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# Optimal Service Planning for a Sustainable Transit System 

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#### Abstract

A mathematical model and methodology are presented in this paper that can be used to determine the sustainability of a bus service. To formulate the optimization model, an entire bus route in a suburban area is considered on which many eligible stop locations are distributed realistically as discrete points such as intersections or entrances to housing developments. The objective total profit function is maximized by optimizing the number and locations of stops, the headway, and the fare. The number of passengers for the service is dependent on passengers’ access distance, wait time, in-vehicle time, and fare. The solution methodology is applied to an example that uses a bus route in suburban Woodbridge, NJ to demonstrate its effectiveness. The sensitivity of the total profit and of the amount of passengers served to various parameters is analyzed.


## INTRODUCTION

Characteristics of a sustainable transit service include its accessibility and costeffectiveness. ${ }^{1}$ A transit service's number and locations of stops, fare, and headway all affect both its accessibility and profit. However, they can cause the accessibility and profit to both improve and degrade. While low fares and short headways with many available stops at which to board can be attractive to passengers, the cost of these characteristics may be too high for the service supplier to provide. In contrast, while high fares and long headways with few stops can be attractive to the service supplier, the costs of these characteristics may be too high for passengers and thus discourage ridership which may reduce the profit of the service provider. Therefore, the determination of the optimal service characteristics for a sustainable transit service is challenging.

The methodology discussed in this paper involves the optimization of a transit service's number and locations of stops, fare, and headway. Unlike in previous research, it incorporates both the amount of demand served and the profit when choosing the optimal service characteristics, it realistically assumes possible stop locations as discrete points along a route, and it considers demand decay due to access distance, wait time, in-vehicle time and fare.

To formulate the optimization model, an entire bus route in a suburban area is considered on which many eligible stop locations are distributed realistically as discrete points such as intersections or entrances to housing developments. The objective total profit function is maximized by optimizing the number and locations of stops, the headway, and the fare subject to a capacity constraint.

The subsequent sections of this article are organized as follows. First, the literature review section summarizes literature relevant to this study. Next, the model formation section states the assumptions made for the study and explains the equations used. Then, the optimization algorithm that can be applied to find the best service characteristics is presented. Finally, the results from the application of the algorithm to a route in suburban Woodbridge, NJ are shown along with a sensitivity analysis.

## LITERATURE REVIEW

Previous research has studied the optimization of bus service characteristics. However, little research has been focusing on stop locations as discrete points while also considering that demand is dependent on the access time, wait time, in-vehicle time, and fare.

A discrete approach was applied by Furth and Rahbee to determine the optimal bus stop locations such that the sum of the net walking time cost, riding delay cost, and operating cost are minimized with given headway and fare. A discrete set of candidate stop locations along the route and a demand distributed realistically to the route were considered. However, their model assumed that demand was fixed and not dependent on the service's characteristics. ${ }^{2}$

Later, Furth et al. developed a method that uses a parcel-level geographic database, the street network, and data on each parcel's land use to realistically estimate demand for a transit service. The increase in passengers' access distance, riding time, and the system's operating cost from changes in the number and locations of stops was studied. The decrease in demand that can be caused by the access distance was considered. However, the loss of passengers because of increases in riding time, wait time and fare was not taking into consideration. ${ }^{3}$

Chien and Qin developed a model to optimize the number and locations of bus stops for a route section that minimized the total cost of the system considering realistic street parameters and spatial boarding/alighting demand distributions. Demand is assumed concentrated at access points where minor streets intersect with the bus route, but may differ spatially. However, the solution method is expensive, in terms of computation time, especially for a route with a large number of entry points, because the problem is a combinatorial optimization problem. The objective total cost function (supplier and user cost) was minimized and demand was assumed fixed regardless of the service characteristics. ${ }^{4}$

## MODEL FORMULATION



Figure 1 A general transit route with entry points, and stop locations
The objective of the research presented in the paper is to develop a mathematical model to optimize the number and locations of bus stops, headway, and fare for a bus service in consideration of the sustainability of the service. An entire bus route is assumed that has many eligible stop locations with different potential demand originating at them. Also, the route terminates at a central business district (CBD) or a transit transfer terminal. The following assumptions are made to formulate the objective maximal profit function:

1. Eligible stop locations along the route have been identified. These include intersections and entrances to housing developments as shown in Figure 1. The consideration of possible stop locations as discrete points along a route is supported by previous research that suggests stops should be located at intersections and not midblock because then buses have an easier time exiting and entering the flow of traffic and pedestrians are less likely to jaywalk. ${ }^{5}$
2. The demand is a many-to-one pattern, with a destination of a CBD or a primary transfer terminal. The number of potential passengers is known through trip generation methods or surveys. A potential passenger's preferred stop at which to board is the one which minimizes a weighted sum of his or her access and in-vehicle time. A potential passenger of the service will become an actual passenger of the service depending on his or her
tolerance of the access time, wait time, in-vehicle time, and fare. Demand is heterogeneously distributed over the route but is constant over a given time period.
3. The service headway is deterministic, and the average passenger wait time is dependant on the headway. Flat fare is applied to all passengers using the service. Also, the bus serves every stop along the route.
4. Eligible stop locations are denoted by $e_{i}$ where $i$ equals 1 through $n$, the total number of eligible stop locations. The number of potential passengers who prefer to board at $e_{i}$ is denoted $y_{i}$ where $i$ denotes the number of the eligible stop locations. Through the application of the optimization algorithm, one can determine which eligible stop locations should become actual stop locations for the service. The actual stop locations are denoted $s_{j}$ where $j$ equals 1 through $m$, the total number of stops. For notation purposes, the CBD is considered both $e_{n}$ and $s_{m}$. These notations are illustrated in Figure 1.

## Total Demand

As stated in the assumptions, the number of potential passengers for the transit service who prefer to board at eligible stop location $i$ and destined for the CBD, is known. However, potential passengers may choose not to use the service. Demand decay can be a result of unacceptable access distance, wait time, in-vehicle time, and fare amounts.

## Demand Decay of Access Distance

Potential passengers have a preferential eligible stop location at which to board. The service will lose passengers for any extra distance passengers have to walk to access the service when their preferred stop has not been chosen as an actual stop location. The distance from passengers' preferential stop location to the actual stop they will have to board at is denoted $l_{i, j}$ with $i$ indexing the preferred eligible stop location and $j$ indexing the actual stop location.

The demand decay function developed by Kimpel et al. shown in equation 1 will be applied. Kimpel et al. empirically estimated and used a distance decay function to find the relationship between the access distance, $l_{i, j}$, and demand for a bus service at the stop level. They found that a negative logistics function best fit their data. In addition, they found that the equation is more suited for distance decay of transit demand than Zhao's exponential function and methods in which a uniform demand density and a one-quarter-mile service area were assumed because their negative logistic function shows a more gradual decline in transit demand for shorter distances and a steeper decline as distances approach one-quarter mile, followed by a more gradual tail. ${ }^{6,7}$

The values of the parameters $p_{a 1}$ and $p_{a 2}$ used in the equation are calibrated so that they may accurately reflect the potential passengers' resistance to walking. Or, if the particular potential passengers' aversion to walking is unknown, they can be chosen such that they maximize the goodness of fit for known access distance decay data for an area with a similar climate and terrain. For example, based on data collected in Portland, Oregon, Kimpel et al. found that the parameters $p_{d 1}$ and $p_{d 2}$ are equal to 2 and 15 , respectively. ${ }^{8}$ The demand decay from access distance for potential passengers who prefer $e_{i}$ but will be boarding at $s_{j}$, is

$$
\begin{equation*}
d_{a i j}=1-\frac{\exp \left(p_{a 1}-p_{a 2} l_{i j}\right)}{1+\exp \left(p_{a 1}-p_{a 2} l_{i j}\right)} \tag{1}
\end{equation*}
$$

