*Working Paper Series*

WORKING PAPER NO. 585

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DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS

BERKELEY

CALIFORNIA AGRICULTURAL EXPERIMENT STATION

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DIVISION OF AGRICULTURE AND NATURAL RESOURCES
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Giannini Foundation of Agricultural Economics
March 1, 1991

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James A. Chalfant and Nancy Wallace

James A. Chalfant is an Associate Professor of Agricultural and Resource Economics at University of California at Berkeley. Nancy E. Wallace is an Assistant Professor in the Haas School of Business at University of California at Berkeley.

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## 1. INTRODUCTION

Restructuring trends in transportation markets have intensified the need for analysis of firm level production technologies. Accurate predictions about the outcomes of deregulation necessarily rely on careful portrayals of the production or cost functions of firms operating in these industries. In particular, the economically relevant information sought in empirical studies of the transportation industry is the estimation of substitution and output elasticities of firms. These elasticities portray the underlying structure of production and thus are usually the focus of policy debates concerning the effects of regulatory intervention.

The purpose of this paper is to compare two strategies for estimating motor carrier cost functions: a semi-nonparametric approach using the Fourier flexible form developed by Gallant (1981, 1982), and the more common locally flexible approximation, the translog (Christensen, Jorgensen, and Lau, 1973). The comparison will be made using a data set for general motor freight carriers observed immediately prior to deregulation and immediately after deregulation. The data set has been constructed to be consistent with previous studies of the highway motor carrier industry, accounting for the output heterogeneity and network structure of these firms (Wang and Friedlaender, 1984; Daughety and Nelson, 1988; Daughety et al., 1986; Caves et al., 1984; Kim, 1984; Friedlaender and Spady, 1981; Ying, 1990).

In every case, the translog restriction is rejected as a special case of the Fourier cost function, indicating that use of the translog may lead to specification bias. This bias can, in turn, lead to incorrect and misleading results for hypothesis tests or estimates of elasticities.

## 2. MOTOR CARRIER COST FUNCTIONS AND THE TRANSLOG

The empirical model draws heavily from assumptions common to many pre-deregulation cost studies. Following numerous recent studies of the trucking industry (e.g. Ying 1990; Daughety and Nelson, 1988; Daughety et al. 1986; Wang and Friedlaender, 1980; Kim, 1984), it is assumed that the spatial network of firms can be represented by a vector of attributes t. The motor carrier firm is assumed to take input prices as exogenous and minimize costs. The unknown cost function is thus represented as

$$g^*(x) = g^*(y,w,t), \tag{1}$$

giving the minimum cost of producing outputs y with factor input prices w and technology attributes t. The objective in estimation is to characterize this function, its first derivatives (thus characterizing marginal costs and factor shares or input levels) and its second derivatives (thus characterizing elasticities).

The typical empirical specification for trucking industry cost functions has been the translog (e.g. Ying, 1990; Friedlaender and Spady, 1981; Spady and Friedlaender, 1978; Wang and Friedlaender, 1980; Daughety et al., 1986; Daughety and Nelson, 1988; Caves et al., 1984; Harmatuck, 1980; Kim, 1984). The translog may be thought of as a valid functional form for cost functions, but is generally assumed to provide a suitable second order approximation to the unknown cost function, $g^*(x)$, where the simple arithmetic means of the vector x are characterized as the point of approximation.

White (1980) has shown, however, that the approximation interpretation is questionable, and Gallant notes (1981, p. 212) that "statistical regression methods essentially expand the true function in a (general) Fourier series -- not in a Taylor's series". Even if the coefficients of a Taylor series were recoverable from empirical methods, Despotakis (1985) and Devezeaux de Lavergne et al. (1989) have noted that the implied behavior of the technology away from an approximation point would behave according to the form chosen, rather than according to the underlying function. Usually, interest is in the properties of technology at more than one data point, so this is of some concern. The translog can provide that information, of course, when it is treated as the correct functional form. As noted by Barnett (1983), however, models based on Taylor approximations have no inherent advantage over other functional forms that satisfy Diewert's (1973) definition of local flexibility. For these reasons, specification tests become important. Such tests generally amount to an examination of goodness-of-fit or checking the properties of concavity or monotonicity, although Berndt and Khaled (1979) tested the translog assumption by nesting it within a generalized Box-Cox model.

Other locally flexible forms can yield different results, so considering alternatives such as the generalized Leontief or generalized Box-Cox makes sense. Monte Carlo evidence (Barnett, Lee and Wolfe, 1985; Guilkey and Lovell, 1980; Guilkey, Lovell and Sickles, 1983; Caves and Christensen, 1980) suggests that each form is likely to perform better in some cases and worse in others. In any event, each alternative remains subject to the problems of locally flexible forms. These observations suggest that an alternative model -- one that approximates the function and its derivatives everywhere in its domain -- is an attractive possibility. Of particular interest are specifications which can emulate the sample cost surface and remain consistent with the global approximation properties of least squares regression methods. The Fourier flexible form introduced by Gallant (1981, 1982) accomplishes this. The Fourier is consistent with Diewert's

(1973) second-order flexibility definition with the additional advantage of "global flexibility" -- it leads to consistent estimates of elasticities throughout the region of approximation (El Badawi, Gallant, and Souza, 1983) and is asymptotically free of specification bias (Gallant, 1982). The Monte Carlo results of Chalfant and Gallant (1985) are encouraging for applications to small samples.

## 3. THE FOURIER FLEXIBLE FORM

Gallant (1982) demonstrates that a flexible form for the cost function based on a Fourier series approximation can avoid the specification errors arising from other flexible forms. He suggests the Sobolev norm as the appropriate measure of distance to measure the error in approximating the unknown cost function $g^*(x)$ with some flexible form $g_K(x \mid \theta)$, where K denotes the number of parameters in the function used to approximate $g^*(x)$. As noted above, the goal in demand analysis is to approximate the cost function and its first and second derivatives; although the local flexibility interpretation involves those properties of the cost function only at a particular price vector, the desired uses almost invariably involve the entire data set. For instance, in a cross-section of firms modeled to examine scale or scope economies, interest is not merely in whether the firm with the price vector corresponding to some hypothesized approximation point behaves a certain way, but what would happen at a different level of output or factor prices.

The Sobolev measure of error is defined by

$$||g^*(x)-g_K(x)|| = ||e||_{m,p} = \left( \sum_{j=0}^{m} \int_{\Psi} [D^i e(x)]^p f(x) dx \right)^{\frac{1}{p}} \qquad 1 < p < \infty. \qquad (2)$$

where m is the largest order derivative of g(x) that is of interest, f(x) is a probability density function describing the data generating process , $D^i$ denotes partial differentiation, and the region of approximation is $\Psi$. The Sobolev norm thus accounts both for errors in approximating the function throughout its domain and errors in approximating derivatives. The importance of this measure for empirical work is that a Fourier series approximation can be found to make the Sobolev measure small. When the cost function is approximated using a Fourier series, then, the approximation has the potential to behave as would the underlying function throughout the data space, where that behavior may be judged by the similarity of costs, factor shares, and elasticities throughout the range of the data. As noted by Gallant (1982, p. 292), the problem of approximating an unknown function becomes one of finding the appropriate Fourier series approximation, rather than searching over an unlimited set of functional forms.

