A NOTE ON SPURIOUS REGRESSION IN PANELS WITH CROSS-SECTION DEPENDENCE

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ABSTRACT

This paper analyses regression of two independent stationary panels with cross-sectional dependence. It is shown that the pooling least squares (PLS) estimator converges to zero in probability while the individual OLS estimator converges to a random variable. However, the PLS-based and the OLS-based t-statistics diverge, so the null hypothesis of no correlation tends to be spuriously rejected.

I. INTRODUCTION

The issue of spurious regression is well documented in econometrics; it was first studied by Granger and Newbold (1974) using simulations and a full analytical explanation was later provided in Phillips (1986). Spurious regressions occur when two independent integrated processes are regressed on each other. It is found that in such occasions: (i) the OLS estimator of the slope coefficient is asymptotically random, so the true slope (zero) fails to be identified and the OLS estimator is inconsistent, and (ii) the t-statistic of the slope does not have a limiting distribution but diverges at a $\sqrt{T}$ rate as the sample size (T) goes to infinity; therefore, the null hypothesis of a zero slope coefficient tends to be spuriously rejected.

Recently, Kao (1999) and Phillips and Moon (1999) examined spurious regressions in panel data when both the cross-section dimension (N) and the time-series span (T) are large. For the case of regression of two independent nonstationary panels, it is found that the pooling least squares (PLS) estimator of the slope converges to zero in probability (so the PLS estimator is consistent), provided that cross-section units within each panel are mutually independent. According to Phillips and Moon (1999), this is because that the strong noise effect, which makes the slope unidentifiable in each individual time-series regression, is attenuated by the inclusion of a large amount of independent cross-section information. On the other hand, the usual t-statistic of the slope diverges (at a $\sqrt{T}$ rate, too), implying that inferences about the slope are wrong with the probability that goes to one asymptotically.

This note studies spurious regression under a different panel setting. In particular, we consider a regression between two independent stationary panels with cross-sectional dependence. To model cross-sectional dependence in panels, we assume a factor model in each panel. We establish the limiting distributions of the PLS estimator and the individual OLS estimator (at any given time) of the slope, and the limiting distributions of the PLS-based and the OLS-based t-statistics. We find that the PLS estimator converges to the true slope value (zero) as in the case of regression in cross-sectionally independent panels (stationary or nonstationary), but at a different convergence rate (as discussed in Section III). On the other hand, the OLS estimator converges to a random variable. We also find that the PLS-based t-statistic and the OLS-based t-statistic diverge (both at a $\sqrt{N}$ rate) and, as a result, spurious rejections of the zero-slope null occur.

The paper is organized as follows. Section II introduces the factor model based panels with cross-sectional dependence. Section III derives the asymptotic distributions of the PLS and the PLS-based t-statistic as well as the asymptotic distributions of the OLS estimator and the OLS-based t-statistic. Section IV concludes. As a matter of notation, throughout the paper, "$(N,T)\to\infty$" denotes N and T go to infinity jointly, "$\Rightarrow$"signify weak convergence, and "$\to_p$" means convergence in probability.

II. CROSS-SECTIONALLY DEPENDENT PANELS

Let x_{it} and y_{it}, for i = 1, ..., N and t = 1, ..., T, be two independent panels defined as

$$\begin{cases} y_{it} = \lambda_i f_t + \mu_{it}, \\ x_{it} = \delta_i g_t + \upsilon_{it}. \end{cases} \tag{1}$$

Here, f_t and g_t are unobservable random factors, λ_i and δ_i are non-random factor loading coefficients and μ_{it} and υ_{it} are idiosyncratic shocks in x_{it} and y_{it}, respectively. Similar to Phillips and Sul (2002), we assume a single-factor structure to model dependence across units. See also Moon and Perron (2003) and Bai and Ng (2003) for a multi-factor panel model. Following Moon and Perron (2003), we make assumptions regarding f_t, g_t, λ_i, δ_i, μ_{it} and υ_{it} as follows.

Assumption 1

(a) $f_t = \sum_{j=0}^{\infty} \theta_j \xi_{t-j}$, where $\xi_t \sim iid(0,1)$ and $\sum_{j=0}^{\infty} j^m |\theta_j| < M$ for some m>1.