## Demand Decay of Wait Time

Wait time, $t_{w}$, is dependant on headway. Fan and Machemehl developed equation 2, shown below, to predict average passenger wait time. Their equation reflects that passengers transition from random to coordinated arrivals as headway increases. In particular, at a 0.167 hour (10 minute) headway, arrivals become coordinated. Because in their paper they only reported data collection for headways up to 1 hour, the authors of this paper will surmise that equation 2 is applicable for headways up to 1 hour. ${ }^{9}$

$$
\begin{equation*}
t_{w}=0.033+0.3 h \tag{2}
\end{equation*}
$$

Assuming demand decay for wait time is linear, let $p_{w}$ be the percentage decrease in demand for every hour increase in wait time. Then, the amount of potential demand lost as a result of the wait time is

$$
\begin{equation*}
d_{w}=p_{w} t_{w} \tag{3}
\end{equation*}
$$

## Demand Decay of In-Vehicle Time

The calculation of the in-vehicle time for passengers who access the service at stop $j$ and then travel to the CBD, denoted $t_{v j}$, consists of three parts.

The first component is the time it takes to travel at cruising speed from stop $j$ to the CBD. The next component is the acceleration/deceleration delay from making stops. The last component is the dwell time spent at stops while passengers board.

The in-vehicle cruising time from stop $j$ to the CBD is found by dividing the distance traveled by the cruising speed, $v$. The distance from $j$ to the CBD is $l_{i m}$ where $i$ is the eligible stop location index of stop $j$ and $m$ is the index of the stop at the CBD.

Deceleration delay is accrued from having to slow down as the bus approaches a stop and acceleration delay is accrued from having to reach cruising speed from being standing when the bus leaves the stop. If $m$ is the total number of stops, including the final stop at the CBD, then passengers boarding at stop $j$ will be on the bus when it accelerates and decelerates $m$ - $j$ times.

The average time it takes a passenger to board the bus, denoted $t_{b}$ (hours/passenger), is assumed and is dependent on the bus's fare collection method and design characteristics such as number of doors. ${ }^{10}$ The in-vehicle time for passengers boarding at stop $j$ will be subjected to passengers boarding at stops $j+1$ to $m-1$ where $Q_{j}$ denotes the total number of passengers boarding at stop $j$. The calculation of $Q_{j}$ is explained later on in a more appropriate section. The total dwell time is then the product of the total demand and the boarding time.

Equation 4 can be used to calculate the in-vehicle time for passengers boarding at stop $j$, that has eligible stop location index $i$. The first part of the sum calculates the cruising time, the second part the acceleration/deceleration time, and the third part the dwell time.

$$
\begin{equation*}
t_{v j}=\frac{l_{i m}}{v}+(m-j)\left(\frac{v}{2 a}+\frac{v}{2 b}\right)+t_{b} \sum_{k=j+1}^{m-1} Q_{k} \tag{4}
\end{equation*}
$$

Assuming demand decay for in-vehicle time is linear, let $p_{v}$ be the percentage decrease in demand for every hour increase in in-vehicle time. Then, the amount of potential demand lost as a result of the in-vehicle time is denoted $d_{v j}$ and is calculated as shown in equation 5.

$$
\begin{equation*}
d_{v j}=p_{v} t_{v j} \tag{5}
\end{equation*}
$$

## Demand Decay of Fare

The fare for the bus service, denoted $f$, is assumed to be flat so it is the same for all passengers. Assuming demand linearly decreases as fare increases, and if $p_{f}$ is the percentage decrease in demand for every dollar increase in fare, then the amount of potential demand lost as a result of fare is denoted $d_{f}$ and is calculated as shown in equation 6 .

$$
\begin{equation*}
d_{f}=p_{f}(f) \tag{6}
\end{equation*}
$$

## Total Demand

To determine how many of the potential passengers who prefer $e_{i}$ will actually use the service, one first needs to determine at which stop they will board, especially if their preferred stop location is not going to be an actual stop location. They will choose the stop that minimizes the weighted sum of their access and in-vehicle time. The weighted sum can be found by calculating, for each stop $j$, the sum of $d_{a i j}$ and $d_{v j}$.

