RISK ANALYSIS UNDER CORRELATED, NON-NORMAL PRICE AND YIELD PROBABILITY DISTRIBUTIONS

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by

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Coffee production throughout the world is carried out using a variety of systems from input-intensive monocultures at near full sun exposure to low density organic plantations under a regulated tropical forest canopy. Consequently, yields can vary ten-fold from over 70 to less than 6 one-hundred-pound bags (cwt.) per hectare. It is commonly accepted that the input-intensive monoculture renders a substantially higher expected net income per unit area, but carries the risk of severe losses during low price periods. Alternatively, it is believed that the diversified shaded-coffee systems are less risky but only yield relatively modest profits in the long run (Sosa).

Coffee growing areas in Central America currently exceed 500,000 hectares, provide permanent or temporary employment for an estimated 25% of the rural population, and account for nearly 10% of the value of the agricultural output. Larger plantings are found in South America's main producing countries, Brazil and Colombia. Many of these areas are considered environmentally sensitive. Consequently, shaded-coffee agroforestry production systems have become of prime interest for ecological reasons.

Tabora shows that agroforestry has the potential of reducing risk through the diversification of income sources that it provides. Price concludes that the tree component of an agroforestry system might be an important risk-reducing factor. Reeves and Lilieholm argue that the relatively lower net income variation characteristic of an agroforestry system is a key factor for small farmers. Binswanger observes that virtually all individuals are moderately risk averse. Risk aversion can be so critical in small farmer decision making that it might become the overriding economic concern (Arnold).

Within this context, the financial risk and return characteristics of the diversified, more ecologically stable shaded coffee production systems found in southwest Costa Rica are of interest (Sosa). They could reduce risk in several ways. The periodical sale and/or consumption of fruits (oranges, plantains, bananas, etc.), fire and sawmill wood and other products could help stabilize farmers' income during periods of low coffee prices. "Service" trees like "poró" (*Erythrina poeppigiana*) help maintain or improve the soil and prevent erosion, extend the useful life of the coffee plantation and, because of their nitrogen fixing properties, reduce fertilizer costs (Somarriba). Shaded-coffee plantations also have lower average variable costs than the input-intensive monocultures. Their yields, although not as high, are less

susceptible to reductions in the use of variable inputs like fertilizer and pesticides. This could help farmers to obtain profits, albeit modest, instead of losses during periods of depressed coffee prices (Ramirez and Gomez).

When simulation is used to assess agricultural systems' risk, estimates of the probability distribution functions (pdf's) that reflect the basic statistical behavior of the key risk-producing variables are required. The issue of non-normal simulation was first addressed in the agricultural economics literature in the early seventies. Anderson stresses the importance of modeling non-normality (skewness and kurtosis) and allowing variances to change with time and location. Gallagher advances a univariate procedure to model and simulate yields using the Gamma distribution. Taylor tackles the problem of multivariate, non-normal simulation. Babcock and Hennessy estimate yield distributions and apply simulation for risk analysis in agriculture. The mathematics/statistics literature also addresses the problem of simulating correlated random variables from given marginal pdf's using "Copulas" (Phelps and Weissfeld, Jouini and Clemen, Shih and Louis, Zheng and Klein).

Ramirez, Moss and Boggess, Ramirez and Ramirez and Somarriba develop a series of techniques that can be combined and used for the joint modeling and simulation of sets of random variables that are correlated among each other, exhibit heteroscedasticity and/or autocorrelation, and have non-normal (kurtotic and right or left skewed) probability distribution functions. Modeling and simulating pdf's that can reflect these potentially key statistical features of commodity prices and yields is important. The reliability of the risk assessment depends on how closely the estimated profit cumulative distribution function (cdf), which is obtained from the simulated price and yield pdf's, resembles the true underlying cdf for profits.

Commodity prices and yields are likely to be correlated with each other, especially for perennial crops like coffee that possess cyclical production patterns and lagged supply responses. Ignoring an existing negative yield-price correlation would result in overestimating the variability of profits and, therefore, risk. Cross-sectional and inter-temporal yields have been found to be heteroscedastic and non-normally distributed, exhibiting both kurtosis and skewness (Ramirez; Ramirez, Moss and Boggess;

Gallagher). Variations in cropping system design such as those encountered in this study can cause differences in expected yields and in yield variability. These should be considered when modeling and simulating the systems' yields.

Ramirez and Somarriba find that time series of international cocoa and Costa Rican plantain prices are autocorrelated and follow non-normal probability distributions during any given year. Specifically, they find that the pdf for cocoa price is right skewed. If a normal distribution is fitted to price (yield) data that actually conforms to a kurtotic and right-skewed distribution, the probabilities of obtaining low prices (yields) are likely to be overestimated while the probabilities of high-price occurrences could be underestimated (Ramirez and Somarriba). This probability prediction error can significantly affect the simulated profit cdf's and the results of the financial risk analysis.

Modeling autocorrelation is important for both short- and long-term financial analysis. If autocorrelation is ignored, the price forecasts are the long-term expected values given by the regression function. These could over or underestimate the correct short-term expected values predicted by an autocorrelated forecast, depending on whether prices are currently in a low or in a high cycle, respectively. As a consequence, the simulated pdf would be centered incorrectly. Identifying the occurrence of relatively long price cycles points to the need of providing separate financial risk information for low, average and high price-period scenarios.

This study estimates the profit cdf's for three alternative agroforestry systems for shaded coffee production and uses them to assess the financial risk and return trade-off among those systems. A time trending, autocorrelated, non-normal model of real coffee prices is estimated using time series data and the Ramirez and Somarriba technique. Methods proposed by Ramirez are modified to jointly estimate non-normal coffee-yield response functions for the three systems using cross-sectional survey data. The model allows for different conditional yield means and variances, but assumes similar slope parameters for the yield response functions and identical degrees of pdf skewness and kurtosis for the three systems under analysis. The alternative of estimating separate non-normal yield-response models for each system is not feasible due to the limited number of observations per system.

Since they are both based on a transformation to normality, the Ramirez and Ramirez and Somarriba likelihood functions can be linked to jointly estimate the price and yield models and correlation. Simulating joint price and yield pdf's that reflect the estimated model characteristics is straightforward. Considering any existing price-yield co-variation is important since a positive correlation is likely to increase profit variability and, therefore, risk, while a negative price-yield correlation could have the opposite effect.

The lack of time series data on these shaded coffee system yields, however, imposes two limitations on the financial risk analysis. First, only the annual within-system yield variability (mainly due to the system's productive status during a given year) can be modeled and simulated. The additional year-to-year yield risk (mainly due to weather) also faced by producers, which could be significant, is not accounted for. As a result, the actual profit variability and financial risk levels faced by farmers are likely higher than this study's estimates. Second, the key price-yield correlation can not be estimated. However, the separately estimated price and yield models are joined for the pdf simulation, assuming realistic levels of co-variation to explore the effect of this factor on risk.

A disadvantage to using a parametric technique, such as Ramirez and Ramirez and Somarriba, for modeling and simulation is that the estimate of the joint probability distribution is only consistent if the assumed pdf's and underlying correlation structure closely represent the statistical process generating the data. The underlying correlation structure may be more complex than what is assumed. Another limitation of this approach is in the modeling of multi-modal distributions, which cannot be represented by a restriction in the parameter space of the Ramirez and Ramirez and Somarriba likelihood functions. As McDonald and White point out, these are better represented by a mixture of two or more densities.

Methods and Procedures

The Shaded-Coffee Agroforestry Systems

Sosa surveyed 57 randomly selected coffee plantations in southwest Costa Rica and used principal component analysis to identify three main production systems differentiated by the characteristics of their shade component. The shade component was classified as: poró (*Erytrhina sp.*),

plantains (*Musa* AAB), fruit trees and other perennial wood trees. For each of these species or groups he collected data on number of plant per unit area. He also included in the analysis other characteristics of the coffee plantation: variety, plant density, average plant and tissue age, number of productive plants and branches per ha and percentage of replanting and pruning. The principal component analysis identified three clearly differentiated agroforestry production systems (AFS): AFS I (19 plantations with a balanced shade of poró and plantains), AFS II (18 plantations with a high-density poró shade) and AFS III (20 plantations with a diversified shade of poró, plantains, fruit and other perennial wood trees) (Table 1.).