(b) $g_t = \sum_{j=0}^{\infty} \gamma_j \zeta_{t-j}$, where $\zeta_t \sim iid(0,1)$ and $\sum_{j=0}^{\infty} j^m |\gamma_j| < M$ for some m>1.

(c) ξ_t and ζ_t are independent.

Assumption 2

(a) $\mu_{it} = \sum_{j=0}^{\infty} d_{ij} \varepsilon_{i,t-j}$, where $\varepsilon_{i,t} \sim iid(0,1)$, across i and over t and with a finite fourth moment, $\inf_i \sum_{j=0}^{\infty} d_{ij} > 0$, and $\sum_{j=0}^{\infty} j^m \bar{d}_j < M$ with $\bar{d}_j = \sup_i |d_{ij}|$.

(b) $\upsilon_{it} = \sum_{j=0}^{\infty} c_{ij} \eta_{i,t-j}$, where $\eta_{it} \sim iid(0,1)$, across i and over t and with a finite fourth moment, $\inf_i \sum_{j=0}^{\infty} c_{ij} > 0$, and $\sum_{j=0}^{\infty} j^m \bar{c}_j < M$ with $\bar{c}_j = \sup_i |d_{ij}|$.

(c) ε_{it} and η_{it} are independent.

Assumption 3

(a) ξ_t and ε_{is} are independent.

(b) ζ_t and η_{is} are independent.

Assumption 4

Define $\sigma_{\mu,i}^2 = \sum_{j=0}^{\infty} d_{ij}^2$ and $\sigma_{\upsilon,i}^2 = \sum_{j=0}^{\infty} c_{ij}^2$, where $\sigma_{\mu,i}^2$ and $\sigma_{\upsilon,i}^2$ signifies the variance of μ_{it} and υ_{it}, respectively. Let $\omega_{\mu}^2 = \lim_{N\to\infty} N^{-1} \sum_{i=1}^{N} \omega_{\mu,i}^2$ and $\sigma_{\upsilon}^2 = \lim_{N\to\infty} N^{-1} \sum_{i=1}^{N} \sigma_{\upsilon,i}^2$. Assume that σ_{μ}^2 and σ_{υ}^2 are both well defined.

Assumption 5

(a) $N^{-1} \sum_{i=1}^{N} \lambda_i \to m_{\lambda}\ (\neq 0)$ and $N^{-1} \sum_{i=1}^{N} \delta_i \to m_{\delta}\ (\neq 0)$.

(b) $N^{-1} \sum_{i=1}^{N} \lambda_i \delta_i \to \sigma_{\lambda\delta}\ (\neq 0)$, $N^{-1} \sum_{i=1}^{N} \lambda_i^2 \to \sigma_{\lambda}^2\ (\neq 0)$, and $N^{-1} \sum_{i=1}^{N} \delta_i^2 \to \sigma_{\delta}^2\ (\neq 0)$.

Assumptions 1-3 assume that the random factors (f_t, g_t) and the idiosyncratic shocks (μ_{it}, υ_{it}) are all zero-mean stationary and they are independent to one another. Under Assumption 1, since f_t and g_t are independent, it is easy to see that, as $T \to \infty$, $T^{-1/2} \sum_{t=1}^{T} f_t g_t \Rightarrow N(0, \omega_{fg}^2)$, where ω_{fg}^2 is the long-run variance of "$f_t g_t$". Note that, under Assumption 1, since the long-run variances of f_t and g_t are well-defined and f_t and g_t are mutually independent, the long-run variance of " $f_t g_t$ " is well-defined. Also, by Assumptions 1 and 2, x_{it} and y_{it} are constructed to be independent to each other. The idiosyncratic shocks are assumed to be independent across units (Assumption 2(c)). The extent of cross-sectional correlation in each panel is given by

$$\begin{cases} corr(x_{it}, x_{jt}) = \dfrac{\delta_i \delta_j E(g_t^2)}{\left(\delta_i^2 E(g_t^2) + E(\upsilon_{it}^2)\right)^{1/2} \left(\delta_j^2 E(g_t^2) + E(\upsilon_{jt}^2)\right)^{1/2}}, \\ corr(y_{it}, y_{jt}) = \dfrac{\lambda_i \lambda_j E(f_t^2)}{\left(\lambda_i^2 E(f_t^2) + E(\mu_{it}^2)\right)^{1/2} \left(\lambda_j^2 E(f_t^2) + E(\mu_{jt}^2)\right)^{1/2}}. \end{cases}$$

Since Assumption 5 does not rule out the possibility that $\lambda_i = 0$ or $\delta_i = 0$ for some i, some cross-section units (in each panel) may be uncorrelated with one another. Assumption 4 assumes the existence of the long-run variances of the idiosyncratic shocks.

