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The Agricultural Polish Technology and Output Mix Adjustments Due to Transition: a Distance Function Approach Using a Restricted Generalised Maximum Entropy Estimator

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Contribution appeared in Arfini, F. (Ed.) (2005) “Modelling Agricultural Policies: State of the Art and New Challenges”, proceedings of the 89th EAAE Seminar, pp. 440 - 457

February 2-5, 2005

Parma, Italy



**UNIVERSITA' DEGLI
STUDI DI PARMA**

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Axel Tonini* and Roel Jongeneel**

Abstract

In this paper an output distance function framework is exploited to analyse the agricultural Polish technology and output mix adjustments due to transition. In order to overcome the limited available data the output distance function is estimated by Generalized Maximum Entropy (GME) for the 1991-2001 time period. Besides the sample data, the estimation procedure exploits prior information e.g. several regularity conditions and a 'weak revenue maximization' assumption. We find that after transition the implicit share of milk and beef and veal output declines over time whereas cereals output and potatoes and rapeseeds and sugar beets output increases. Our estimates suggest a clear complementarity relationship between chicken and pig meats output and cereals output. The EU accession resulted in a relative increase of cow milk/beef and veal meats output over chicken and pig meats output, and in a relative increase of cereals output over potatoes, rapeseeds and sugar beets output.

JEL classification: C14; Q1.

Keywords: Distance Function, Generalized Maximum Entropy, Poland, Prior Information, Transition.

1. Introduction

Data availability and the related reliability are one of the major limitations in modelling the agricultural sector of CEECs. Reliable information, especially with respect to prices, is only sparsely available and when available frequently affected by measurement errors partly due to the different data collecting systems and the way in which data were generated. The empirical

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evidence shows that in CEECs consumption and production have been hugely overstated before the nineties and partly understated afterward (Hallam, 1998). Therefore when modelling CEECs it is important to select an opportune methodology in order to deal with a poor data environment.

The focus of this paper is on providing an empirical framework able to analyze multi-output agricultural technologies when data are sparsely available requiring minimal assumptions. In doing so we exploit for Poland a distance function approach which is estimated relying on a Generalized Maximum Entropy (GME) econometric approach. The GME estimator has the advantage that it can handle ill-posed estimation problems (lack of degrees of freedom) and allows for the imposition of several theoretical constraints and other sources of prior information following from assuming a form of ‘weak revenue maximization’. The paper models the agricultural technology and output mix allocation of Poland after transition and also retrieves important substitutability relationships between output variables. Additionally, several economic conjectures are made with respect to the effect of the Polish EU entry on the output mix allocation.

The paper is divided in five sections. In Section 2 the paper proceeds by introducing the theoretical model. In Section 3 the empirical model and estimation are treated. Section 4 discusses results. Finally the paper closes with conclusions and qualifications.

2. Theoretical model

This section begins by introducing the main theoretical advantages of a distance function approach for our analysis and then moves to introduce it theoretically presenting the most important relationships exploited in the analysis. Distance function approaches have been already applied in several empirical studies particularly for efficiency and productivity analysis (see Coelli and Perelman, 1996; Morrison-Paul and Johnston, 2002; Morrison-Paul, Johnston et al., 2002; among others). Still the application of a distance function approach may be more generally exploited to handle multi-output technologies especially when the below listed advantages are relevant (see for further insights Coelli, 2002). First, a distance function approach easily allows multi-output and multi-input representation of technology. The typical single output production function cannot theoretically handle the possibility in which a production unit produces more than one output, constituting an important drawback when modelling multi-outputs in the agricultural sector (i.e. livestock production and arable production). This has been usually solved, without free costs, by aggregating outputs or by recurring to a dual profit function framework. The former fails to represent important relations between multi-outputs and suffers from aggregation problems, whereas the latter requires price information for both output and input variables. Reliable price information is only to a limited extent available for the CEECs and is frequently of unsure quality. Second, an output (input) distance function solves the simultaneous equation bias related to output (input) variables, because the output (input) variables are treated in a ratio-form and therefore assumed to be exogenous and constant across observations. As known the distance function considers a radial expansion of out-

puts given certain levels of inputs (a similar argument holds for the input variables when an input orientation is defined). Third, a distance function does not a-priori require any price information, since it only relies on physical quantities (output and input quantities) without necessarily requiring the imposition of strict behavioural assumptions (e.g. profit maximization) that may be questioned for CEECs.

Following Morrison-Paul, Johnston et al. (2002) we define the output set $P(\mathbf{x})$ that represents the locus of all feasible output vectors $\mathbf{y} \in \mathfrak{R}_+^M$ given an input vector $\mathbf{x} \in \mathfrak{R}_+^K$ so that $P(\mathbf{x})$ can be written formally as $P(\mathbf{x}) = \{ \mathbf{y} \in \mathfrak{R}_+^M : \mathbf{x} \text{ can produce } \mathbf{y} \}$. An output distance function is defined as

$$D_o(\mathbf{x}, \mathbf{y}) = \min \{ \theta : (\mathbf{y}/\theta) \in P(\mathbf{x}) \} \quad (1)$$

where the subscript “o” indicates an output-oriented distance function and θ a scalar representing the maximum output expansion. In the output orientation $D_o(\mathbf{x}, \mathbf{y}) \leq 1$ if $\mathbf{y} \in P(\mathbf{x})$ and $D_o(\mathbf{x}, \mathbf{y}) = 1$ if \mathbf{y} is on the outer boundary of $P(\mathbf{x})$, underlining that the production unit is operating on the production surface. An output distance function has to respect several regularity conditions, it has to be homogeneous in output, convex in outputs, non-decreasing in outputs, quasi convex in inputs and non-increasing in inputs (Shepard, 1970).

