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UNDER EXPECTED PROFIT MAXIMISATION

DAVID J PANNELL

Agricultural Economics  
Discussion Paper 13/89

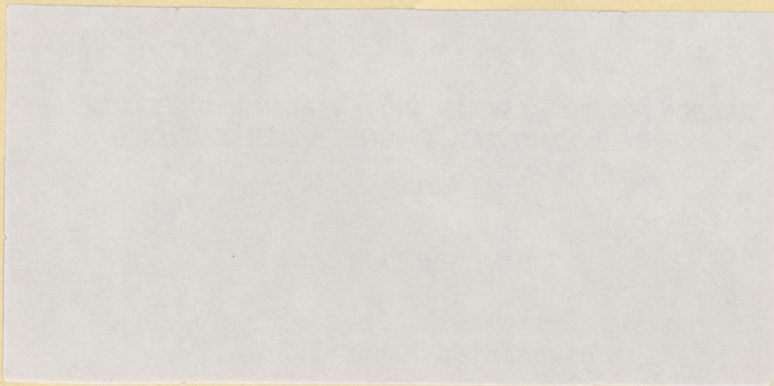
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AGRICULTURAL ECONOMICS

School of Agriculture



Perth, Western Australia

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## RESPONSES TO RISK IN WEED CONTROL DECISIONS UNDER EXPECTED PROFIT MAXIMISATION

### Abstract

Risk is an important characteristic of decisions about weed control in crops. In this paper it is shown that risk can affect weed control decisions even if the objective of the decision maker is to maximise expected profits: that is, even if the decision maker is "risk neutral" in the usual economic sense. This is shown for two decision frameworks: the optimal rate approach and the economic threshold approach. Empirical results are presented for control of ryegrass in wheat in Western Australia. It is found that, in general, risk reduces the optimal level of herbicide use under expected profit maximisation. Although individual sources of risk have a small impact on the optimal decision rules, combinations of uncertain variables can have a relatively large effect.

### Introduction

In many studies of the economics of pest or weed control, the assumed objective of decision makers is maximisation of expected profit (eg, Marra and Carlson 1983, Moffitt et al. 1984, Taylor and Burt 1984, Gold and Sutton 1986, Johnston and Price 1986, Zacharias et al. 1986). There is evidence that, in some circumstances, optimal pesticide decisions under expected profit maximisation differ little to decisions made under risk aversion (Webster 1977, Thornton 1984).

Of those studies which assume risk neutrality, the majority adopt a deterministic decision framework. This approach can sometimes be defended on the basis that the decision which maximises expected profit in a stochastic framework corresponds to the profit maximising decision in a deterministic framework using expected values of parameters. This is why expected profit maximisers are often referred to as "risk neutral".

Nevertheless there are several ways in which risk can affect the decisions of individuals whose objective is to maximise expected profit. Tisdell (1986) showed that uncertainty about a parameter value can affect the optimal level of pest control by affecting expected profit. He argued that:

"in many cases the expected level of application is greater under uncertainty than under full information but . . . this depends on convexity conditions of relevant functions" (p.161)

and that

"convexity conditions may sometimes be such as to give rise to the opposite consequence" (p.159).

He did not discuss which parameters are likely to increase and which to decrease treatment levels under uncertainty. Auld and Tisdell (1986, 1987,

1988) showed that because of convexity of the relationship between weed density and crop yield, uncertainty about weed density reduces expected yield loss. Auld and Tisdell (1987) argued that this increases the economic threshold, reducing the overall level of pesticide use. They noted that this does not seem consistent with comments in the literature that risk increases pesticide use. They attributed the difference to the influence of risk aversion dominating the effect of risk on expected profit.

Another circumstance where risk can affect the decisions of "risk neutral" decision makers is where the problem is dynamic (Antle 1983). Zacharias et al. (1986) tested this hypothesis in their dynamic programming study of soybean cyst nematode. They found modest support for the hypothesis, with very small differences between the results of their deterministic and stochastic models.

A third possibility is where the decision maker is subject to a progressive marginal taxation rate. Taylor (1986) showed that the effect of this on decision making is essentially the same as the effect of risk aversion; it makes the decision maker behave in a more risk averse manner than they otherwise would have.

This paper is an examination of the effects of risk on weed control decisions made by "risk neutral" farmers. Attention is focused on effects due to non-linearities in the response model rather than dynamic or tax induced effects. Sources of risk which affect expected profit are identified and analysed in the context of a theoretical response model for herbicide application. The directions of response to different sources of risk are derived using the theoretical model and the magnitudes of the responses are estimated for a particular empirical example: ryegrass (*Lolium rigidum*) control in wheat in Western Australia.

