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Equilibrium?**

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Abstract

This paper examines the extent to which speculation against the French franc in 1992-93 was motivated by the fundamentals or resulted from a sunspot phenomenon. We develop a model of currency crises which encompasses both hypotheses about the origin of speculation. The estimation shows that the model with sunspots not only better tracks the episodes of speculation, but also gives a better account of the relationship between the fundamentals and the devaluation expectations.

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1 Introduction

The crises in the exchange rate mechanism of the European Monetary System provide a fertile garden for testing existing strains of exchange rate determination and for developing new strains. In particular, they have forced researchers to confront the issue of whether or not exchange rate movements, or at least pressures in exchange markets, are dictated by economic fundamentals. Looking at speculative attacks (identified by large movements in exchange rates, interest rates, and international reserves) between 1967 and 1992, Eichengreen, Rose and Wyplosz (1994) find that though crises outside the ERM exhibit differences in key macroeconomic variables (budget deficits, inflation, export/import ratios, credit growth, and international reserves) relative to non-crisis periods, the same is not true of the ERM observations. This raises the issue, therefore, of what caused the crises.

In fact, these events have led some to question whether existing models of speculative crises were adequate. Extensions of those models have included modeling the devaluation decision as being the result of optimizing behavior by the monetary authorities and including in the set of fundamentals the unemployment rate, since it can be an important element of the cost of maintaining an exchange rate peg in the face of speculative pressures (Obstfeld, 1994). Appealing to high unemployment helps, in principle, explain why the French franc came under attack in 1992-93, despite a strong balance of payments position and no obvious problem of price competitiveness–indeed, inflation had been lower in France than in Germany for several years (Caramazza, 1993; Drazen and Masson, 1994). Similarly, taking into account the cost of high interest rates, including their effect on unemployment, helps to throw light on the problems faced by the United Kingdom in maintaining its ERM parity (Ozkan and Sutherland, 1995; Masson, 1995). Thus the crises have shifted the traditional focus of attack models away from the adequacy of reserves and the rate of growth of domestic credit toward other fundamental factors, at least for countries, like those in Europe, which have ready access to private sources of financing in foreign currency.

However, even a wider set of economic fundamentals does not necessarily provide convincing explanations of why attacks occurred, nor of their timing (Eichengreen and Wyplosz, 1993; Obstfeld and Rogoff, 1995). The crisis in the ERM of September, 1992, occurred suddenly, despite the fact that unemployment had been rising steadily throughout the year in a number of European countries (indeed, the recession in the United Kingdom may already have ended when sterling was forced off its ERM peg on September 17). Moreover, the attacks on the French franc subsided in November, 1992, resumed again in January-March, 1993, and seemed to have been laid to rest by the election of the strongly pro-European government of Edouard Balladur in April.

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Indeed, French short-term interest rates went below those in Germany in June, as the franc strengthened markedly (see Moutot, 1994, for a chronology of the crises). However, a new attack erupted in July, 1993, affecting almost all ERM currencies, including otherwise strong currencies like the Belgian franc, and this led to the general widening of the bands of fluctuation from 2.25 percent to 15 percent on August 2.

The absence of a clear correlation with macroeconomic variables is not, of course, an indication that exchange rate movements are unconnected to fundamental factors, since some of these fundamentals may not be measured by the usual set of macroeconomic time series. For instance, market commentary in the financial and popular press often points to events that are supposed to explain the movements in interest rates, exchange rates, or stock prices. Statements by officials or company spokesmen are often cited. In the limit, the absence of "news" can be interpreted as news, and hence be a reason to move the market (Frenkel, 1981). During the ERM crises, for instance, a major role in igniting speculation against the pound sterling and lira, forcing them off their ERM pegs, seems to have been played by negative opinion polls leading up to the French referendum on September 20 – even though the voters in fact approved the Maastricht Treaty. Another example is the report of friction between French and German officials early in July, 1993, which seems to have ignited speculation against the French franc.

The fact that large financial events can be triggered by press speculation or expected events that do not occur lends plausibility to the argument that there is not a simple relationship between economic fundamentals and interest and exchange rates. Some economists have put forward the hypothesis that financial markets are subject to multiple equilibria linking them to fundamentals, and that what are commonly called "sunspot equilibria" exist, where an extrinsic variable (such as market sentiment) might influence the equilibrium outcomes (Obstfeld, 1996; Obstfeld and Rogoff, 1995). The subject of this paper is the attempt to test for the existence of these phenomena.

Another approach which is similar in many respects to the structure estimated here involves assuming that there are shifts in regimes, and estimating the transitions between them (Hamilton 1988, 1989, 1990; Kaminsky and Peruga, 1990). However, in these models the reasons for the multiplicity of regimes are typically not specified, nor are restrictions imposed on the estimated parameters–an exception is Van Norden (1996). We would argue that the additional structure of our model and the restrictions imposed in estimation provide good economic intuition and a tighter test of the model's ability to explain the underlying data.

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An alternative explanation for financial market volatility unrelated to volatility in fundamentals is that exchange rates and interest rates are subject to non-linear, and, in particular, chaotic, dynamics. For instance, De Grauwe *et al.* (1993) construct a model of foreign exchange markets in which some investors trade on the basis of fundamentals while others only respond to movements in exchange rates. Postulating particular parameter values, they find that chaotic movements in exchange rates may result–that is, movements which seem erratic and are extremely sensitive to initial conditions. While we shall see below that a model in which investors' expectations take into account the policy tradeoffs of the authorities may produce cyclic and chaotic dynamics, we prefer at this stage of our research to highlight the role of arbitrary shifts in sentiment in influencing the equilibrium outcome. Boldrin and Woodford (1990) survey the literature and explain the connections between chaos and endogenous fluctuations.

Sunspot equilibria are already the subject of a large, and rapidly growing literature, though empirical studies are few. The theoretical literature on sunspot equilibria has been largely concerned with overlapping generations models, physical investment, the demand for fiat money and business cycles, and it includes articles by Azariadis (1981), Durlauf (1991), Woodford (1987, 1990a, 1990b), several papers in a special issue of the *Journal of Economic Theory* on growth fluctuations and sunspots (1994, vol. 63, 1) and a survey by Chiappori and Guesnerie (1991). Empirical studies have focused mainly on the question of whether sunspots can explain the fluctuations of output and the dynamics of unemployment (Manning, 1992; Farmer and Guo, 1994; Dagsvik and Jovanovic, 1994).

Given the volatility of financial markets, it is perhaps surprising that the role of extraneous variables in triggering different equilibria has so far received relatively little attention in this context. While the theory of speculative bubbles involves multiple equilibria and has been applied to the determination of the price of financial and monetary assets, including floating exchange rates, it is not generally regarded as a convincing explanation for the excessive volatility of currency exchange rates (Mussa, 1990), and anyway does not account for the volatility of devaluation expectations in fixed exchange rates systems. More recently, models of fixed exchange rate systems exhibiting multiple equilibria have been developed by, among others, Obstfeld (1986, 1994, 1996), Velasco (1996) and Bensaid and Jeanne (1996). The common characteristic of these models is to derive the multiplicity of equilibria from the interaction between the market and an optimizing policymaker. In a related paper, Jeanne (1996) examines whether the fundamentals in the ERM during 1987-93 were in a range permitting multiple equilibria to occur. To our knowledge, there have been no other attempts at direct estimation of models implying sunspot equilibria in

currency crises.

The plan of the paper is as follows. The next section looks at the chronology and possible causes of the speculation against the French franc. The third section introduces a theoretical model of the decision to stay in the ERM, highlighting the possibility of multiple equilibria that depend on market expectations. Section 4 presents estimates of that model, showing that jumps between two equilibria explain the data considerably better than a model with a single relationship between fundamentals and interest differentials. Section 5 concludes.

2   The French Franc in 1992-93

The events in the ERM during 1992-93 have been widely analysed elsewhere (see, for instance, IMF 1993a, 1993b). Lack of economic convergence among participating countries led to tensions within the ERM which culminated in the September 1992 float of the lira and pound sterling and subsequent devaluations of the peseta, escudo, and the Irish pound. After a period of relative calm, renewed pressures arose in July 1993, became generalized to almost all currencies, and led to the widening of the bands of fluctuation from 2.25 percent on either side of central parities to 15 percent, effective August 2.

Though the deterioration of external competitiveness and the incompatibility of exchange rate pegs with the requirements of internal balance made some countries, e.g. Italy, Spain, and the United Kingdom, clear candidates for devaluation, the timing of the speculative attacks was, in most cases, sudden and difficult to relate to a worsening of economic fundamentals. Moreover, though these countries were widely viewed as having overvalued currencies, their willingness to pay the cost in terms of higher unemployment without devaluing over an extended period (since January 1987, except for the pound sterling, which joined the ERM in October 1990), made it increasingly likely that they would continue to defend their pegs in order not to lose acquired credibility. What undoubtedly changed in September 1992 was that the costs of maintaining those parities rose dramatically. Thus, an exchange rate arrangement which might be sustainable if markets judged it credible (therefore not exacting large interest rate premiums relative to German rates) could become unsustainable when its credibility was thrown into question.

The link to economic fundamentals was even more tenuous in the case of the French franc. French competitiveness was generally quite good, as proxied by unit

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labor costs relative to all other industrial countries (given lower inflation than in Germany over the previous several years) or as judged by the trade balance, which had changed from a deficit to a surplus. Even the depreciations of the pound sterling and lira in September 1992 did not cause a large deterioration of French competitiveness (see Chart 1). It is true that French unemployment was rising to a record high level and output was falling (Chart 2), but this was not obviously an exchange rate problem. Again, the problem was that defense of the franc was likely to be difficult in the face of strong lack of credibility, since raising interest rates would exacerbate the unemployment problem. For the franc there seemed to be clear evidence of multiple equilibria: if investors believed that the rate was sustainable, then it would be maintained, since the costs of doing so were bearable given the strong external position. However, in the face of strong enough devaluation expectations, the Bank of France might have to give in, because unwilling to bear the extra costs of high unemployment and recession.

The timing of the crises, though it can in some cases be linked to political events, is difficult to relate to economic fundamentals that could reasonably be expected to affect the equilibrium exchange rate commensurately with the exchange market pressures observed. Pressure on the French franc became strong in the few days leading up to the French referendum on the Maastricht Treaty, held on September 20 – especially after the lira and pound were forced out of the ERM (Chart 3). However, a narrow "yes" vote in the referendum did not cause exchange market pressures to abate. As many commentators have noted, speculation was emboldened by the success in picking off the weaker currencies. In effect, investors had come to realize that they could increase the costs of maintaining pegs to an extent that might make them unsustainable. However, on September 23 the Bank of France raised its day-to-day intervention rate by 2.5 percentage points, while discontinuing its 5-10 day repos, and this calmed the market (Chart 4). By November, the central bank had recouped its foreign exchange reserves and the first battle of the franc was over. Its success moreover led to the expectation that credibility would be enhanced by the evident determination of the authorities.

However, speculation resumed near the turn of the year, in part linked to electoral uncertainties. The Bank of France once again raised interest rates and discontinued longer-term repos, and this crisis, though longer, was overcome in March 1993. With the election of a new right-of- center parliamentary majority in April 1993, and the naming of a strongly pro-European government led by prime minister Edouard Balladur, concerns about the commitment of the authorities to the franc's central parity seemed to have been laid to rest. Indeed, after a period of franc appreciation, the Bank of France allowed short-term interest rates to go below those in Germany. This

was consistent with economic fundamentals since French inflation continued to be lower than that in Germany, and there was more economic slack in France.