When approximating a non-periodic function by a Fourier series expansion, linear and quadratic terms are often appended (Gallant, 1982). The Fourier cost function can thus be written as

$$\begin{aligned} g_k(x|\theta) = u_0 + b'x + \tfrac{1}{2}x'Cx + \\ \sum_{\alpha=1}^{A} \{u_{0\alpha} + 2[u_\alpha \cos(\lambda k_\alpha' x) - v_\alpha \sin(\lambda k_\alpha' x)]\} \end{aligned} \tag{3}$$

where x denotes a vector that includes output, input prices, and any technology covariates. Output and the prices of inputs are measured in natural logarithms. The x vector is scaled by $\lambda$ so that the data will fall within the $(0, 2\pi)$ interval, following the procedure described by Gallant (1982). The matrix C has the structure

$$C = -\sum_{\alpha=1}^{A} u_{0\alpha} \lambda^2 k_\alpha k_\alpha' \tag{4}$$

and the vector of parameters is $\theta = (u_0, b, u_{01}, u_1, v_1, ..., u_{0A}, u_A, v_A)'$.

The $k_\alpha$ vectors, $\alpha = 1,2, ..., A$, are the set of multi-indices that determine the directions along which the Fourier expansions are taken. The particular set of $k_\alpha$ vectors used in this paper is defined in the Appendix. The Fourier cost function can thus be thought of as the sum of a second-order approximation using the translog, plus the sum of A univariate Fourier series expansions determined by the $k_\alpha'$s. Linear homogeneity in terms of prices is imposed on the cost function by requiring that the elements corresponding to prices in each $k_\alpha$ sum to zero and that $\sum b_i = 1$. A final set of parametric restrictions relates to the fact that some of the $u_{0\alpha}$ parameters are redundant in the C matrix, as detailed in the Appendix.

Since the translog is nested within the Fourier, a test of the significance of the extra terms in the latter is a test of whether there is anything systematic in the residuals of the estimated translog model. Finding such significance, of course, means that one of the implications of the maintained translog hypothesis -- the residuals do not depend systematically on factor prices or other exogenous variables -- is violated. It is then of interest to examine how the incorrect restriction to the translog case -- essentially an omitted variables problem -- biases the estimates of coefficients or elasticities. Gallant (1982) suggested this nesting, and it was also exploited by Chalfant (1987) in the context of generalizing the almost ideal demand system and by Devezeaux de Lavergne et al. (1989) in estimating aggregate cost functions for sectors of the French economy. In all three cases the added Fourier terms were statistically significant.

## 4. ELASTICITIES AND STANDARD ERRORS

Differentiation of the logarithmic Fourier flexible form provides share equations of the form

$$S_i(x|\theta) = b - \lambda \sum_{\alpha=1}^{A} \{u_{0\alpha} k_\alpha' x + 2[u_\alpha \sin(\lambda k_\alpha' x) + v_\alpha \cos(\lambda k_\alpha' x)]\} k_\alpha . \tag{5}$$

An expenditure system is formed for estimation with the cost function and n - 1 of the share equations. The system is linear in the parameters, and so can be estimated with the usual iterated seemingly-unrelated regressions estimator (Zellner, 1962; Barten, 1969) to obtain invariance to the deleted equation.

Following Gallant (1982), the Hessian matrix of the Fourier cost function is the N by N matrix

$$H = -\lambda^2 \sum_{\alpha=1}^{A} \{u_{0\alpha} + [u_\alpha \cos(\lambda k_\alpha' x) - v_\alpha \sin(\lambda k_\alpha' x)]\} k_\alpha k_\alpha' . \tag{6}$$

The own-price elasticities of substitution can thus be written as

$$\sigma_{ii} = 1 + \frac{h_{ii}}{S_i(x|\theta)^2} - \frac{1}{S_i(x|\theta)} , \tag{7}$$

where $h_{ii}$ is the $i,i^{th}$ element of the Hessian, evaluated at the data point of interest and $S_i$ is the estimated share for the $i^{th}$ factor input. Similarly, the Allen-Uzawa elasticity of substitution between factors i and j can be written as

$$\sigma_{ij} = 1 + \frac{h_{ij}}{S_i(x|\theta)S_j(x|\theta)} . \tag{8}$$

First-order approximations to the standard errors for the elasticities have been proposed by Gallant (1982), although recent studies (Anderson and Thursby, 1986; Eakin et al., 1990) have questioned the precision of estimates obtained in this manner from the cost function, as well as the normality of the underlying distribution. For these reasons, bootstrap sampling methods (e.g. Freedman and Peters, 1984; Eakin et al.,1990) could be used to obtain suitable standard errors for the statistics of interest.

## 5. EMPIRICAL ANALYSIS

The cost functions were estimated using data for 60 Class I and II less-than-truckload general freight carriers that did not purchase transportation from other carriers during 1979 and 1981. Most recent empirical studies of highway motor carrier focus either exclusively on firms that purchase transportation or do not distinguish between firms that do and do not purchase transportation services (Daughety and Nelson, 1986; Daughety et al., 1988; Rose, 1987). There is, therefore, little information available on the production technology of firms that solve the linehaul deployment problem internally rather than purchase linehaul transportation either through brokers or on long term contracts from other firms.

A period immediately prior to deregulation was chosen for two reasons. First, it was desirable to have the analysis be comparable with previous empirical studies of trucking costs, without the confounding effects of deregulation. Second, the cost functions were estimated under the assumption that output and network structure are exogenously determined. This assumption may not be valid in the deregulated environment. Accordingly, we tested the endogeneity of output for both sets of observations. This was done using a Hausman-Wu test (Hausman, 1978; Wu, 1973) in which separate estimates of parameters were obtained using iterated SUR and an iterated three-stage least-squares technique using instrumental variables. The exogeneity of output was not rejected, so we report iterated SUR results below. Because the test is not exact and is dependent on instrument choice, the failure to reject exogeneity cannot be considered definitive. In any event, the translog results do not seem to depend on this choice. We find the same pattern of results for 1979 or for 1981, and preliminary experience with corrections for possible simultaneity suggest that the results are not sensitive to the estimation technique.

*Data*

There are four input prices, one output, and one variable to represent the attributes of motor carrier spatial networks. The variables and their labels are as follows:

**Factor Prices:**

$x_1$: Price of fuel ($/Gallon)

$x_2$: Price of labor ($/Employee/Year)

$x_3$: Price of terminal capital ($/terminal unit)

$x_4$: Price of vehicle capital ($/vehicle unit)

**Output:**

$x_5$: Total ton-miles

Although it would be ideal to represent the output of a firm operating over a network as ton-miles of attributes carried between origin-destination pairs, data limitations make such a characterization impossible. Instead, the standard output measure, ton-miles, is used. Thus, we examine the cost of producing ton-miles as a function of factor prices and firm characteristics.

The theory of the firm underlying cost function estimation dictates including only factor prices and the level of output in the cost function and share equations. If the particular application suggests that there are other variables that should be included as "shifters", essentially indexing the cost function and technology, these can be treated in a manner similar to output. First, they might only be included to adjust the level of costs but not interacted with prices, in which case they would not show up in the equations that determine factor shares, but only in the cost function itself. This is analogous to the homotheticity restriction for output. If the particular variable also affects optimal input ratios, it should also appear in the share equations. Gallant (1982) likens these choices to allowing for interaction effects in an experimental design.

We constructed a network variable to serve as such a shift variable. A measure of network structure can explain variation in the cost of producing a particular level of ton-miles, holding factor prices constant, by capturing the cost of operating a particular firm's network arrangement. The network variable is defined as

$$NETWORK = \left[ \frac{\sum_{i=1}^{n} d_{ig}^{2}}{n} \right]^{1/2} \tag{9}$$

where $d_{ig}$ is the straight line distance in miles from the terminal to the main terminal and n is the number of terminals. The network variable is a "standard distance measure" (Bachi, 1963) indexing the dispersion of points (terminals) in Euclidean space. The measure is the geometric average of distance from the center of gravity (the main terminal address). It is an aggregate measure of the spatial distribution of terminals and varies with distance between terminals and the number of terminals. The minimum value of the network variable is zero for firms with only one terminal, so very small values (.01) were used for such firms. Assuming an isotropic surface and equal interaction propensities, it may be presumed that a greater aggregate amount of movement would occur when the standard distance is large since pairwise distances are great. As with any measure of central tendency, this measure can be biased by extreme locations.