Färe (1988) and Färe and Grosskopf (1990) define the revenue-deflated relative output shadow prices, exploiting a distance-function-oriented Shepard’s lemma referring to the output distance function duality with the revenue function, as given by

$$r_g^*(\mathbf{x}, \mathbf{y}) = D_{o,g}(\mathbf{x}, \mathbf{y}) \quad (2)$$

where the subscript g indicates the partial derivative of the output distance function with respect to output g . The ratios of these revenue-deflated relative output shadow prices represent the slope of the production possibility curve (PPC) and therefore the marginal rate of transformation (MRT) between two outputs as given by $MRT_{g,h} = (r_g^*/r_h^*)$. The $MRT_{g,b}$ can be compared in case of revenue maximization to the output price ratio p_g/p_h . As risen by Grosskopf, Margaritis et al. (1995) since the $MRT_{g,b}$ between two outputs will not be invariant to the choice of the output ratio a more interpretable measure of the $MRT_{g,b}$ can be derived by normalizing the $MRT_{g,b}$ by the relative output ratio given by

$$sub_{g,h} = D_{o,g}(\mathbf{x}, \mathbf{y})/D_{o,h}(\mathbf{x}, \mathbf{y}) \cdot y_g/y_h = (r_g^*/r_h^*) \cdot y_g/y_h = \varepsilon_{D_{o,g}}/\varepsilon_{D_{o,h}} \quad (3)$$

where $sub_{g,b}$ represents the ratio of the $MRT_{g,b}$ relative to the output mix, hence a normalized (i.e. unit less) marginal rate of transformation and $\varepsilon_{D_{o,g}}$, $\varepsilon_{D_{o,h}}$ the output distance function elasticity with respect to output g and h respectively. When $sub_{g,b} > 1$ it is difficult to move away from output g indicating that the outputs g and h are difficult to substitute, the opposite holds when $sub_{g,b} < 1$. Therefore from an output distance function it is possible to retrieve output substitutability measures and output composition changes by only relying on simple relation-

ships. Additionally the $sub_{g,h}$ ratio can be directly compared to the relative revenue values $RV_g/RV_h = p_g y_g / p_h y_h$ in order to evaluate discrepancies from revenue maximization (i.e. allocative inefficiency).

Additional information can be recovered with respect to the PPC curvature or output substitutability via the distance function Morishima elasticity (Blackorby and Russell, 1989) that explains the degree of substitutability through relative shadow values as given by

$$\begin{aligned} M_{g,h} &= -d \ln(D_{O,g}/D_{O,h})/d \ln(y_g/y_h) = y_g \cdot [(D_{O,gh}/D_{O,h}) - (D_{O,gg}/D_{O,g})] \\ &= (y_g/D_{O,h}) \cdot \partial D_{O,h} / \partial y_g - (y_g/D_{O,g}) \cdot \partial D_{O,g} / \partial y_g \end{aligned} \quad (4)$$

where the subscripts indicate first and second order partial derivatives of the output distance function. The Morishima substitution elasticity $M_{g,h}$ represents the relative adaptation of r_h^* and r_g^* to a change in y_g given y_h . These elasticities cannot be symmetric because they depend on the input which is held fixed. Similar measures (i.e. marginal rate of technical substitution and input Morishima substitution elasticities) can also be recovered in a similar way for the input variables.

3. Empirical model and estimation

This section introduces the empirical model and its estimation. The most encountered functional forms for the distance function are the so-called Cobb-Douglas and transcendental logarithmic (abbreviated in translog). Since this study has to deal with a small number of observations we specify a restricted translog output distance function. Second order terms including squared and cross terms for the output variables are kept in order to gain flexibility and recover time varying statistics about the output mix composition, which is one of the main focuses of the paper. All second order terms for the input variables are restricted to zero, this result in having the following specification with G outputs and K inputs

$$\ln D_{O_i} = \alpha_0 + \sum_{g=1}^4 \alpha_g \ln y_{gi} + 0.5 \sum_{g=1}^4 \sum_{h=1}^4 \alpha_{gh} \ln y_{gi} \ln y_{hi} + \sum_{k=1}^4 \beta_k \ln x_{ki} + e_i \quad (5)$$

where i represents the i^{th} observation in the sample and e_i is a random error. Data are based on FAO statistics (FAO, 2004) and the observation period is 1991-2001. Hallam (1998) documents that measurement errors in production before transition are frequent because during the Communist regime no allowance was made for losses, intermediate consumptions, stocks, etc. However they are known to constitute an important component, estimated for several CEECs at percentages up till 30 per cent. The selected reference period takes into account only the after-transition period and as such avoids the measurement errors caused by the central plan-

ning regime. Four outputs (cow milk/beef and veal meats (y_1), chicken and pig meats (y_2), cereals (y_3), potatoes and rapeseeds and sugar beets (y_4)) and four inputs (fertilisers (x_1), labour (x_2), livestock (x_3) and machinery (x_4)) are distinguished. The aggregate output variables are selected according to their importance in the agricultural production value of Poland and are obtained with a discrete approximation to the Divisia index (Tornqvist or translog) (SHAZAM, 2004) with base year fixed for 1998. Cereal output is obtained from the aggregation of barley, mixed grain, rye and wheat. Livestock input is obtained from the aggregation in livestock unit (LU) of cattle, chickens and pigs using opportune conversion factors (Hayami and Ruttan, 1985:450). Fertilizers input is defined as quantity of aggregate nitrogenous, potash and phosphate fertilizers of plant nutrient consumed in agriculture. Labor input is measured as the economically active population in agriculture, i.e. people engaged in or seeking work in agriculture, hunting, fishing or forestry. Machinery is measured in number of tractors in use. Land input, also available from FAO statistics, was not included because of its low informative power related to its small sample variance.