#### Response Model

Lichtenberg and Zilberman (1986) showed that yield response to pesticides must be represented as an indirect response; pesticides kill pests and it is the removal of pests which increases yield. Use of a response model which does not recognise this two stage process leads to biased predictions of response and erroneous conclusions about the optimal pesticide strategy.

In this paper, crop yield (Y) is represented using the following general form.

$$(1) \quad Y = Y_0 [1 - D(W)]$$

where  $Y_0$  is production with no weeds present and D is the damage function representing the proportion of production lost at weed density W.

Cousens (1985) conducted tests of a wide range of functional forms for the damage function. He found that the following hyperbolic form best fitted published data on weed competition:

$$(2) \quad D(W) = a/[1 + a/(bW)] .$$

The parameter  $a$  can be interpreted as the asymptotic yield loss as  $W \rightarrow \infty$ . Crops typically give some positive yield even at very high weed densities, so  $a$  is normally less than one. The parameter  $b$  is the yield loss per weed as  $W \rightarrow 0$ .

$W$  is a function of pre-treatment weed density ( $W_0$ ) and  $K(H)$ , the proportion of weeds killed at herbicide rate  $H$ .

$$(3) \quad W = W_0 [1 - K(H)] .$$

The kill function must be bounded by zero and one. It is usually represented in the literature by the following exponential function (eg, Feder 1979, Doyle et al 1984, Moffitt et al 1984, Auld et al 1987):

$$(4) \quad K(H) = 1 - \exp(-kH) .$$

Substituting (2), (3) and (4) into (1) gives the response function:

$$(5) \quad Y = Y_0 \left[ 1 - a / \left( 1 + a / [bW_0 \exp(-kH)] \right) \right] .$$

Profit ( $\pi$ ) is given by:

$$(6) \quad \pi = P_y Y - P_h H - A - F$$

where  $P_y$  is output price,  $P_h$  is herbicide cost,  $A$  is herbicide application cost (which is independent of the application rate,  $H$ ) and  $F$  represents costs from all other inputs which are assumed to be fixed at optimal levels.

### Theoretical Analyses

In the following analyses, two decision frameworks are used. Firstly the herbicide rate which maximises expected profit is derived. Although it selects economically efficient control strategies, this approach is not widely used in the weed economics literature. The usual approach is to fix the herbicide rate at some fixed "recommended" rate and to calculate the threshold weed density above which it is worth applying that rate. The threshold density approach is the second framework used here.

### Stochastic pre-treatment weed density

There are two sources of stochasticity related to the initial weed density ( $W_0$ ). One is uncertainty about the mean value of  $W_0$ . The other is spatial variability in weed density on the ground. Weeds do not grow in uniformly spaced formation. Rather they tend to grow in "clumps" of high density mixed with areas of relatively low density. Greater variability of  $W_0$  leads to less precision in inferences about the mean value of  $W_0$ . The variability also means that under the usual practice of applying a uniform herbicide rate to a large area of crop, some regions will receive less than their optimal dosage and others more. Auld and Tisdell (1986, 1987, 1988) have noted that uncertainty about, or variability in  $W_0$  can affect expected profit. They suggested that this would increase the threshold density, although they did not rigorously prove this claim. They also did not consider the question in the context of a variable herbicide rate.

Figure 1 illustrates the way in which a stochastic initial weed density affects expected yield and, consequently, expected profit.

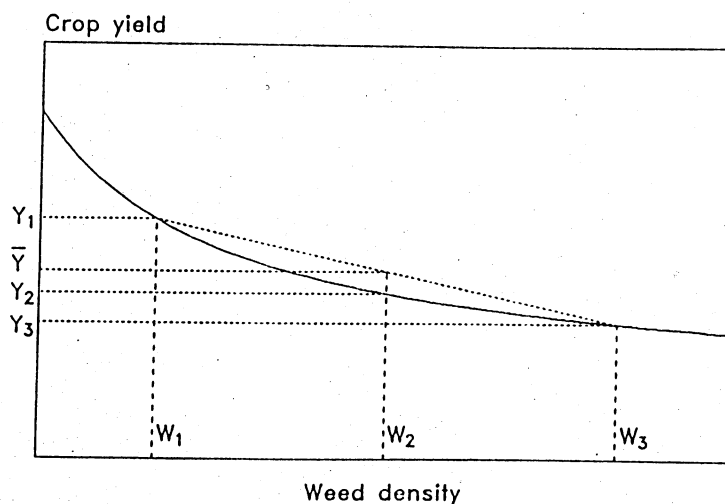


Figure 1: Effect of stochastic weed density on expected crop yield

Consider a crop containing a uniform distribution of weeds at density  $W_2$ . The crop yield function shows that the crop would give a yield of  $Y_2$ . Now consider a similar crop in which half the area of crop is infested with weeds uniformly distributed at density  $W_1$  while the other half is infested at density  $W_3$ . The mean weed density in this mixed crop is  $W_2$  but the mean yield is  $\bar{Y}$ , the average of  $Y_1$  and  $Y_3$ . Note that, because of the convexity of the yield function,  $\bar{Y}$  is greater than  $Y_2$ . In other words the same number of weeds causes less yield loss if they are unevenly distributed than if they are evenly distributed. The biological explanation for this



is that in the uneven distribution, those weeds clumped together at high densities are forced to compete with each other for resources as well as with the crop plants. In these weeds, the competitive ability per weed plant is reduced so that, on average, yield loss is reduced.