A non-parametric test for whether a network variable should be included was performed in the following fashion. We made use of Varian's (1984) nonparametric test for cost-minimizing behavior. In brief, one calculates for firm i the cost of buying the inputs of any firm j that was observed to have a larger output. If firm i could have purchased firm j's input bundle for less than it actually spent, then either firm i is not as successful at minimizing costs or it does not have as good a technology as firm j. One reason for such outcomes might be a less favorable network structure. We made all possible comparisons for the 60 firms in our sample -- T*(T-1)/2 or 1770 in all -- noting which firms were ever revealed to be inefficient in this manner and which were the more efficient ones. We performed this test for each data set.

Every violation of Varian's Weak Axiom of Cost Minimization produces data on an inefficient firm i and an efficient firm j. In the absence of any network effects, it would be just as common for firm i's network variable to exceed firm j's as the reverse. It was found, however, that the firms with the smaller networks tended to appear inefficient relative to other firms. Of the 228 violations where the network variable differed between firm i and firm j, 131 of these occurred when the network variable of firm i was less than that of firm j. A simple $\chi^2$ test for independence rejects the hypothesis that the probability of a violation is unaffected by differences in the network variable ($\chi^2 = 5.07$ compared to a critical value of $\chi^2_{1,.05} = 3.84$). Similar patterns were also observed for the number of terminals operated, average length of haul, and average load size. The network variable has the potential to capture these effects in a relatively parsimonious fashion; in any cost function, including several covariates and their interactions with prices and output leads to an infeasible number of parameters to estimate. Accordingly, we made use of the network variable as a covariate in the cost function.

*The Fourier Flexible Form*

We estimated four versions of the Fourier cost function. Our first objective is to test the translog restrictions. Our second objective is to examine the trade-off that exists with the Fourier between good properties concerning curvature restrictions and the number of parameters estimated. We first estimate a model referred to as Model I in which we restrict the multi-indexes to a length of less than or equal to 2 (i.e. multi-indexes $k_1$ through $k_9$ as shown in the Appendix). This model implies a homothetic technology underlying the cost function, since interactions between prices and output will not appear in factor share equations. Network enters similarly--only in the cost function itself.

Models II - IV broaden the set of multi-indexes to those of length less than or equal to 3 (i.e. multi-indexes $k_1$ through $k_{21}$ in the Appendix). Models II and III are only briefly discussed below. Model II allows interactions of price terms with output only (multi-indexes $k_{10}$ through $k_{15}$ are added to Model I) and Model III allows interactions with network only (multi-indexes $k_{16}$ through $k_{21}$ are added to Model I). Model IV contains both network and output interactions.

*Model I: The Homothetic Model*

The cost function was augmented by the equations for factor shares of fuel, labor, and capital invested in vehicles. For both years, the system was estimated using the ITSUR option of the SYSNLIN procedure of SAS (Version 6.03) with the symmetry and homogeneity restrictions imposed. As already noted, this procedure produces maximum likelihood estimates that are invariant to which share equation is dropped. Parameter estimates obtained using the 1979 observations are shown in Table 1 and those from 1981 in Table 2.

Both models perform reasonably well from the point of view of having a number of statistically significant parameter estimates. Concavity of the cost function is satisfied for 48 of the 60 firms for 1979 and for all 60 for 1981. As shown in Tables 1 and 2, the output, network, and output/network interaction covariates have a statistically significant effect on costs in both the regulated and deregulated environments. The sine and cosine parameter estimates for the covariates are also statistically significant and these terms would not appear in the translog-restricted version of Model I. The pattern of significance of the main input price ratio effects is not the same across the two years; however, in both years, the translog restriction would be binding, given the statistically significant parameter estimates on the fuel/vehicle interaction terms for 1979 and the labor/vehicular capital interaction terms for 1981.

*A Test of the Translog Restriction*

For each model, we tested the translog restriction conditional on the homotheticity restriction, by testing whether the $u_\alpha$ and $v_\alpha$ parameters could be set equal to zero. Each estimated system involves 33 parameters, a subset of which corresponds to the parameters of the translog cost function. The 18 $u_\alpha$ and $v_\alpha$ ($\alpha = 1,\ldots,9$) parameters are all equal to zero if the translog restriction is correct. Thus, testing whether the added terms -- the coefficients of sine and cosine terms -- are statistically significantly different from zero is a test of whether the translog would be a proper restriction to impose on the model. When the likelihood-ratio statistic was calculated to test the hypothesis that all of these parameters were zero, we obtained a value of 55.41 for 1979, as compared with the $\chi^2$ value with $\alpha$ = .05 and 18 degrees of freedom (corresponding to the 18 coefficients $u_\alpha$ and $v_\alpha$ that would be deleted) of 28.87. For 1981, the test statistic was 34.62, again leading us to reject the translog hypothesis in favor of the Fourier cost function.

What is the gain from using the Fourier? The natural characteristics to focus on are the elasticities of substitution, since the translog is relatively limited in that it dictates their behavior away from the "point of approximation". The local flexibility interpretation is correct for the translog, in that it can allow arbitrary values for these elasticities at some point. This is evident from the familiar translog formula for the substitution elasticity:

$$\sigma_{ij} = 1 + \frac{\gamma_{ij}}{S_i S_j}.$$

What if we evaluate the elasticity at different price vectors? It varies only because shares vary, and not because the curvature of the cost function (i.e. the second derivatives of the logarithm of total costs with respect to logarithms of prices) varies. In that sense, the translog is relatively inflexible; unless firms have very different shares, they will not have very different elasticities of substitution. While this corresponds to a plausible cost function and is sufficient to obtain the local flexibility property, there is no theoretical reason why the second derivatives have to be constant.

In fact, substitution elasticities do not seem to vary much throughout the sample for the translog, as can be seen in Table 3 for 1979 data and Table 4 for 1981. For the Fourier, the averages of the same elasticities differ from the translog and they also exhibit considerably more variation throughout the samples. The range of observed Fourier elasticities *always* includes the range for the translog values.

*Interaction Effects between Prices and Output or Network*

The next set of models we considered allowed share equations to depend on output and/or network variables. If the level of shares depends on relative prices *and* output, then the cost function is non-homothetic. Network, similarly, can affect the level of shares. A summary of results of likelihood ratio tests for the Fourier is shown in Table 5.

In 1979, Model I was rejected in favor of II, implying a non-homothetic technology. When Model I was tested against Model III, we found that adding interactions between relative prices and the network variable also contributed sufficient explanatory power to cause the restriction to be rejected. In 1981, Model I is not rejected as a special case of either Model II or Model III. However, testing whether both sets of interactions combined would improve over Model I led us to reject the homotheticity restriction for 1981, as in 1979.

We then tested the hypotheses that network alone or output alone would suffice in the share equations (each variable was retained in the cost function itself). For both years, however, Model IV with both variables included in the share equations dominates the others. Models II and III are each rejected as special cases of Model IV for both years. Thus, Model IV is the preferred model on this basis for both 1979 and 1981.