Financial Risk and Return Analysis

The income/costs from the sale/purchase of all products/inputs, or their opportunity values when they were consumed/provided by the farm family, were considered in the financial risk and return analysis. Opportunity values were estimated by adjusting comparable market values. The net benefits for each AFS were then calculated as:

(1) $\qquad NBi = P_C Y_{Ci} + \mathbf{P_O}\mathbf{Y_{Oi}} - VC_i; \; i = 1,2,3;$

where *NBi* denotes the net benefit per hectare for AFS i, P_C the price paid to the farmer per hundred-pound bag (cwt.) of coffee, Y_{Ci} the coffee yield per hectare from AFS i, $\mathbf{P_O}$ the vector of prices paid to the farmer for the sale of other products, $\mathbf{Y_{Oi}}$ the vector of quantities of other products sold from AFS i, and VC_i the variable costs per hectare for AFS i.

To assess the financial risk resulting from the variability of coffee prices and of the yields of the different AFS, the pdf's for these variables are modeled and simulated. The price and yield pdf's are used to derive the pdf for the net benefits from each AFS through equation (1). Specifically, 5,000 simulations of P_C and Y_{Ci} are utilized to obtain an equal number of probable net benefits for each AFS during 1998. The resulting pdf's for the *NBi* are accumulated to construct the cdf's used to evaluate the risk levels associated with each of the AFS.

In diversified systems, coffee yields and prices are still the main determinants of risk. Additional profit-determining factors such as plantain, other fruit and timber and firewood prices and yields, and the within-system production costs are believed to be less volatile (Ramirez and Gomez). Plantain price

variability, for example, is relatively small (Ramirez and Somarriba). Plantain yields in the low-density systems considered in this study (less that 300 clusters/ha vs. 4,000-6,500 coffee plants/ha) are also very stable: one bunch/cluster every 17-18 months. Plantain production accounts for less that 10% of total revenues in a typical year. Therefore, their main effect on risk is modeled through a simple shifting of expected profits. This leads to a potentially significant underestimation of the variability of profits and risk, which could tip the analysis in favor the systems with more diversified (non-coffee) revenue sources.

Price and Yield Modeling and Simulation

Coffee in Costa Rica is mainly an export commodity, and the international market determines its domestic price. Data on the annual average FOB price for Costa Rica's export coffee from the 1914 to the 1996-97 producing cycles are available (Sosa). Average annual prices paid to farmers in southwest Costa Rica from 1967-68 to 1996-97 are also available. FOB export prices (in real 1997 U.S.$/cwt. calculated using the U.S.$ consumer price index) are used for the modeling and simulation and then transformed to prices paid to farmers by a conversion factor based on the average relation among the two price series during the last 20 years, which has been fairly stable. Sosa measured yields during the 1997 production cycle and collected data on the management, plantation and shade-component characteristics of the 57 farms considered for the study. Table 2 provides the means of the variable management costs and of the plantation-structure variables.

Real coffee prices (**Y**) could be decreasing through time, as in the case of other agricultural commodities. An autoregressive process is expected considering the cyclical nature of production. Non-normality in the form of right-skewness in the yearly pdf's is also expected because of the extremely high prices observed occasionally. Therefore, coffee prices are modeled using the technique developed by Ramirez and Somarriba. The concentrated log-likelihood function to maximize is:

$$(2) \qquad L = 0.5 \times \ln(1-\rho^2) + \sum_{t=1}^{T} \{ \ln(g_t) - 0.5 \times (R_{3t}^2) \},$$

where if the dependent variable is autocorrelated and not normally distributed:

$$g_t = R_1/(\sigma\Theta[1+\{(R_1/\sigma)(Y^*_t - R_{2t})\}^2]^{1/2});$$

$R_1 = \exp(0.5\Theta)\{\exp(\Theta\mu) - \exp(-\Theta\mu)\}/2$ and $R_{2t} = -\sigma + \mathbf{X^*_t}\Gamma$;

Y^*_t and $\mathbf{X^*_t}$ are the tth row of $\mathbf{PY}$ and $\mathbf{PX}$; where $\mathbf{Y}$ is (Tx1) and $\mathbf{X}$ is a (Txk) matrix of explanatory variables.

$\mathbf{P}$ is (TxT) matrix such that $(\mathbf{P'P})^{-1} = \mathbf{\Phi}$, the covariance matrix for the error term that expresses the assumed autocorrelation process (Judge, et al., 1985);

ρ is the model's first order autocorrelation coefficient; and

$R_{3t} = [\ln\{R_{4t} + (1+R_{4t}^2)^{1/2}\}/\Theta] - \mu$ and $R_{4t} = \{(R_1/\sigma)(Y^*_t - R_{2t})\}$.

Alternatively, if the dependent variable coffee price ($\mathbf{Y}$) is normally distributed, the parameters Θ and μ will approach zero during estimation making $g_t = \sigma^{-1}$ and $R_{3t} = (Y^*_t - \mathbf{X^*_t}\Gamma)/\sigma$ and (2) becomes the well-known normal first-order autoregressive likelihood function (Judge, et al.). If coffee prices are not autocorrelated, ρ will be statistically insignificant; if ρ is set to zero, maximizing (2) under normality is equivalent to a standard OLS regression. The model above does not force non-normality or autocorrelation, but it allows for their testing and modeling.

Crop yields have also been found to be non-normal (Ramirez; Toure, Major and Lindwall; Taylor; Gallager). Since cross-sectional data from three different systems are available in this case, it is also important to estimate the yields from each AFS given the values of the coffee plantation structure and management variables. The possibility that yields may have different variances depending on the system must be considered as well. Ramirez's technique can be modified to jointly model and simulate those conditions. The concentrated log-likelihood function to be maximized is:

$$(3) \qquad L = \sum_{i=1}^{3} \sum_{n=1}^{N_i} \{ \ln(g_{in}) - 0.5 \times (R_{4in}^2) \},$$

where if YI_i (coffee yields) is not normally distributed:

$g_{in} = R_1/(\sigma_i\Theta [1+\{(R_1/\sigma_i)(YI_{in} - R_{2in})\}^2]^{1/2})$ (i=1,2,3 represents the three AFS and n =1,…N$_i$, the number of observations on each AFS);

$$R_1 = [e^{0.5\Theta}(e^{\Theta\mu} - e^{-\Theta\mu})]/2 \text{ and } R_{2in} = -\sigma_i + \beta_{0i} + \mathbf{Z_{in}\beta};$$

$$R_{4in} = (\ln(R_{3in}+(1+(R_{3in}))^{1/2})/\Theta) - \mu; \text{ and } R_{3in} = \{(R_1/\sigma_i)(YI_{in}-R_{2in})\}.$$

Alternatively, if the YIi (i=1,2,3), i.e. coffee yields from all three agroforestry systems, are normally distributed, the parameters Θ and μ will approach zero during estimation making

$$g_i = \sigma_i^{-1} \text{ and } R_{4in} = (YI_{in} - \beta_{0n} - \mathbf{Z_{in}\beta})/\sigma_i.$$

The price and yield models rely on the same basic technique: an inverse hyperbolic sine transformation to normality (Ramirez, Moss and Boggess). In both cases if Θ and μ are statistically different from zero, it is concluded that the dependent variable exhibits a kurtotic and asymmetric distribution. If $\mu \neq 0$ a positive σ indicates right skewness and a negative σ left skewness; as μ approaches zero the pdf becomes symmetric, but it is still kurtotic. If both Θ and μ are statistically insignificant they can be set to zero and a normal regression model is obtained. The parameter σ^2 is proportional to and controls the variance of the dependent variable. Therefore, the second model (eq. (3)) allows for different yield variability in each of the AFS analyzed. Also, $E[Y_t^*] = \mathbf{X^*_t\Gamma}$ which implies $E[Y_t] = \mathbf{X_t\Gamma}$ in the coffee price model and $E[YIi] = \beta_{0i} + \mathbf{Z_{in}\beta}$ in the agroforestry systems yield model.