Once the stop location for passengers who prefer $e_{i}$ has been determined, one calculates how many of these potential passengers will become actual passengers. This is done by subtracting from the total number of potential passengers the people who will not use the service because of the access distance, wait time, in-vehicle time and fare as shown in equation 7.
where

$$
q_{i j}=\left\{\begin{array}{lc}
y_{i}\left(1-d_{a i j}-d_{w}-d_{v j}-d_{f}\right) & \text { if }\left(1-d_{a i j}-d_{w}-d_{v j}-d_{f}\right) \geq 0  \tag{7}\\
0 & \text { otherwise }
\end{array}\right.
$$

$i$ : index of eligible stop location;
$j$ : index of actual stop location;
$q_{i j}$ : actual demand that prefers eligible stop location $i$ and boards at stop $j$ for the CBD;
$y_{i}$ : potential demand from eligible stop location i destined for the CBD;
$d_{a i j}$ : demand decay from access distance from eligible stop location i to actual stop location j;
$d_{w}$ : demand decay from wait time;
$d_{v j}$ : demand decay for in-vehicle time from stop j ;
$d_{f}$. demand decay from fare.
The total number of passengers who board at $s_{j}$ is denoted $Q_{j}$. As the passengers from multiple eligible stop locations may be boarding at $s_{j}$, a calculation of the sum of $q_{i j}$ for different $i$ needs to be computed to find $Q_{j}$ as shown in equation 8.

$$
\begin{equation*}
Q_{j}=\sum_{i} q_{i j} \tag{8}
\end{equation*}
$$

The total demand for the transit service is then found by summing the number of passengers who board at each stop j along the route.

$$
\begin{equation*}
Q=\sum_{j=1}^{m-1} Q_{j} \tag{9}
\end{equation*}
$$

## Supplier Cost

The transit supplier's cost is incurred by the fleet size, $F$, and the bus operating cost, $u$. The fleet size, in turn, is dependant on the round trip travel time, $t_{r}$. If the bus only makes stops in one direction, $t_{r}$ equals the one-way travel time with stop delay plus the one-way travel time without stop delay. If the bus makes stops in both directions, then $t_{r}$ equals twice the one-way travel time. As this study is only considering serving demand in one direction, the former will be used for this study. As the round trip travel time is dependant on the number of passengers and number of stops, it can not be calculated until after the stop locations and number of passengers have been determined.

$$
\begin{equation*}
t_{r}=2(L / v)+m[(v / 2 a)+(v / 2 b)]+t_{b} Q \tag{10}
\end{equation*}
$$

The fleet size is chosen so that the service headway is achieved. Thus,

$$
\begin{equation*}
F=\frac{t_{r}}{h} \tag{11}
\end{equation*}
$$

After the fleet size is found, the transit supplier cost, denoted $C$, is determined by multiplying the fleet size by the hourly bus operating cost $u$ :

$$
\begin{equation*}
C=F u \tag{12}
\end{equation*}
$$

## The Objective Total Profit Function

The transit supplier's total revenue, $R$, equals the fare per passenger multiplied by the total demand.

$$
\begin{equation*}
R=F Q \tag{13}
\end{equation*}
$$

The total profit, denoted as $P$, equals the total revenue less the total cost of supplying the transit service. Thus,

$$
\begin{equation*}
P=R-C \tag{14}
\end{equation*}
$$

## Capacity Constraint

To ensure that the service capacity satisfies the demand, a maximum headway, $h_{\max }$, is calculated. The maximum headway is found by dividing the capacity of a bus, $g$, by the total demand. Thus,

$$
\begin{equation*}
h_{\max }=\frac{g}{Q} \tag{15}
\end{equation*}
$$

In summary, the model describes how to calculate the total profit for a bus service. The total profit is dependent on the number and location of stops, fare, and headway. It is also dependant on potential passengers’ sensitivity to the access distance, wait time, in-vehicle time, and fare.

## OPTIMIZATION ALGORITHM

The algorithm was designed to consider both the social and economic sustainability of the service. It first considers for a specified number of stops, where the stops should be located such that the maximal number of passengers are served. Then, for the configuration of the specified number of stops that serves the greatest demand, it optimizes the values of headway and fare such that profit is maximized. Finally, after repeating these previous steps for 2 through $n$ possible numbers of stops, it chooses the number of stops and its associated configuration of stop locations and headway and fare that produces the maximal profit. The option of 1 stop is not considered as this would just be a stop located at the CBD.

The optimization algorithm is presented below and shown in Figure 2.
Step 1: Locate all eligible stop location $e_{i}$ along the route and for each obtain its potential demand $y_{i}$.