Before turning to a discussion of those results, and an examination of the tradeoff between the number of parameters and good properties concerning concavity, we return to the translog restriction. In each of the models estimated, the translog restriction amounts to deletion of the $u_\alpha$ and $v_\alpha$ parameters; the $u_{0\alpha}$ parameters remain, as described in the Appendix. Thus, the translog is easily tested as a special case of these models. We did so, as in Model I, by reestimating the models subject to the appropriate zero restrictions. Table 6 contains the results of likelihood ratio tests of the translog restriction for all four models. For both years, the translog is rejected as a special case of each of the four models estimated.

*The Results for Models II - IV: The Nonhomothetic Models*

Parameter estimates for Model IV are given for 1979 in Table 7 and for 1981 in Table 8. Estimated elasticities of substitution are summarized in Table 9 for 1979 and Table 10 for 1981. Once more, we calculated elasticities of substitution for the 60 firms in our sample, using estimated parameters from Model IV for each year. In 1979, 31 of the observations were consistent with the concavity restriction while 47 of 60 firms were consistent with concavity from the 1981 results. Neither of these numbers is as high as one would like, reflecting possible heterogeneity of firms that has not been captured fully by including characteristics of output such as the network structure. It may also reflect the fact that there are many insignificant parameter estimates included in these models; some of these may correspond to redundant parameters that are reducing the precision of our elasticity estimates. Almost surely, our Model IV is not the "right" model, in the sense that an approximation using a different set of multi-indexes -- some other Fourier cost function -- could perform better. We have not attempted any systematic search over which of any other possible multi-indexes one might include -- those with coefficients of -1 instead of +1 on the covariates, for instance.

In any event, if the results for the firms not violating concavity are examined, the same result concerning variability of elasticities emerges. The translog elasticities do not vary much from observation to observation, while the Fourier counterparts do; again, the range for the Fourier estimates for each elasticity of substitution includes the entire range of translog values.

Further evidence of the fact that there may be some redundant parameters can be found in Tables 7 and 8, where the parameter estimates themselves are reported. A large number are not statistically significantly different from zero. Nevertheless, the results are similar to those of Model I, and here again the output, network, and output/network interaction covariates have statistically significant parameter estimates. The 1979 and 1981 estimates for Model IV again do not reveal consistent patterns of the input price ratio effects on costs; however, many of the main effect parameters are statistically significant and they also correspond to parameters on the sine and cosine terms that would not appear in the translog. As already noted, the translog model is rejected as a special case of the Fourier.

A final point concerning curvature restrictions should be made. There is clearly a trade-off between flexibility of the cost function and the number of violations of concavity. Everyone is familiar with the problem that going from a Cobb-Douglas to a translog can cause -- concavity violations become possible. None occurred with our translog results. As we introduced more $u_\alpha$ and $v_\alpha$ parameters, we tended to find more violations. The final column in Table 6 reports the number of firms for which the concavity restriction was violated for all 4 Fourier models. Model II, which relaxes the homotheticity restriction but does not introduce the network variable into share equations, turns out to have 10 more observations consistent with concavity than did Model IV. For 1981, Model II never exhibits a violation, similar to Model I. Model III behaves similarly in 1981, although curvature fares no better in 1979 with that model than with Model

IV.

These observations reveal the trade-off inherent in imposing restrictions on flexible functional forms. When shares are constant, substitution elasticities are the same for all firms and the Cobb-Douglas cost function that is implied is globally concave. When we fit the relatively more flexible translog, there was some variation in elasticities permitted but no violations of concavity resulted. As we added parameters by relaxing restrictions, elasticities became more variable and curvature violations more frequent. Thus, there is a trade-off between good global properties and variability of estimated elasticities. Likelihood ratio tests do not support the translog restriction. These results imply that there is probably an intermediate case, obtainable by deleting some of the Fourier parameters, that would capture both the improved explanatory power of the Fourier and still have fewer violations of the concavity restriction. Of course, this assumes that violations are due to the form estimated, rather than firm behavior; some of the firms tend to exhibit curvature violations more than others, so it may be the individual observations that are responsible and not the functional form itself.

Suppose one does not care much about elasticities of substitution, or that the variability they exhibit in the Fourier is of no particular interest. Would the use of the translog have any other economic implications? Subject to the restrictions on the Fourier cost functions to obtain the translog case, we reexamined the tests between Models I to IV described earlier, to see whether input shares depended only on relative prices, or whether output and network interactions should be retained in share equations. Some interesting reversals of our previous tests occurred.

For 1979, the homotheticity restriction is rejected using either Model II (against Model I) or Model IV (against I or III). However, the network variable does not contribute a significant amount of explanatory power, in that Model I is not rejected as a special case of Model III, nor is Model II rejected as a special case of Model IV. Thus, use of the translog would lead to the erroneous conclusion that a firm's network structure was unnecessary in explaining factor shares. The same results were obtained for 1981.

## 6. CONCLUSIONS

Using two cross sections of firms from the trucking industry, we have tested the translog assumption for cost functions. Almost invariably, that functional form is used to model industry structure. While it is a plausible cost function, and its convenience and familiarity have helped make it widely used, it does not follow from the "local approximation" interpretation that it will not be subject to specification errors. We have made use of the Fourier cost function to test the translog specification; essentially, to see if there remains any correlation with prices in the error terms. We find that Fourier cost functions lead to rejection of the translog restriction, regardless of the specification of output and shifters (i.e. whether or not homotheticity is assumed). As the Fourier cost function is not much harder to estimate than is the translog, the translog assumption should be tested in future applications in this manner.

There seems to be a trade-off between explanatory power and good global properties concerning concavity of the cost function. This is not a new observation, since many studies have reported that some observations violate concavity using a translog or generalized Leontief, for instance. The generalization those forms provided, away from inflexible but globally well-behaved forms, such as the Cobb-Douglas or CES, is sufficient to permit arbitrary values for substitution elasticities at any particular price vector. The results in this paper illustrate that this property does not guarantee that the estimated model is a good approximation to the unknown cost function throughout the range of the data.

If the homotheticity restriction is not maintained, and Model IV is taken as the correct model, imposing curvature restrictions might be desirable. This could be done using inequality constrained estimation and a nonlinear programming method (e.g. Gallant and Golub, 1985) or in a Bayesian fashion (e.g. Chalfant, Gray, and White, 1991). The translog restriction should be retested, of course, once the restrictions are imposed. Our results for Model I and Model II, which exhibit few violations, suggests that the translog will still prove a strong restriction. In any event, this paper has shown that a maintained hypothesis of the translog model, that the residuals are well-behaved, can be rejected for this industry.

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## APPENDIX: STRUCTURE OF THE C MATRIX

The estimation of a Fourier flexible form cost function requires that a set of multi-indexes be chosen to determine the sine and cosine terms that are included in the approximation. No particular model selection rule has been shown to dominate, but selecting the 'low-order' vectors seems sensible and has given good results in the past. The entire set we considered are those multi-indexes of length less than or equal to 3; essentially the same set without the network effects was used by Gallant (1982). In order to keep the number of parameters to be estimated from expanding too rapidly, when interaction effects were included, we required that the output or network variables have a coefficient of +1 in each multi-index. Multi-indexes with a -1 associated with those variables could also have been included.

The 21 multi-indexes that result are given in Table A.1. As shown, there are three multi-indexes ($k_1$ - $k_3$) for the output, network, and output/network interaction covariates; six multi-indexes represent the main effects of input price ratios on cost ($k_4$ - $k_9$); the remaining 12 multi-indexes represent the main effects interacted with output ($k_{10}$ - $k_{15}$) and network ($k_{16}$ - $k_{21}$).