III. SPURIOUS PANEL REGRESSIONS

Consider a simple panel regression model

$$y_{it} = \beta x_{it} + \varepsilon_{it}, \text{ i=1, ..., N; t=1, …, T} \tag{2}$$

The PLS estimator of β is given by

$$\hat{\beta} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it} y_{it}}{\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}^2},$$

and the PLS residuals $\hat{e}_{it} = y_{it} - \hat{\beta} x_{it}$. Then, to test the null hypothesis of $\beta = 0$, we define the usual t statistic:

$$t_\beta = \frac{\hat{\beta}}{\hat{s}_\beta}, \text{ where } \hat{s}_\beta = \left[\frac{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} \hat{e}_{it}^2}{\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}^2}\right]^{1/2}$$

For comparison, we also consider the OLS estimator of the slope in (2) for any given t,

$$\hat{\beta}_t = \frac{\sum_{i=1}^{N} x_{it} y_{it}}{\sum_{i=1}^{N} x_{it}^2}$$

and the OLS residuals $\hat{e}_{it} = y_{it} - \hat{\beta}_t x_{it}$. And, define the OLS-based t-statistic as

$$t_{\beta_t} = \frac{\hat{\beta}_t}{\hat{s}_{\beta_t}}, \text{ where } \hat{s}_{\beta_t} = \left[\frac{\frac{1}{N}\sum_{i=1}^{N} \tilde{e}_{it}^2}{\sum_{i=1}^{N} x_{it}^2} \right]^{1/2}.$$

Theorem 1

Let $\sigma_f^2 = E(f_t)$ and $\sigma_g^2 = E(g_t)$. Under Assumptions 1-5, we have the following.

(i) Let $(N,T) \to \infty$,
$$\begin{cases} (a)\ T^{1/2}\hat{\beta} \Rightarrow N\left(0, \dfrac{\sigma_{\lambda\delta}^2 \omega_{fg}^2}{\left(\sigma_\delta^2 \sigma_g^2 + \sigma_\upsilon^2\right)^2}\right), \\ (b)\ N^{-1/2} t_\beta \Rightarrow N\left(0, \dfrac{\sigma_{\lambda\delta}^2 \omega_{fg}^2}{\left(\sigma_\lambda^2 \sigma_f^2 + \sigma_\mu^2\right)\left(\sigma_\delta^2 \sigma_g^2 + \sigma_\upsilon^2\right)}\right). \end{cases}$$

(ii) For any t, let $N \to \infty$,
$$\begin{cases} (a)\ \hat{\beta}_t \Rightarrow \left(\dfrac{\sigma_{\lambda\delta}}{\sigma_\delta^2}\right)\left(\dfrac{f_t g_t}{g_t^2}\right) \equiv \beta_t^*, \\ (b)\ N^{-1/2} t_{\beta_t} \Rightarrow \left[\dfrac{\sigma_\delta^2 g_t^2}{\Lambda_t^*}\right]^{1/2} \beta_t^*, \end{cases}$$

where $\Lambda_t^* = \sigma_\lambda^2 f_t^2 - 2\beta_t^* \sigma_{\lambda\delta} f_t\, g_t + \beta_t^{*2} \sigma_\delta^2 g_t^2$.

Remarks

1. The PLS estimator of the slope is $\sqrt{T}$ -consistent. This contrasts with the well-known fact that the PLS estimator is $\sqrt{NT}$ -consistent in the conventional panel regression that assumes over-time stationarity and cross-unit independence. This also contrasts with the $\sqrt{T}$ -consistency achieved in nonstationary panel regression when cross-sectional independence is assumed (Kao (1999) and Phillips and Moon (1999)). On the other hand, the time-specific individual OLS estimator of the slope (for any t) is not consistent. It is also worth noting that the PLS estimator, once correctly scaled, converges to a normal distribution. On the contrary, the OLS estimator converges to a random variable that depends on the random factors.