Step 2: Initialize the values of $h$ and $f$. Do not chose values for which demand or profit will be less than or equal to 0 .

Step 3: Determine all combinations of stop locations when there are just 2 stops along the route. One stop will be located at the CBD.

Step 4: For each combination of stop locations, calculate the total demand, $Q$.
Step 5: From Step 4, identify the stop combination that produced the greatest $Q$.
Step 6: Using the total profit equation, optimize the values of $h$ and $f$ so that profit is maximized for the stop location configuration found in Step 5. The authors of this paper used, and recommend using, commercially available optimization software to optimize the values.

Step 7: Recalculate $Q$ for the stop location configuration using the new values of $h$ and $f$.
Step 8: Check the capacity constraint. If it has been breached, substitute $h_{\max }$ for $h$ and recalculate $Q$ with it.

Step 8: Calculate the total profit, $P$, using the optimized values of $h$ and $f$ and the stop locations found in Step 4.

Step 9: Repeat steps 3 through 8 for 3 through $n$ stops along the route.
Step 9: For the transit service, choose the number of stops and its corresponding stop locations, $h$, and $f$ that produced the greatest $P$ found in Step 7.


Figure 2 Optimization Procedure

Table 1 Notations Used for Constants and Variables and Their Values

| Symbol | Description | Unit | Baseline Values |
| :---: | :---: | :---: | :---: |
| a | vehicle acceleration rate | $\mathrm{ft} / \mathrm{s}^{2}$ | 4.80 |
| $b$ | vehicle deceleration rate | $\mathrm{ft} / \mathrm{s}^{2}$ | 4.80 |
| C | total supplier cost | \$/hr | - |
| $d_{\text {aj }}$ | decay from access distance from $e_{i}$ to $s_{j}$ | - | - |
| $d_{f}$ | decay from fare | - | - |
| $d_{v j}$ | decay from in-vehicle time | - | - |
| $d_{w}$ | decay from wait time | - | - |
| $e_{i}$ | eligible stop location I $1 \leq j \leq n$ | - | - |
| $F$ | fleet size | buses/hr | - |
| $f$ | fare | \$/pass | - |
| $g$ | bus capacity | pass/bus | 50 |
| $h$ | headway | hr | - |
| $h_{\text {max }}$ | maximum allowable headway | hr | - |
| $i$ | index of entry points | - | - |
| $j$ | index of stops | - | - |
| L | length of route | mi | 4.56 |
| $I_{i, j}$ | distance from eligible stop location $i$ to actual stop location $j$ | mi | - |
| $m$ | number of stops | stops | - |
| $n$ | number of eligible stop locations | stops | - |
| $P$ | total supplier profit | \$/hr | - |
| $p_{\text {a }}$ | access distance decay parameter | - | 2 |
| $p_{\mathrm{a} 2}$ | access distance decay parameter | 1/mi | 15 |
| $p_{f}$ | fare decay parameter | 1/\$ | 0.08 |
| $p_{v}$ | in-vehicle time decay parameter | 1/hr | 0.38 |
| $p_{w}$ | wait time decay parameter | 1/hr | 0.75 |
| $Q$ | total demand | pass/hr | - |
| $q_{i j}$ | passengers who prefer eligible stop $i$ and board at stop $j$ | pass/hr | - |
| $Q_{j}$ | total number of passengers who board at stop $j$ | pass/hr | - |
| $R$ | total revenue | \$/hr | - |
| $s_{j}$ | stop j $1 \leq j \leq m$ | - | - |
| $t_{\text {a }}$ | access time | hr/pass | - |
| $t_{b}$ | boarding time | hr/pass | 0.0011 |
| $t_{r}$ | round trip travel time | hr | - |
| $t_{v j}$ | in-vehicle time from stop $j$ to the CBD | hr | - |
| $t_{w}$ | average wait time | hr | - |
| $u$ | hourly cost of operating a bus | \$/veh-hr | 50 |
| $v$ | average cruising speed | $\mathrm{mi} / \mathrm{hr}$ | 25 |
| $y_{i}$ | potential passengers who prefer eligible stop location i | pass/hr | - |