The translog restriction reduces the parameter set in Equation (3) to $u_0$, b, and C. The matrix C contains the (logarithmic) second derivatives of the translog cost function. One way to see that the translog is a restrictive case when applied to more than one data point is to note that the translog requires that the elements in C be constant; $C_{ij}$ does not vary with the level of prices. The addition of the sine and cosine parameters relaxes this assumption, and one could always allow the second derivatives to depend only on those, and include no C matrix. However, in order for the translog to be nested within the Fourier, C must be included.

There is no direct correspondence between A, the number of terms chosen in the Fourier series that augments the translog, and the number of parameters in C. If A is too small, the translog is not nested, and if A is larger than necessary, some of the redundant $u_{0\alpha}$ parameters must be deleted. For the latter case, there is more than one collection of restrictions on the $u_{0\alpha}$'s that would do; the choice is arbitrary. However, all results except for the estimated $u_{0\alpha}$'s themselves are invariant to that choice.

As Devezeaux de Lavergne et al. (1989) note, any choice for the matrix P will suffice:

$$vec\ C = P \cdot U_{0\alpha}, \tag{11}$$

where vec C = $(C_{11},\ldots,C_{1N}, C_{22},\ldots,C_{2N},\ldots,C_{NN})'$ is the vector of the N(N+1)/2 unconstrained coefficients of C and $U_{0\alpha}$ denotes a vector of length A containing all $u_{0\alpha}$ terms. Our particular choice of $u_{0\alpha}$'s was made by taking the multi-indexes in the order they were generated by FORTRAN code (Monahan (1981)) and retaining each $u_{0\alpha}$ in turn if it represented new information in C.

Recall that the vector x is of length 6, containing 4 input prices, output, and the network variable. When homotheticity is imposed on the technology, there are no interactions between the prices and output or the network variable; this restriction can be imposed on the cost function by including only the first 9 multi-indexes. As can be seen from its structure, this implies certain zero restrictions in the matrix C:

$$
\begin{bmatrix}
u_{04}+u_{05}+u_{06} & -u_{04} & -u_{05} & -u_{06} & 0 & 0 \\
-u_{04} & u_{04}+u_{07}+u_{08} & -u_{07} & -u_{08} & 0 & 0 \\
-u_{05} & -u_{07} & u_{05}+u_{07}+u_{09} & -u_{09} & 0 & 0 \\
-u_{06} & -u_{08} & -u_{09} & u_{06}+u_{08}+u_{09} & 0 & 0 \\
0 & 0 & 0 & 0 & u_{01}+u_{03} & u_{03} \\
0 & 0 & 0 & 0 & u_{03} & u_{02}+u_{03}
\end{bmatrix} \quad \text{(A1)}
$$

Only 9 $u_{0\alpha}$ terms are present; denoting by $C_{ij}$ the second partial derivative of the cost function with respect to the $i^{th}$ and $j^{th}$ elements of x, $u_{01}$ and $u_{02}$ contribute to the second derivatives $C_{55}$ and $C_{66}$, while $u_{03}$ measures the additional effect $C_{56}$. Similarly, the coefficient of the square of the first price, $C_{11}$, is equal to the sum of the parameters $u_{04}$, $u_{05}$, and $u_{06}$, each associated with a multi-index that includes the first price.

The interactions between two prices, measured by $C_{12}$, $C_{13}$, etc., are each uniquely determined by the $u_{0\alpha}$ term corresponding to the multi-index that measures such interactions. For instance, $k_4 = (1\ {-1}\ 0\ 0\ 0\ 0)'$, so $u_{04}$ is the derivative $C_{12}$. Finally, notice that terms of the form $C_{ij}$ or $C_{ji}$, where i=1,2,3,4 and j=5,6, are all zero to imply no interactions between prices and output or network. This implies that share equations do not include those variables as "shifters".

*The Nonhomothetic Case:*

When interactions with output are allowed, there are six more multi-indexes included in the model; a large number of parameters appear to be added to the C matrix, but not all are free parameters. Some are redundant and may be set equal to zero. To simplify notation, consider below the matrix $C_y$ that measures the added terms that would appear in C, following the introduction of multi-indexes $k_{10}$ through $k_{15}$, corresponding to interactions with output of the price combinations appearing in $k_4$ through $k_9$.

$$\begin{bmatrix} \{u_{010}+u_{011}+u_{012}\} & -u_{010} & -u_{011} & -u_{012} & \{u_{010}+u_{011}+u_{012}\} & 0 \\ -u_{010} & \{u_{010}+u_{013}+u_{014}\} & -u_{013} & -u_{014} & \{-u_{010}+u_{013}+u_{014}\} & 0 \\ -u_{011} & -u_{013} & \{u_{011}+u_{013}+u_{015}\} & -u_{015} & \{-u_{011}-u_{013}+u_{015}\} & 0 \\ -u_{012} & -u_{014} & -u_{015} & \{u_{012}+u_{014}+u_{015}\} & \{-u_{012}-u_{014}-u_{015}\} & 0 \\ \{u_{010}+u_{011}+u_{012}\} & \{-u_{010}+u_{013}+u_{014}\} & \{-u_{011}-u_{013}+u_{015}\} & \{-u_{012}-u_{014}-u_{015}\} & \{u_{010}+u_{011}+u_{012}+u_{013}+u_{014}+u_{015}\} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (A2)$$

Proceeding down the 5th column of the matrix, first with element $C_{15}$, note first that $u_{010}$ contributes new information, namely, the interaction between the first price and output. There is no need to have $u_{011}$ or $u_{012}$ for the same interaction, making the latter two parameters redundant. These are zeroed out. Next, for $C_{25}$, along with $-u_{010}$, the new parameter $u_{013}$ appears, also representing new information. $u_{014}$ is redundant and is set equal to zero. Finally, $u_{015}$ is needed for $C_{35}$. By homogeneity of the cost function in input prices, those 4 effects must sum to zero, which is accomplished in row 4, with element $C_{45}$. Symmetry accounts for the first 4 elements of the 5th row. For $C_{55}$, the $u_{0\alpha}$'s that were added for column 5 appear accordingly.

Thus, only three of the parameters are necessary to incorporate into C; these enter in the following way:

$$\begin{bmatrix} u_{010} & -u_{010} & 0 & 0 & u_{010} & 0 \\ -u_{010} & u_{010}+u_{013} & -u_{013} & 0 & -u_{010}+u_{013} & 0 \\ 0 & -u_{013} & u_{013}+u_{015} & -u_{015} & -u_{013}+u_{015} & 0 \\ 0 & 0 & -u_{015} & u_{015} & -u_{015} & 0 \\ u_{010} & -u_{010}+u_{013} & -u_{013}+u_{015} & -u_{015} & u_{010}+u_{013}+u_{015} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A3)}$$

A similar number of interaction effects are added to the model when price-network interactions are also permitted:

$$\begin{bmatrix} u_{016} & -u_{016} & 0 & 0 & u_{016} & 0 \\ -u_{016} & u_{016}+u_{020} & -u_{020} & 0 & -u_{016}+u_{020} & 0 \\ 0 & -u_{020} & u_{020}+u_{021} & -u_{021} & -u_{020}+u_{021} & 0 \\ 0 & 0 & -u_{021} & u_{021} & -u_{021} & 0 \\ u_{016} & -u_{016}+u_{020} & -u_{020}+u_{021} & -u_{021} & u_{016}+u_{020}+u_{021} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A4)}$$