2. The PLS-based t-statistic and the OLS-based t-statistic are both divergent, so the spurious results appear. Interestingly, the divergence rate of the PLS-based test is determined by N, the cross-section dimension, only. This is opposite to the nonstationary panel regression case (with cross-sectional independence) studied in Kao (1999) and Phillips and Moon (1999), in which the t-statistic diverges at a rate that depends on T, the time-series span, only.

3. There is no need to put any restriction on the relative growing rate between N and T to obtain the joint asymptotic distributions of the PLS estimator and the PLS-based t-statistic. In contrast, the result obtained in Phillips and Moon (1999) requires the assumption that N grows slowly than T.

VI. CONCLUSIONS

In this note, we consider a spurious regression of two independent stationary, cross-sectionally correlated panels. To model cross-sectional dependence, a single-factor model for each panel is assumed. We find that the PLS estimator converges to zero in probability so it is consistent. On the other hand, the OLS estimator converges to a random variable. We also find that both the PLS-based t-statistic and the OLS-based t-statistic diverge and consequently spurious results occur.

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Appendix - Proof Of Theorem 1

Part (i)-(a)

$$T^{1/2}\hat{\beta} \Rightarrow N\left(0, \frac{m_{\lambda\delta}^2 \omega_{fg}^2}{\left(\sigma_\delta^2 \sigma_g^2 + \sigma_\upsilon^2\right)^2}\right).$$

We first claim that the numerator of $\hat{\beta}$:

$$N^{-1}T^{-1/2}\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it} y_{it} \Rightarrow N\left(0, m_{\lambda\delta}^2 \omega_{fg}^2\right), \tag{A1}$$

as $(N,T)\to\infty$. Note that

$$\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it} y_{it} = \sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i \delta_i f_t g_t + \sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i f_t \upsilon_{it} + \sum_{i=1}^{N}\sum_{t=1}^{T} \delta_i g_t \mu_{it} + \sum_{i=1}^{N}\sum_{t=1}^{T} \upsilon_{it} \mu_{it} \tag{A2}$$

Under Assumption 1, f_t and g_t are two independent stationary processes, it follows that

$$\begin{aligned} &N^{-1}T^{-1/2}\sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i \delta_i f_t g_t = \left(N^{-1}\sum_{i=1}^{N} \lambda_i \delta_i\right)\left(T^{-1/2}\sum_{t=1}^{T} f_t g_t\right) \\ &\Rightarrow m_{\lambda\delta}\omega_{fg} B(1) \equiv N\left(0, m_{\lambda\delta}^2 \omega_{fg}^2\right), \end{aligned} \tag{A3}$$

where $B(1)$ is the standard Brownian motion and ω_{fg}^2 is the long-run variance of "$f_t g_t$". For the second term on the right hand side of (A2), we first note that

$$\begin{aligned} &E\left(\sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i f_t \upsilon_{it}\right)^2 \\ &= \sum_{s=1}^{T}\sum_{t=1}^{T} E\left[f_s f_t \left(\sum_{i=1}^{N} \lambda_i \upsilon_{is}\right)\left(\sum_{j=1}^{N} \lambda_j \upsilon_{jt}\right)\right] \\ &= \sum_{s=1}^{T}\sum_{t=1}^{T} E[f_s f_t] E\left[\left(\sum_{i=1}^{N} \lambda_i \upsilon_{is}\right)\left(\sum_{j=1}^{N} \lambda_j \upsilon_{jt}\right)\right] \\ &= \sum_{s=1}^{T}\sum_{t=1}^{T} E[f_s f_t] \sum_{i=1}^{N} \lambda_i^2 E\left(\upsilon_{is}\upsilon_{it}\right), \end{aligned} \tag{A4}$$