The third phase of attack on the franc however began early in July. Given the franc's strength, the French finance minister Edmond Alphandéry was emboldened to make some comments in a radio interview that suggested that the franc might take over the leadership of the ERM from the deutsche mark, and he also invited his German counterpart, Theo Waigel, to a meeting in Paris to discuss exchange rate issues. The cool reception to the idea by the latter was apparently interpreted by the markets as calling into question the Franco-German partnership. Why this should have provoked a run on the franc, in circumstances where the France remained committed to the franc's central parity and was unlikely to run a more expansionary long- run monetary policy than Germany, is unclear. In any case, the crisis widened and spread, leading to downward pressures on almost all other ERM currencies against the DM. When, on July 29 the Bundesbank lowered the Lombard rate but not the discount rate, speculative flows became enormous, swamping the attempts of central banks to intervene to keep the rates within the ERM bands. The decision to widen the bands on the weekend resulted.

Though the above chronology suggests some events that may have provoked the crises, a number of questions remain about their timing. First, why did speculation against the franc not subside after the September 20, 1992, vote in favor of the Treaty? Second, if the financial markets had doubts about the determination of French policymakers to defend the currency, why did not these doubts subside once the Bank of France showed the willingness to take the appropriate measures, and once the character of the new government was made clear? Third, why should rumors of minor disagreement between France and Germany - something that occurs relatively frequently - have provoked a major shift in sentiment with respect not only to the franc but also with respect to other currencies?

We would argue that it is most useful to interpret the above events from the perspective of multiple equilibria, in which expectations of devaluation influence the costs that the authorities must bear to defend the currency. In the case of France, where there was no reason to believe that the exchange rate was overvalued, credibility of the parity would have been consistent with interest rates equal to those in Germany, that would not have imposed large economic or political costs on the authorities since German rates by 1993 had already declined considerably. However, lack of credibility could be self validating, since it imposed much larger costs. In addition, we would argue that it was essentially extrinsic shocks-including rumors of disagreement, poll results, and nuances in official statements-that triggered jumps between equilibria. In

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this interpretation, such "news" would not be part of the "fundamentals," but would rather be a trigger for sunspot equilibria. A model with these properties is developed below.

An alternative interpretation, argued by Krugman (1996), is that political factors need to be included in the fundamentals, and that given that such factors are very hard to measure, one should not interpret the absence of clear signs in the data of a deterioration of fundamentals at the time of speculative attacks as evidence that they were self-fulfilling or based on sunspots: "... one may question whether any of the quantitative measures available is a good proxy for the true fundamentals implied by a realistic model of the decision whether to defend a fixed rate: since the decision is essentially political, it is likely to be influenced strongly by the exhaustion of hard-to-measure reserves of public patience and political capital rather than tangible measures like financial reserves." (Krugman, 1996, p. 27). However, in the case of France we would argue that the political fundamentals were, if anything, improving over the period, and, at a broader level, there was ample evidence of Franco-German solidarity. The problem with maintaining that the fundamentals are essentially unobservable is that there is no way to resolve such disagreements of interpretation. A more useful approach in our view is to limit the set of fundamentals to economic variables, and, since these are not sufficient to explain either the broad movements or precise timing of the attacks on the franc, to model short-run dynamics in a way that introduces other sources of fluctuations. Though it is not conclusive proof of the existence of self-fulfilling expectations, we will see below that a modelling strategy which assumes that extrinsic uncertainty explains jumps between equilibria that are dictated by the varying costs imposed by private sector expectations does a good job in tracking the movements in the French-German interest rate differential.

3   The Model

In this section, we present a model of a fixed exchange rate system with an optimizing policymaker, in which we study the form of the relationship between the devaluation expectations and the fundamentals. After a statement of the assumptions (3.1), we first study the equilibria in which devaluation expectations are determined solely by the fundamentals (3.2), before examining the role that sunspots might play in the model (3.3). Finally, subsection 3.4 considers procedures to estimate the model.

3.1 Assumptions

We consider a country which is in a fixed exchange rate system. At each period, the domestic policymaker decides to stay in the system or opt out, i.e., devalue.[1] The decision to devalue or not depends on which policy gives the lower loss function value, the latter being assumed to be a quadratic in the deviations of some variables y_{kt} relative to some target values $\overline{y}_k$, plus an opting out cost C:

$$L_t = \sum_{k=1}^{K} w_k (y_{kt} - \overline{y}_k)^2 + \delta_t C \tag{1}$$

Variable δ_t is a dummy variable characterizing the policymaker's decision ($\delta_t = 1$ if there is a devaluation in period t, and 0 otherwise). The opting out cost C may represent a lump-sum reputation cost or the political cost of a devaluation, while the y_k are economic variables that matter for the policymaker's decision to opt out or not, the weights of which are given by the w_k.

We assume that a devaluation changes variable y_k by a net amount Δ_k and that in addition, the probability of a devaluation expected for next period, π_t, affects y_{kt} negatively by an amount $a_k \pi_t$.[2] We write variable y_{kt} therefore as the sum of these two effects plus the exogenous fundamentals f_{kt} that exclude them:

$$y_{kt} = f_{kt} - a_k \pi_t + \delta_t \Delta_k \tag{2}$$

There are several ways to interpret equation (2). For example, variable y_k may represent output, in which case equation (2) could be interpreted as saying that output is affected negatively by devaluation expectations because of the rise in interest rates, and positively by a devaluation because of the improvement in competitiveness.[3] While the output level is obviously an important variable for the policymaker's decision, y_k may reflect other factors, like the health of the banking system, the trade balance, inflation, anti-inflationary credibility, public debt, etc. In general, these variables matter to different degrees for the policymaker, and are not affected by devaluation expectations or a devaluation in the same way. For example, one may expect inflation to be increased by a devaluation and decreased by a restrictive shift

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[1] Opting out may be followed by a floating regime or the fixation of a new parity. It is assumed that in either case there is an initial depreciation of a fixed amount against the foreign currency. This allows us to use interest differentials as a proxy for devaluation expectations.

[2] Note that expectations of the future enter here, as in Krugman (1996), rather than expectations formed in the past of current variables, as is assumed by Obstfeld (1994, 1996) or Jeanne (1995). For a discussion of the implications of the timing assumption for multiple equilibria, see Kehoe (1996).

[3] Krugman (1996) presents a simple macromodel of currency crisis in which the loss function of the policymaker has the same form as (1) where y is output, and the reduced form of output is analogous to (2).

in monetary policy ($a_k > 0$, $\Delta_k > 0$). For the real burden of the public debt or real competitiveness, the expected signs of the coefficients would be $a_k < 0$, $\Delta_k < 0$ and $a_k \simeq 0$, $\Delta_k > 0$ respectively.

The sequence of events at period t is as follows. First, the market representative agent forms its devaluation expectations in a rational way on the basis of all available information. Second, the policymaker decides to opt out or stay in the fixed exchange rate system given π_t. A devaluation will occur when opting out lowers the value of the loss function, i.e.:

$$\sum_{k=1}^{K} w_k (f_{kt} - a_k \pi_t + \Delta_k - \overline{y}_k)^2 + C < \sum_{k=1}^{K} w_k (f_{kt} - a_k \pi_t - \overline{y}_k)^2$$

or

$$\sum_{k=1}^{K} w_k \Delta_k (2 f_{kt} - 2 a_k \pi_t + \Delta_k - 2 \overline{y}_k) + C < 0 \qquad (3)$$

Let us denote $b_t = \sum_{k=1}^{K} w_k \Delta_k (2 f_{kt} + \Delta_k - 2 \overline{y}_k) + C$ the <u>gross</u> benefit of the fixed exchange rate system and $\alpha = 2 \sum_{k=1}^{K} w_k \Delta_k a_k$. Then the policymaker decides to devalue when the <u>net</u> benefit

$$B_t = b_t - \alpha \pi_t \qquad (4)$$

is strictly negative, and otherwise keeps the rate fixed.

Equation (4) shows that at any given point in time, the benefit of belonging to the fixed exchange rate system depends not only on the objective current fundamentals but also on the credibility of the policymaker's commitment to the system. Other things equal, a lower credibility (i.e. a higher π) means that the monetary authorities must set the interest rate at a higher level in order to counterbalance the devaluation expectations, which reduces the benefit of the system through a number of channels (lower economic activity, fragilization of the banking sector, higher interest burden on the public debt, etc.).[4]

The dynamics of the system are driven by the exogenous stochastic fundamentals f_{kt}. For the sake of notational simplicity, we specify the stochastic process followed by these fundamentals in terms of the gross benefit b_t, to which we shall sometimes refer as the "fundamental". We assume that the dynamics of the gross benefit can

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[4]Eichengreen and Wyplosz (1993) discuss the different channels through which raising the interest rate was costly for European policymakers in 1992-3.

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be approximated by an AR(1) process:

$$b_t = (1 - \rho)\bar{b} + \rho b_{t-1} + \varepsilon_t \tag{5}$$

where ε is an i.i.d. normal shock of variance σ^2 and $\rho \in]0, 1[$.

3.2 The fundamental based regime

Because of the rationality of expectations, the devaluation probability at time t must be equal to the probability that next period the net benefit $B_{t+1} = b_{t+1} - \alpha\pi_{t+1}$ is negative, *i.e.*:

$$\pi_t = Prob_t[b_{t+1} < \alpha\pi_{t+1}] \tag{6}$$

The defining characteristic of the fundamental-based regime is that the state of the system is uniquely determined by the exogenous fundamental b_t. In particular, the devaluation probability can be written as a univariate function of b:

$$\pi_t = \pi(b_t)$$

The fundamental based regime is characterized by the critical level of the gross benefit which triggers devaluation, i.e., the level b^* such that the policymaker opts out at any date t if and only if $b_t < b^*$. In a rational expectations equilibrium, b^* is determined as a fixed point in the reciprocal relationship between the private sector expectations and the policymaker's policy. For a given level of b^*, the private sector estimates the devaluation probability at time t as:

$$
\begin{aligned}
\pi_t &= Prob_t[b_{t+1} < b^*] \\
&= Prob_t[(1 - \rho)\bar{b} + \rho b_t + \varepsilon_{t+1} < b^*] \\
&= F_\sigma[b^* - (1 - \rho)\bar{b} - \rho b_t]
\end{aligned} \tag{7}
$$

where $F_\sigma(.)$ is the cumulative distribution function of a normal distribution with variance σ^2. Conversely, b^* must be the optimal triggering level for the policymaker given this expectation function, i.e., the net benefit $b - \alpha\pi(b)$ must be equal to zero for $b = b^*$:

$$b^* = \alpha F_\sigma[(1 - \rho)(b^* - \bar{b})] \tag{8}$$

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Once this equation is solved for b^*, one obtains the devaluation probability as a non-linear function of the current fundamental (equation (7)). This function simply equates the devaluation probability to the probability that the fundamental next period will exceed b^*, as given by the cumulative distribution of the normal. This is depicted in Figure 1, for $b^* = \bar{b} = 100$.

3.3 The sunspot regimes

As both sides of equation (8) are increasing with b^*, it may have multiple solutions. To illustrate, Figure 2 shows a case where there are three critical benefit thresholds $b_I^* < b_{II}^* < b_{III}^*$. The multiplicity of solutions is intuitive. It relies on the fact that high devaluation expectations can validate themselves because they decrease the membership benefit of the fixed exchange rate system.