# Table A.1

Multi-Indexes Included in the Fourier Cost Function

| | Covariates | | | Main Effects of Price Ratios | | | | | | Interactions with Output | | | | | | Interactions with Network | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $X_i$ $k_\alpha$: | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | $k_7$ | $k_8$ | $k_9$ | $k_{10}$ | $k_{11}$ | $k_{12}$ | $k_{13}$ | $k_{14}$ | $k_{15}$ | $k_{16}$ | $k_{17}$ | $k_{18}$ | $k_{19}$ | $k_{20}$ | $k_{21}$ |
| P fuel | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| P labor | 0 | 0 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 |
| P veh. cap. | 0 | 0 | 0 | 0 | -1 | 0 | -1 | 0 | 1 | 0 | -1 | 0 | -1 | 0 | 1 | 0 | -1 | 0 | -1 | 0 | 1 |
| P term. cap. | 0 | 0 | 0 | 0 | 0 | -1 | 0 | -1 | -1 | 0 | 0 | -1 | 0 | -1 | -1 | 0 | 0 | -1 | 0 | -1 | -1 |
| Output | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Network | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

## Table 1: Parameter Estimates–Fourier Cost Function, 1979 Model I

| Description | Parameter | Estimate | Std. Error |
|---|---|---|---|
| Intercept | $u_0$ | 12.6291* | 0.6871 |
| | $b_1$ | 0.0965* | 0.0245 |
| | $b_2$ | 0.6520* | 0.0209 |
| b'x | $b_3$ | 0.0978* | 0.0186 |
| | $b_4$ | | |
| | $b_Q$ | 1.2528* | 0.6750 |
| | $b_{net}$ | -0.6517 | 0.6738 |
| | $u_{01}$ | 0.2284 | 0.1744 |
| | $u_1$ | 0.0554 | 0.1353 |
| | $v_1$ | 0.2003* | 0.0852 |
| | $u_{02}$ | -0.0600 | 0.1506 |
| Covariates | $u_2$ | -0.0383 | 0.1437 |
| | $v_2$ | -0.2926* | 0.0994 |
| | $u_{03}$ | -0.0708* | 0.0294 |
| | $u_3$ | -0.0417 | 0.0462 |
| | $v_3$ | -0.1156* | 0.0393 |
| | $u_{04}$ | 0.0012 | 0.0110 |
| | $u_4$ | -0.0043 | 0.0052 |
| | $v_4$ | -0.0077 | 0.0052 |
| | $u_{05}$ | -0.0304* | 0.0110 |
| | $u_5$ | 0.0089* | 0.0032 |
| Main Effects | $v_5$ | 0.0166* | 0.0063 |
| of | $u_{06}$ | 0.0014 | 0.0014 |
| Price Ratios | $u_6$ | -0.0007 | 0.0011 |
| | $v_6$ | -0.0029* | 0.0012 |
| | $u_{07}$ | 0.0142 | 0.0118 |
| | $u_7$ | -0.0119* | 0.0066 |
| | $v_7$ | -0.0171* | 0.0047 |
| | $u_{08}$ | -0.0090* | 0.0046 |
| | $u_8$ | 0.0053 | 0.0036 |
| | $v_8$ | -0.0046 | 0.0042 |

*Continued on next page*

**Table 1** *(continued)*

| Description | Parameter | Estimate | Std. Error |
|---|---|---|---|
| Main Effects | $u_{09}$ | 0.0007 | 0.0019 |
| of | $u_9$ | -0.0030* | 0.0016 |
| Price Ratios | $v_9$ | -0.0016 | 0.0015 |

## Table 2: Parameter Estimates--Fourier Cost Function, 1981 Model I

| Description | Parameter | Estimate | Std. Error |
|---|---|---|---|
| Intercept | $u_0$ | 10.1750* | 0.8563 |
| b'x | $b_1$ | 0.0671* | 0.0351 |
| | $b_2$ | 0.6504* | 0.0352 |
| | $b_3$ | 0.1240* | 0.0110 |
| | $b_4$ | | |
| | $b_Q$ | 2.2902* | 0.8304 |
| | $b_{net}$ | 0.3104 | 0.8151 |
| Covariates | $u_{01}$ | 0.7610* | 0.2531 |
| | $u_1$ | 0.4024* | 0.1868 |
| | $v_1$ | -0.0863 | 0.1043 |
| | $u_{02}$ | 1.0055* | 0.2855 |
| | $u_2$ | 0.1456 | 0.1769 |
| | $v_2$ | 0.2361 | 0.1791 |
| | $u_{03}$ | -0.3472* | 0.0765 |
| | $u_3$ | 0.0404 | 0.0578 |
| | $v_3$ | 0.2312* | 0.0739 |
| Main Effects of Price Ratios | $u_{04}$ | 0.0436 | 0.0447 |
| | $u_4$ | -0.0361 | 0.0227 |
| | $v_4$ | 0.0066 | 0.0115 |
| | $u_{05}$ | 0.0111 | 0.0137 |
| | $u_5$ | -0.0053 | 0.0075 |
| | $v_5$ | -0.0027 | 0.0047 |
| | $u_{06}$ | 0.0026 | 0.0022 |
| | $u_6$ | -0.0027* | 0.0014 |
| | $v_6$ | -0.0017 | 0.0023 |
| | $u_{07}$ | -0.0248 | 0.0182 |
| | $u_7$ | 0.0078 | 0.0113 |
| | $v_7$ | -0.0099* | 0.0037 |
| | $u_{08}$ | -0.0183* | 0.0066 |
| | $u_8$ | 0.0001 | 0.0047 |
| | $v_8$ | 0.0070 | 0.0056 |

*Continued on next page*

**Table 2** *(continued)*

| Description | Parameter | Estimate | Std. Error |
|---|---|---|---|
| Main Effects | $u_{09}$ | 0.0019 | 0.0019 |
| of | $u_9$ | -0.0019 | 0.0016 |
| Price Ratios | $v_9$ | -0.0027 | 0.0017 |

**Table 3: Elasticities of Substitution**
**Model I: Homothetic Model**
**1979 Data**

| Substitution Elasticity | Translog Mean | Translog Min. | Translog Max. | Fourier Mean | Fourier Min. | Fourier Max |
|---|---|---|---|---|---|---|
| $\sigma_{11}$ | -9.09 | -9.67 | -7.21 | -6.90 | -13.96 | -0.96 |
| $\sigma_{12}$ | 0.40 | 0.15 | 0.55 | 0.21 | -0.03 | 1.30 |
| $\sigma_{13}$ | 1.55 | 1.26 | 2.14 | 1.96 | -1.49 | 4.18 |
| $\sigma_{14}$ | 0.68 | 0.42 | 0.81 | 0.53 | -0.39 | 2.00 |
| $\sigma_{22}$ | -0.31 | -0.46 | -0.20 | -0.30 | -0.59 | -0.20 |
| $\sigma_{23}$ | 0.57 | 0.34 | 0.64 | 0.51 | 0.18 | 1.31 |
| $\sigma_{24}$ | 0.81 | 0.78 | 0.86 | 0.94 | 0.58 | 1.12 |
| $\sigma_{33}$ | -5.35 | -7.07 | -4.13 | -5.19 | -10.41 | -1.71 |
| $\sigma_{34}$ | 1.06 | 1.04 | 1.09 | 0.76 | 0.00 | 1.84 |
| $\sigma_{44}$ | -5.16 | -6.19 | -3.43 | -5.67 | -8.03 | -2.63 |

NOTE: Fourier mean values include the 48 (of 60) firms for which the concavity restriction was satisfied.