A linear time-trending process is assumed for expected coffee prices: $\mathbf{X_t\Gamma} = \Gamma_0 + \Gamma_1 t$ (t=1,...,T). In the yield model, $\mathbf{Z_n}$ includes variable costs (VC), VC^2 and VC^3, and the other five variables in Table 2, for the nth farm. Therefore, it estimates an inverse cost function adjusted by the values undertaken by plantation structure variables that differentiate the AFS. The parameter vector $\mathbf{\beta}$ measures the marginal impact of those variables on coffee yields, which in the final model is assumed to be constant across AFS. The possibility that the three AFS are not characterized by the same inverse cost function is considered through a different intercept (β_{0i}) for each system.

Once the models' parameters are estimated, simulation can be conducted modifying the techniques described by Ramirez and Ramirez and Somarriba. For coffee prices, generate a vector $\mathbf{z}$

containing draws from a normal random variable with mean μ and a variance σ^2. The price pdf for the future

time period j is simulated using:

(4) $\quad \mathbf{Y_{S(T+j)}} = (\sigma/R_1)(e^{\Theta\mathbf{z}} - e^{-\Theta\mathbf{z}})/2 + Y_{F(T+j)} - \sigma,$

where R_1 is as specified in (2) and $\mathbf{Y_{S(T+j)}}$ represents the simulated prices around the autocorrelated forecast

$Y_{F(T+j)}$. A similar process is used for simulating the yields of the agroforestry systems: Generate three

vectors $\mathbf{z_i}$, each containing draws from a normal random variable with mean μ (the same for all systems) and

variance σ_i^2 (different for each system), and calculate:

(5) $\quad \mathbf{YI_{iS}} = (\sigma_i/R_1)(e^{\Theta\mathbf{z_i}} - e^{-\Theta\mathbf{z_i}})/2 - \sigma_i + \beta_{0i} + \mathbf{X_{in}\beta}$ (i=1,2,3 for AFS I, II and III)

These techniques could also be adapted for the joint estimation of the price and yield models,

which would be more efficient statistically, and provide an estimate of the price-yield correlation. That is

not feasible in this case because time-series data on yields are not available. However, given the

importance of a potential price-yield covariation for the financial risk and return analysis, different

positive or negative correlation levels can be assumed when simulating the price and yield vectors. This is

done by joining the standard normal $\mathbf{z}$ and $\mathbf{z_i}$ vectors in a matrix $\mathbf{M}$, and multiplying it by the Cholesky

decomposition of a covariance matrix $\mathbf{\Sigma}$ with unit diagonal elements and the estimated or desired

correlations as off-diagonal elements (Ramirez). The transformed $\mathbf{z}$ and $\mathbf{z_i}$ vectors are substituted in (4)

and (5) to conduct the simulation.

Results

Price and Yield Modeling and Simulation

The models estimate that real FOB Costa Rican export coffee prices have decreased at an average

of $\Gamma_1 = 0.49223$ U.S.\$/cwt. per year during the last 84 years (Table 3 and Figure 1). An estimate of

$\rho=0.50574$, that is statistically different from zero at the 99% level, indicates that they are autocorrelated.

This can be seen in Figure 1, which also shows that coffee prices are currently in a period of depression,

below their expected long-term trend. The autocorrelated prediction of expected coffee prices is given by (Ramirez and Somarriba):

(6) $Y_{F(T+j)} = \Gamma_0 + \Gamma_1(T+j) + \{ \rho^j (Y_{(T)} - \Gamma_0 - \Gamma_1(T)) \}$,

where $Y_{F(T+j)}$ is the autoregressive forecast for period T+j, $Y_{(T)}$ is the last (1997) observed price, T is the last value undertaken by the independent variable time (84 in this case), and j is the number of years into the future for which the prediction is desired. The model forecasts that real coffee prices should experience a recovery during the next five years, returning to their long-term trend of about U.S.$170 /cwt. in 2002. Thereafter, they are expected to decrease at an average rate of U.S.$0.492/cwt. per year.

The non-normality of coffee prices can also be perceived in Figure 1. Extremely high prices occurred in 1954-57 and 1977-78, while the lowest real price observed is in 1920. This suggests that the probability density function (pdf) of coffee prices for any given year could be kurtotic and right skewed. Kurtosis and skewness are recognized by the model since both Θ and μ are statistically different from zero at the 99.9% level (Table 3).

The estimated pdf's for 1997 and 2002 coffee prices, based on 5,000 simulations each, are presented in Figure 2. The intricacies of the model are reflected on these pdf's. Because of autocorrelation, their expected values increase from E_{1997} = U.S.$142.5/cwt. in 1997 (compared to an actual 1997 average price of U.S.$160/cwt.) to E_{2002} = U.S.$170/cwt. in 2002, before beginning a decreasing long-term trend. By design, they exhibit the same variance, kurtosis and degree of asymmetry through time. They reflect the previously discussed peculiarities of the data, for example, that real prices in excess of U.S.$350 can occur, but real prices below U.S.$60 are highly unlikely.

The parameter estimates and related statistics for the agroforestry systems yield model are presented in Table 4. First, an unrestricted model with different intercepts, slope parameters and variances but equal kurtosis and skewness coefficients for each AFS is estimated. This is not a highly reliable model, since its 32 parameters are estimated with only 57 observations. Most of the slope coefficients are not statistically different from zero. Restricted model 1 estimates equal slope, kurtosis (Θ) and skewness

(μ) parameters, but different intercepts (β_{01}, β_{02}, β_{03}), variances (σ_1, σ_2, σ_3) and for each AFS. The maximum likelihood function value is only reduced by 2.55, and the corresponding likelihood ratio test (Judge et al.) for Ho: all slope coefficients are equal across the three AFS yields a $\chi^2_{(16)}$ statistic of 5.10 (p>.90) hence it is concluded that these restrictions are consistent with the data generating process.

In restricted model 1 the estimates for the intercepts of the AFS II and the AFS III equations (β_{02} and β_{03}) are similar. Restricted model 2 estimates a common intercept for those two AFS. A likelihood ratio test for Ho: $\beta_{02} = \beta_{03}$ yields a $\chi^2_{(1)}$ statistic of 0.04 that does not lead to rejection of Ho at any conventional significance. A model with the same intercept for all three AFS under analysis has a maximum likelihood function value of –215.523. The likelihood ratio test for Ho: β_{02} & $\beta_{03} = \beta_{01}$ rejects Ho at the 0.10 level, with a $\chi^2_{(1)}$ statistic of 2.766 (p=.096). When all other factors are held constant, AFS I exhibits significantly lower average yields than AFS II and III.

Those are interesting findings. Fertilizer use represents an important share of the variable production costs. However, $\beta_{02} = \beta_{03}$ implies that the higher densities of poró (*Erytrhina* sp), a widely recommended nitrogen-fixing tree, observed in AFS II (339 trees/ha vs. 179 in AFS III) do not result in higher yields when variable production costs are held constant across the two systems. This is consistent with most recent expert opinion that poró densities of about 200 trees/ha are sufficient to provide yield-maximizing nitrogen levels in this type of coffee plantation. Additional poró trees and nitrogen applications are likely useless in AFS II.

In contrast, the higher densities of plantains grown in AFS I (291 clusters/ha vs. 17 in AFS II and 156 in AFS III) appear to affect average coffee yields, when coffee plantation density and all other factors considered in the yield-response models are held constant. This is likely due to increased competition for space and water. A more diverse agroforestry system (AFS III), with lower poró densities, some musaceas (plantain and bananas) and other fruit trees, and wood-producing species like laurel (*Cordia alliadora*), may be a good strategy.