The multiplicity of solutions makes it possible to construct equilibria in which the critical benefit b^* can jump from one level to another, or, to put it in other terms, the economy can jump across states with different levels of devaluation expectations. *A priori*, the jumps between states may be related to the fundamentals, but this is not necessarily the case; they may also be driven by extrinsic uncertainty. In the latter case, the dynamics of the devaluation probability are driven by two independent processes: the gross benefit, which depends on the economic fundamentals, and a sunspot variable, which coordinates the private sector expectations on one state or the other. We call this type of equilibria *sunspot regimes* in what follows.

In order to construct sunspot regimes, it is important first to note that the states between which the economy jumps do not correspond to the fundamental based regimes characterized in the preceding section. If b^* can change, the possibility of state shifts, which is rationally anticipated by the agents, changes the whole structure of their expectations and consequently of the policymaker's optimization problem. Thus, one needs to redefine and characterize the equilibria in a way that takes into account the possibility of state shifts.

For this purpose, we need to introduce precisely the notion of state. Let us assume that the economy can be in n states $s = 1, ..., n$. The states differ from each other through the level of the gross benefit which triggers devaluation. We assume that if the state at time t is s, the policymaker opts out at t if and only if $b_t < b_s^*$. The threshold benefit levels are ranked by increasing order, i.e. $b_1^* \leq b_2^* \leq ... \leq b_n^*$, which means that for given fundamentals, devaluation is less likely in state 1 than in state 2, in state 2 than in state 3 and so on. Like in Dagsvik and Jovanovic (1994) or Jeanne (1996), the transition across states is assumed to follow a markov process independent of the fundamentals, characterized by the transition matrix $\Theta = [\theta(i,j)]_{1 \leq i,j \leq n}$.

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At time t, if the state is s_t, the devaluation probability can be written:

$$\pi_t = \sum_{s=1}^{n} \theta(s_t, s) Prob_t[b_{t+1} < b_s^*]$$

$$= \sum_{s=1}^{n} \theta(s_t, s) F_\sigma[b_s^* - (1-\rho)\overline{b} - \rho b_t]$$

The devaluation probability now depends jointly on the state and the gross benefit. The expectation function takes the form:

$$\pi_t = \pi_{s_t}(b_t) \tag{9}$$

with:

$$\pi_s : b \rightarrow \sum_{s'=1}^{n} \theta(s, s') F_\sigma[b_{s'}^* - (1-\rho)\overline{b} - \rho b]$$

Conversely, b_s^* must be the optimal triggering level for the policymaker when the state is s. This is the case if $b_s^* = \alpha \pi_s(b_s^*)$ for all states s, i.e.:

$$\forall s = 1, ..., n, \quad b_s^* = \alpha \sum_{s'=1}^{n} \theta(s, s') F_\sigma[b_{s'}^* - (1-\rho)\overline{b} - \rho b_s^*] \tag{10}$$

Since these equations summarize all that the model implies for the relationship between the fundamentals and the devaluation probability, we can characterize sunspot regimes by vectors $(b_1^*, ..., b_n^*)'$, and markov matrices Θ satisfying the n constraints (10). One can note that according to this definition, the fundamental based regimes may be viewed as a degenerate case of the sunspot regime, corresponding to Θ equal to the identity matrix. In that case, equations (10) all reduce to equation (8), and the b_s^* are necessarily equal to b_I^*, b_{II}^* or b_{III}^*. The economy never jumps, but deterministically remains in its initial state.[5]

Of course, we are more interested in non-degenerate cases, in which the economy actually jumps between different states. In order to characterize the latter type equilibria, we shall say that a sunspot regime is non degenerate of order n iff the matrix Θ is truly stochastic, i.e. it has some elements which are different from zero and one, and the states are strictly different, i.e. $b_1^* < ... < b_n^*$.[6] The following

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[5]Another case in which the transition between states is deterministic is when the matrix Θ is cyclic, *i.e.* there is a way to order states such that Θ sends the economy from state 1 to state 2, from state 2 to state 3, and so on. It is possible to show that cyclic equilibria are always degenerate in our model, *i.e.* if Θ is cyclic, then necessarily $b_1^* = ... = b_n^*$. In a slightly generalized version of the model (in which the net benefit depends not only on the current devaluation probability but also on the lagged one), non-degenerate cyclic equilibria can arise. When this is the case, it is also possible to build equilibria in which the dynamics of the devaluation probability follow a deterministic chaotic process.

[6]This terminology is borrowed from Chiappori and Guesnerie (1991).

proposition states formally the relationship between the multiplicity of solutions to (8) and the existence of non degenerate sunspot regimes.

Proposition 1 . *Non degenerate sunspot regimes exist if and only if there are multiple solutions to equation (8), which is the case iff the following two conditions are satisfied:*

$$z \equiv \frac{(1 - \rho)}{\sqrt{2\Pi}} \frac{\alpha}{\sigma} > 1 \tag{11}$$

$$\overline{b} \in]\overline{b}_{\min}, \overline{b}_{\max}[\tag{12}$$

where $\overline{b}_{\min}$ and $\overline{b}_{\max}$ are two bounds.

Moreover, if these conditions are satisfied, it is possible to construct non-degenerate sunspot regimes of any order n.

The proof of the proposition, as well as explicit formula for the bounds $\overline{b}_{\min}$ and $\overline{b}_{\max}$, may be found in the Appendix. The Proposition first says that non-degenerate sunspot regimes exist if two conditions are satisfied. Equation (11) may be viewed as a condition on coefficient α, which reflects the sensitivity of the net benefit to the devaluation probability. It is clear that in the extreme case where $\alpha = 0$, there is one unique equilibrium in which the policymaker devalues if and only if the gross benefit b is negative. The multiplicity of equilibria can arise only if coefficient α is large enough. The second condition - equation (12) - relates to the fundamental process. It states that the long term values of the gross benefit $\overline{b}$ must be neither too high nor too low for the multiplicity of equilibria to arise. If the gross benefit $\overline{b}$ is too high (low), there is one unique equilibrium in which the devaluation expectations are low (high). The devaluation expectations cease to be uniquely determined when $\overline{b}$ belongs to the intermediate zone $]\overline{b}_{\min}, \overline{b}_{\max}[$.

One might have expected the number of states to be limited to three, which is the maximum number of solutions that equation (8) can have. The last part of the Proposition shows that this conjecture is wrong. In fact, the number of states can be arbitrarily large. This implies that we can take the states arbitrarily close to each other, and in the limit define the set of states as a continuum. This property is in sharp contrast with other models, e.g. Obstfeld (1994, 1996), Jeanne (1996), Vélasco (1996), where the number of states is two or three. The source of the difference lies in the assumptions concerning the timing of devaluation expectations. In other papers, the net benefit of the fixed exchange rate system at a given period depends

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on the devaluation expectations formed in the preceding period. In our notations, this corresponds to the assumption $B_t = b_t - \alpha\pi_{t-1}$, from which it directly follows that $\pi_t = Prob_t[b_{t+1} < \alpha\pi_t] = F_\sigma[\alpha\pi_t - \rho b_t]$. This equation has the same form as (8) and cannot have more than three solutions. In contrast, we assume here that the net benefit of the sytem at a given period depends on the expectations about the future. This assumption enlarges considerably the set of equilibria, according to a logic that is not without analogy with the Folk theorem (Fudenberg and Maskin, 1986).

3.4   Estimation procedure

We assume that the devaluation expectations are generated by the following model:

$$\pi_t = \sum_{s'=1}^{n} \theta(s, s') F_\sigma[b_{s'}^* - (1 - \rho)\overline{b} - \rho b_t] + \eta_t \tag{13}$$

$$b_t = \beta_0 + \beta_1 x_{1t} + \ldots + \beta_K x_{Kt} \tag{14}$$

The first equation is simply equation (9) to which we have added an i.i.d. normal stochastic term η that can be interpreted as the model prediction error. The second equation is a linear specification of the gross benefit of the fixed rate, where the $(x_k)_{k=1}^K$ are the fundamentals, i.e., a set of macroeconomic variables which are *a priori* relevant for the determination of the devaluation. Since our sample does not include a devaluation, the beneficial effect of a devaluation on the fundamentals is not captured by the observed $(x_k)_{k=1}^K$, but is included in the constant term β_0. (In the next section, we shall take the x_k as the unemployment rate, the trade balance, the real exchange rate, and time.)

We estimate this model taking into account the constraints of the structural model of the previous section. In particular, the coefficients ρ and $\overline{b}$ used in the first equation are taken as the estimates resulting from a regression of b_t, as defined in the second equation, on its lagged value and a constant. Moreover, we estimate the model under the n constraints (10) linking the transition probabilities to the benefit thresholds $b_1^*, \ldots, b_n^*$.

We do not adopt the restriction, however, that σ^2 is the estimated variance of the process b_t. The reason is that by adding a white noise to the opting out cost parameter σ^2 can be set at any value larger than the variance of b_t. In the case of the fundamental based regime this point can be established formally in the following way. Let us add an i.i.d. stochastic term ϵ_C to the opting out cost, so that (4) becomes $B_t = b_t + \epsilon_{Ct} - \alpha\pi_t$. Assume further that the triggering level of the gross benefit may

be written $b_t^* = b^* + \epsilon_{b^*t}$, where ϵ_{b^*t} is i.i.d. normal of variance $\sigma_{b^*}^2$. Then it follows that the expression of π_t is that given in the text, with $\sigma^2 = \sigma_b^2 + \sigma_{b^*}^2$. For the generalized model to be self-consistent, the process ϵ_C must satisfy $\epsilon_{Ct} = \alpha\pi(b^* + \epsilon_{b^*t}) - (b^* + \epsilon_{b^*t})$, which actually defines an i.i.d. stochastic process. While the argument does not generalize to the sunspot regime, we think it provides a sufficient basis for not taking into account the restriction on σ in the estimation.

Like in Dagsvik and Jovanovic (1994) and Jeanne (1996), the estimation is implemented through the Maximum Likelihood method. For a given set of observations $(\pi_t)_{t=1}^T$, $(x_{kt})_{t=1,...,T}^{k=1,...,K}$, the likelihood of the model can be written:

$$L = L_\eta L_\theta$$

with:

$$L_\eta = \frac{1}{(2\Pi)^{T/2}\sigma_\eta^T} \exp\left(-\frac{\sum_1^T \eta_t^2}{2\sigma_\eta^2}\right)$$

$$L_\theta = \prod_2^T \theta(s_{t-1}, s_t)$$

Maximizing over σ_η and leaving aside an unimportant constant, the likelihood can be written in logarithms:

$$\log L = -\frac{T}{2}\log\left(\frac{\sum_1^T \eta_t^2}{T}\right) + \sum_2^T \log\theta(s_{t-1}, s_t)$$

This function must be maximized over n, $(\beta_k)_{k=0}^K$, $(b_s^*)_{s=1}^n$, Θ, $(s_t)_{t=1}^T$, α and σ under the constraints (10), and taking ρ and $\bar{b}$ as the estimates derived from the regression of b_t on b_{t-1} and a constant. Because of some indeterminacy in the parameters,[7] one can adopt the normalization $\bar{b} = 100$.

In practice, this maximization problem is difficult to solve because: 1) the states are discrete variables; and 2) the likelihood function is not continuous in the parameters because the model is subject to bifurcations.[8] In order to tackle the first

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[7]The indeterminacy comes from the fact that the likelihood function does not change when one multiplies $(\beta_k)_{k=0}^K$, $(b_s^*)_{s=1}^n$ and σ by a positive constant. One might have adopted other, maybe more natural normalization, like fixing the value of σ. We found, however, that the maximization algorithm tended to converge better when $\bar{b}$ was fixed.