Now consider what effect this reduction in yield loss has on the optimal herbicide rate. The aim of the following analysis is to determine whether greater variance of  $W_0$  increases or reduces the optimal herbicide rate ( $H^*$ ), i.e. to determine the sign of  $\partial H^*/\partial \text{var}(W_0)$ . Assume that a farmer has a subjective distribution of initial weed density such that  $W_0 = \bar{W}_0 + \epsilon_0$  where  $\bar{W}_0$  is the mean of the distribution and  $\epsilon_0$  is a random variable with mean zero. After application of herbicide, the weed density is reduced from  $W_0$  to  $W$ . The variance in  $W_0$  leads to variance in  $W$ . That is,  $W = \bar{W} + \epsilon_w$  where  $\epsilon_w$  is another random variable with mean zero. Note that  $\epsilon_w$  is simply a transformation of  $\epsilon_0$ . In this analysis it does not incorporate uncertainty about the level of weed kill which is analysed separately later. Although herbicide application reduces the variance of  $W$  to less than that of  $W_0$  it is not reduced to zero. For convenience, it will be assumed that post-treatment weed density is normally distributed, i.e.  $\epsilon_w \sim N(0, \sigma_w^2)$ . The distribution of weed density may not be normal in practice (indeed it cannot be since it must be truncated at zero) but examination of weed count data from field trials in Western Australia shows that the normal distribution gives a reasonable approximation of the actual distribution. From (1) and (6), profit ( $\pi$ ) is given by:

$$(7) \quad \pi = P_y Y_0 [1 - D(W)] - P_h H - A - F .$$

An approximation will simplify the analysis. The damage function,  $D(W)$ , is approximated by a second order Taylor series approximation about  $\bar{W}$ , the expected value of  $W$ :

$$(8) \quad D(W) \approx D(\bar{W}) + (W - \bar{W})D'(\bar{W}) + 0.5(W - \bar{W})^2 D''(\bar{W})$$

where primes denote derivatives. This can be rearranged to give:

$$(9) \quad D(W) \approx \alpha + \beta W + \gamma W^2$$

where  $\alpha = D(\bar{W}) - \bar{W} D'(\bar{W}) + 0.5 \bar{W}^2 D''(\bar{W})$

$$\beta = D'(\bar{W}) - \bar{W} D''(\bar{W})$$

$$\gamma = -D''(\bar{W}) .$$

Note that  $\beta > 0$  since

$$D'(W) = \left[ a^2 / \left[ bW^2 [1 + a/(bW)]^2 \right] \right] > 0 \quad \text{and}$$

$$D''(W) = D'(W)(a^2/b) \left[ \left[ -2/W \right] - \left[ a/bW^2 [1 + a/(bW)] \right] \right] < 0$$

Also note that this latter result implies that  $\gamma$  is negative. The use of this approximation is defended on the grounds that the important features of the damage function for this analysis are preserved, in particular that  $D'(W) > 0$  and  $D''(W) < 0$ . Now substituting (9) into the profit function (7) gives:

$$(10) \quad \pi \approx P_y Y_o \left[ 1 - \alpha - \beta W - \gamma W^2 \right] - P_h H - A - F.$$

The objective is to maximise expected profits. Since it is assumed that weed density is the only uncertain variable and that it is normally distributed, expected profit is given by:

$$(11) \quad E(\pi) \approx P_y Y_o \left[ 1 - \alpha - \beta \bar{W} - \gamma (\bar{W}^2 + \sigma_w^2) \right] - P_h H - A - F.$$

The first and second order conditions for selecting the herbicide rate which maximises expected profit are:

$$(12) \quad \partial E(\pi) / \partial H = 0$$

and

$$(13) \quad \partial^2 E(\pi) / \partial H^2 < 0$$

The response model presented above can have regions where (13) is not satisfied as well as regions where it is. We will assume here and in the subsequent discussion that the optimal herbicide rate is correctly determined in a region where (13) is satisfied. This means that (12) becomes a necessary and sufficient condition for expected profit maximisation. From (11) the first order condition is:

$$(14) \quad \partial E(\pi) / \partial H = P_y Y_o \left[ -\beta \partial \bar{W} / \partial H - \gamma (2\bar{W} \partial \bar{W} / \partial H + \partial \sigma_w^2 / \partial H) \right] - P_h = 0.$$