because $E(f_s \upsilon_{jt}) = 0$ for any s, t and j, and $E(\upsilon_{is}\upsilon_{jt}) = 0$ if $i \neq j$. Let $E[f_t f_{t-h}] = \Gamma^{(f)}(h)$ and $E\left(\upsilon_{i,t}\upsilon_{i,t-h}\right) = \Gamma_i^{(\upsilon)}(h)$, we have

$$\begin{aligned} \text{(A4)} &= \sum_{s=1}^{T}\sum_{t=1}^{T} \Gamma^{(f)}(t-s) \sum_{i=1}^{N} \lambda_i^2 \Gamma_i^{(\upsilon)}(t-s) \\ &\leq \sum_{s=1}^{T}\sum_{t=1}^{T} \left|\Gamma^{(f)}(t-s)\right| \left|\sum_{i=1}^{N} \lambda_i^2 \Gamma_i^{(\nu)}(t-s)\right| \end{aligned} \tag{A5}$$

Note that, under Assumption 2(a),

$$\left|\Gamma_i^{(\upsilon)}(h)\right| \leq \sup_i \left|\Gamma_i^{(\upsilon)}(h)\right| = \sup_i \sum_{j=o}^{\infty} \left|d_{ij} d_{i,j+h}\right| \leq \sum_{j=0}^{\infty} \left|\bar{d}_j \bar{d}_{j+h}\right| \equiv \bar{\Gamma}^{(\upsilon)}(h),$$

so that,

$$\text{(A5)} \leq \left(\sum_{i=1}^{N}\lambda_i^2\right)\left(\sum_{t=1}^{T}\sum_{s=1}^{T}\left|\Gamma^{(f)}(t-s)\right|\left|\bar{\Gamma}_i^{(v)}(t-s)\right|\right)$$
$$\leq \left(\sum_{i=1}^{N}\lambda_i^2\right)\left(T\sum_{h=0}^{\infty}\left|\Gamma^{(f)}(h)\right|\left|\bar{\Gamma}^{(v)}(h)\right|\right)$$
$$\leq T\left(\sum_{i=1}^{N}\lambda_i^2\right)\sqrt{\sum_{h=0}^{\infty}\left|\Gamma^{(f)}(h)\right|^2}\sqrt{\sum_{h=0}^{\infty}\left|\bar{\Gamma}_i^{(\upsilon)}(h)\right|^2},$$

by the Cauchy inequality. Due to the summability conditions of Assumptions 1(a) and 2(b), $\sum_{h=0}^{\infty}\left|\Gamma^{(\upsilon)}(h)\right|^2 < M$ and $\sum_{h=0}^{\infty}\left|\Gamma^{(f)}(h)\right|^2 < M$, for some finite M (>0). And, since $\sum_{i=1}^{N}\lambda_i^2 = O(N)$, we conclude

$$E\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\lambda_i f_t \upsilon_{it}\right)^2 = O(NT).$$

Therefore,

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\lambda_i f_t \upsilon_{it} = O_p(N^{1/2}T^{1/2}). \tag{A6}$$

Similarly,

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\delta_i g_t \mu_{it} = O_p(N^{1/2}T^{1/2}). \tag{A7}$$

Also, since μ_{it} and υ_{it} are stationary, cross-sectionally independent, and independent to each other, it can be shown that

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\upsilon_{it}\mu_{it} = O_p(N^{1/2}T^{1/2}). \tag{A8}$$

By (A3) and (A6) ~ (A8), the result of (A1) directly follows.

We next claim that the denominator of $\hat{\beta}$:

$$N^{-1}T^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}x_{it}^2 \to_p \sigma_\delta^2\sigma_g^2 + \sigma_\upsilon^2, \tag{A9}$$

as $(N,T)\to\infty$. Write

$$\sum_{i=1}^{N}\sum_{t=1}^{T}x_{it}^2 = \sum_{i=1}^{N}\sum_{t=1}^{T}\delta_i^2 g_t^2 + 2\sum_{i=1}^{N}\sum_{t=1}^{T}\delta_i g_t \upsilon_{it} + \sum_{i=1}^{N}\sum_{t=1}^{T}\upsilon_{it}^2. \tag{A10}$$

Since $N^{-1}T^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\delta_i^2 g_t^2 \to_p \sigma_\delta^2\sigma_g^2$, $\sum_{i=1}^{N}\sum_{t=1}^{T}\delta_i g_t \upsilon_{it} = O_p(N^{1/2}T^{1/2})$, and $N^{-1}T^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\upsilon_{it}^2 \to_p \sigma_\upsilon^2$, the result (A9) directly follows. By (A1) and (A9), we complete the proof of Theorem 1(i).