[8]To see where the bifurcations come from, let us assume, for example, that starting from a configuration of parameters in which sunspot regimes exist, we continuously decrease parameter α leaving the other parameters unchanged. At some point, a bifurcation in the sense of Azariadis (1993, chapt. 8) will occur: the conditions for the existence of the sunspot regimes (11) and (12) will cease to be satisfied, and the set of equilibria contracts to one unique equilibrium.

problem, we have taken s_t at each period t as the state which minimizes the absolute value of the prediction error η_t.[9] The second problem is difficult to avoid because the existence of bifurcations is inevitable in models in which the existence of multiple equilibria is conditional on the parameters. It can be overcome by using sufficiently robust maximization algorithms[10] and simplifying the set of parameters under which bifurcations arise, for example by putting some restrictions on the transition matrix Θ. In the estimation presented in the next section, for example, we restrict ourselves to the case when there are three states and the transition matrix has the form:

$$\Theta = \begin{pmatrix} \theta(1,1) & \theta(1,2) & 0 \\ \theta(2,1) & \theta(2,2) & \theta(2,3) \\ 0 & 0 & 1 \end{pmatrix}$$

This restriction means that there are two states, 1 and 2, between which the economy can effectively switch, while the third state is perfectly absorbing. The critical threshold benefit of state 3 is the same as in the less stable fundamental based regime, i.e., $b_3^* = b_{III}^*$.

The structure of the model, which involves estimation of a markov transition matrix, is similar in many respects to the setup pioneered by Hamilton (1988, 1989, 1990) designed to account for data drawn from more than one regime. However, it should be noted that the structure estimated here is more closely linked to an economic model and is more tightly parameterized. In particular, the set of fundamentals and the variance of the shocks to the gross benefit of the fixed exchange rate system are the same across regimes. Moreover, the existence of multiple regimes is not an assumption, but rather emerges from the structure of the model, when parameters have values that fall in a certain range.

4   Empirical results

The model was estimated by an iterative maximum likelihood procedure that imposed the relations discussed above between the parameters and the possibility of multiple equilibria. We consider monthly data between February 1987 and July 1993, which is the longest sample period without change in regime for the franc (it starts after the last franc devaluation, that took place in January 1997, and ends before the ERM bandwidth was widened to 15% in August 1993).

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[9]By yielding a likelihood which may be lower than the true level in the sunspot regimes, this simplification may bias the estimation of the model against the existence of multiple equilibria.

[10]In particular, by using algorithms that do not rely too sensitively on the gradient of the likelihood function.

The data on devaluation probabilities were calculated from the one-month interest differential between Euro-franc and Euro- DM instruments, after correcting for expected movement toward the center of the band using Svensson's method, and assuming a devaluation size of 5 percent. Two variants were estimated, one in which the fundamentals included only a time trend (t) and the unemployment rate (ur), and a second one in which the trade balance (as ratio to GDP, $trbal$) and the percentage deviation of the real effective exchange rate from its 1990 level (rer) were also included among the fundamentals.[11] The real exchange rate is computed on the basis of unit labor costs in production; an increase in this index corresponds to a real appreciation of the franc. While the unemployment rate, the trade balance and real competitiveness influence the benefit of maintaining the fixed parity for obvious reasons, including time can be justified by reputational considerations. One of the main motivation to maintain a fixed parity, that was put forward especially in the case of the EMS (Giavazzi and Giovannini, 1989), is the desire of the policymaker to acquire an anti-inflationary reputation. This reputation builds gradually through time as private agents revise their beliefs about the policymaker, through a learning process that has been modeled, e.g., by Masson (1995). From this point of view, including time in the definition of b may be viewed as a short cut to take into account the reputational dimension of the benefit of the fixed peg.

The equation for the gross benefit was specified as:

$$b_t = \beta_0 + \beta_u ur + \beta_t t + \beta_b trbal + \beta_r rer$$

where β_0 was chosen so as to make $\bar{b}$ equal to 100.

Estimation proceeded by first estimating the model without multiple equilibria (i.e. the fundamentals-based model), which as discussed above is a special case of the model with a low enough value of α. If there is only a single equilibrium, then the value of that parameter is not identified, so it was imposed arbitrarily to equal 20 (it was verified that other values of α below the critical level for multiple equilibria, which is 154, gave the identical likelihood).

The results are presented in columns (1) and (2) of the following table, and the

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[11]Data are taken from the International Financial Statistics (IMF). The set of fundamentals could of course be widened further, in particular to include fiscal variables, which are critical in many speculative attack models because they explain domestic credit and hence monetary growth. However, as in most developed economies, there is no automatic mechanism in France linking deficits to money creation, and seigniorage over this period was negligible. Moreover, the deterioration of the deficit over our sample period was largely due to cyclical factors (which are also reflected in the unemployment rate), and the public debt ratio, which remained below 50 percent of GDP, was not likely to have been a factor in explaining interest rates in France (unlike in Italy, where it rose to 120 percent of GDP).

forecast values for the probability to devalue are plotted against the data in Chart 5. Table 1 presents the P-values of our estimates, i.e., the level of confidence at which one can reject that they are equal to zero. These P-values were estimated using the Likelihood Ratio test for each coefficient. (The standard errors computed by Gauss from the Hessian are unrealistically small, which we suspect reflects the fact that because of the discontinuity of the likelihood function the computed Hessian is very far from the true one). Because σ and ρ cannot be set to zero in the estimation, the P-values are not reported for these coefficients.

TABLE 1. Maximum Likelihood Estimates of Parameters

(p-values in Parentheses)

	(1)	(2)	(3)	(4)
α	20	20	200	200
β_u	-2.495	.6326	-.0105	-.0434
	(0.01)	(0.39)	(0.00)	(0.00)
β_t	.2916	.4492	.00274	.0060
	(0.00)	(0.23)		
β_b	-	-6.871	-	.0384
		(0.22)		(0.20)
β_r	-	-.3812	-	-.2306
		(0.29)		(0.66)
σ	46.54	48.47	.3905	1.0130
ρ	0.9798	0.8398	0.9897	0.9807
$(\lg L)/T$	3.9991	4.0457	4.3240	4.3344

The results for the restricted set of fundamentals are sensible but those for the larger set are not, since they attribute the wrong signs to the variables except for the real exchange rate and time (for instance, higher unemployment increases b_t, making it more likely that the authorities would stay in the system). Therefore, the chart only plots the fitted devaluation probabilities for the more restricted fundamental. It can be seen that though the fitted values capture the broad trend of π_t, (even showing some upward curvature at the end of the period, because of the increase in the unemployment rate), they do not capture any of the movements associated with episodes of speculation. As a matter of fact, the model does not perform significantly

better than a linear regression of π on the fundamentals (which gives a per period log likelihood of 3.995 with the restricted set of fundamentals, and 4.037 with the extended one).

The model was then estimated with a higher value of α, such that multiple equilibria exist. In this range it also was evident that α was not very well determined, so that its value ($\alpha = 200$) was simply imposed. Note that as a result of multiplying α by 10, the estimated value for σ is smaller by a factor of almost 100. The value of β_t was chosen optimally subject to the constraint that it produced a value of $\rho < 1$ and $\bar{b} = 100$. Otherwise, the estimation program gave erratic results and could easily wander into inadmissible regions producing the log of negative numbers. Though the estimation of α would be desirable, for our purposes here it is sufficient simply to show that there is a solution with multiple equilibria that gives a significantly higher likelihood than for the fundamentals based model. Columns (3) and (4) of the above table give the parameter estimates, and Chart 6 plots actual and fitted values of π_t, corresponding to column (4). The plot for column (3) was very similar.

It can be seen that the estimates are more satisfactory in a number of respects. First, the mean log likelihood reported is higher. A formal test, calculated as twice the difference in likelihood multiplied by the number of observations, 78, when compared to a chi-square with 1 degree of freedom, is significant at the one-percent level. Comparisons both of columns (1) and (3) and of (2) and (4) give test statistics in excess of 45, vs. a one-percent critical value of 6.63. Second, when the full set of fundamentals is included (column (4)), each coefficient has its expected sign, unlike in column (2), (though only that for unemployment is different from zero at a high degree of significance). Third, the plot in Chart 6 shows that the model with multiple equilibria seems to capture well several of the episodes of sharp movements in interest differentials. In particular, the sharp upward moves around $t = 10$, $t = 20$, and $t = 35$, as well as the upticks after $t = 67$ (August 1992), are modelled by a jump to the second equilibrium with a higher threshold value for the fundamentals needed to maintain the parity.

The following Θ matrices (for columns (3) and (4), respectively) show the estimated transition probabilities.

$$\Theta^{(3)} = \begin{pmatrix} .879 & .121 & 0 \\ .249 & .719 & .031 \\ 0 & 0 & 1 \end{pmatrix}, \Theta^{(4)} = \begin{pmatrix} .876 & .124 & 0 \\ .238 & .731 & .031 \\ 0 & 0 & 1 \end{pmatrix}$$

It can be seen that the first state (with lower threshold value of the benefits from

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staying in the system) is fairly stable, as is the second one. The estimated threshold benefits $(b_1^*, b_2^*, b_3^*) = (97.846, 97.961, 192.081)$ show that while states 1 and 2 are extremely close, the third state is characterized by a very high threshold benefit. In fact, the threshold benefit in the third state remained always higher than the estimated gross benefit during the sample period, so that switching to the third state would have triggered an immediate devaluation. This event, however, is not observed in the data. The essential difference between states 1 and 2 is that in the latter one, there is a three percent probability to jump to state 3. Thus, the model explains the deterioration of the interest differential on various occasions as a shift in devaluation expectations, corresponding to a change in the perceived probability of devaluation of around 3 percent. Even though the likelihood function penalizes non-unitary values of the Θ matrix, the model with multiple equilibria still does significantly better in terms of likelihood.

5   Concluding comments

We have presented a model of a fixed exchange rate system which, though it is very stylised, can encompass several hypotheses about the relationship between the fundamentals and the devaluation expectations. At the empirical level, we have found that the model gives a substantially better account of the French franc crisis when it gives a role to sunspots. First, it tracks better the episodes of speculation, by interpreting them as self-fulfilling jumps in the beliefs of foreign exchange market participants. Second, and more unexpectedly, it also improves the empirical relationship between the fundamentals and the devaluation expectations. While in the absence of sunspots the coefficients of the unemployment rate, the trade balance or the real exchange rate are not significantly different from zero and often exhibit the wrong signs, the same coefficients take on the right signs when sunspots are introduced, and that of unemployment becomes very significantly different from zero.

To some extent, this paper illustrates how the study of fixed exchange rate systems can benefit from a different way of articulating models and performing empirical work. Most of the existing empirical work about currency crises consists in linear regressions of devaluation expectations on a set of relevant economic variables. The results of these regressions are then interpreted in the light of models that are more structural than the one we have used here, in the sense that they are grounded in an explicit representation of the economy including assumptions about money demand, the determination of output, purchasing power parity, etc.. By contrast, we estimate our theoretical model literally, taking into account its structural constraints

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and in particular its non-linearities. Such an estimation may be implemented with some hope of success because reducing the structure of the underlying model to the minimum makes it very flexible and does not make it dependent on assumptions that would themselves be contestable at the empirical level. Despite this simplicity, the model captures some essential features of the data that are not explained by purely fundamental based models.