Although there is no closed form solution for  $H^*$  available from (14), Hey (1981, p.38) demonstrated how the comparative static properties of a solution like this can be found by total differentiation.

$$(15) \quad \left[ \partial H^* / \partial X \right] \approx 0 \text{ as } \left[ \partial^2 E(\pi) / \partial H \partial X \right] / \left[ \partial^2 E(\pi) / \partial H^2 \right]$$

where  $X$  is a variable in (14) and the partial derivatives on the right hand side of (15) are evaluated at  $H^*$ . But to ensure that  $H^*$  is a maximum, we have already assumed in (13) that  $\partial^2 E(\pi) / \partial H^2$  is negative. Therefore the

sign of  $\partial H^*/\partial X$  depends on the sign of the cross partial derivative,  $\partial^2 E(\pi)/\partial H \partial X$ , evaluated at  $H^*$ .

$$(16) \quad \left[ \frac{\partial H^*}{\partial H} \right] \begin{matrix} \geq \\ \leq \end{matrix} 0 \text{ as } \left[ \frac{\partial^2 E(\pi)}{\partial H \partial X} \right] \begin{matrix} \geq \\ \leq \end{matrix} 0$$

This result will be used in this and the next section.

Return now to the problem of finding the direction of response to an increase in  $\text{var}(W_0)$  (i.e. the sign of  $\partial H^*/\partial \sigma_0^2$ ). From (16) we seek the sign of  $\partial^2 E(\pi)/\partial H \partial \sigma_0^2$  and from (14),

$$(17) \quad \frac{\partial^2 E(\pi)}{\partial H \partial \sigma_0^2} = -P_y Y_0 \gamma \cdot \frac{\partial^2 \sigma^2}{\partial H \partial \sigma_0^2}.$$

To clarify what this means,  $\partial \sigma_w^2/\partial H$  is the rate at which the variance of weed density changes in response to changes in herbicide rate. Examination of field trials shows that variance of weed density decreases with increased herbicide rate. The issue here is whether  $\sigma_w^2$  decreases at a higher or lower absolute rate if  $\sigma_0^2$  is increased; i.e. is  $\partial^2 \sigma_w^2/\partial H \partial \sigma_0^2$  positive or negative. In addressing this, recall that  $W = W_0(1 - K)$ . Assuming that  $W_0$  is normally distributed [ $W_0 \sim N(\bar{W}_0, \sigma_0^2)$ ],

$$(18) \quad \sigma_w^2 = \text{var}(W) = \text{var}[W_0(1 - K)] = \sigma_0^2(1 - K)^2$$

$$(19) \quad \frac{\partial \sigma_w^2}{\partial H} = \sigma_0^2(2)(1 - K)(-\partial K/\partial H).$$

Now taking the partial derivative with respect to  $\sigma_0^2$

$$(20) \quad \frac{\partial^2 \sigma_w^2}{\partial H \partial \sigma_0^2} = (2)(1 - K)(-\partial K/\partial H)$$

which, when substituted into (17), gives:

$$(21) \quad \frac{\partial^2 E(\pi)}{\partial H \partial \sigma_0^2} = P_y Y_0 \gamma (2)(1 - K)(\partial K/\partial H)$$

which is negative since  $\gamma$  is the only negative term. So from (16)  $\partial H^*/\partial \sigma_0^2$  is also negative. As the variance of initial weed density increases, the herbicide rate which maximises expected profit decreases. This is counter to the usually presumed result that risk increases herbicide use. At least for decision makers whose objective is to maximise expected profits, uncertainty about  $W_0$  reduces  $H^*$ . It does so because increasing the variance of weed density reduces the expected yield loss at a given mean weed density (after treatment). Expected crop yield at each herbicide rate is increased such that the yield response function moves upwards to the

left, reducing the herbicide rate at which the price line is tangent to the response function.

Now consider the effect of  $\sigma^2$  on the traditional economic threshold,  $W_o^T$ . The derivation of the traditional economic threshold is quite different to the derivation of the optimal dosage. The latter requires marginal analysis, as demonstrated above, whereas the threshold is determined by comparing profits from two discrete input levels. The result of the marginal analysis is an input level, whereas the result of the threshold analysis is a pest density at which a fixed input level should be used. The threshold is derived as follows.