In order to impose the required linear homogeneity in outputs, the following set of restrictions in the output distance function equation (5) must apply

$$\sum_{g=1}^4 \alpha_g = 1, \quad \sum_{h=1}^4 \alpha_{gh} = 0 \quad (6)$$

Another common possibility imposing the linear homogeneity restriction in equation (5) follows by noting that

$$D_0(\mathbf{x}, \xi \mathbf{y}) = \xi D_0(\mathbf{x}, \mathbf{y}) \quad (7)$$

Therefore by simply selecting a g_{th} output (y_1) in the output set G and substituting for $\xi = 1/y_1$ into (7) we get

$$D_0(\mathbf{x}, \mathbf{y}/y_1) = D_0(\mathbf{x}, \mathbf{y})/y_1 \quad (8)$$

Applying (8) to equation (5) it leads the following specification

$$\ln(D_{0i}/y_{1i}) = \alpha_0 + \sum_{g=1}^3 \alpha_g \ln y_{gi}^* + 0.5 \sum_{g=1}^3 \sum_{h=1}^3 \alpha_{gh} \ln y_{gi}^* \ln y_{hi}^* + \sum_{k=1}^4 \beta_k \ln x_{ki} + e_i \quad (9)$$

where $y_{gi}^* = y_{gi}/y_{1i}$. From that it follows that with homogeneity in output imposed the summation sign over g implies now only summation over three outputs. Using the logarithmic property, equation (5) can be rewritten as

$$\ln(D_{0i}) - \ln(y_{1i}) = \alpha_0 + \sum_{g=1}^3 \alpha_g \ln y_{gi}^* + 0.5 \sum_{g=1}^3 \sum_{h=1}^3 \alpha_{gh} \ln y_{gi}^* \ln y_{hi}^* + \sum_{k=1}^6 \beta_k \ln x_{ki} + e_i \quad (10)$$

or

$$-\ln(y_{li}) = \alpha_0 + \sum_{g=1}^3 \alpha_g \ln y_{gi}^* + 0.5 \sum_{g=1}^3 \sum_{h=1}^3 \alpha_{gh} \ln y_{gi}^* \ln y_{hi}^* + \sum_{k=1}^6 \beta_k \ln x_{ki} - \ln(D_{0i}) + e_i \quad (11)$$

Symmetry in the output cross effects was also imposed by setting

$$\alpha_{gh} = \alpha_{hg}, \quad g, h = 1, \dots, 3 \quad (12)$$

Additionally linear homogeneity in inputs was also tested for and imposed during estimation by adding the following restriction on parameters

$$\sum_{k=1}^6 \beta_k = 1, \quad k = 1, \dots, 6 \quad (13)$$

The contemporaneous linear homogeneity in output and input variables implies the implicit imposition of the hypothesis of constant return to scale (CRS). This is a frequently maintained hypothesis for country level analysis, where the interpretation of variable return to scale (VRS) has no sound interpretation.

Since the obtained estimates have to provide a consistent interpretation with the expected sign of conventional intermediate production theory, the negative sign on the dependent variable (y_{li}) during the estimation can be ignored in order to have a more convenient way to assess the model, making the estimates more comparable to traditional production function models. This reverses the sign of the estimated coefficients without affecting the overall results (Coelli and Perelman, 1996).

During estimation we assumed technical efficiency assuring that at country level the observed output mix lies on the PPC of the technology. This results in having $D_{0i} = 1$ (i.e. full technical efficiency) and consequently $\ln(D_{0i}) = 0$. A similar approach was also applied by Grosskopf, Hayes et al. (1995). This assumption is necessary first because only one country and not a cross-section of countries is considered. This makes it impossible to distinguish a frontier, different from the production possibility curve of Poland itself. Secondly the assumption allows us to completely attribute deviations from the maintained 'weak revenue maximization' to allocative inefficiencies.

The restricted translog output distance function in (11) is estimated using a GME approach (Golan, Judge et al., 1996; Mittelhammer, Judge et al., 2000). When models are *ill-posed* and/or *ill-conditioned*, GME constitutes a valuable estimation procedure where the application of traditional econometric methods is infeasible (for example due to an insufficient number of observations). In our case the selected reference period implied that only eleven time series observations are available. In comparison to Bayesian approaches, a GME approach does not use a regularized likelihood function and requires minimal assumptions on the underlying error properties. Moreover the estimation method is relatively easy to implement. Bayesian approaches usually require detailed information about the statistical properties of the parameter vector, information that is often not available. In addition subjective probabilities are required

to regularize the likelihood function. The latter is usually selected for convenience without an empirically based knowledge.

GME is more efficient and robust than traditional econometric approaches when samples are small (Golan, Judge et al., 1996). First GME is an efficient estimator when sample are limited because it considers all the information in the data constraint (11) for each observation rather than to rely only on sample moment conditions. Second GME is a more robust estimator when sample data are limited because of the implicit weighting in the objective function (22) between prediction and precision so that outlying observations have lower impact in the estimations. Additionally estimating technological relationships with aggregate time series data (hard data) frequently bring near exact linear dependencies among the explanatory variables causing the matrix of the explanatory variables to not have a full column rank or a stable inverse matrix (i.e. *ill-conditioned* problem). When problems are *ill-conditioned* Golan, Judge et al. (1996: 135) show through simulation experiments that GME provides more efficient estimates than traditional methods. Moreover one of the most interesting advantages of GME is that it can handle *ill-posed* and *ill-conditioned* problems by taking into account non-sample information (soft data), *viz.* theoretical restrictions on parameters and prior information. The inclusion of non-sample information is attractive because it increases the efficiency of the inference procedure, as well as the reliability of the obtained estimates when problems are *ill-conditioned* (Jongeneel, 2000; Dorfman and McIntosh, 2001).