In both models 1 and 2, the estimates of the yield variance in AFS I and AFS III are similar, but relatively different from the estimate for AFS II. Model 3 incorporates the additional restriction that the yield variance is equal in AFS I and III. A likelihood ratio test for Ho: $\sigma_1 = \sigma_3$ yields a $\mathcal{X}^2_{(1)}$ statistic of 0.76 (p=0.38). The yield variance appears to be the same in the two systems. The further restriction Ho: σ_1 & $\sigma_3 = \sigma_2$ is rejected at the .05 level, through a $\mathcal{X}^2_{(1)}$ statistic of 3.928 (p=.047). Coffee yield variability in AFS II is significantly higher than in AFS I and III. AFS II is almost exclusively oriented to coffee production, and includes few plantain clusters, fruit and other perennial wood trees. These serve an alternative purpose of providing a partial shade to the coffee plants. Less shade can increase average yields under optimal weather and management conditions, but can also result in higher yield variability.

The parameters of the third-degree polynomial inverse cost function cannot be estimated with an acceptable degree of statistical precision (Table 4). A simpler linear relation between variable costs and yields is evaluated in model 4, which also excludes the variable "plantation age" that was not statistically significant in the first three models. Most of the plantations surveyed were less than 18 years old, and all were less than 22. It is commonly believed that coffee yields do not start to significantly decline until the plantation is 20 years old, and a plantation of this type is seldom renewed before it reaches that age.

The likelihood ratio test for Ho: $\beta_2 = \beta_3 = \beta_4 = 0$ yields a $\mathcal{X}^2_{(3)}$ statistic (p=.896), indicating that model 4 is as appropriate as model 3, statistically. Model 4 is used for simulation. According to model 4, the key plantation structure variables affecting yields are plant tissue age (i.e. the number of years after a pruning) (+6.65cwt./year), coffee plant population density (–5.018cwt./ 1000 plants) and the number of coffee producing plants per ha (+18.715cwt./1000 plants) and of producing branches per ha (+2.155cwt./1000 branches). These results are compatible with expectations. For example, a higher coffee plant population density should reduce yields if the numbers of coffee producing plants and branches per ha (which are more direct determinants of yields) are held constant. It is believed that coffee yields steadily increase during the first four years after a pruning, at which time the plantation is customarily pruned again. For

the range of yields and variable costs in the sample, every dollar spent on the plantation management is estimated to increase coffee yields by 0.061cwt., with a standard error of 0.019.

Statistically, Θ and μ are highly significant in all the models, indicating that coffee yields are both kurtotic and right skewed. This suggests that they are considerably less variable on the low side than they are on the high side. Downward variability is limited near zero. On the other hand, a well-structured plantation on the last year of a pruning cycle and on the second (high) year of its natural biannual production cycle can render extremely high yields. Kurtosis and skewness are reflected in the simulated pdf's for the 3 AFS yields (Figure 3). Since model 4 was used for simulation, the expected and, therefore, the overall yields of AFS I are lower than those of AFS II and III, while the variance of the yield pdf for AFS II is over 100% higher than the others. The degrees of kurtosis and skewness are the same, by design. The pdf's for AFS I and III are identical except for their expected value, that shifts them along the horizontal axis.

Financial Risk and Return Analysis

To determine farm-level net benefits, the simulated values of Costa Rica's FOB export coffee prices were multiplied by the previously mentioned conversion factor, and the per/unit harvesting costs (U.S.$16.67/cwt.) were subtracted; then 5,000 probable net benefits for each AFS and year (1997 and 2002) were calculated using equation (1) and the 5,000 adjusted price and yield simulations. The simulated pdf's for the net benefits from the 3 AFS during 1997 and 2002 are presented in Figure 4, and the corresponding cdf's in Figure 5.

The yield advantage of AFS III over AFS I (Figure 3) is reduced when the systems are evaluated in terms of their annual net benefits. This is mainly due to the higher annual income generated by AFS I from the sale of the plantain production (U.S.$768/ha vs. U.S.$412.5/ha by AFSIII), without increased variable costs. AFS II, in contrast, is affected by its low plantain production (U.S.$44/ha) and higher variable costs. Since the coffee price and the agroforestry systems yield pdf's are all right skewed, and no correlation among them is assumed, the pdf's for the AFS net benefits are also asymmetric. For example, for 1997 the expected net benefit from AFS II is of U.S.$2,272.32/ha; the probability of a negative net

benefit (U.S.$2,272.32/ha below the mean) is 4%, equal to the probability of having net benefits in excess of U.S.$8,000/ha (U.S.$5,727.68/ha above the mean). AFS I carries a very low probability of yielding a negative net benefit during 1997 (less than 0.5%), but presents lower expected net benefits (U.S.$2,070.16/ha). AFS III offers the best risk protection and the highest expected net benefits (U.S.$2,600.56/ha).

These risk and return values are more favorable in 2002 because of the increase in expected international coffee prices (from the actual 1997 price of U.S.$142.5 to U.S.$170) predicted by the model. This scenario is useful in evaluating the performance of the AFS during periods of more favorable prices. The expected net benefit for AFS II increases to U.S.$3,000.01 /ha, vs. U.S.$2,482.66 /ha for AFS I and U.S.$3,287.78/ha for AFS III, and the probability that it yields a negative net benefit decreases to 2.6%. AFS II becomes clearly superior to AFS I in regards of expected net benefits because of its higher coffee production, but remains inferior in terms of risk, which is still infinitesimal (less than 0.5%) in the case of AFS I and III.

The initial risk and return values are favorable for all three AFS, but they do not take into account the fixed costs. They are useful to assess short-term profitability and risk. The survey data do not allow for a direct calculation of the fixed costs of the different AFS. However, studies of the Costa Rican Coffee Institute (ICAFE) indicate that the variable input, labor, harvesting and transportation costs in a typical agroforestry system of that country represent about 50% of the total costs. The remaining 50% includes the opportunity cost of the land and capital invested in establishing and renovating the system, the depreciation of infrastructure and equipment and of the system itself, short-term credit, managerial and administrative costs, etc. These are indirectly estimated at U.S.$1,500/ha (on average for the three AFS) using the survey data and the assumption that although variable costs are somewhat different, fixed costs are likely similar within this type of system. The following results should be evaluated considering that the per ha total-to-fixed cost relation in a typical Costa Rican coffee plantation might not hold in this case. It is also possible that the fixed costs are not the same across systems.

The effect of considering both fixed and variable costs on expected net benefits and risk could be depicted by shifting the horizontal axis of Figures 4 and 5 by the amount of the fixed costs. Alternatively, that amount can be subtracted from the previously calculated expected net benefits, and require minimum net benefits of U.S.$1,500 (i.e. a recovery of the fixed costs) instead of U.S.$0 when assessing risk. Then, the expected 1997 net benefits of the three AFS decrease to U.S.$570.16/ha, U.S.$772.32/ha and U.S.$1,100.56/ha, while their risk levels increase to 26.0%, 34.0% and 17.2%, respectively.

Because of the price autocorrelation cycles, however, the risk levels are not independent from year to year. For example, if an excess supply shock depresses prices to U.S.$75/cwt. in 1998, the expected international coffee price for 1999 will be only U.S.$125/cwt. [by equation (6)]. The cdf for the expected net benefits from AFS II during 1999 will be centered below zero and imply a risk level of nearly 60%, in addition to the U.S.$2,375 loss experienced in 1998. Price autocorrelation cycles such as those observed during 1936-46 and 1989-1996 represent a concomitant source of risk. Overall, AFS III is better protected against risk and, given the most common farm size of 8-12 hectares, would yield an attractive excess profit for the average Costa Rican small farmer in the long run.