However, we would acknowledge that there remains scope for further development. Even those economists who support the thesis of self-fulfilling speculation express some dissatisfaction with the state of the art of modeling multiple equilibria (see, e.g., Obstfeld, 1994, 1995). In particular, the assumption that the economy jumps from one equilibrium to another following the realization of an extraneous shock raises a number of questions. To the extent that the sunspot variable instantaneously coordinates the expectations of all market participants, one would like to relate this variable to an event that is publicly observable. It would be interesting, in this respect, to see whether the transitions between states that we identify are correlated with political events or other news, but this would require extending our analysis to higher data frequency than monthly. A more radical criticism is that the selection of the equilibria should not be based on an hypothetical variable, but rather on an explicit modeling of the dynamics of the beliefs of market participants. From this point of view, it would be interesting to see what the literature about eductive and evolutive learning (Chiappori and Guesnerie, 1991) can teach us about the determination of equilibria in our setting.

Finally, some extensions of our model have potentially interesting properties, like cyclical or chaotic dynamics, that we have only touched upon in this paper. It is noteworthy that in our model, these chaotic dynamics are perfectly consistent with the rationality of the foreign exchange market participants, and in particular do not require some of them to follow *ad hoc* rules. Whether these non-linear dynamics give a good account of devaluation expectations is an interesting question for future research.

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Appendix

Proof of Proposition 1

Let us consider a non-degenerate sunspot regime characterized by a vector $(b_1^*, ..., b_n^*)'$ satisfying $b_1^* < ... < b_n^*$ and a markov matrix Θ. Then using equation (10), one can write:

$$
\begin{aligned}
b_1^* &= \alpha \sum_{s=1}^n \theta(1,s) F_\sigma[b_s^* - (1-\rho)\overline{b} - \rho b_1^*] \\
&> \alpha \sum_{s=1}^n \theta(1,s) F_\sigma[b_1^* - (1-\rho)\overline{b} - \rho b_1^*] = \alpha F_\sigma[(1-\rho)(b_1^* - \overline{b})]
\end{aligned}
$$

Similarly, one can show that $b_n^* < \alpha F_\sigma[(1-\rho)(b_n^* - \overline{b})]$. Figure 2 makes clear that these inequalities can be consistent with $b_1^* < b_n^*$ if and only if there are multiple solutions to equation (8) and $b_1^* \in]b_I^*, b_{II}^*[,\ b_n^* \in]b_{II}^*, b_{III}^*[$.

The derivation of conditions (11) and (12) is based on the representation of equation (8) given in Figure 2. The multiplicity arises when the curve $\mathcal{C}_{\overline{b}}$ representing the r.h.s. of equation (8) intersects the 45° line in three points. This is possible only if the maximum of the slope of $\mathcal{C}_{\overline{b}}$, which is equal to z, is strictly larger than 1. The curve $\mathcal{C}_{\overline{b}}$ must also be neither too much to the right nor to the left of the 45° line, which requires $\overline{b}$ to be in some range $]\overline{b}_{\min}, \overline{b}_{\max}[$. The thresholds $\overline{b}_{\max}$ and $\overline{b}_{\min}$ satisfy the tangency conditions:

$$
\begin{cases}
b^* &= \alpha F_\sigma[(1-\rho)(b^* - \overline{b}_{\max/\min})] \\
1 &= \alpha(1-\rho) F_\sigma'[(1-\rho)(b^* - \overline{b}_{\max/\min})]
\end{cases}
$$

More precisely, $\overline{b}_{\max}(\overline{b}_{\min})$ is the solution of that system which satisfies $\overline{b}_{\max} < b^*(\overline{b}_{\min} > b^*)$. In order to find $\overline{b}_{\max}$, one can rewrite the second equation as:

$$
(1-\rho)(b^* - \overline{b}_{\max}) = \sigma\sqrt{2\lg z}
$$

Noting that, $\forall x,\ F_\sigma(x/\sigma) = F_1(x)$, the first equation then implies $b^* = \alpha F_1(\sqrt{2\lg z})$. Substituting out b^* then yields:

$$
\overline{b}_{\max} = \alpha \left[F_1(\sqrt{2\lg z}) - \frac{1}{z}\sqrt{\frac{\lg z}{\Pi}} \right]
$$

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One can derive the expression for $\overline{b}_{min}$ in a similar way:

$$\overline{b}_{min} = \alpha \left[F_1(-\sqrt{2\lg z}) + \frac{1}{z}\sqrt{\frac{\lg z}{\Pi}} \right]$$

In order to prove the last part of the Proposition, let us show that if there are multiple fundamental based regimes, any vector $(b_1^*, ..., b_n^*)'$ satisfying $b_1^* < ... < b_n^*$ and $b_1^* \in]b_I^*, b_{II}^*[$, $b_n^* \in]b_{II}^*, b_{III}^*[$ can generate a non-degenerate sunspot regime of order n. In order to find a markov matrix Θ for which this is the case, we restrict the attention to matrices Θ satisfying $\theta(i,j) = 0$ for $j \neq 1$ or $j \neq n$. Equation (10) then implies, for all s:

$$\begin{cases} b_s^* = \alpha \left[\theta(s,1)F_\sigma[b_1^* - (1-\rho)\overline{b} - \rho b_s^*] + \theta(s,n)F_\sigma[b_n^* - (1-\rho)\overline{b} - \rho b_s^*] \right] \\ 1 = \theta(s,1) + \theta(s,n) \end{cases}$$

which determines one unique pair $\theta(s,1)$, $\theta(s,n)$ for all states $s = 1,...,n$. It remains to check that the matrix Θ thus determined is a markov matrix, *i.e.* that $\forall s = 1,...,n$, $\theta(s,1)$ and $\theta(s,n) \in]0,1[$. Substituting $\theta(s,n)$ out of the two equations above gives:

$$\theta(s,1) = \frac{F_\sigma[b_n^* - (1-\rho)\overline{b} - \rho b_s^*] - b_s^*/\alpha}{F_\sigma[b_n^* - (1-\rho)\overline{b} - \rho b_s^*] - F_\sigma[b_1^* - (1-\rho)\overline{b} - \rho b_s^*]} \tag{15}$$

It is strictly positive because:

$$b_s^*/\alpha \leq b_n^*/\alpha < F_\sigma[(1-\rho)(b_n^* - \overline{b})] \leq F_\sigma[b_n^* - (1-\rho)\overline{b} - \rho b_s^*] \tag{16}$$

and it is strictly smaller than 1 because:

$$b_s^*/\alpha \geq b_1^*/\alpha > F_\sigma[(1-\rho)(b_1^* - \overline{b})] \geq F_\sigma[b_1^* - (1-\rho)\overline{b} - \rho b_s^*] \tag{17}$$

Q.E.D.

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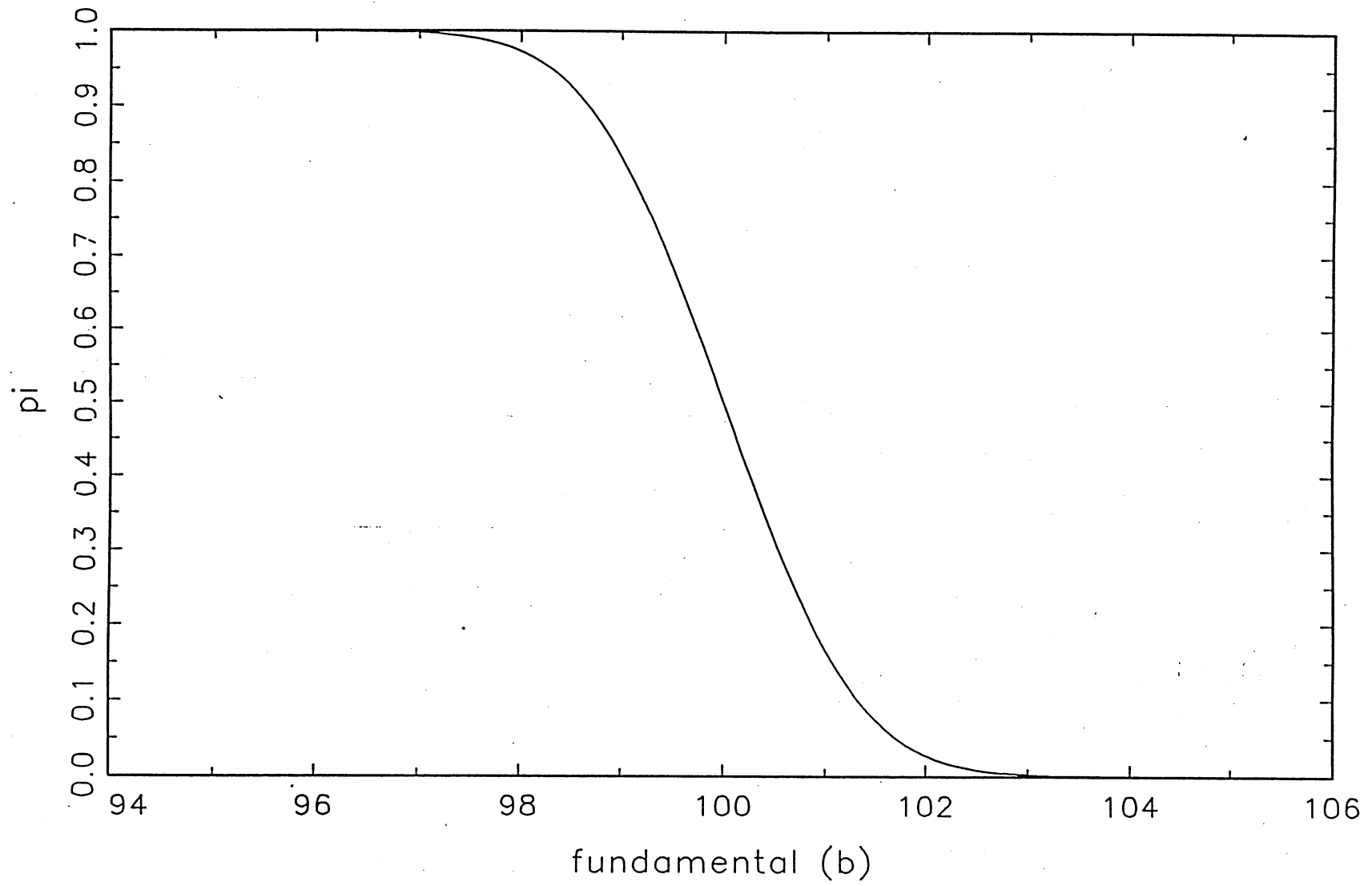## Figure 1: Probability of Devaluation
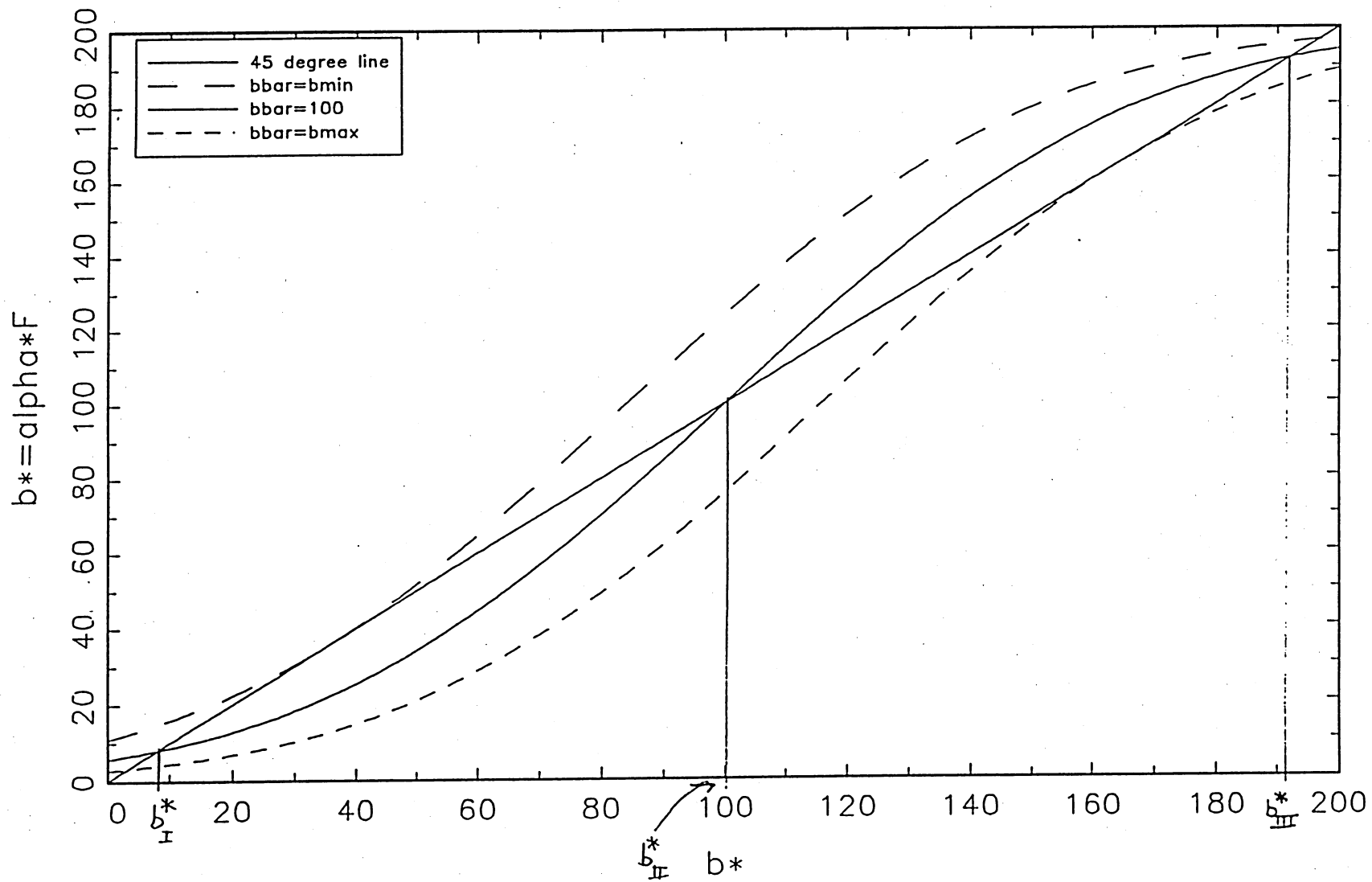
Figure 2: Solutions for b*