GME reparameterizes the output distance function model in (11) by defining for each parameter and error a set of support points so that parameters and error are specified in form of probabilities. Considering Y a $N \times (1 + G + (G^2 - G))$ matrix of G output variables including the intercept term α_0 , X a $N \times K$ matrix of input variables, $y_{1i} \in \mathfrak{R}^N$ a $N \times 1$ vector for the numéraire output and $e_i \in \mathfrak{R}^N$ a $N \times 1$ noise vector, the unknown parameters can be re-written in a reparameterized form as

$$\alpha = \mathbf{z}\mathbf{p} = \begin{bmatrix} \mathbf{z}_1^\circ & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{z}_2^\circ & \cdots & \mathbf{0} \\ 0 & 0 & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{z}_{(1+G+(G^2-G))}^\circ \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_{(1+G+(G^2-G))} \end{bmatrix} \quad (14a)$$

$$\beta = \mathbf{z}\mathbf{p} = \begin{bmatrix} \mathbf{z}_1^\circ & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{z}_2^\circ & \cdots & \mathbf{0} \\ 0 & 0 & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{z}_K^\circ \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_K \end{bmatrix} \quad (14b)$$

where, $\alpha \in \mathfrak{R}^N$ is a $(1 + G + (G^2 - G)) \times 1$ vector of unknown output parameters and $\alpha \in \mathfrak{R}^N$ is a $K \times 1$ vector of unknown input parameters, $\mathbf{z}_g = [z_{g1}, z_{g2}, \dots, z_{gM}]$ is a $M \times 1$ vector of

parameter supports such that $z_{g1} < z_{g2} < \dots < z_{gM}$ and M is a fixed integer, $\mathbf{z}_k = [z_{k1}, z_{k2}, \dots, z_{kL}]$ is a $L \times 1$ vector of parameter supports such that $z_{k1} < z_{k2} < \dots < z_{kH}$ and H is a fixed integer, $\mathbf{p}_g = [p_{g1}, p_{g2}, \dots, p_{gM}]$ is a $M \times 1$ vector of unknown probabilities such that $\mathbf{p}_g \in [0,1]$ and $\mathbf{p}_k = [p_{k1}, p_{k2}, \dots, p_{kL}]$ is a $L \times 1$ vector of unknown probabilities such that $\mathbf{p}_k \in [0,1]$. The parameter support bounds are defined in such a way that $\alpha_{g,gh} \in [z_{g1}, z_{gM}]$ and $\beta_k \in [z_{k1}, z_{kL}]$, $\forall g, gh = 1, 2, \dots, (1 + G + (G^2 - G))$ and $\forall k = 1, 2, \dots, K$ where g, gh and h denote parameters. Similarly for the error term we have

$$\boldsymbol{\varepsilon} = \mathbf{v}\mathbf{w} = \begin{bmatrix} \mathbf{v}_1^\circledast & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{v}_2^\circledast & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{v}_N^\circledast \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_N \end{bmatrix} \quad [15]$$

where $\mathbf{v}_i = [v_{i1}, v_{i2}, \dots, v_{ij}]$ is a $J \times 1$ row vector of error supports such that $v_{i1} < v_{i2} < \dots < v_{ij}$ and J is a fixed integer, $\mathbf{w}_i = [w_{i1}, w_{i2}, \dots, w_{ij}]$ is a $J \times 1$ vector of unknown probabilities such that $\mathbf{w}_i \in [0,1]$. The error support bounds are defined in such a way that $e_i \in [v_{i1}, v_{ij}]$, $\forall i = 1, 2, \dots, N$ where i denotes observations. The reparameterized GME estimator may now be written using a marginal probability notation in such a way that the parameters in equation (11) are reparameterized as follows:

$$\alpha_g = \sum_{s=1}^5 z_{gs} [s] p_g [s] \quad g = 0, 1, \dots, 3 \quad (16a)$$

$$\alpha_{gh} = \sum_{s=1}^5 z_{ghs} [s] p_{gh} [s] \quad g, h = 0, 1, \dots, 6 \quad (16b)$$

$$\beta_k = \sum_{s=1}^5 z_{ks} [s] r_k [s] \quad k = 1, \dots, 4 \quad (16c)$$

where z_g, z_{gh} and z_k represent respectively the support space for the output variables, second order output terms and input variables, p_g, p_{gh} and r_k are the convexity weights or unknown probabilities respectively for the output variables, second order output terms and input variables. The error term is also reparameterized as given by

$$e_i = \sum_{s=1}^5 v_i [s] w_i [s] \quad i = 1, \dots, 11 \quad (17)$$

where v_i is the support for the error term and w_i the convexity weights or unknown probabilities for the error term. The number of support points both for the parameters and error supports is equally defined for $S = 5$, since following Golan, Judge et al. (1996:138-140) the greatest empirical improvement in precision is achieved by setting five support points. The z and v

vectors span up a uniform and symmetric parameter support space. The lower bound of the parameter support space is \underline{z}_j and the upper bound \bar{z}_j and for the error term the lower and upper bounds are respectively ν_j and ν_s . The parameter support space should be properly chosen in order to include the real values for $\tilde{\alpha}_g$, $\tilde{\alpha}_{gh}$ and $\tilde{\beta}_k$. Following Golan, Judge et al. (1996:110) the error support space has to be strictly related to the properties of the error term, so that the ‘3 σ -rule’ of Pukelsheim (1994) based on the Chebychev’s inequality is the usual reference when defining the error support space. The ‘3 σ -rule’ requires to define the error support in relation to the estimated sample standard deviation of the dependent variables.

In order to increase the efficiency of the inference procedure and the accuracy of the estimates, prior information is included. First output monotonicity is imposed by centring the expected value of the distance function-based output elasticities in accordance with the imposed output homogeneity condition given by (8). This resulted in having the following constraint

$$\varepsilon_{D_{o,m}} = \varepsilon_{y_{1,g}} = \alpha_g + \sum_h \alpha_{gh} \ln y_{hi} = E\left(\alpha_g + \sum_h \alpha_{gh} \ln y_{hi}\right) + epa_{g,i} \quad (18)$$

where $E(\cdot)$ represent the expected value of the distance function-based output elasticities and the error term is reparameterized as follow

$$epa_{g,i} = \sum_{s=1}^3 vpa_{g,i}[q] wpa_{g,i}[q] \quad (19)$$

where vpa is the support for the error term and wpa the convexity weights or unknown probabilities for the error term. The number of support points for the (epa) prior error is equally defined for $Q = 3$. The support space vpa is consistently defined in order to respect the homogeneity condition as given by (8) so that vpa span up a uniform support space in such way that $-1 < \varepsilon_{y_{1,g}} < 0$. Additionally we allowed a ‘weak revenue maximization’ condition because slight departures from revenue maximization were permitted by incorporating a stochastic term in the output substitutability relationship defined in equation (3). This resulted in exploiting external data (soft data) and adding the following condition during estimation