The 1997 coffee price distribution (Figure 2), with an expected value of U.S.$142.5/cwt., is not the most favorable. It implies a 3% probability of a real price under U.S.$70/cwt., which has occurred only once during this century, in 1993. The 2002 distribution characterizes price conditions that are closer to the long-term trend, such as the 1924-1934, 1948-49, 1959-1975 and 1981-1987 periods which encompass nearly half of the years in the analysis. During these periods prices are not expected to go below U.S.$100/cwt., but can be as high as U.S.$300/cwt. (Figure 2). Upward autocorrelation cycles that take expected prices well above their long-term trend (such as in1950-1958 and 1976-1980) are necessary for the extremely high real prices of U.S.$400/cwt. and U.S.$520/cwt. observed in 1954 and 1977 to become possible. Under the more optimistic 2002 price distribution and considering both fixed and variable costs, the expected net benefits from the three AFS are U.S.$982.66/ha, U.S.$1,500.01/ha and U.S.$1,787.78 /ha, while their risk levels were 18.5%, 22.3% and 7.2%, respectively. The risk levels associated to AFS I and II are still considerable relative to AFS III.

Another factor that can affect risk and returns is a correlation between prices and yields. Coffee production is cyclical at the farm level. The cycles can be controlled and accentuated through lengthening or shortening the different pruning and replanting activities, shade management and, to a certain extent, variable input use, all of which have lagged effects (Sosa). Coffee growers often attempt to time the periods of better prices and place the highest possible production in the market during those years. Correlated price-yield series are simulated recognizing that a single correlation coefficient and two marginal densities might not precisely represent the joint price-yield distribution. These simulations are used explore the impact of successfully implementing that strategy vs. the alternatives of no timing and of pursuing the opposite tactic, for the case of AFS III. The expected 1997 price and modest correlations of 0.35, and -0.35 are assumed and compared with the previously discussed scenario of no correlation. The simulated variables are slightly adjusted (linearly) to ensure that expected prices and yields are exactly U.S.$142.5 and 57cwt./ha in all three cases.

The cdf's for the net benefits of AFS III under the three price-yield correlation scenarios are presented in Figure 6. A positive correlation shifts the function clockwise at the 0.50 probability point, while a negative correlation shifts it counterclockwise in a more pronounced manner. Risk, determined by the height of the cdf below the 0.50 probability level, is directly proportional to the degree of correlation, i.e. risk can be decreased by synchronizing high yields with low price periods and vice versa. Expected net benefits are also directly proportional to the degree of correlation and thus risk, since the magnitude of the cdf shift is larger above the 0.50 probability point than below it, due to the right-skewness of the cdf.

After subtracting the fixed costs of U.S.$1,500/ha, imposing a positive price-yield correlation increases expected net benefits by more than 20%, from U.S.$1,100.56/ha to U.S.$1,322.14/ha. Risk levels also rise, but marginally, from 17.2 to 19.4%. The opposite strategy of imposing a negative correlation decreases expected net benefits by exactly the same amount, to U.S.$878.98/ha, but significantly reduces risk to 11.5%. Clearly, the farmers' strategy is founded in a rational choice to tolerate a modest amount of additional risk in exchange for a significant increase in expected net income.

Conclusions and Recommendations

International coffee prices are autocorrelated. The much more pronounced upward cycles indicate a severe right skewness in their probability distribution for any given year. Currently, they are in a downward cycle, and are expected to recover to a long-term trend value of U.S.$170/cwt. by the year 2002. Their long-term trend, however, has been to decline, in real terms, at a rate of approximately U.S.$0.50/year during the last 83 years.

The pdf's for the yields of the three shaded coffee production systems evaluated are also non-normal, specifically kurtotic and right-skewed. The relatively high density of plantain in AFS I reduces coffee yields an estimated 14.1cwt./ha. The high densities of poró found in AFS II do not appear to increase yields in relation to AFS III, which, with intermediate poró densities, produces a statisticaly similar standardized (average) yield of 56.4cwt./ha. AFS II also showed substantially higher yield variability than other AFS's.

The three AFS for shaded coffee production found in southwest Costa Rica are profitable, on average, in the short as well as the long run when both fixed and variable costs are considered. However, only the more diversified system (AFS III), found in 35% of the farms, provides adequate risk protection, especially during low-price cycles. The expected long-term profits from AFS III of between U.S.$1,100 and U.S.$1,800/ha during relatively normal price periods (1997 and 2002) are sufficient for a typical 8-12 ha Costa Rican small farmer to support a family. However, the more favorable conclusions about AFS III must be interpreted in the light of the previously discussed simulation assumptions, which could lead to a significant underestimation of the variability of profits and risk and tip the analysis in favor this system because of its more diversified (non-coffee) revenue sources.

An interesting risk-return trade off is observed between the plantain and the poró intensive systems (AFS I vs. AFS II). AFS I carry a lower risk (18.5-26% vs. 22.3-34% for AFS II), but also reduced expected returns (U.S.$200/ha in 1997 and U.S.480/ha in 2002). The diversified poró/plantain/fruit & wood-producing tress system (AFS III) provides superior risk protection (7.2-17.2%) and the highest expected net benefits under any coffee price condition.

Another empirical finding of this study is that changes in the correlation among the non-normal price and yield variables used to calculate the cdf of net benefits causes a risk-return trade off. In the case of AFS III, a correlation of 0.35 increased expected net benefits by about 20% and risk by 2.2% with respect to the base-line scenario of no correlation; while a correlation of –0.35 decreased them by 20% and 5.7%, respectively. This explains the economic rationale of the farmers that are attempting to time their plantations' high-yield periods with the years of more favorable coffee prices. Their rationale is consistent with Binswanger observation that virtually all individuals are moderately risk averse.

In regards to other methodological issues, the combination of non-normal price and yield models resulted in severely kurtotic and skewed simulated cdf's for the net benefits of the three AFS under analysis. It is clear that using normal price and yield models would have forced a significant departure from reality in this case, and resulted in a flawed financial risk and return analysis. A normal distribution could not accommodate the asymmetries observed. Instead, it would distribute the price and yield variances symmetrically, causing a source of error in the prediction of probabilities and risk levels.

The joint modeling of non-normality and autocorrelation, versus the alternative of only accounting for either one of those two basic characteristics of the coffee-price time series, was also key to the analysis. Accounting for autocorrelation improved the quality of the model's predictions and pointed to the need of conducting financial analysis for different scenarios of depressed, normal and above-normal expected prices. Also, since there were only between 18 and 20 observations available for each AFS, and eight independent variables, a model that assumed similar degrees of kurtosis and skewness for all yields, but different conditional means and variances was helpful econometrically. All critical statistical attributes could be modeled with a limited number of parameters. The possibility of estimating the covariance among the variables of interest, jointly with all other price and yield model parameters, and to incorporate it in the simulated pdf's and cdf's, improves the precision and opens new dimensions to be explored in the financial analysis.

Table 1. Averages per hectare for the variables measuring the characteristics of the shade components that differentiate the 3 AFS identified.

System	# of Species in the Shade Component	# of Poró Trees	# of Plantain Clusters	# of Fruit Trees	# of other Perennial Wood Trees
AFS I	2.68	159.21	290.79	17.11	13.16
AFS II	1.89	338.89	16.67	9.72	4.17
AFS III	3.25	178.75	156.25	40.00	43.75

Table 2. Average of the variable costs and the plantation-structure variables.

Variable	AFS I	AFS II	AFS III
Variable Costs ($/ha)	556.61	672.52	628.74
Plantation Age (years)	8.50	11.6	9.00
Tissue Age (# of years after last pruning)	4.56	3.09	3.37
Plantation Density (# of coffee plants per ha)	5311.84	5186.11	6000.00
Number of Producing Coffee Plants per ha	4627.63	3843.06	4550.00
Number of Producing Branches per ha	12809.08	12009.4	18215.2

Table 3. Parameter estimates and related statistics for the autocorrelated non-normal coffee price model[a].