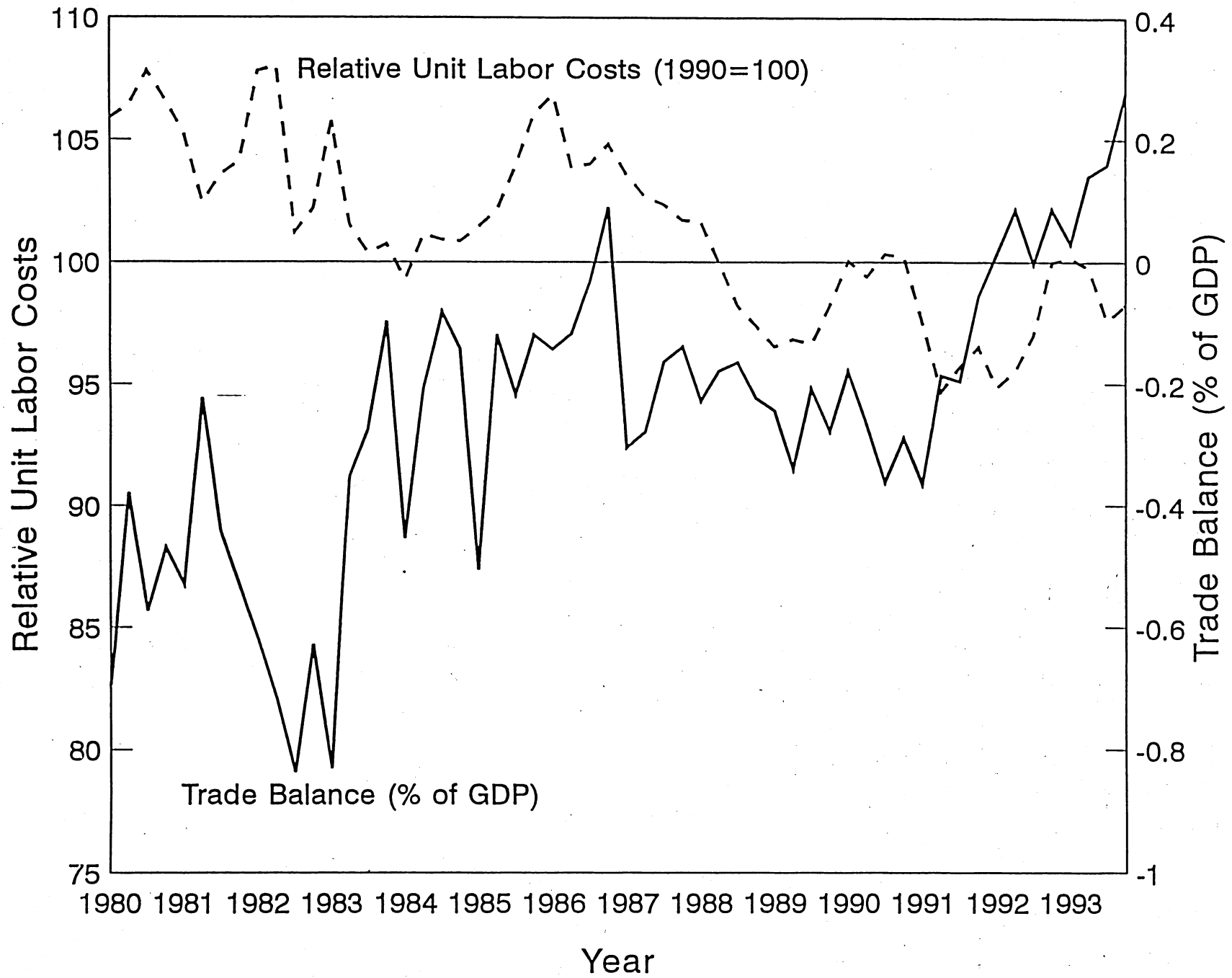# Chart 1. Relative unit labor costs and trade balance, 1980-93

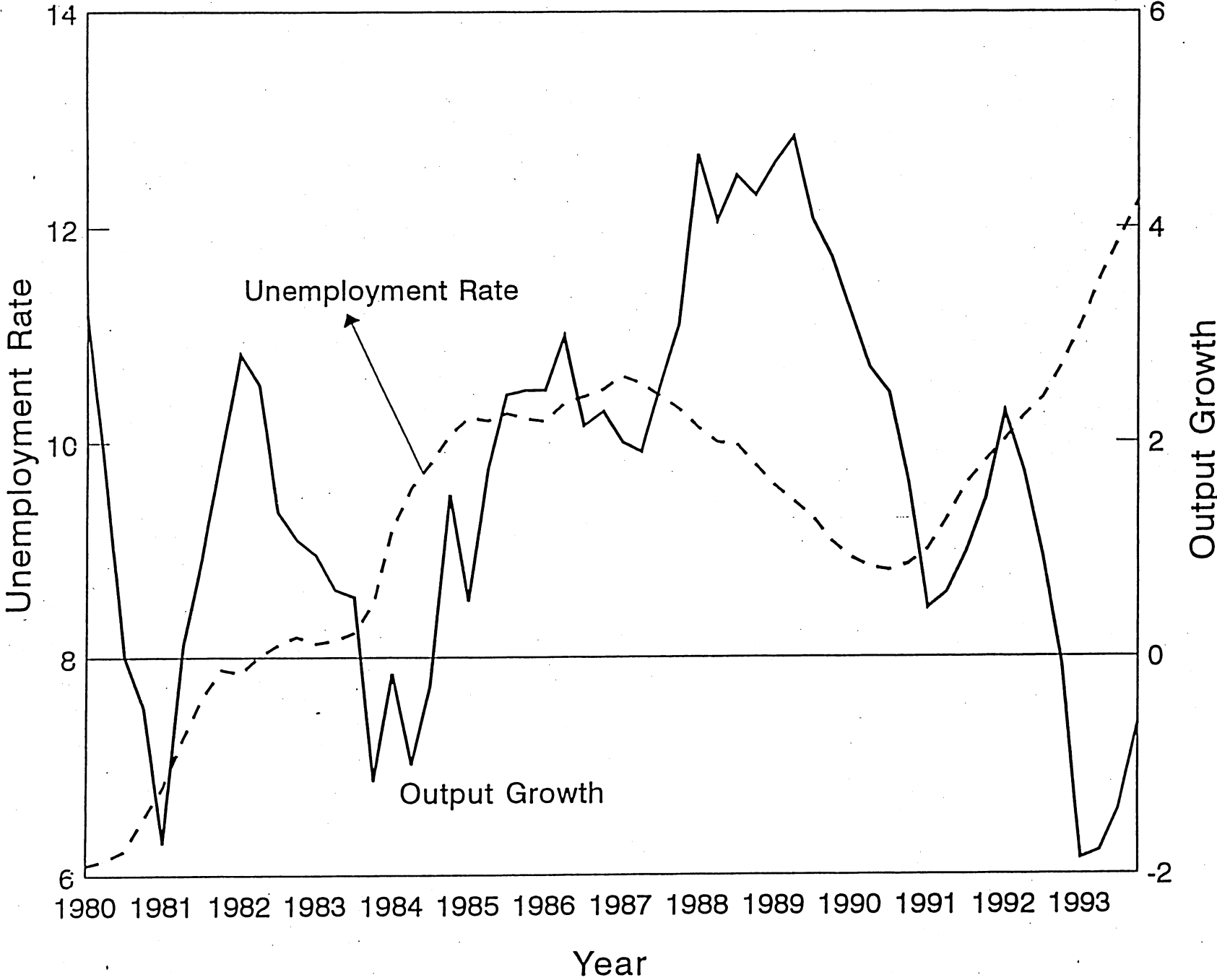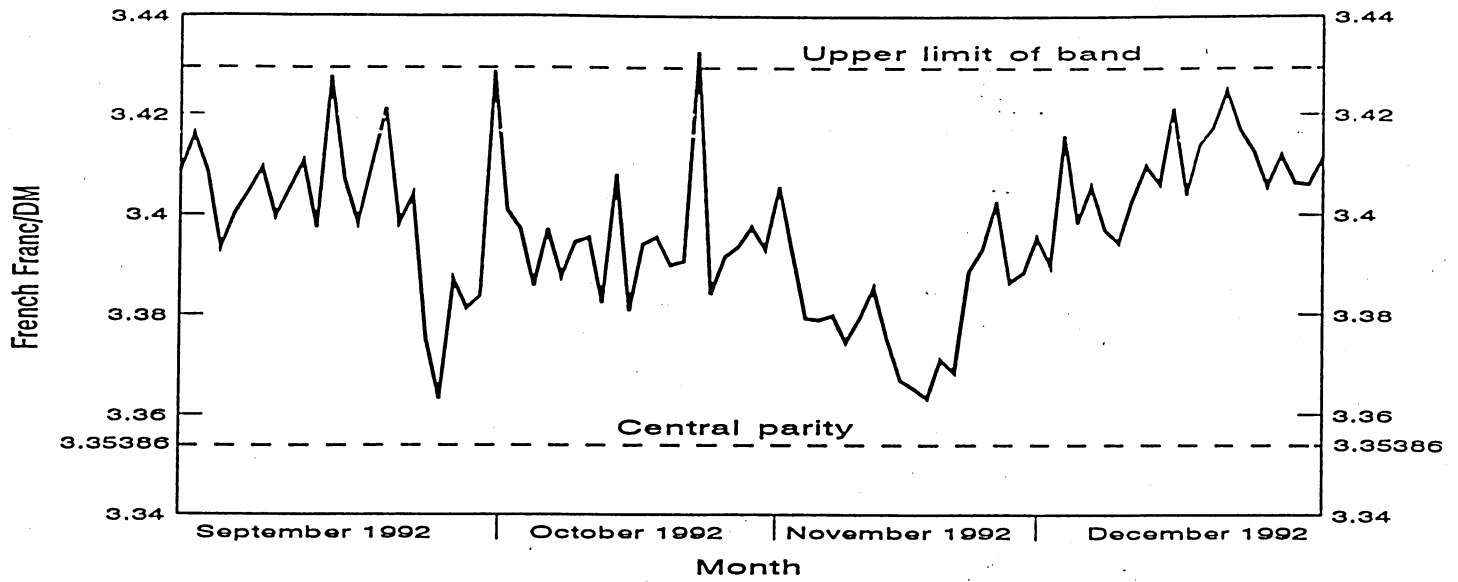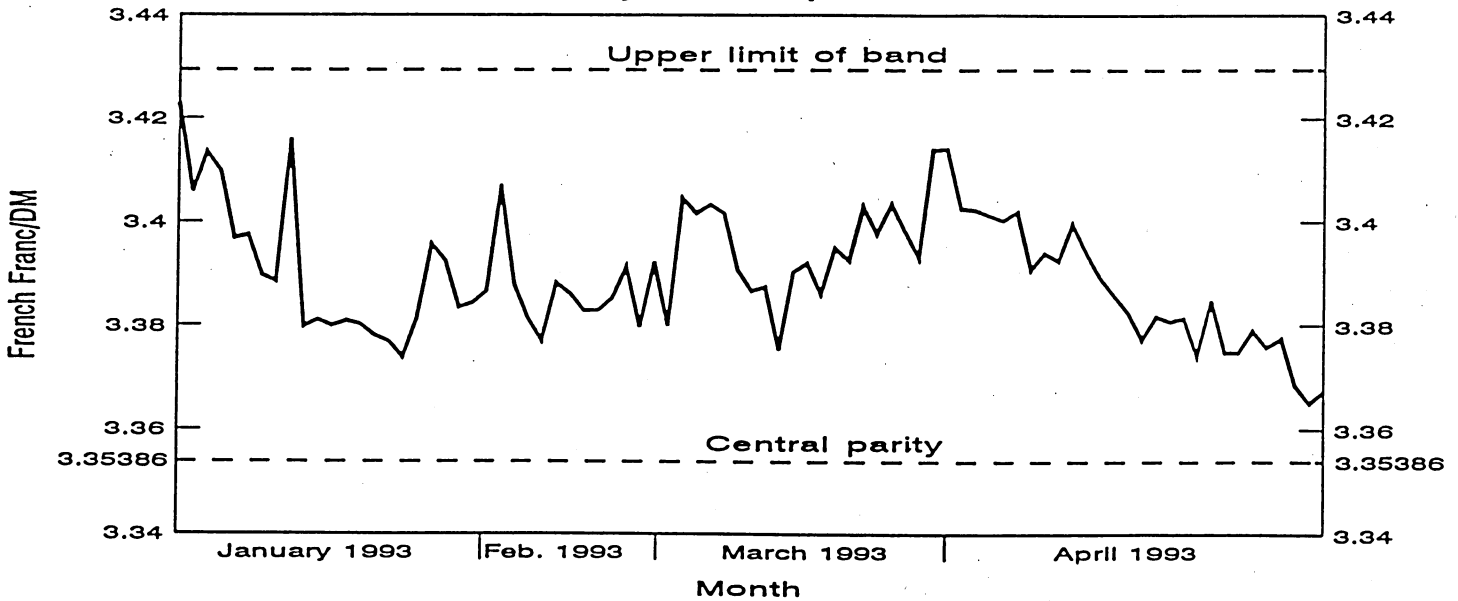# Chart 2. Unemployment rate and output growth, 1980-93
(in percent)