$$\left(sub_{g,h}\right)_i = \left(r_g^*/r_h^*\right)_i \cdot \left(y_g/y_h\right)_i = \left(\varepsilon_{D_{o,g}}/\varepsilon_{D_{o,h}}\right)_i = p_{gi}y_{gi}/p_{hi}y_{hi} + epb_{g,h/i} \quad (20)$$

where the error term is reparameterized as follow

$$epb_{g,h/i} = \sum_{s=1}^3 vpb_{g,h/i}[x] wpb_{g,h/i}[x] \quad (21)$$

where vpb is the support for the error term and wpb the convexity weights or unknown probabilities for the error term. The number of support points for the (epb) prior error is equally

defined for $X = 3$. In order to not introduce redundant information only the strictly independent relationships are introduced via equation (20).

The model was finally estimated by jointly maximising the cumulative entropy of all probabilities associated with all the parameters (α, β) , the error term (e) and the stochastic priors (epa, epb) as in

$$\begin{aligned}
 H(p, w) = & -\sum_{g=1}^3 \sum_{s=1}^5 p_g[s] \ln p_g[s] - \sum_{gg=1}^6 \sum_{s=1}^5 p_{gh}[s] \ln p_{gh}[s] - \sum_{k=1}^4 \sum_{s=1}^5 r_k[s] \ln r_k[s] + \\
 & - \sum_{i=1}^{11} \sum_{s=1}^5 w_i[s] \ln w_i[s] - \sum_{epa=1}^{22} \sum_{s=1}^3 wpa_{g/i}[x] \ln wpa_{g/i}[x] + \\
 & - \sum_{epb=1}^3 \sum_{s=1}^3 wpb_{g,h/i}[q] \ln wpb_{g,h/i}[q]
 \end{aligned} \tag{22}$$

subject to the data consistency constraint in equation (11) and the prior information in term of additional constraints as given by (18 and 20) reparameterized according to (16a-16c, 17, 19 and 21), and finally adding the following proper probability additive constraints

$$\sum_{s=1}^5 p_g[s] = 1, \quad \forall g \tag{23a}$$

$$\sum_{s=1}^5 p_{gh}[s] = 1, \quad \forall gh \tag{23b}$$

$$\sum_{s=1}^5 r_k[s] = 1, \quad \forall k, \tag{23c}$$

$$\sum_{s=1}^5 w_i[s] = 1, \quad \forall i, \tag{23d}$$

$$\sum_{s=1}^3 wpa_{g/i}[s] = 1, \quad \forall g/i \tag{23e}$$

$$\sum_{s=1}^3 wpb_{g,h/i}[s] = 1, \quad \forall g,h/i \tag{23f}$$

where $p_g, p_{gh}, r_k, w_i, wpa_{g/i}$ and $wpb_{g,h/i}$ should be non-negative. The solution to this problem gives estimated probabilities for $\tilde{p}_g, \tilde{p}_{gh}, \tilde{r}_k, \tilde{w}_i, \tilde{wpa}_{g/i}$ and $\tilde{wpb}_{g,h/i}$ which can be substituted in (16a-16b, 17), and (19, 21) to obtain the parameter estimates $\tilde{\alpha}_0, \tilde{\alpha}_g, \tilde{\alpha}_{gh}, \tilde{\beta}_k$ and error terms $\tilde{e}_i, \tilde{epa}_{g/i}$ and $\tilde{epb}_{g,h/i}$.

The components of the Morishima substitution elasticity $M_{h,g} = \epsilon_{h,g} - \epsilon_{g,g}$ are recovered by exploiting the following relationship from Morrison-Paul, Johnston et al. (2002) according to which

$$A_{h,g} = \partial S_h / \partial \ln y_g = S_h \cdot (\varepsilon_{h,g} - \varepsilon_{y_{1,g}}) = \varepsilon_{y_{1,h}} \cdot (\varepsilon_{h,g} - \varepsilon_{y_{1,g}}) = \partial \varepsilon_{y_{1,h}} / \partial \ln y_g = \alpha_{h,g} \quad (24)$$

and S_g is defined as a revenue share and $\varepsilon_{y_{1,g}}$, $\varepsilon_{y_{1,h}}$ are simply the output distance function elasticities with respect to output h and g , since $\varepsilon_{D_{o,g}} = \partial \ln D_o / \partial \ln y_g = \partial D_o / \partial y_g \cdot (y_g / D_o) = \partial y_1 / \partial y_g \cdot (y_g / y_1) = \varepsilon_{y_{1,g}}$ and $\varepsilon_{y_{1,g}} = \partial \ln y_1 / \partial \ln y_g^* = \partial \ln y_1 / \partial \ln y_g = \partial y_1 / \partial y_g \cdot (y_g / y_1)$. The first part of the Morishima substitution elasticity explains the contribution of a change in y_g to the productivity or shadow valuation of y_h ($\varepsilon_{h,g} = \partial \ln r_h^* / \partial \ln y_g$) capturing composition changes in the output mix. Moreover $\varepsilon_{h,g}$ provides information about net substitutability and complementarity relationships, where $\varepsilon_{h,g} > 0$ indicates net complements and $\varepsilon_{h,g} < 0$ net substitutes (Grosskopf, Hayes et al. 1995:293). The second part of the Morishima substitution elasticity ($\varepsilon_{g,g} = \partial \ln r_g^* / \partial \ln y_g$) explains the impact of a change in $(\ln y_g)^2$ providing information about the curvature of the distance function.

4. Results and Discussion

This section presents the obtained parameter estimates for the output distance function resulting from GME and discuss them. The estimation was carried out using the GAMS (Generalized Algebraic Modelling System) which is a nonlinear-optimization program selecting the PATHNLP solver. Table 1 presents the GME estimates for the restricted translog output distance function.