## NUMERICAL EXAMPLE



Figure 3 Bus Route 803 in Woodbridge, NJ with 16 hypothetical eligible stop locations
This section demonstrates the use of the model developed by applying it to New Jersey Transit Route 803 in Woodbridge, New Jersey. The route connects the Metropark Rail Station on the Northeast Corridor Line with the Woodbridge Train Station on the North Jersey Coast Line. The Metropark Rail Station is designated stop location 1 and the Woodbridge Train Station the final stop location. Demand was assumed to be many-to-one with the Woodbridge Train Station as the final destination. The bus route is 4.56 miles long through a suburban area with $n=16$ (including the Woodbridge Train Station). Potential stop locations 2 through 12 are at the entrances to housing developments and potential stop locations 13 through 15 are in a neighborhood on a grid network. A map of the route can be seen in Figure 2.

The authors acknowledge that because the route is anchored by two train stations demand could be in both directions along the route but to illustrate the methodology presented in this paper, this possibility is not being considered. However, it can be used for future studies. Also, as this paper does not focus on determining the amount of potential demand for a route, as discussed earlier, the authors arbitrarily assigned potential demand amounts to the eligible stop locations.

Table 2 Eligible Stop Locations, Optimal Stop Locations, and Potential and Actual Demand Served

| Eligible <br> Stop No. | Eligible Stop Name | Actual <br> Stop <br> Location | Actual <br> Stop <br> No. | Distance from <br> Beginning of <br> Route | Potential <br> Demand <br> $Y_{i}$ <br> (pass/hr) | Actual <br> Demand <br> $q_{i j}$ <br> (pass/hr) | Percentage <br> Served <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Metropark STN | yes | 1 | 0 | 20.00 | 7.42 | 37.10 |
| 2 | Gill Ln and 657 | yes | 2 | 0.51 | 18.00 | 6.90 | 38.33 |
| 3 | Barbara CT | no | - | 0.73 | 4.00 | 1.52 | 38.00 |
| 4 | Laurie PL | yes | 3 | 0.82 | 8.00 | 3.12 | 39.00 |
| 5 | Tammy PL | no | - | 0.88 | 8.00 | 3.09 | 38.63 |
| 6 | Ronson RD | no | - | 1.54 | 16.00 | 6.46 | 40.38 |
| 7 | Wdbg Commons Way | yes | 4 | 1.58 | 20.00 | 8.13 | 40.65 |
| 8 | Highview DR | no | - | 2.05 | 30.00 | 11.88 | 39.60 |
| 9 | Woodbridge TER | yes | 5 | 2.18 | 30.00 | 12.66 | 42.20 |
| 10 | Maple Hill DR | no | - | 2.25 | 30.00 | 12.49 | 41.63 |
| 11 | Plaza DR N | yes | 6 | 2.49 | 18.00 | 7.76 | 43.11 |
| 12 | Plaza DR S | no | - | 2.61 | 16.00 | 6.57 | 41.06 |
| 13 | St. James AVE | yes | 7 | 3.91 | 10.00 | 4.56 | 45.60 |
| 14 | Columbus AVE | yes | 8 | 4.13 | 20.00 | 9.30 | 46.50 |
| 15 | Amboy AVE | no | - | 4.27 | 10.00 | 4.32 | 43.20 |
| 16 | Wdbg Train STN | yes | 9 | 4.56 | - | - | - |
| Total |  | - | - | $\mathbf{9}$ | $\mathbf{4 . 5 6}$ | $\mathbf{2 5 8 . 0 0}$ | $\mathbf{1 0 6 . 1 8}$ |

The optimal number of stops, fare, and headway were found to be $9, \$ 5.45$, and 0.29 hrs , respectively. Stops were optimally located at eligible stop locations $1,2,4,7,9,11,13,14$, and 16, the Woodbridge Train Station. The average distance between stops was 0.59 miles. The total profit with these service characteristics is $\$ 490 / \mathrm{hr}$ and the amount of demand served is 106.18 pass/hr. Table 2 shows the amount of passengers served from each eligible stop location.


Figure 4 Max Profit and Total Actual Demand vs. Number of Stops
Figure 4 shows for all the possible number of actual stops for the route, the maximized profit and amount of demand served. One can see that the optimal number of stops is 8 as the profit is the greatest for this number of stops. However, the total profit for having 7 through 12
stops varies by less than $\$ 10 / \mathrm{hr}$. This is because for this set of stops, the optimal headway only ranges by .005 hr , the optimal fare only ranges by $\$ 0.06$, and the demand served only ranges by 5 passengers/hr.