Table 4: Elasticities of Substitution
Model I: Homothetic Model
1981 Data

| | Translog | | | Fourier | | |
|---|---|---|---|---|---|---|
| Substitution Elasticity | Mean | Min. | Max. | Mean | Min. | Max |
| $\sigma_{11}$ | -6.63 | -7.15 | -5.33 | -4.65 | -27.57 | -1.02 |
| $\sigma_{12}$ | 0.19 | -0.13 | 0.35 | 0.10 | -0.24 | 2.30 |
| $\sigma_{13}$ | 2.21 | 1.60 | 3.06 | 1.40 | 0.72 | 4.13 |
| $\sigma_{14}$ | 1.09 | 1.05 | 1.14 | 0.90 | 0.26 | 1.99 |
| $\sigma_{22}$ | -0.29 | -0.48 | -0.18 | -0.26 | -0.64 | -0.08 |
| $\sigma_{23}$ | 0.42 | 0.16 | 0.52 | 0.49 | -0.12 | 1.01 |
| $\sigma_{24}$ | 0.72 | 0.67 | 0.79 | 0.64 | 0.16 | 0.93 |
| $\sigma_{33}$ | -6.23 | -7.85 | -4.85 | -6.36 | -12.42 | -1.21 |
| $\sigma_{34}$ | 1.16 | 1.09 | 1.25 | 1.15 | 0.38 | 2.39 |
| $\sigma_{44}$ | -4.04 | -5.05 | -2.77 | -3.53 | -4.43 | -2.52 |

NOTE: Both translog and Fourier mean values include all 60 firms, since the concavity restriction was always satisfied.

## Table 5: Tests of Restrictions for the Fourier Cost Functions

1979

| Restricted Model | Unrestricted Model | $\chi^2$ | #d.f. | Reject? |
|---|---|---|---|---|
| II | IV | 33.00 | 15 | Yes |
| III | IV | 33.21 | 15 | Yes |
| I | IV | 60.95 | 30 | Yes |
| I | II | 27.94 | 15 | Yes |
| I | III | 28.02 | 15 | Yes |

1981

| Restricted Model | Unrestricted Model | $\chi^2$ | #d.f. | Reject? |
|---|---|---|---|---|
| II | IV | 50.17 | 15 | Yes |
| III | IV | 25.57 | 15 | Yes |
| I | IV | 50.17 | 30 | Yes |
| I | II | 22.29 | 15 | No |
| I | III | 22.60 | 15 | No |

NOTE: $\chi^2_{15,.05} = 25.0$, $\chi^2_{30,.05} = 43.77$.

Table 6: Tests of the Translog Restriction

1979

| Model | $\chi^2$ | #d.f. | Reject? | # Consistent with Concavity |
|---|---|---|---|---|
| I | 55.41 | 18 | Yes | 48 |
| II | 73.26 | 30 | Yes | 41 |
| III | 81.33 | 30 | Yes | 31 |
| IV | 101.42 | 42 | Yes | 31 |

1981

| Model | $\chi^2$ | #d.f. | Reject? | # Consistent with Concavity |
|---|---|---|---|---|
| I | 34.62 | 18 | Yes | 60 |
| II | 43.60 | 30 | Yes | 60 |
| III | 54.44 | 30 | Yes | 56 |
| IV | 67.06 | 42 | Yes | 47 |

NOTE: $\chi^2_{18,\,.05} = 28.87$, $\chi^2_{30,\,.05} = 43.77$, $\chi^2_{42,\,.05} = 58.12$.

**Table 7: Parameter Estimates–Fourier Cost Function, 1979 Model IV**

| Description | Parameter | Estimate | Std. Error |
|---|---|---|---|
| Intercept | $u_0$ | 12.6449* | 0.6727 |
| | $b_1$ | 0.1255* | 0.0369 |
| | $b_2$ | 0.6225* | 0.0294 |
| b'x | $b_3$ | 0.0709* | 0.0305 |
| | $b_4$ | | |
| | $b_Q$ | 1.2354* | 0.6601 |
| | $b_{net}$ | -0.5850 | 0.6685 |
| | $u_{01}$ | 0.2592 | 0.1719 |
| | $u_1$ | 0.1147 | 0.1373 |
| | $v_1$ | 0.2428* | 0.0953 |
| | $u_{02}$ | -0.0643 | 0.1495 |
| Covariates | $u_2$ | -0.0089 | 0.1464 |
| | $v_2$ | -0.3380* | 0.1046 |
| | $u_{03}$ | -0.0652* | 0.0306 |
| | $u_3$ | -0.0491 | 0.0458 |
| | $v_3$ | -0.0913* | 0.0408 |
| | $u_{04}$ | 0.0158 | 0.0115 |
| | $u_4$ | -0.0157* | 0.0055 |
| | $v_4$ | -0.0089 | 0.0053 |
| | $u_{05}$ | -0.0483* | 0.0143 |
| | $u_5$ | 0.0123* | 0.0039 |
| Main Effects | $v_5$ | 0.0282* | 0.0083 |
| of | $u_{06}$ | -0.0036* | 0.0018 |
| Price Ratios | $u_6$ | 0.0019 | 0.0015 |
| | $v_6$ | -0.0013 | 0.0014 |
| | $u_{07}$ | 0.0370* | 0.0155 |
| | $u_7$ | -0.0168* | 0.0074 |
| | $v_7$ | -0.0209* | 0.0053 |
| | $u_{08}$ | -0.0267* | 0.0074 |
| | $u_8$ | 0.0001 | 0.0037 |
| | $v_8$ | -0.0018 | 0.0047 |

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Table 7 *(continued)*

| Description | Parameter | Estimate | Std. Error |
|---|---|---|---|
| Main Effects | $u_{09}$ | 0.0144* | 0.0051 |
| of | $u_9$ | -0.0028* | 0.0016 |
| Price Ratios | $v_9$ | -0.0026 | 0.0017 |
| | $u_{010}$ | -0.0065* | 0.0023 |
| | $u_{10}$ | -0.0010 | 0.0018 |
| | $v_{10}$ | -0.0044* | 0.0020 |
| | $u_{11}$ | -0.0040* | 0.0015 |
| | $v_{11}$ | 0.0004 | 0.0018 |
| | $u_{12}$ | 0.0009 | 0.0010 |
| | $v_{12}$ | -0.0018 | 0.0014 |
| Interactions | $u_{013}$ | -0.0158* | 0.0048 |
| with | $u_{13}$ | 0.0017 | 0.0024 |
| Output | $v_{13}$ | -0.0011 | 0.0022 |
| | $u_{14}$ | -0.0001 | 0.0030 |
| | $v_{14}$ | -0.0009 | 0.0037 |
| | $u_{015}$ | -0.0173* | 0.0038 |
| | $u_{15}$ | -0.0015 | 0.0013 |
| | $v_{15}$ | 0.0006 | 0.0011 |
| | $u_{016}$ | 0.0023 | 0.0015 |
| | $u_{16}$ | 0.0027 | 0.0021 |
| | $v_{16}$ | -0.0029 | 0.0022 |
| | $u_{17}$ | -0.0046* | 0.0018 |
| | $v_{17}$ | 0.0048* | 0.0017 |
| | $u_{18}$ | 0.0002 | 0.0011 |
| | $v_{18}$ | 0.0003 | 0.0014 |
| Interactions | $u_{19}$ | -0.0008 | 0.0025 |
| with | $v_{19}$ | -0.0086* | 0.0022 |
| Network | $u_{020}$ | 0.0111* | 0.0043 |
| | $u_{20}$ | 0.0057 | 0.0037 |
| | $v_{20}$ | -0.0028 | 0.0032 |
| | $u_{021}$ | 0.0014 | 0.0019 |
| | $u_{21}$ | 0.0017 | 0.0012 |
| | $v_{21}$ | -0.0003 | 0.0013 |