Table 1. GME estimates for the restricted translog output distance function

α_0	α_1	α_2	α_3	α_4	α_{11}	α_{22}	α_{33}	α_{44}	α_{12}
-0.06834	-0.20186	-0.28587	-0.29565	-0.21662	-0.91010	0.21620	-0.23102	-0.22750	0.10706
α_{13}	α_{14}	α_{23}	α_{24}	α_{34}	β_1	β_2	β_3	β_4	
0.50049	0.30255	-0.25884	-0.06442	-0.01063	0.14783	0.47397	0.26670	0.13150	

Note: The estimated Entropy Value is 87.77580 and $\hat{\sigma}_{GME}^2 = 0.00291$.

Parameters of a translog specification usually have no direct interpretation, however the contributions of an input to overall production can be directly recovered from Table 1. It appears that labor is the single most important input with an input elasticity of about 0.47 (see parameter β_2) followed by livestock at 0.27 (see parameter β_3), by fertilizers at 0.15 (see parameter β_1) and machinery at 0.13 (see parameter β_4). Due to the trending nature of the labor input variable, the labor parameter should be interpreted with care. The labor coefficient may in fact

pick up the downward trend of several other variables and its estimate may be inaccurate. Additionally from Table 1 it is possible to analyze the cross-terms ($\alpha_{g,b}$) which explain the contribution of output (y_g) in total output from an increase in the associated output variable (y_b) suggesting information with respect to net substitutes and net complements. For example the negative cross-term $\alpha_{2,3}$ on chicken and pig meats output (y_2) and cereals output (y_3), suggests that an increase in cereals output leads to an increase in the chicken and pig meats output relative to milk and beef production. The positive and remarkably high cross-term $\alpha_{1,3}$ on cow milk and beef and veal meats output (y_1) and cereals output (y_3) suggests that an increase in cereals output leads to a decrease in the cow milk and beef and veal meats output.

The linear homogeneity constraint in inputs, as given by the constraint (13), was tested through the entropy ratio statistics E that behaves like a likelihood-ratio statistics under particular condition described in Golan, Perloff et al. (2001:Appendix) where for r restrictions we have that $E = 2(S_U - S_R) \xrightarrow{d} \chi^2(r)$. The hypothesis of linear homogeneity in inputs was accepted at 5 per cent significance level with a calculated entropy ratio statistics of $E = 0.2419$ which is less than the critical value of the $\chi^2(1) = 3.8410$.

In Table 2 the average distance function output elasticities are reported. The output elasticities evidence that the most important contribution to total production comes from cow milk and beef and veal meats output (y_1) followed by chicken and pig meats output (y_2), cereals output (y_3), potatoes and rapeseeds and sugar beets output (y_4) respectively. The sample average output elasticity with respect to cow milk and beef and veal meats output (y_1) is about 0.30, which means that a 1 per cent increase in cow milk and beef and veal meats output (y_1) will increase by 30 per cent total output. The implicit share of milk and beef and veal output (y_1) declines over time whereas cereals output (y_3) and potatoes and rapeseeds and sugar beets (y_4) increase. The implicit share for chicken and pig meats output (y_2) is relatively stable over time. In Figure 1 the time varying distance function output elasticities are traced for completeness. The output elasticity with respect to cow milk and beef and veal meats output (y_1) is the most volatile output elasticity evidencing a sharp decline until 1995.

Table 2. Average sample output distance function elasticities (1991-01).

	$\epsilon_{DO,1}$	$\epsilon_{DO,2}$	$\epsilon_{DO,3}$	$\epsilon_{DO,4}$
1991-1995	0.35657	0.26657	0.19839	0.17847
1996-2001	0.23589	0.26493	0.29730	0.20189
Sample Average	0.29074	0.26568	0.25234	0.19124

Note: Output distance function elasticities are reported for convenience inverting their sign for a more convenient interpretation.

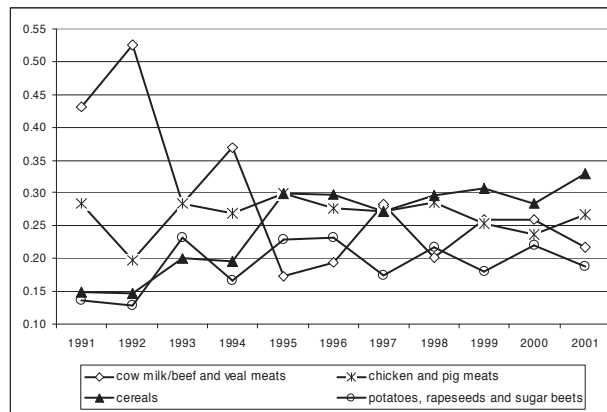


Figure 1. Time varying distance function output elasticities.

In Table 3 the substitutability measures as recovered from relationship (3) are presented. By looking at the 1991-95 values of $sub_{1,b}$ there appears a difficulty of substituting away from cow milk and beef and veal meats output (y_1) as evidenced by the $sub_{1,b}$ values greater than one. The apparently difficult substitution away from cow milk and beef and veal meats output (y_1) decreases over time in relation to chicken and pig meats output (y_2) and cereals output (y_3). These results are in accordance with the decrease in $\epsilon_{DO,1}$ (output elasticity with respect to cow milk/beef and veal meats) and in the increase in $\epsilon_{DO,3}$ (output elasticity with respect to cereals) particularly in the second part of the mid ninety (see Table 2). The revenue ratios $RV_{2,i}/RV_b$ presented in Table 3 indicates the values to which the substitutability measures $sub_{g,b}$ have to converge in case of revenue maximization. By observing the squared deviations between the output distance function substitutability measures ($sub_{g,b}$) and the market revenue ratios ($RV_{2,i}/RV_b$) it is possible to envisage discrepancies due to adjustment costs and therefore allocative market inefficiencies. From the figures in Table 3 there appears a higher discrepancy from market revenue maximization particularly for ratios associated with cow milk/beef and veal output ($sub_{1,b}$) suggesting that the valuation of cow milk/beef and veal output (y_1) was different from its market price. Discrepancies were relatively small during the sample period for the revenue ratios RV_2/RV_3 and the substitutability measure $sub_{2,3}$ associated with chicken and pig meats output (y_2) and cereals output (y_3).