If the recommended herbicide rate ( $H_r$ ) is used, profit is given by:

$$(22) \quad \pi(H_r) = P_y Y_o [1 - D(W_r)] - P_h H_r - A - F$$

where  $W_r$  is weed density surviving application of the recommended herbicide rate. If no herbicide is applied, profit is:

$$(23) \quad \pi(0) = P_y Y_o [1 - D(W_o)] - F.$$

Without herbicide application, weed density is higher ( $W_o > W_r$ ) so the level of yield loss is greater. On the other hand, savings are made on herbicide costs ( $P_h H_r$ ) and application costs ( $A$ ). The threshold ( $W_o^T$ ) is the lowest density at which application of  $H_r$  is at least as profitable as application of no herbicide, i.e. where  $\pi(H_r) = \pi(0)$ . Setting (22) equal to (23) and simplifying gives:

$$(24) \quad D(W_o^T) = \left[ P_h H_r + A \right] / \left[ P_y Y_o \right] + D(W_r)$$

This expression for the threshold applies to a deterministic model but here,  $W_o$  is a random variable so the interpretation of  $W_o^T$  is problematic. The most reasonable solution seems to be to redefine the threshold in terms of the mean weed density,  $\bar{W}_o$ . Thus:

$$(25) \quad D(\bar{W}_o^T) = \left[ P_h H_r + A \right] / \left[ P_y Y_o \right] + D(\bar{W}_r)$$

If  $\sigma^2 = 0$ , equations (24) and (25) are equivalent and  $\bar{W}_o^T = W_o^T$ . However, as we have seen in the previous discussion, if  $\sigma^2$  is increased to a value greater than zero, the level of yield loss at a given mean weed density will be reduced. That is, both  $D(\bar{W}_o)$  and  $D(\bar{W}_r)$  will be reduced by the increase in  $\sigma^2$ . However application of high rates of herbicide substantially reduces the variance of weed density, so  $\text{var}(W_r)$  will be very much less than  $\text{var}(W_o)$ . This also means that any change in  $\text{var}(W_r)$

following an increase in  $\sigma_0^2$  is small. Thus the main effect of an increase in  $\sigma_0^2$  is a reduction in  $D(\bar{W}_0)$ . But if the left hand side of (25) is reduced while the right hand side is almost unchanged, the equality will not be satisfied. To maintain the equality it is necessary to increase the left hand side by increasing  $\bar{W}_0$ .  $\bar{W}_0^T$  is the new, higher value of  $\bar{W}_0$  which reinstates the equality. Thus if the decision maker maximises expected profit, the traditional economic threshold is increased by uncertainty about  $W_0$ . This is consistent with the findings above for  $H^*$  in that a higher threshold results in a lower expected level of herbicide use. Again it contradicts the conventional wisdom in the literature that risk increases use of inputs for damage control.

### Stochastic weed competitiveness

The actual level of yield loss per weed is unknown until the end of the growing season. In this analysis, uncertainty about weed competitiveness is characterised by uncertainty about the value of parameter  $b$  in the damage function. All other parameters and variables are assumed to be deterministic.

Consider the actual yield function:

$$(26) \quad Y = Y_0 \left[ 1 - D(W) \right] = Y_0 \left[ 1 - \frac{a}{1 + a/(bW)} \right].$$

The damage function can be approximated by a second order Taylor series as follows

$$(27) \quad D = \alpha + \beta(bW) + \gamma(bW)^2$$

where  $\beta > 0$  and  $\gamma < 0$ . Note that  $\alpha$ ,  $\beta$  and  $\gamma$  are different in this example than in the previous example. Now let the parameter  $b$  be a random variable  $b = \bar{b} + \xi$  where  $\xi \sim N(0, \sigma_b^2)$ . The problem is to find the direction of response to an increase in  $\text{var}(b)$  (i.e. the sign of  $\partial H^* / \partial \sigma_b^2$ ). From (16) we seek the sign of  $\partial^2 E(\pi) / \partial H \partial \sigma_b^2$ . Expected damage is:

$$(28) \quad E(D) = \alpha + \beta \bar{b} W + \gamma W^2 (\bar{b}^2 + \sigma_b^2)$$

and expected profit is:

$$(29) \quad E(\pi) = P_y Y_0 \left[ 1 - \alpha - \beta \bar{b} W - \gamma W^2 (\bar{b}^2 + \sigma_b^2) \right] - P_h H - A - F.$$

It follows that:

$$(30) \quad \partial E(\pi) / \partial H = P_y Y_0 \left[ -\beta \bar{b} \frac{\partial W}{\partial H} - \gamma 2W \frac{\partial W}{\partial H} (\bar{b}^2 + \sigma_b^2) \right] - P_h = 0$$

and

$$(31) \quad \partial^2 E(\pi) / \partial H \partial \sigma_b^2 = -2P_y Y_o \gamma W \partial W / \partial H$$

which is negative. Given equation (31), equation (16) implies that an increase in  $\text{var}(b)$  reduces the optimal herbicide rate. The result is similar to that for uncertainty about initial weed density and it again runs counter to conventional wisdom about the effect of risk on herbicide use.