Parameters	ρ	Γ_0	Γ_1	σ	Θ	μ
Estimates	0.506	216.686	-0.492	41.555	0.837	0.898
Std. Errors	0.055	17.256	0.298	15.065	0.182	0.378
P-values	0.000	0.000	0.052	0.004	0.000	0.001

[a] Estimation and simulation was conducted using the GAUSS 2.01 matrix algebra language, specifically, the OPTMUM procedure was used for maximum likelihood estimation.

Table 4. Parameter estimates and related statistics for the restricted AFS yield models[a].

Par.	Model 1			Model 2			Model 3			Model 4		
	Est.	S.E.	P.V.	Est.	S.E.	P.V.	Est.	S.E.	P.V.	Est.	S.E.	P.V.
Θ	0.220	0.086	0.007	0.220	0.089	0.009	0.223	0.059	0.000	0.228	0.104	0.016
μ	51.637	8.635	0.000	30.706	6.166	0.000	30.734	4.391	0.000	35.622	5.686	0.000
β_1	0.442	0.460	0.171	0.440	0.396	0.136	0.350	0.388	0.186	0.061	0.010	0.001
β_2	-2.598	3.162	0.208	-2.593	2.737	0.174	-1.925	2.652	0.236	0.000	--	--
β_3	5.490	6.864	0.214	5.504	6.012	0.183	3.996	5.743	0.245	0.000	--	--
β_4	0.648	0.932	0.245	0.686	1.077	0.264	0.481	0.907	0.299	0.000	--	--
β_5	6.479	2.662	0.010	6.349	2.876	0.016	6.484	2.827	0.013	6.650	2.598	0.007
β_6	-16.155	4.087	0.000	-16.101	4.771	0.001	-15.945	4.169	0.000	-15.018	4.357	0.001
β_7	21.306	4.227	0.000	21.188	4.531	0.000	20.001	4.394	0.000	18.715	4.444	0.000
β_8	2.320	0.785	0.003	2.257	0.837	0.005	2.239	0.894	0.008	2.155	0.845	0.007
β_{01}	-129.64	94.167	0.088	-127.83	79.440	0.057	-108.42	80.649	0.093	-47.801	25.451	0.033
β_{02}	-115.35	92.732	0.110	-115.30	79.318	0.077	-93.730	79.182	0.121	-33.690	24.309	0.086
β_{03}	-117.55	93.078	0.107	-115.30	79.318	0.077	-93.730	79.182	0.121	-33.690	24.309	0.086
σ_1	112.80	45.526	0.009	112.54	46.534	0.010	99.387	25.522	0.000	99.141	44.781	0.016
σ_2	159.05	72.863	0.017	157.00	72.341	0.018	154.34	42.369	0.000	145.45	75.267	0.030
σ_3	87.965	36.375	0.010	88.621	38.810	0.014	99.387	25.522	0.000	99.141	44.781	0.016
MVLF	-214.12			-214.14			-214.52			-214.82		

[a] MVLF refers to the maximum value reached by the concentrated likelihood function, β_i (i=1,...,3) to the parameters associated to CV, $CV^2/1000$, $CV^3/1000000$, β_i (i=4,...,8) are the parameters related to the coffee plantation structure variables (X_i) in the order listed in Table 2, Est., S.E. and P.V. denote the corresponding parameter estimates, standard errors and P-values from asymptotic t-tests, X_6, X_7, and X_8 are expressed in thousands.

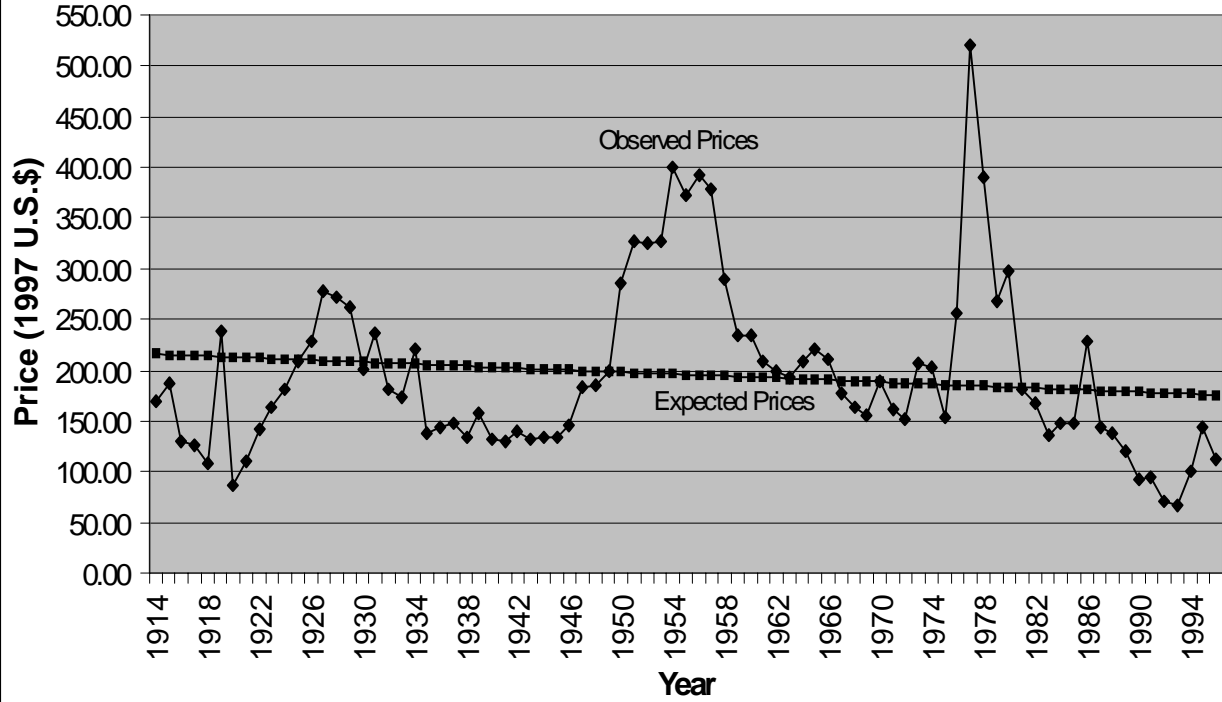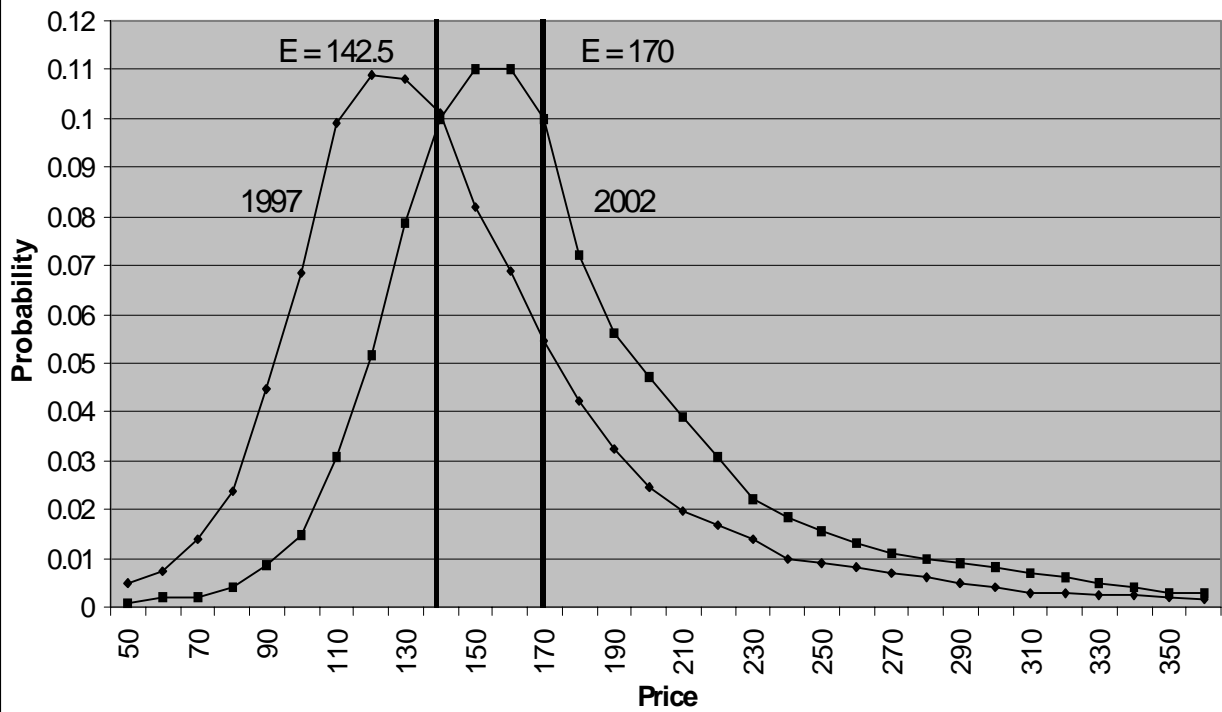**Figure 1: Observed and Expected Coffee Prices (1914-1996)**