Table 3. Output distance function substitutability measures and market revenue ratios (1991-01).

	<i>sub</i> _{1,2}	<i>sub</i> _{1,3}	<i>sub</i> _{1,4}	<i>sub</i> _{2,3}	<i>sub</i> _{2,4}	<i>sub</i> _{3,4}
1991-1995	1.42640	2.07147	2.28656	1.40718	1.55134	1.11624
1996-2001	0.89817	0.79991	1.19474	0.89409	1.32739	1.49068
Sample Average	1.13827	1.37789	1.69102	1.12731	1.42918	1.32048
	<i>RV</i> ₁ / <i>RV</i> ₂	<i>RV</i> ₁ / <i>RV</i> ₃	<i>RV</i> ₁ / <i>RV</i> ₄	<i>RV</i> ₂ / <i>RV</i> ₃	<i>RV</i> ₂ / <i>RV</i> ₄	<i>RV</i> ₃ / <i>RV</i> ₄
1991-1995	0.8488	1.2114	1.2827	1.4405	1.5350	1.0764
1996-2001	1.0100	1.0795	1.3856	1.0337	1.3786	1.3423
Sample Average	0.9368	1.1229	1.3388	1.2186	1.4497	1.2214
	(<i>Sqr.</i> <i>Dev.</i>) _{1,2}	(<i>Sqr.</i> <i>Dev.</i>) _{1,3}	(<i>Sqr.</i> <i>Dev.</i>) _{1,4}	(<i>Sqr.</i> <i>Dev.</i>) _{2,3}	(<i>Sqr.</i> <i>Dev.</i>) _{2,4}	(<i>Sqr.</i> <i>Dev.</i>) _{3,4}
1991-1995	0.33358	0.73980	1.00782	0.00111	0.00027	0.00159
1996-2001	0.01251	0.07817	0.03644	0.01949	0.00263	0.02203
Sample Average	0.04061	0.06504	0.12404	0.00833	0.00042	0.00981

In Table 4 we report the estimated Morishima substitution elasticities as defined by

$M_{h,g} = \epsilon_{h,g} - \epsilon_{g,g}$ by presenting also the single components $\epsilon_{h,g}$ and $\epsilon_{g,g}$. The first component $\epsilon_{h,g}$ of the Morishima substitution elasticity provides information on whether pairs of output are net substitutes or net complements.

Table 4. Output distance function components of the Morishima substitution elasticities (1991-01)

	$\epsilon_{1,2}$	$\epsilon_{1,3}$	$\epsilon_{1,4}$	$\epsilon_{2,1}$	$\epsilon_{2,3}$	$\epsilon_{2,4}$
1991-1995	-0.08139	-1.42833	-0.80485	-1.32058	1.19086	0.42546
1996-2001	-0.19805	-1.86712	-1.10650	-1.31572	1.27801	0.44595
Sample Average	-0.14502	-1.66767	-0.96939	-1.31793	1.23840	0.43664
	$\epsilon_{3,1}$	$\epsilon_{3,2}$	$\epsilon_{3,4}$	$\epsilon_{4,1}$	$\epsilon_{4,2}$	$\epsilon_{4,3}$
1991-1995	-3.89008	1.66202	0.23579	-2.71148	0.65011	0.26169
1996-2001	-3.41895	1.13865	0.23778	-2.42668	0.58783	0.35060
Sample Average	-3.63310	1.37655	0.23687	-2.55613	0.61614	0.31019
	$\epsilon_{1,1}$	$\epsilon_{2,2}$	$\epsilon_{3,3}$	$\epsilon_{4,4}$		
1991-1995	3.31459	-0.56240	1.44390	1.53301		
1996-2001	4.17167	-0.55423	1.07714	1.34227		
Sample Average	3.78208	-0.55794	1.24385	1.42897		
	$M_{1,2}$	$M_{1,3}$	$M_{1,4}$	$M_{2,1}$	$M_{2,3}$	$M_{2,4}$
1991-1995	-3.23067	-5.80017	-4.62156	0.71134	2.22442	1.21251
1996-2001	-3.22581	-5.32904	-4.33677	0.70152	1.69288	1.14206
Sample Average	-3.22802	-5.54319	-4.46622	0.70598	1.93449	1.17408
	$M_{3,1}$	$M_{3,2}$	$M_{3,4}$	$M_{4,1}$	$M_{4,2}$	$M_{4,3}$
1991-1995	-1.79544	-0.25304	-1.18220	-1.68697	-1.10754	-1.29722
1996-2001	-1.32978	0.20087	-0.72654	-1.47282	-0.89632	-1.10450
Sample Average	-1.54144	-0.00545	-0.93366	-1.57016	-0.99233	-1.19210

Net complementarity relationships are found for example between chicken and pig meats output (y_2) and cereals output (y_3) where $\varepsilon_{2,3} > 0$, and between chicken and pig meats output (y_2) and potatoes and rapeseeds and sugar beets output (y_4) where $\varepsilon_{2,4} > 0$. The strongest complementarity relationship is found by looking at sample average values for cereals output (y_3) and chicken and pig meats output (y_2), where $\varepsilon_{3,2} = 1.38$. Net substitutability relationships are found for example between cow milk/beef and veal meats output (y_1) and chicken and pig meats output (y_2) where $\varepsilon_{1,2} < 0$, and between cow milk/beef and veal meats output (y_1) and cereals output (y_3) where $\varepsilon_{1,3} < 0$. The strongest substitutability relationship is found by looking at sample average values for cereals output (y_3) and cow milk/beef and veal meat output (y_1) where $\varepsilon_{3,1} = -3.63$. Over time we note a decrease in the complementarity between cereals output (y_3) and chicken and pig meats output (y_2) and in the substitutability between cereals output (y_3) and cow milk/beef and veal meat output (y_1) whereas the complementarity between chicken and pig meats output (y_2) and cereals output (y_3) and the substitutability between cow milk/beef and veal meats output (y_1) and cereals output (y_3) increase. Moving to the interpretation of the output Morishima substitution elasticities we observe for example that by looking at sample average values $M_{2,3} = 1.93$ indicating that the relative adaptation of r_3^* and r_2^* is increasing for a change in the chicken and pig meats output (y_2) given cereals output (y_3). From our estimates there appears a violation in the output distance function curvature condition for chicken and pig meats output (y_2) as underlined by the negative estimates for $\varepsilon_{2,2}$, this result requires further investigations.