**Table 8: Parameter Estimates--Fourier Cost Function, 1981 Model IV**

| Description | Parameter | Estimate | Std. Error |
|---|---|---|---|
| Intercept | $u_0$ | 9.9042* | 0.8596 |
| | $b_1$ | 0.0697* | 0.0381 |
| | $b_2$ | 0.6054* | 0.0429 |
| b'x | $b_3$ | 0.1296* | 0.0154 |
| | $b_4$ | | |
| | $b_Q$ | 1.9934* | 0.8204 |
| | $b_{net}$ | 0.8952 | 0.8433 |
| | $u_{01}$ | 0.7063* | 0.2523 |
| | $u_1$ | 0.3570* | 0.1899 |
| | $v_1$ | -0.1166 | 0.1106 |
| | $u_{02}$ | 1.1405* | 0.3021 |
| Covariates | $u_2$ | 0.2914 | 0.1854 |
| | $v_2$ | 0.2771 | 0.1906 |
| | $u_{03}$ | -0.3440* | 0.0791 |
| | $u_3$ | 0.0591 | 0.0606 |
| | $v_3$ | 0.2271* | 0.0741 |
| | $u_{04}$ | 0.0156 | 0.0469 |
| | $u_4$ | -0.0223 | 0.0240 |
| | $v_4$ | 0.0031 | 0.0114 |
| | $u_{05}$ | 0.0126 | 0.0142 |
| | $u_5$ | -0.0049 | 0.0071 |
| Main Effects | $v_5$ | -0.0028 | 0.0046 |
| of | $u_{06}$ | -0.0036 | 0.0033 |
| Price Ratios | $u_6$ | 0.0014 | 0.0022 |
| | $v_6$ | -0.0022 | 0.0030 |
| | $u_{07}$ | -0.0078 | 0.0232 |
| | $u_7$ | 0.0042 | 0.0132 |
| | $v_7$ | -0.0093* | 0.0041 |
| | $u_{08}$ | -0.0200 | 0.0121 |
| | $u_8$ | -0.0102 | 0.0068 |
| | $v_8$ | 0.0090 | 0.0065 |

*Continued on next page*

## Table 8 *(continued)*

| Description | Parameter | Estimate | Std. Error |
|---|---|---|---|
| Main Effects of Price Ratios | $u_{09}$ | 0.0085 | 0.0069 |
| | $u_9$ | -0.0002 | 0.0028 |
| | $v_9$ | -0.0027 | 0.0023 |
| Interactions with Output | $u_{010}$ | -0.0080* | 0.0033 |
| | $u_{10}$ | 0.0011 | 0.0024 |
| | $v_{10}$ | 0.0010 | 0.0022 |
| | $u_{11}$ | -0.0018 | 0.0019 |
| | $v_{11}$ | -0.0016 | 0.0019 |
| | $u_{12}$ | -0.0023 | 0.0016 |
| | $v_{12}$ | 0.0001 | 0.0017 |
| | $u_{013}$ | -0.0110* | 0.0062 |
| | $u_{13}$ | 0.0054* | 0.0029 |
| | $v_{13}$ | 0.0013 | 0.0026 |
| | $u_{14}$ | 0.0074* | 0.0040 |
| | $v_{14}$ | 0.0005 | 0.0042 |
| | $u_{015}$ | -0.0151* | 0.0053 |
| | $u_{15}$ | -0.0006 | 0.0016 |
| | $v_{15}$ | -0.0007 | 0.0017 |
| Interactions with Network | $u_{016}$ | 0.0050* | 0.0027 |
| | $u_{16}$ | 0.0067* | 0.0030 |
| | $v_{16}$ | 0.0002 | 0.0035 |
| | $u_{17}$ | -0.0031 | 0.0029 |
| | $v_{17}$ | -0.0013 | 0.0024 |
| | $u_{18}$ | 0.0000 | 0.0022 |
| | $v_{18}$ | 0.0046* | 0.0022 |
| | $u_{19}$ | -0.0013 | 0.0033 |
| | $v_{19}$ | -0.0052 | 0.0035 |
| | $u_{020}$ | 0.0046 | 0.0059 |
| | $u_{20}$ | 0.0054 | 0.0051 |
| | $v_{20}$ | -0.0098* | 0.0050 |
| | $u_{021}$ | 0.0052* | 0.0028 |
| | $u_{21}$ | -0.0007 | 0.0025 |
| | $v_{21}$ | 0.0017 | 0.0020 |

Table 9: Elasticities of Substitution
Model IV: Output and Network Effects
1979 Data

| | Translog | | | Fourier | | |
|---|---|---|---|---|---|---|
| Substitution Elasticity | Mean | Min. | Max. | Mean | Min. | Max |
| $\sigma_{11}$ | -8.70 | -9.10 | -7.05 | -9.41 | -26.37 | -1.72 |
| $\sigma_{12}$ | 0.44 | 0.20 | 0.58 | 0.48 | -0.13 | 1.57 |
| $\sigma_{13}$ | 1.42 | 1.20 | 1.87 | 1.27 | -3.70 | 4.78 |
| $\sigma_{14}$ | 0.43 | -0.05 | 0.66 | 0.64 | -1.26 | 2.17 |
| $\sigma_{22}$ | -0.29 | -0.43 | -0.18 | -0.29 | -0.63 | -0.14 |
| $\sigma_{23}$ | 0.57 | 0.34 | 0.64 | 0.70 | -0.12 | 1.63 |
| $\sigma_{24}$ | 0.69 | 0.65 | 0.78 | 0.70 | 0.40 | 1.01 |
| $\sigma_{33}$ | -5.23 | -6.80 | -4.07 | -6.12 | -13.27 | -2.40 |
| $\sigma_{34}$ | 1.03 | 1.02 | 1.05 | 0.81 | -0.48 | 1.81 |
| $\sigma_{44}$ | -4.43 | -5.15 | -3.10 | -4.71 | -7.35 | -2.59 |

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NOTE: Fourier mean values include the 31 (of 60) firms for which the concavity restriction was satisfied.

## Table 10: Elasticities of Substitution
## Model IV: Output and Network Included
## 1981 Data

| | Translog | | | Fourier | | |
|---|---|---|---|---|---|---|
| Substitution Elasticity | Mean | Min. | Max. | Mean | Min. | Max |
| $\sigma_{11}$ | -6.15 | -6.44 | -5.11 | -6.04 | -17.53 | -0.77 |
| $\sigma_{12}$ | 0.24 | -0.06 | 0.39 | 0.46 | -0.14 | 1.88 |
| $\sigma_{13}$ | 1.86 | 1.43 | 2.47 | 1.21 | -2.04 | 4.86 |
| $\sigma_{14}$ | 0.88 | 0.81 | 0.93 | 0.13 | -1.03 | 1.30 |
| $\sigma_{22}$ | -0.26 | -0.43 | -0.15 | -0.32 | -0.68 | -0.10 |
| $\sigma_{23}$ | 0.42 | 0.15 | 0.51 | 0.58 | -0.29 | 1.41 |
| $\sigma_{24}$ | 0.58 | 0.51 | 0.69 | 0.64 | -0.03 | 1.07 |
| $\sigma_{33}$ | -5.92 | -7.26 | -4.69 | -6.26 | -14.85 | -1.19 |
| $\sigma_{34}$ | 1.16 | 1.10 | 1.26 | 0.73 | -0.04 | 1.82 |
| $\sigma_{44}$ | -3.38 | -4.02 | -2.45 | -2.79 | -4.90 | -0.75 |

NOTE: Fourier mean values include the 47 (of 60) firms for which the concavity restriction was satisfied.