Proof of Theorem 1 (i)-(b):

$$N^{-1/2}t_{\beta} \Rightarrow N\left(0, \frac{\sigma_{\lambda\delta}^{2}\,\omega_{fg}}{\left(\sigma_{\lambda}^{2}\sigma_{f}^{2}+\sigma_{\mu}^{2}\right)\left(\sigma_{\delta}^{2}\sigma_{g}^{2}+\sigma_{\upsilon}^{2}\right)}\right).$$

Write

$$t_{\beta} = \frac{\hat{\beta}}{s_{\beta}} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}y_{it}}{\left(\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}^{2}\right)^{1/2}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}x_{it}^{2}\right)^{1/2}}.$$

Note that

$$\begin{aligned}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}^{2} &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left[y_{it}-\hat{\beta}x_{it}\right]^{2} \\ &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it}^{2} - 2\hat{\beta}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}x_{it}y_{it} + \hat{\beta}^{2}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}x_{it}^{2}.\end{aligned} \tag{A11}$$

Following the proof of (A9), it is easy to show that $N^{-1}T^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it}^{2} \rightarrow_{p} \sigma_{\lambda}^{2}\sigma_{f}^{2}+\sigma_{\mu}^{2}$. Also, by the proof of part (i) in Theorem 1, $\hat{\beta}=O_{p}(T^{-1/2})$, $\sum_{i=1}^{N}\sum_{t=1}^{T}x_{it}^{2}=O_{p}(NT)$ and $\sum_{i=1}^{N}\sum_{t=1}^{T}x_{it}y_{it} = O_{p}(NT^{1/2})$. Therefore, the first term on the right hand side of (A11) dominates the other terms in the same equation and we conclude

$$\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}^{2} \rightarrow_{p} \sigma_{\lambda}^{2}\sigma_{f}^{2}+\sigma_{\mu}^{2}. \tag{A12}$$

By (A12) and Theorem 1(i)-(a), the result follows.

Proof of Theorem 1 (ii)-(a)

$$\hat{\beta}_{t} \Rightarrow \left(\frac{\sigma_{\lambda\delta}}{\sigma_{\delta}^{2}}\right)\left(\frac{f_{t}g_{t}}{g_{t}^{2}}\right) \equiv \beta_{t}^{*}.$$

It is easy to show that $N^{-1}\sum_{i=1}^{N}x_{it}y_{it} \Rightarrow \sigma_{\lambda\delta}g_{t}\,f_{t}$ and $N^{-1}\sum_{i=1}^{N}x_{it}^{2} \Rightarrow \sigma_{\delta}^{2}g_{t}^{2}$. The result follows.

Proof of Theorem 1 (ii)-(b)

$$N^{-1/2}t_{\beta_{t}} \Rightarrow \left[\frac{\sigma_{\delta}^{2}g_{t}^{2}}{\Lambda_{t}^{*}}\right]^{1/2}\beta_{t}^{*}.$$

Similar to the proof of (ii)-(a), it is easy to show

$$\begin{aligned}N^{-1}\sum_{i=1}^{N}\tilde{e}_{it}^{2} &= N^{-1}\sum_{i=1}^{N}y_{it}^{2} - 2N^{-1}\hat{\beta}_{t}\sum_{i=1}^{N}x_{it}y_{it} + N^{-1}\hat{\beta}_{t}^{2}\sum_{i=1}^{N}x_{it}^{2} \\ &\Rightarrow \sigma_{\lambda}^{2}f_{t}^{2} - 2\beta_{t}^{*}\sigma_{\lambda\delta}f_{t}\,g_{t} + \beta_{t}^{*2}\sigma_{\delta}^{2}g_{t}^{2} \equiv \Lambda_{t}^{*}.\end{aligned} \tag{A13}$$

By the Theorem 1 (ii)-(a) and (A13), the result follows.

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