Uncertainty about  $b$  also affects the traditional economic threshold. From (28) it can be seen that an increase in  $\sigma_b^2$  decreases yield loss ( $\gamma$  is negative). The subsequent reduction in yield loss associated with untreated weeds will be much greater than the reduction in yield loss for the few weeds surviving treatment with the recommended dosage. In that case an increase in  $\sigma_b^2$  causes a larger decrease in the left hand side of the threshold equation (24) than the right hand side. To maintain the equality it is necessary to increase the left hand side by increasing  $W_o$ .  $W_o^T$  is the new, higher value of  $W_o$  which reinstates the equality.

Again these conclusions are counter to the usual claims made about risk and herbicide/pesticide use. Uncertainty about  $b$  is another possible cause of reductions in herbicide use.

### Stochastic weed kill

As well as uncertainty about the damage caused at a particular weed density, there can also be uncertainty about the number of weeds killed by a particular herbicide dosage. This seems to loom large in the minds of Western Australian wheat farmers. However its effects on  $H^*$  and  $W_o^T$  are ambiguous. A direct effect of introducing stochasticity to the kill function is to reduce the level of weed mortality (in a similar fashion to that illustrated in Figure 1). This tends to increase the optimal herbicide rate. There is, however, a further impact of stochastic weed kill; it leads to uncertainty about  $W$  and from the first part of this subsection, uncertainty about  $W$  reduces  $H^*$ . Thus uncertainty about weed kill has two effects on  $H^*$ : (a) expected weed survival is increased, which tends to increase  $H^*$  and (b) weed density is made uncertain, decreasing  $H^*$ . The net effect depends on the balance of forces; numeric examples are presented later.

The total effect of stochastic kill on the traditional economic threshold is also ambiguous. The direct effect of uncertainty about kill is a tendency to reduce  $W_o^T$ , but stochastic kill also leads to uncertainty about the weed density which tends to increase  $W_o^T$ . The net effect depends on the convexity properties of the kill and damage functions.

### Stochastic herbicide rate

Although the farmer selects the herbicide dosage to apply, he or she does not have perfect control over dosage. In particular there is likely to be spatial variation in the dosage, especially with some of the modern herbicides which are applied at rates of just a few grams active ingredient per hectare. Although the farmer controls the mean dosage applied to a crop, dosage received by different areas of the crop will follow some distribution. Chiao and Gillingham (1989) analysed the implications for optimal fertilizer practices of spatial variation in fertilizer rate.

The effects of stochastic herbicide rate on  $H^*$  and  $W_0^T$  are similar to the stochastic weed kill example above. Again there are two responses with an ambiguous net effect. Numerical examples are presented below.

### Empirical Results

There are several reasons for considering empirical examples of the principles derived in the previous subsection. Firstly, the theoretical results indicate only the directions of response to the changes considered. There may be considerable variation in the magnitudes of the responses for different parameters. An empirical analysis will suggest the variables to which  $H^*$  and  $W_0^T$  are most sensitive. Secondly, two of the results were ambiguous due to tendencies to move in both directions. Empirical results may indicate which direction of response tends to dominate in practice. Thirdly, the response model was simplified to make theoretical analyses more tractable. Empirical results can help by indicating whether these simplifications have affected results. Finally, the theoretical results were derived under the assumption that all variables other than the one under consideration were deterministic. Even with this simplification some of the analyses were not straightforward and produced ambiguous results. In this circumstance, theoretical analyses for combinations of uncertain variables are likely to be equivocal and unproductive. However it is much easier to include combinations of uncertain variables in an empirical/numerical analysis. In the following discussion, numerical results are presented for uncertain variables individually and in combination.

### Method

The problem selected for analysis was control of ryegrass in wheat by application of Hoegrass (active ingredient diclofop-methyl). This problem was selected because of its economic importance in Western Australia where farmers consider ryegrass to be one of their most important crop weeds

(Roberts et al 1988). The basic biological relationships were taken from Pannell (1989). Weed survival is given by

$$(32) \quad W = W_0 / [1 + \exp(F)]$$

where

$$(33) \quad F = -2.85 - 0.995 \ln(H) - 0.00559 W_0 - 0.00366 \ln(H)W_0.$$

This function differs from the kill function in equation (4) in that the functional form is logistic and the proportion of weeds killed at a given herbicide dose is not independent of the weed density. The simplified version in (4) was adopted to facilitate theoretical analysis. Pannell (1989) estimated the following yield function:

$$(34) \quad Y = Y_0(1 - 0.149H) \left[ 1 - [0.544] / [1 + 0.544 / (bW)] \right]$$

where:

$$(35) \quad b = 0.0172 \cdot \exp(-0.801Y_0) \cdot \exp(-5.70H).$$

This differs from the damage function given in equation (2) in two respects. Firstly, the parameter  $b$  is not fixed but depends on the weed-free yield and herbicide rate and, secondly, there is an additional term representing direct damage to the crop by herbicide.