Chart 3. French Franc/DM

a. September 1992 - December 1992

b. January 1993 - April 1993

c. May 1993 - August 1993
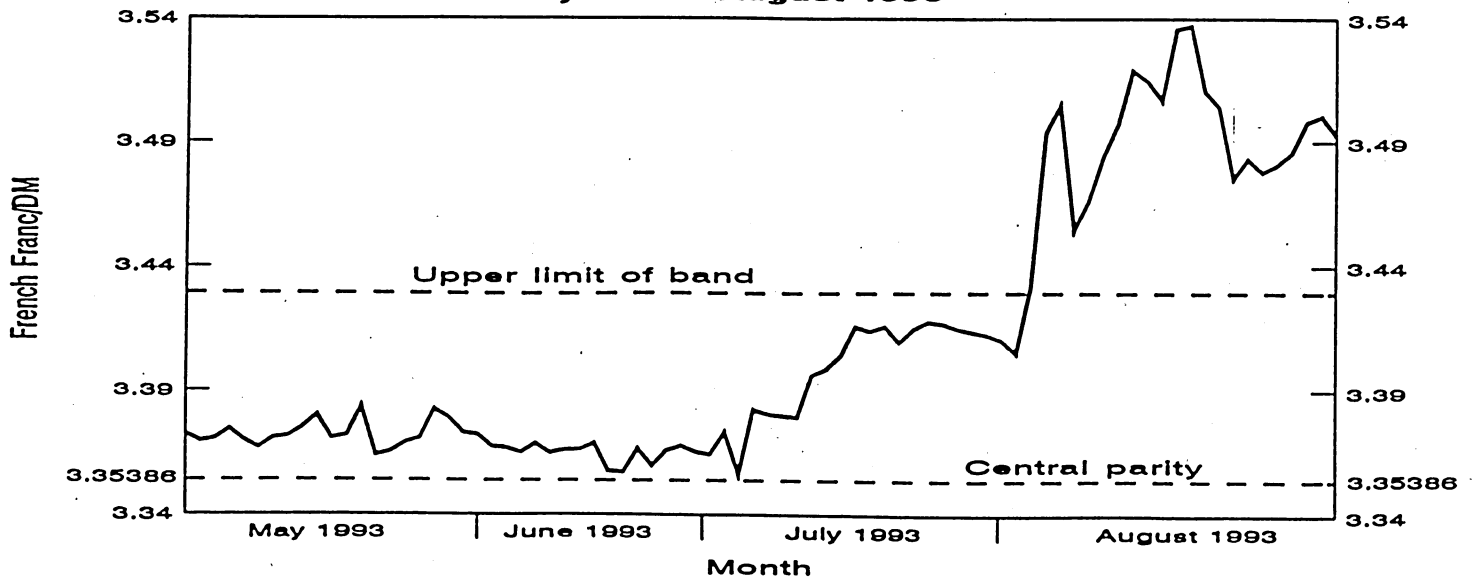
Chart 4. Euro-deposit rates

a. September 1992 - December 1992
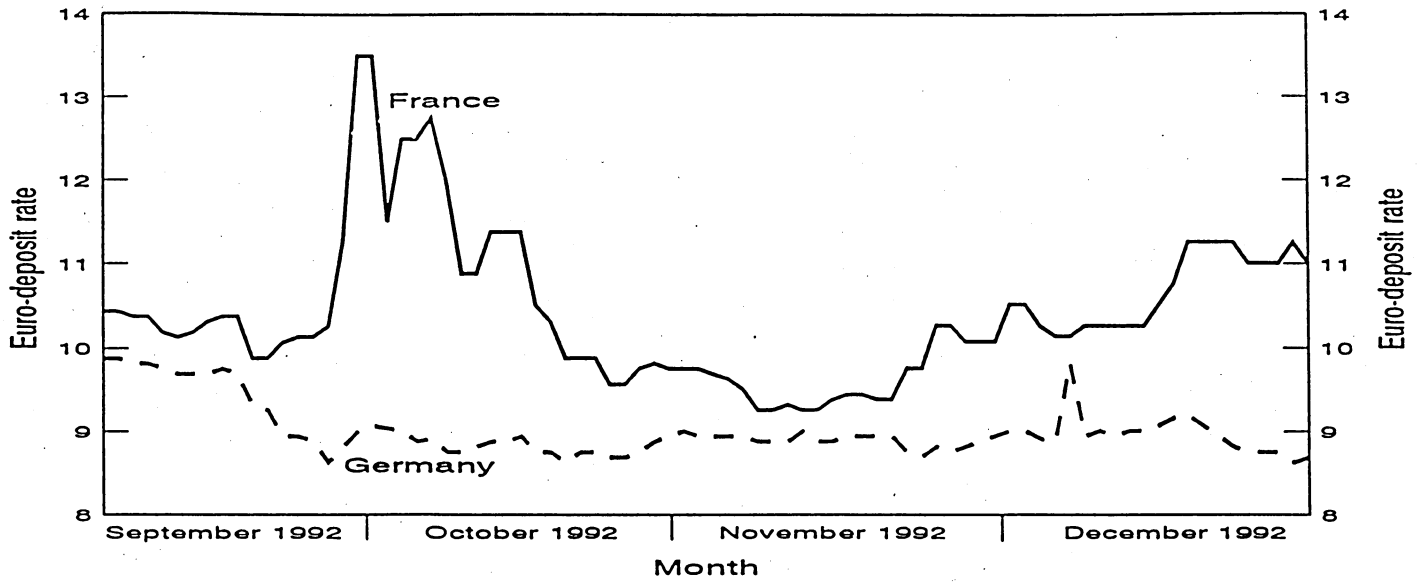
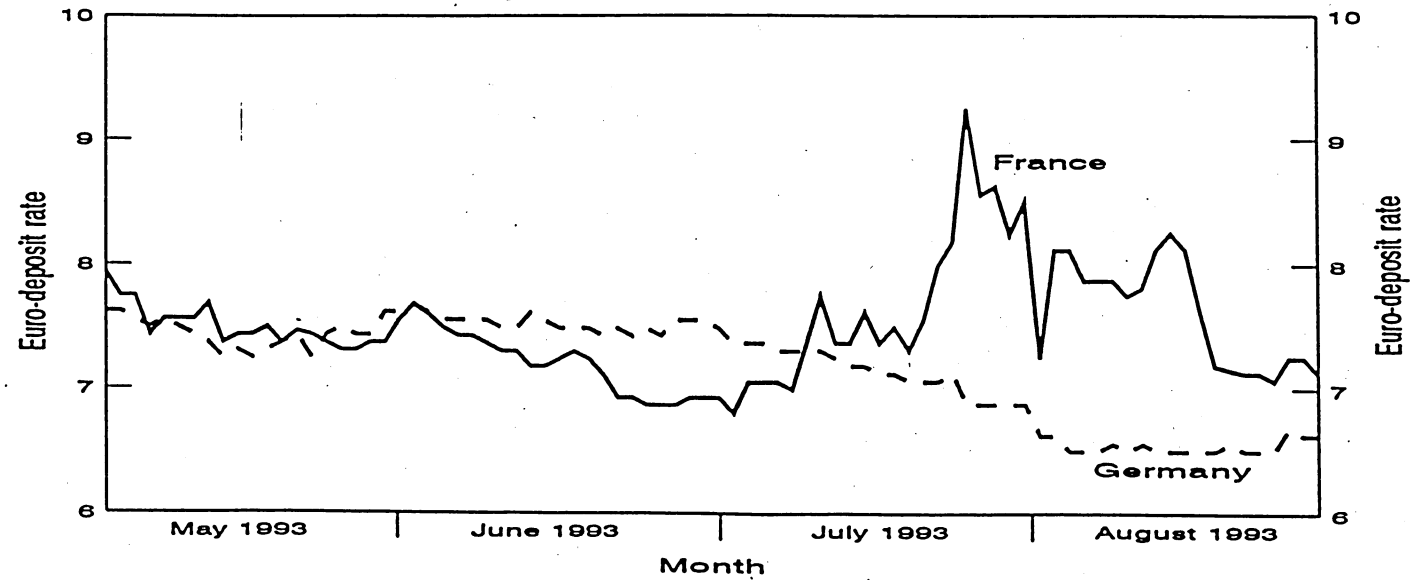## b. January 1993 - April 1993
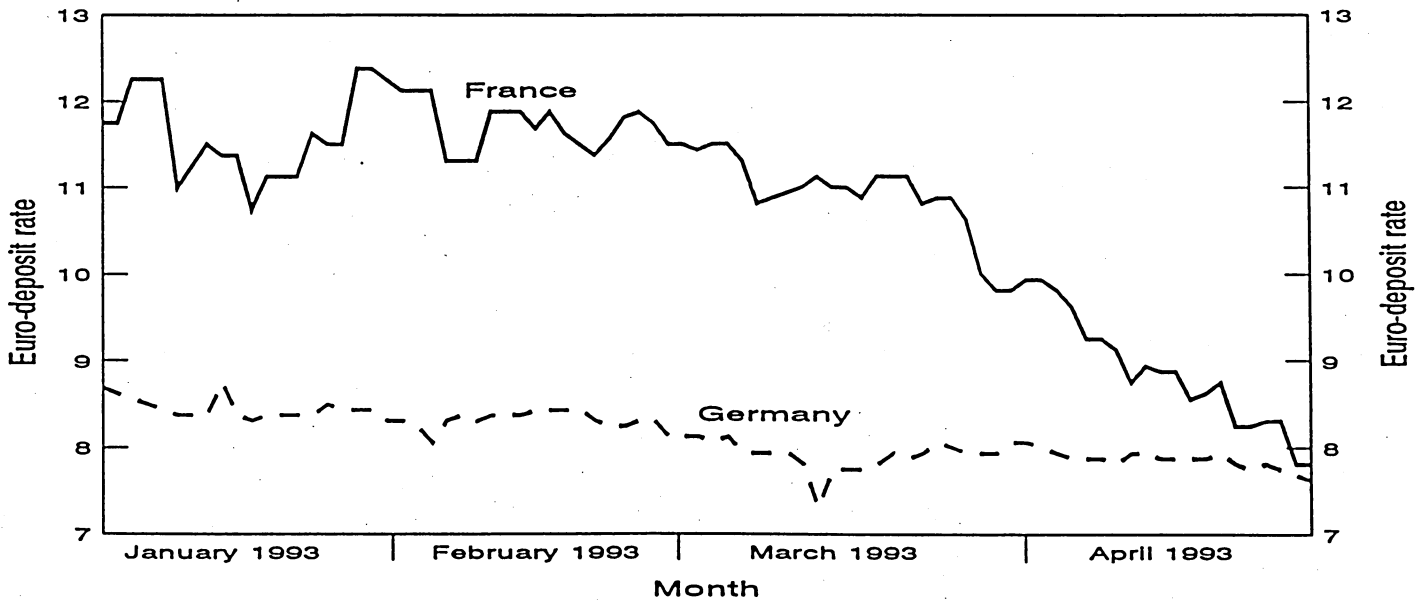
c. May 1993 - August 1993

