{2t-1}$	0.680	4.379	0.819	9.174
$\ln W_{3t-1}$	0.850	4.348	0.804	9.193
$\ln W_{4t-1}$	-0.150	-1.072	-0.190	-5.959
$\ln W_{5t}$	-0.584	-4.598	-0.671	-7.479
$\ln D_1$	0.576	4.610	0.662	5.893
$\ln D_2$	-10.707	-4.487	-12.310	-7.462

[a]Variables defined as (ln=natural log): W_{1t-1}=water (January, February and March); W_{2t-1}=water (April and May); W_{3t-1}= water (June, July, August and September); W_{4t-1}=water (October, November and December); W_{5t}=water (January, immediately before harvest in late harvest years); $\ln D_1$=1 before the freeze, 0 otherwise; $\ln D_2$=1 for early harvest, 0 otherwise.

improved from -1.072 to -5.959 (Table 1). Thus, more confidence (in a statistical sense) can be placed in the GLS regression coefficients and the hypothesis was not rejected.

The need to include dated inputs in production relations for citrus crops is supported by the wide divergence in estimated values for the regression coefficients. The marginal product (MP) of water is, indeed, variable across growth stages. This has obvious significance for economic optimization. Also, the coefficient for $\ln W_{5t}$ was found to be significant, indicating there are inter-temporal connections and affects on yields. That is, W_{5t} affects q_t but also affects q_{t+1} (as W_{5t} is a part of W_{1t}).

Economic Optimization

The response relations in the Jensen model are such that the MP in any growth stage is related to the MP in other growth stages. Consider the case for the MP of W_{2t-1} (from the GLS estimates):

$$MP_{2t-1} = \frac{\partial q_t}{\partial W_{2t-1}} = \frac{0.819 q_t}{W_{2t-1}} \qquad (3)$$

Variable q_t is a function of all water variables. Thus, the economic optimum amount of W_{2t-1} is a function of past and future water levels.

The optimization process is complicated by the time interconnectedness of the yield response to water in stage 5, which is a part of stage 1 for the following years crop. Using the Frisch model [4, pp. 269-281], this can be described as a "factorially determined production process" with respect to the W_{5t} and W_{1t} variables. The tree allocates the water applied during growth stages 5 and 1 between yield in t and (t+1). This "internal" allocation process is beyond the direct control of the manager.[5]

The factorially determined relationship (over time) nessitates simultaneous maximization of profit over the life of the tree, as the optimum level of W_{5t} is affected by W_{1t}. The optimum level of W_{1t} (in turn) is affected by the level of W_{5t+1}. This interrelation continues over the life (L) of the tree. The total profit (Π) over L becomes

$$\Pi = \sum_{t=1}^{L} p_t q_t - \sum_{t=1}^{L} \sum_{i=1}^{4} r_{it-1} W_{it-1} - \sum_{t-1}^{L} r_{5t} W_{5t} \qquad (4)$$

where, the variables are defined as before, except

p_t = price of oranges (in price/pound solids) in t, and

r_{it} = price (and/or cost) of water during stage i and time t.

The resultant first order conditions for any given year t are:

$$\frac{\partial \Pi_t}{\partial W_{1t-1}} = p_t \frac{\partial q_t}{\partial W_{1t-1}} - r_{1t-1} = 0 \tag{5}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\frac{\partial \Pi_t}{\partial W_{4t-1}} = p_t \frac{\partial q_t}{\partial W_{4t-1}} - r_{4t-1} = 0 \tag{6}$$

$$\frac{\partial \Pi_t}{\partial W_{5t}} = p_t \frac{\partial q_t}{\partial W_{5t}} + p_{t+1} \frac{\partial q_{t+1}}{\partial W_{5t}} - r_{5t} = 0 \tag{7}$$

There are (5xL) such equations to be solved simultaneously to define the optimum levels of water. This could be accomplished only under conditions of perfect knowledge regarding prices, both present and future, which is not likely to represent the "real world" situation. The practical difficulties involved in using the results from the simultaneous solution of (5xL) equations are obviously nearly insurmountable when L is large (such as for a tree with a 40-year life).

Another interpretation of the optimization process is also possible, however, which reduces the problem to manageable proportions. While this process requires consideration of all future (and past) water application levels and prices, the problem can also be stated in the context of "expectations". At any given point in time the producer has certain expectations about future price and water availability conditions. In Florida, for example, water permits are assigned for 20 years [2]. Except for short term (intra-crop year) shortages, the producer is guaranteed a quantity of water over that period. This institutional arrangement, plus the producers own expectations regarding future short term drought conditions, could allow calculation of the appropriate probabilities regarding water availability. Of course, expectations regarding r_t and p_t would also have to be established.

Interestingly, because of the time interconnection in Equation (7) the extent of resource misallocation in time period t attributable to incorrect expectations of future periods could then be directly examined. Stated more explicitly, the magnitude of $p_{t+1}\frac{\partial q_{t+1}}{\partial W_{5t}}$ (which affects resource allocation in t) is a function of the producers expectations regarding future prices and water availability in periods beyond t. Various scenarios regarding the future could then be used to test for the sensitivity and affects on current resource allocation.

Conclusions

The following appear apropos:

1. Contemporaneous and serially correlated errors may prevail in crop-water response experimental data; the Park's method can be used to improve the efficiency of the estimators.

2. The economic optimization process for continuously growing crops may require consideration for expectations regarding future growth and economic conditions.

Researchers must continue to improve on crop-water models. The Jensen model, with some modification, should be considered more in dated input response analysis.

FOOTNOTES

[1]Only after examples are referenced due to space limitations.

[2]Over 40 articles were reviewed. Nearly all of the articles having actual, estimated production function equations were in the WP group.

[3]The crop year is defined to include (t-1) and t, with harvest in t.

[4]Crop growth stages cannot be defined exactly over time because variations in climatic conditions. The above is representative, however, of time periods for major parts of the growth cycle.

[5]This statement is true short of using a different rootstock for the pineapple variety, which may allocate the water differently.

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