Observed Prices

Expected Prices

Price (1997 U.S.$)

Year

Figure 2: Probability Density Functions of 1997 and 2002 Coffee Prices

E = 142.5

E = 170

1997

2002

Probability

Price

Figure 3: Probability Density Functions of Coffee Yields

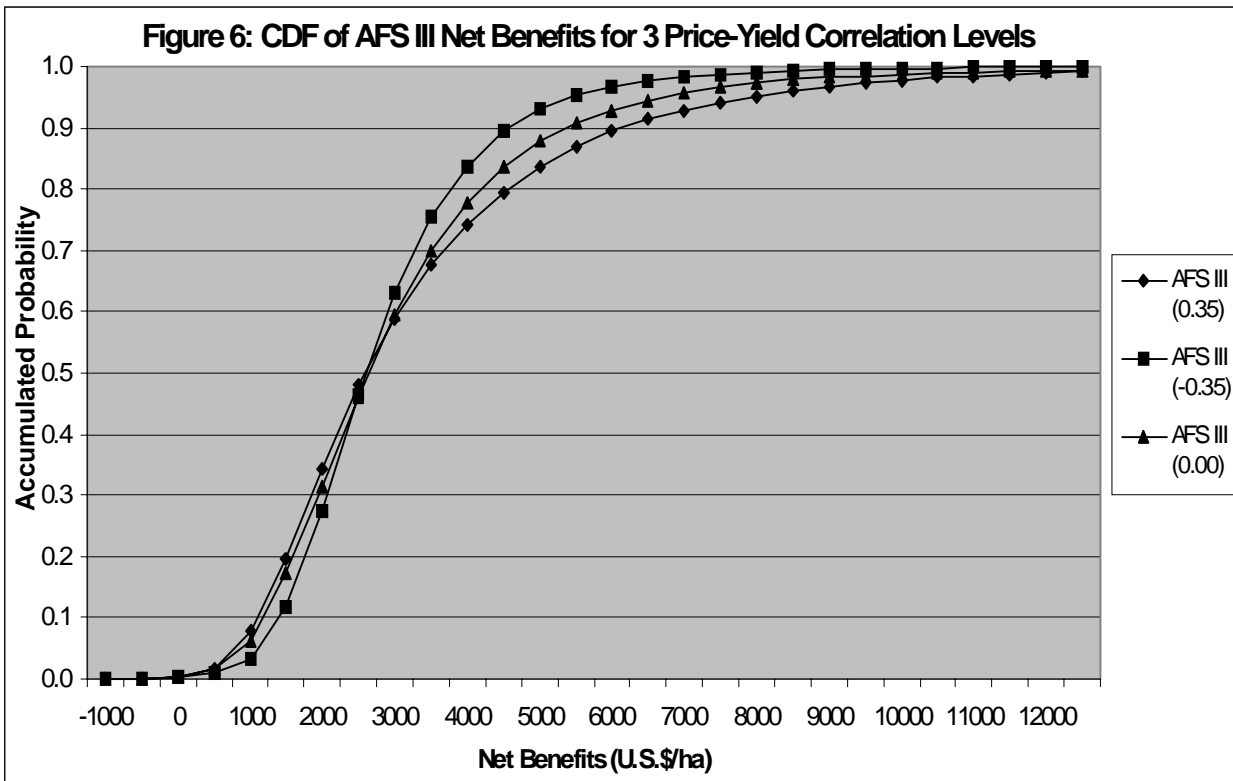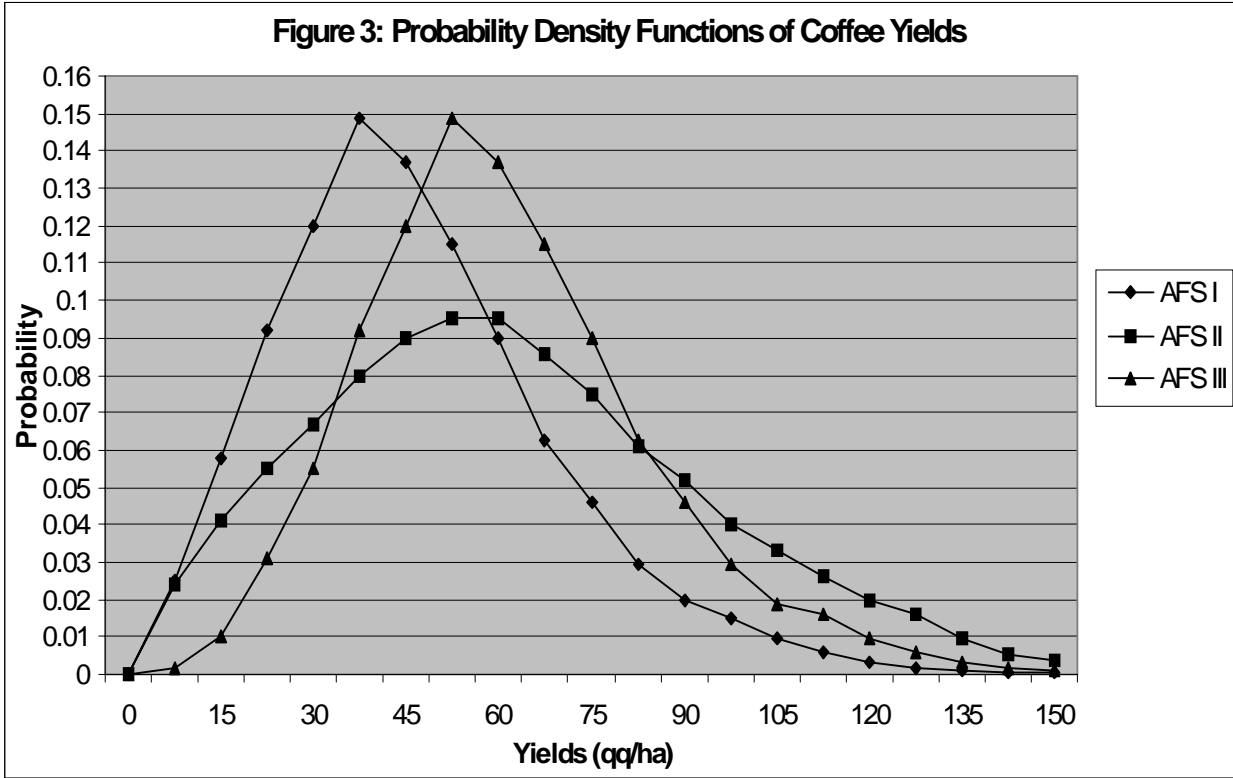Figure 6: CDF of AFS III Net Benefits for 3 Price-Yield Correlation Levels