Templates for a microcomputer spreadsheet program were developed for deriving optimal herbicide rates and density thresholds under uncertainty. The empirical analyses were conducted using mean values for costs, prices, weed densities and yields considered reasonable for the shire of Merredin in Western Australia's eastern wheatbelt: wheat price \$144 tonne<sup>-1</sup>, Hoegrass cost \$48 per kg a.i., weed-free yield 1.14 tonnes ha<sup>-1</sup>, initial weed density 200 m<sup>-2</sup> and recommended herbicide rate 0.375 kg a.i. ha<sup>-1</sup>.

Risk was included in the spreadsheets in two ways. In the case of the weed-free yield, the probability distribution was estimated by solving a biological simulation model of wheat growth for each of 76 years for which the required rainfall data were available for Merredin (1912 to 1987). The discrete distribution of 76 points obtained by this procedure was used directly in the spreadsheet. Each of the 76 values of weed-free yield were used to calculate actual yield for a particular set of assumptions regarding herbicide rate, weed density etc. These were used to calculate the distribution of profit for given prices and costs. Finally expected profit was calculated based on the assumption that each of the 76 observations was equally likely to occur. The distribution of weed-free



yields obtained from the simulation model had a mean of 1.14 tonne ha<sup>-1</sup> and a standard error of 0.626 tonne ha<sup>-1</sup>.

Uncertainty about variables other than weed-free yield was represented in a similar way except that the probability distributions were generated by a normal random number generator. For consistency with the yield data, each randomly generated distribution consisted of 76 observations. The coefficient of variation of weed density was estimated from field trials as 40 per cent. Coefficients of variation used for other variables were: herbicide rate, weed kill, and weed competitiveness, 40 per cent; output price, 20 per cent.

In the spreadsheet used to calculate optimal herbicide rates, the effect of risk on decision making was evaluated by directly calculating the solution which maximised expected profit. The herbicide rate was increased by discrete increments until expected profit started to decline. This was repeated with progressively smaller increments until an arbitrarily accurate solution was obtained. Checks were included to ensure that the solution obtained was a global optimum.

A somewhat similar approach was taken in the threshold spreadsheets except that the variable being solved for was the weed density and the criterion for stopping was when expected profits from treatment with the recommended dose just exceeded expected profits from non-treatment. In both spreadsheets, solutions were obtained using a combination of spreadsheet formulae to calculate expected profit for a given set of parameter values and macro programs to control changes in parameter values and tests for optimality/thresholds.

## Results

Results are shown in Table 1. First consider the results for the variables considered individually in the earlier theoretical analyses. Results for  $W_0$  and  $b$  are consistent with the theoretical findings that uncertainty decreases  $H^*$  and increases  $W_0^T$ . However despite this consistency with the theory, it can be seen that the magnitudes of the effects are very small. At least for this problem and this set of assumptions it seems that the effect of uncertainty about  $W_0$  or  $b$  on expected profit is unlikely to significantly affect farmer behaviour.

Theoretical results for the effects on  $H^*$  of uncertainty about weed kill and herbicide rate were ambiguous but in each empirical example  $H^*$  is reduced by uncertainty. At the same time density thresholds are increased. However, as with  $W_0$  and  $b$ , the effects are quite small.

**Table 1:** Optimal herbicide rates ( $H^*$ ) and density thresholds ( $W_0^T$ ) for risk neutral decision makers under different sources of uncertainty

Stochastic variable	$H^*$ (kg a.i. ha )	$W_0^T$ (plants m )
Nil	0.265	40
Initial weed density ( $W_0$ )	0.264	43
Weed competitiveness (b)	0.263	43
Weed kill (K)	0.262	41
Herbicide rate (H)	0.259	43
Weed-free yield ( $Y_0$ )	0.249	51
$Y_0$ and $W_0$	0.248	53
$Y_0$ and H	0.237	57
$Y_0$ , H and $W_0$	0.241	58

Table 1 also includes results for weed-free yield. This was not included in the theoretical analysis because the simplified response model did not include the mechanism by which uncertainty about  $Y_0$  affects expected profit. In the more detailed model of equations (32) to (35),  $Y_0$  affects weed competitiveness but it does so non-linearly. The result is that uncertainty about  $Y_0$  increases expected yield loss, reducing expected profit. In the empirical model, uncertainty about  $Y_0$ , like all the other variables, reduced  $H^*$  and increased  $W_0^T$ . The impact of  $Y_0$  on  $H^*$  and  $W_0^T$  was greater than any of the other individual variables.

The final three sets of results in Table 1 show that even if individual sources of risk have a small impact on decision making, in the realistic case of multiple sources of risk the effects can be more substantial. If the effect of multiple sources of risk on expected profit are considered,  $H^*$  can be reduced by over 11 per cent and  $W_0^T$  increased by as much as 45 per cent.