Based on the estimated substitutability measures (see relationship (3)) we also simulate the effect of the EU entry on the Polish agricultural output mix. This simplified analysis is only made with the purpose to illustrate the possibility of using a distance function framework to predict changes in the output mix allocation following a change in relative prices. Since information about the agricultural output prices of Poland for the year 2004 was not directly available for the output category considered in this analysis we recover such information by forecasting the missing price information with a linear model based on 1995-2001 data. We then define as a status quo the year 2004 and simulate the effect of the May 2004 Polish accession to the EU. The accession to the EU of Poland is simulated in a simplified way for illustrative purposes, by a 20 per cent rise in the price of cow milk/beef and veal meats output, by a 25 per cent rise in the price of chicken and pig meats output and by a rise of 6 per cent in the price of potatoes and rapeseeds and sugar beets output. The price of cereal is kept invariant since cereal prices in Poland are almost aligned to world prices. The forecasted price indexes for the agricultural output categories considered in this analysis are presented in Table 5.

Table 5. Agricultural output price indexes for Poland (2001-04).

	2001	2001	2002	2003	2004
Cow milk/beef and veal output (y_1)	100.00	95.16	97.87	97.43	97.00
Chicken and pig meats (y_2)	100.00	114.15	109.00	112.26	115.52
Cereals (y_3)	100.00	98.74	101.37	103.86	106.35
Potatoes, rapeseeds and sugar beets (y_4)	100.00	77.08	79.56	75.07	70.58

Note: 2002-2004 values are forecasts.

In Figure 2 we present the simulated effect of the EU accession for Poland on the agricultural output mix composition. Figure 2 is built in order to show percentage changes in the relative output mix composition from the status quo of 2004. It appears that the relative importance of cow milk/beef and veal meat output (y_1) over chicken and pig meats output (y_2) increases with respect to the status quo of 2004. Similarly we observe an increase in the relative importance of cereals output (y_3) over potatoes and rapeseeds and sugar beets output (y_4). This is due to the multiplicative effect on cereals output (y_3) of an increased price ratio (p_4/p_3) between potatoes and rapeseeds and sugar beets output (y_4) and cereals output (y_3) after the accession and a substitutability measure $sub_{3,4} > 1$. It is expected that after the Poland's accession to the EU, large part of arable crop production will benefit of wider financial supports favouring the contraction of potato area in favour of cereals (AgraEurope, 2004).

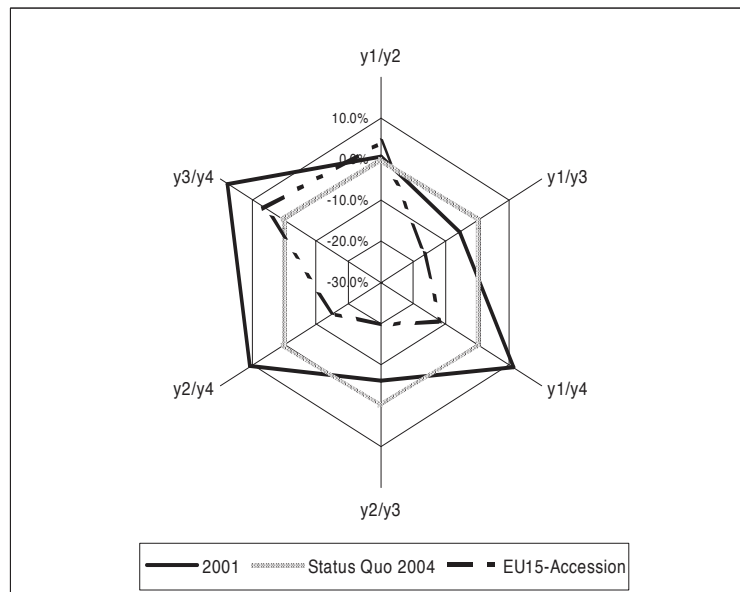


Figure 2. Percentage output mix changes from the 2004 status quo after EU accession.

5. Concluding Remarks

In this study we provided an empirical framework able to analyze multi-output agricultural technologies when data are sparsely available requiring minimal assumptions. This empirical framework is of easy implementation and can be extended to analyze the agricultural sector of other CEECs. The paper modelled the agricultural technology and output mix allocation of Poland after transition and also retrieves important substitutability relationships between out-

put variables. Additionally, several economic conjectures were made with respect to the effect of the Polish EU entry on the output mix allocation.

In particular, after transition there appears a decline in the implicit share of milk and beef and veal output over time and an increase for the cereals output and potatoes and rapeseeds and sugar beets output. This underlines that after transition during a period of severe economic uncertainty less capital intensive agricultural productions were favoured over the most capital intensive. With the relative increase in the implicit share of cereals over time, substitutability movements away from cereals worsened over time. From our estimate there appears to be allocative output inefficiencies particularly for cow milk/beef and veal output at the beginning of the 1990. The Polish accession to the EU according to our estimates increased the relative importance of cow milk/beef and veal meat (y_1) output over chicken and pig meats (y_2) output with respect to the status quo of 2004.

Points of further research are the introduction of a weighting maximum entropy objective function balancing data constraints against priors, the introduction of a continuous support parameter space and third a qualitative assessment of the external data included in a form of priors through the entropy ratio.

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