Chart 5: Probability of Devaluation: actual and fitted
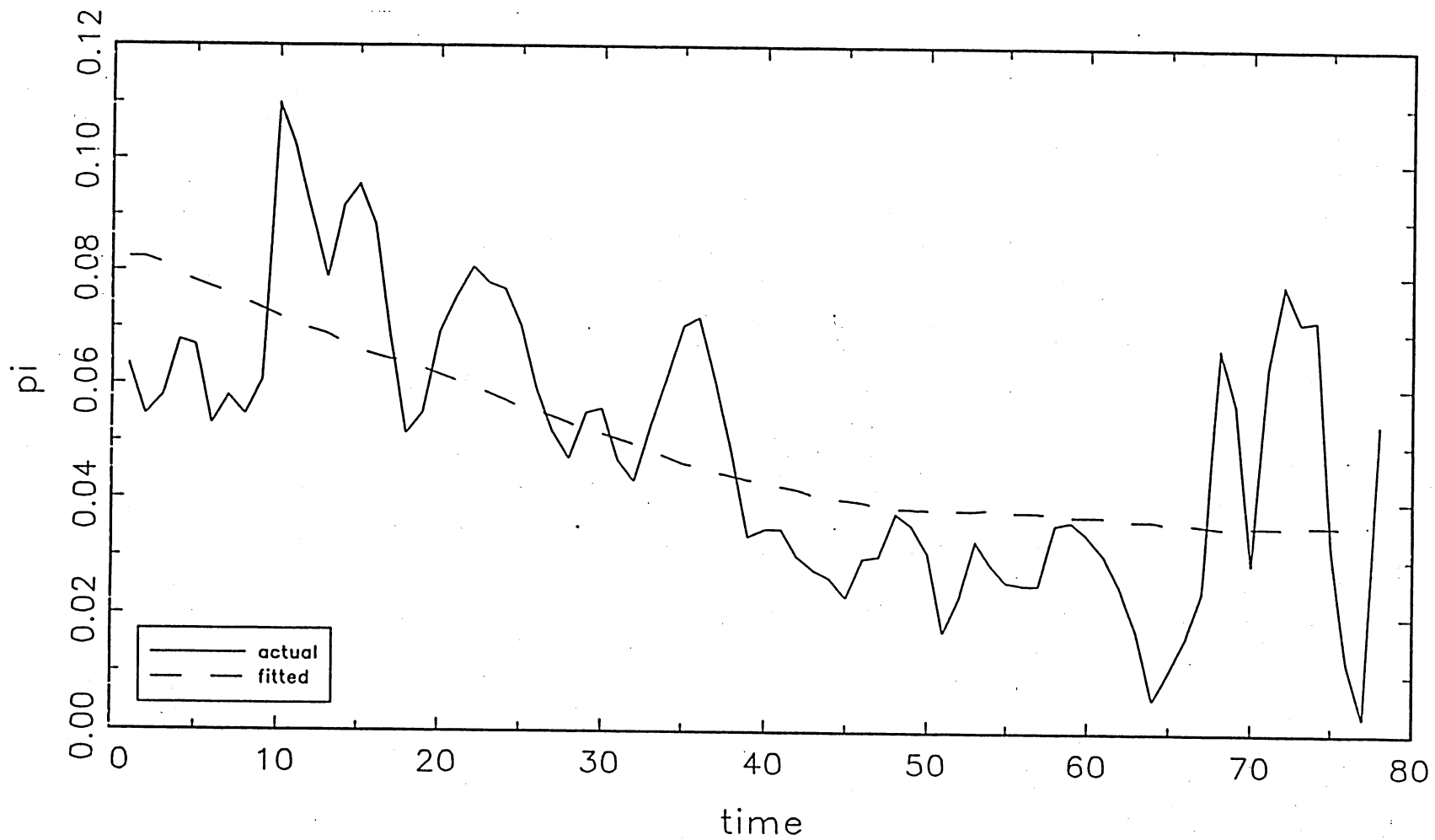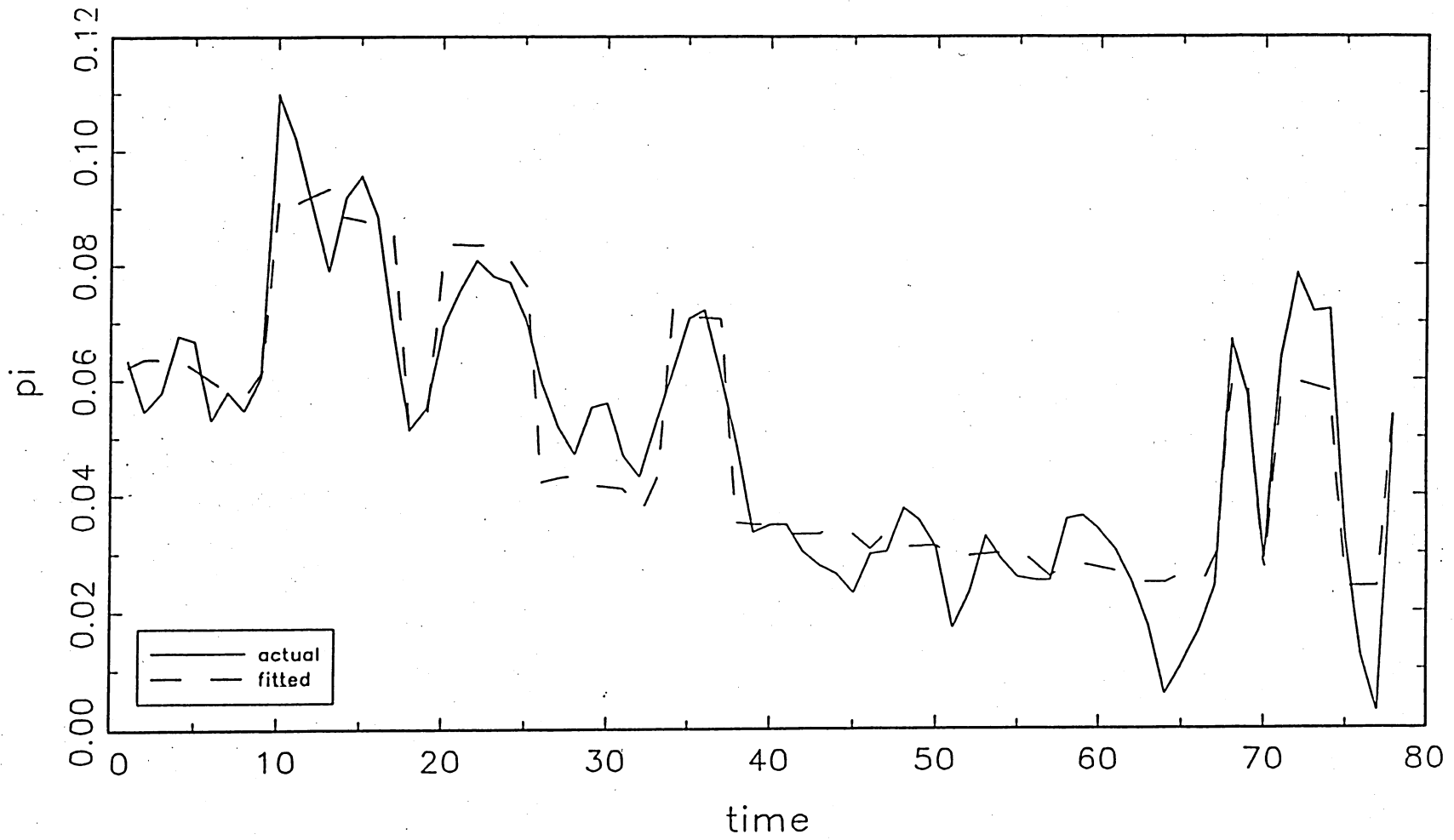
Feb 1987–July 1993

Chart 6: Probability of Devaluation: actual and fitted

Feb 1987–July 1993

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