### Conclusion

Results presented above highlight the importance of using biologically realistic relationships for analysis of response. Without them, the implications of risk for decision making under expected profit maximisation would not have been apparent.

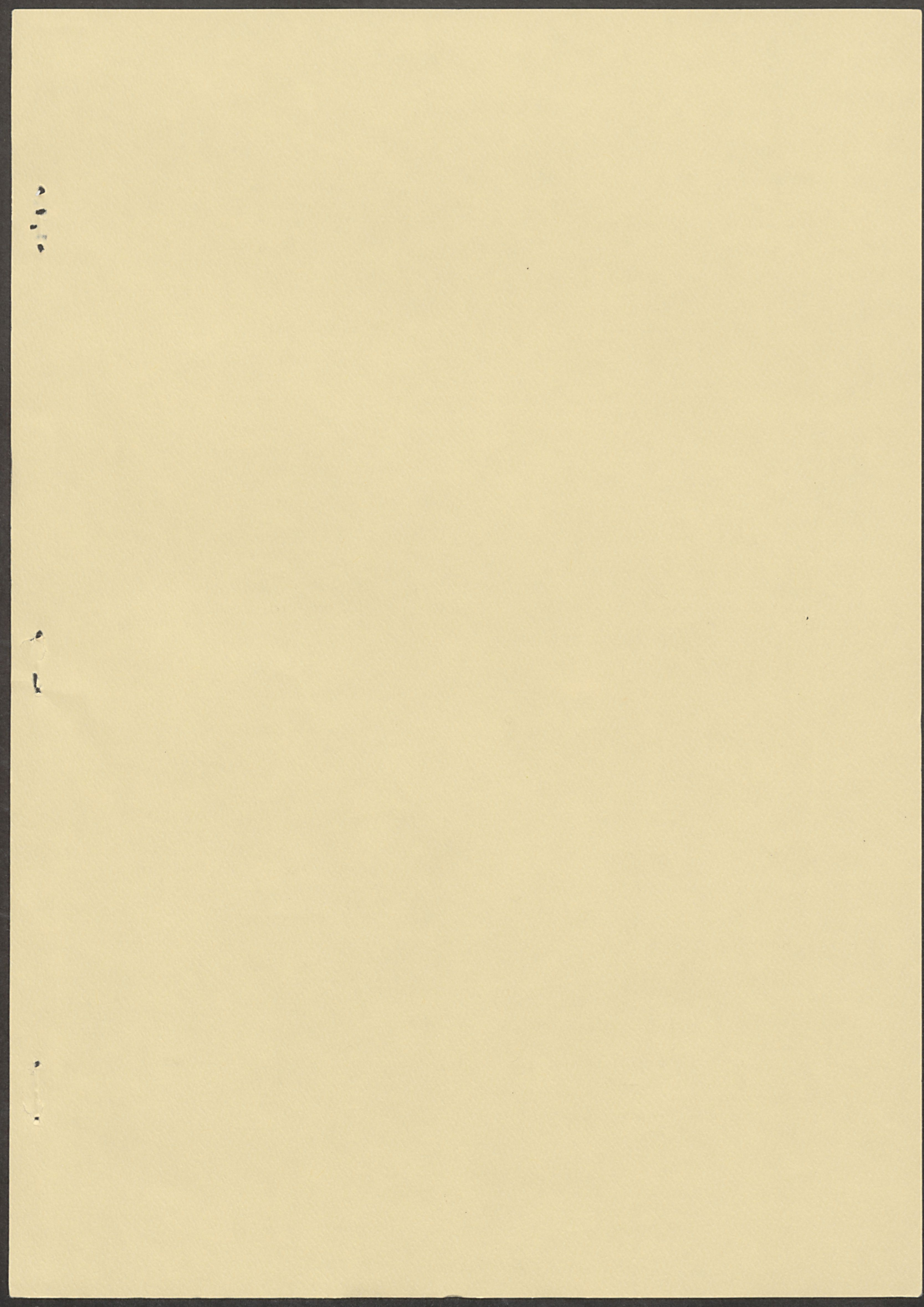
A second general conclusion to be drawn here is that it can be important to consider multiple sources of risk in estimating the magnitudes of effects of risk on decision making. Although individual

sources of risk had small effects on optimal decision making for weed control, combinations of risky variables had relatively large impacts.

Finally, it is remarkable that in all the numerical results presented here, the effect of risk was to reduce herbicide use, either by reducing the optimal herbicide rate or by increasing the threshold for herbicide use. This runs directly counter to the usual presumption about the impact of risk on use of herbicides and other types of pesticides. Whether the effect of risk aversion on herbicide use is sufficient to counter these effects is a subject for further investigation.

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