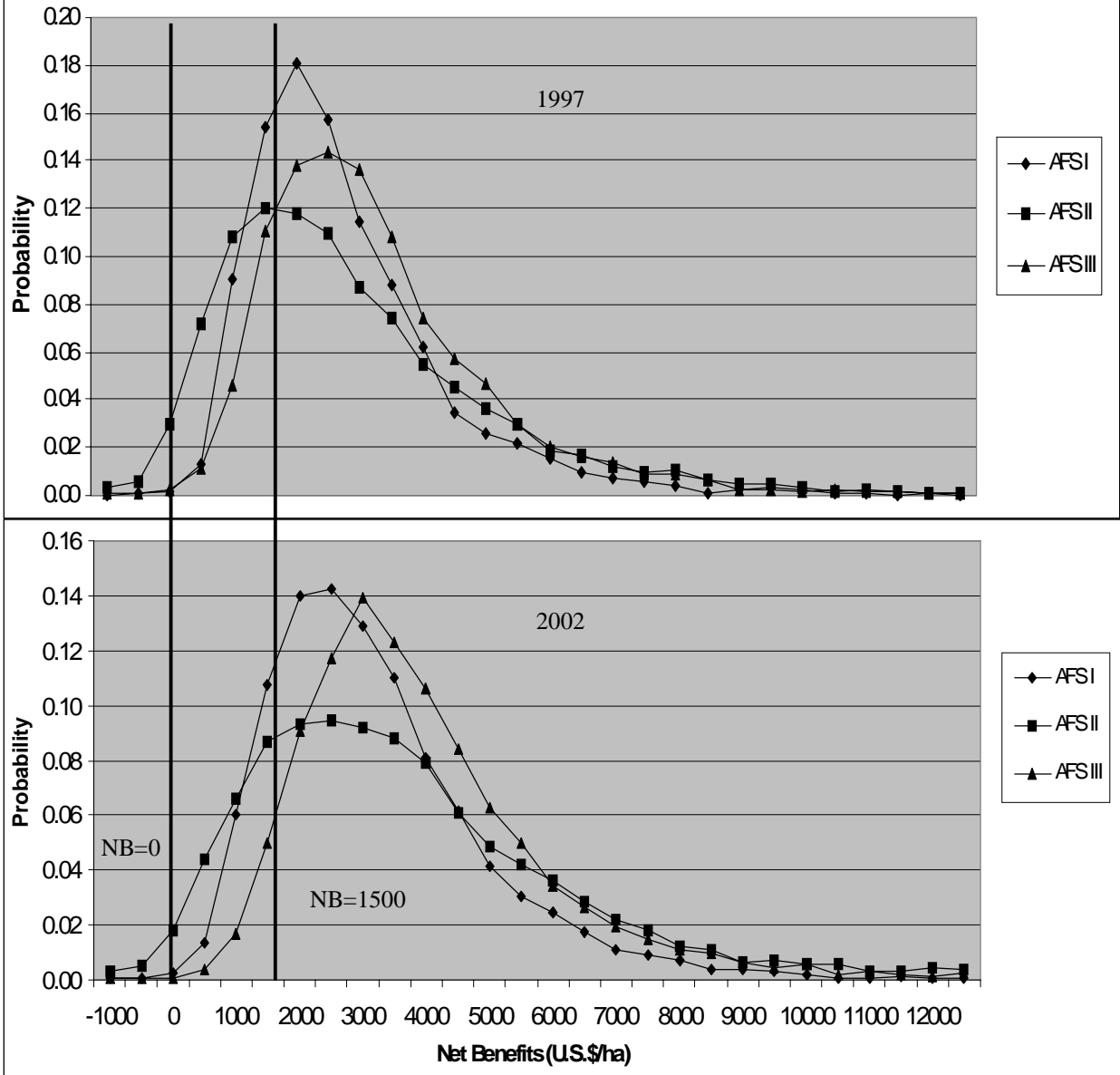Figure 4: Simulated pdf's for the AFS Net Benefits (1997 & 2002)
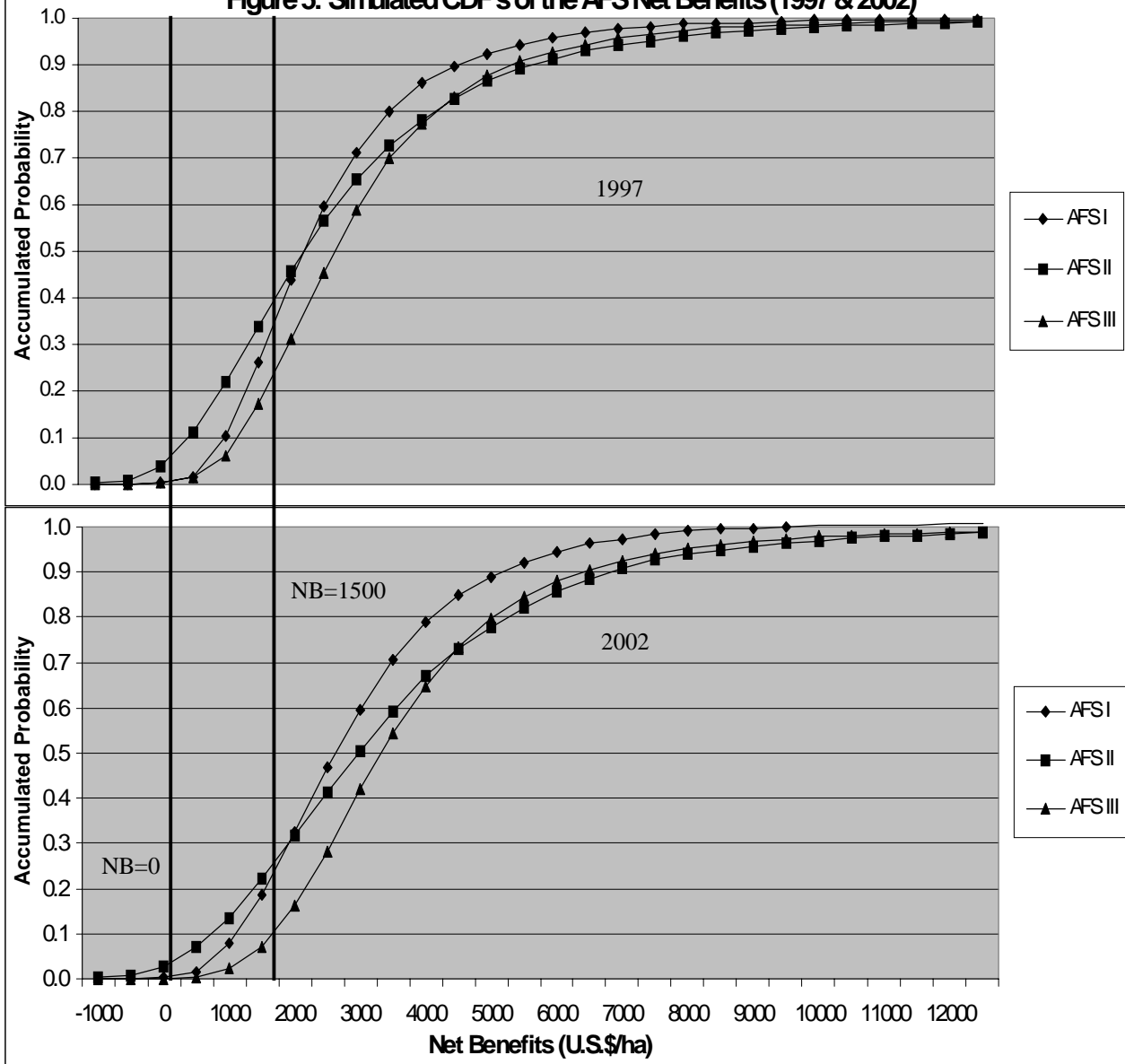
Figure 5: Simulated CDF's of the AFS Net Benefits (1997 & 2002)

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