In addition, one can see that the demand served decreases after 9 stops. This is because the additional in-vehicle time accrued from serving more than 9 stops causes more inconvenience to passengers in terms of in-vehicle time addition then it causes more convenience in terms of access distance deduction. From the map and Table 2 one can see that when eligible stop locations are located within walking distance of one another, usually not all of them have stops located at them. This observation will be studied further in research to immediately follow the completion of this paper. In particular, the tradeoff between the cost to both the service supplier and passengers already on a bus from the additional time of decelerating/accelerating for a stop versus the additional cost to passengers for the increased access distance will be studied.

## SENSITIVITY ANALYSIS

The study of a transit service's profit and the number passengers it serves is an integral part in the research of sustainable transportation. Both profit and number of passengers are dependent on potential passenger's sensibility to different service characteristics. A sensitivity analysis was conducted to determine how the value of the fare decay parameter, wait time decay parameter, and in-vehicle time decay parameter affect the transit service's profit and the number passengers it serves. In addition, a sensitivity analysis was conducted to determine how values other than the optimal ones for fare and headway affect the profit and number of passengers served. For all the analyses, stop location was assumed fixed at the 9 optimal stops found in the example. Therefore, consideration was not given to how changes in the parameter's values might affect the number or locations of stops.


Figure 5 Optimal Fare and Headway vs. Fare Decay Parameter


Figure 6 Max Profit and Total Demand vs. Fare Decay Parameter

Figure 5 shows that as the fare decay parameter increases from the example's value of 0.08 , the optimal fare exponentially decreases while the optimal headway linearly increases. As the service is trying to maximize profit, headway needs to increase to save supplier costs while
revenue decreases. Figure 6 shows that both the maximal profit and the demand for the service decrease as potential passengers become more sensitive to the value of fare.


Figure 7 Optimal Fare and Headway vs. Wait Time Decay Parameter


Figure 8 Max Profit and Total Demand vs. Wait Time Decay Parameter

Figure 7 shows that as the wait time decay parameter increases, both the optimal headway and fare only decrease slightly. However, as figure 8 illustrates, maximal profit decreases more severely.


In figure 9 one can see that as the in-vehicle time decay parameter increases the optimal fare decreases while the optimal headway increases. However, neither have a significant change as the range for headway is only 0.021 hours which is 1.26 minutes and the range for fare is only $\$ 0.49$. Both the profit and number of passengers served decrease as the wait time decay parameter increases as shown in figure 10.


Figure 11 Max Profit and Total Demand vs. Fare
Figure 11 shows the maximal profit and total demand for different fare values, holding all other parameter values constant. Within the fare range of $\$ 4.45$ to $\$ 6.45$ the maximal profit is within $\$ 20 /$ hr. In addition, as the demand varies by about 40 passengers within this range, with the greatest demand for the lowest fare, the service provider could consider decreasing the fare so that both an objective of maximizing profit and maximizing the number of people service is provided to are met.


Figure 12 Max Profit and Total Demand vs. Headway
Figure 12 shows that the maximal profit is pretty flat between headways of 0.2 and 0.4 hours.

## CONCLUSION

A methodology was developed to optimize bus service planning for a sustainable bus service. The methodology can be applied to determine the optimal number and locations of stops, headway, and fare. Demand decay and the realistic, discrete locations of possible bus stops are considered.

The methodology was applied to a route in suburban Woodbridge, NJ to demonstrate its effectiveness.

The model contributes to research of sustainable transportation because it considers the profit and number of passengers served for different stop location configurations along a route. In addition, this model can be applied to future research that studies the accessibility of a service with potential passengers from different socio-economic groups along the same route. Finally, the sensitivity analysis demonstrates how the profit and demand for the service vary for different values of headway and fare and for different passenger characteristics.

Future research includes considering different objective functions such as minimizing total cost. Also, as the methodology is exhaustive, in that all combinations of possible stop locations are considered, the application of a heuristic algorithm should be developed.

## Notes

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