Computing the Socially Optimal Forest Stock for the Ivory Coast

Simeon K. Ehui and Thomas W. Hertel[1]

Abstract: A two-sector dynamic model is used to determine the optimal steady-state forest stock for the Ivory Coast. The optimal steady-state forest stock is shown to be most sensitive to changes in the discount rate and the expected technological change. When agricultural technology is assumed to be stagnant, the forest stock is not completely exhausted in the optimal steady-state situation. However, with continual technological change, eventually clearing all the forest lands is optimal.

Introduction

In spite of widespread concern about deforestation in the tropics, little formal analysis of the socially optimal allocation of land between forest and agricultural use is available. This is compounded by a lack of knowledge about the relationship between deforestation, erosion, and productivity in tropical soils. This paper uses an optimal control model to estimate the optimal steady-state forest stock for the Ivory Coast, which currently has the highest rate of deforestation in the world (300,000 ha or 6.5 percent per year) (Bertrand, 1983; OTA, 1984; Allen and Barnes, 1985; and Bene *et al.*, 1977). By "mining" its forest resources, the Ivory Coast has achieved the fastest agricultural growth rate (5 percent per year) in sub-Saharan Africa (Spears, 1986).

The theoretical framework for this paper is outlined in the next section. It is followed by the specification of a quadratic agricultural yield function. The optimal steady-state forest stock is derived, and comparative static results regarding the impacts of prices, the social rate of discount, and technology are developed. This is followed by numerical estimates of optimal steady-state forest stocks for the Ivory Coast under a variety of scenarios. The paper closes with a summary and some concluding qualifications and comments.

Analytical Framework

This section presents a theoretical model for optimal control of forest lands in the tropics. The social objective is assumed to be to maximize the utility derived from aggregate profit, subject to changes in forest stocks over time. Both forested and deforested lands are considered as a source of future profits, and this relationship is a nonlinear one that highlights the interplay between deforestation and agricultural productivity.

Formally, the control problem, over an infinite horizon, is stated as follows:

$$(1)\ \max_{F_t, D_t} W = \int_0^\infty e^{-\delta t}[V(\pi_t)]dt, \text{ subject to:}$$

$$(2)\ \pi_t = P_{Ft}F_t + (L-F_t)(P_{At})Z(D_t,\ F_0-F_t,\ X_t) - P_{Xt}X_t,$$

$$(3)\ \dot{F}_t = -D_t = 0, \text{ if } F_t = 0,$$

$$(4)\ F_t,\ D_t,\ X_t \geq 0, \text{ and}$$

$$(5)\ F_0 = \bar{F}_0,\ L = \bar{L}.$$

Here, W is a measure of the present value of society's welfare; δ is the social rate of discount and thus provides an indication of how future-generation welfare is discounted; $V(\cdot)$ is a twice differentiable instantaneous utility function depending on aggregate profit, π_t;

profit is equal to the sum of net returns in both agriculture and forestry [equation *(2)*]; L represents total arable land; and F_t represents the land area covered by forest at time t (i.e., the forest "stock," in hectares). Thus L-F_t represents total land area devoted to agriculture at time t.

$Z(\cdot)$ is a concave aggregate agricultural yield function and is assumed to depend on current period purchased inputs, X_t, the current rate of deforestation, D_t, and the cumulative amount of deforested lands, F_0-F_t. Average yield is assumed to be increasing in purchased inputs and current period deforestation. The latter effect is attributable to the nutrient content of the ash left after burning the forests (Cordero, 1984; and Sanchez, 1981). Yield is assumed to decline with increases in cumulative deforestation. This is because of productivity losses due to increased erosion and leaching of nutrients (Sanchez, 1981; and Lal, 1981 and 1985).

The variables P_{A_t} and P_{F_t} are used to denote per-kilogram returns to agriculture and per-hectare returns to forestry at time t. P_{X_t} is the per-kilogram cost of purchased inputs. These are assumed to be exogenously determined in the international marketplace. Constraint *(3)* describes the changes in forest "stock" over time. It also dictates that if, over some interval, F_t equals zero, then the rate of deforestation must be constrained so that $\dot{F}_t$ equals zero over that interval as well.[2] Constraint *(4)* is the nonnegativity conditions on D_t, F_t, and X_t. Finally, constraint *(5)* defines the initial endowment of forest lands and the total arable land available at time equals zero.

Defining $V[\pi(X, D, F)]$ as equal to $U(X, D, F)$, the current value Hamiltonian associated with the control problem described by *(1)*-*(5)* is given by:[3]

$$(6)\ H(D, F, X, 4) = U(D, F, X) - \psi D,$$

where ψ is the current value costate variable associated with the equation of motion *(3)*. Assuming an interior solution the maximum principle requires that the following hold:

$$(7)\ 0 = U_X = U_\pi(L\text{-}F)(P_A Z_X\text{-}P_X),$$

$$(8)\ \psi = U_D = U_\pi[P_A Z_D(L\text{-}F)],$$

$$(9)\ \delta\psi - \dot{\psi} = U_F = U_\pi[P_F + P_A Z_F(L\text{-}F)\text{-}P_A Z + P_X X], \text{ and}$$

$$(10)\ \lim_{t\to\infty} e^{-\delta t}\, \psi_t F_t = 0.$$

Equation *(7)* indicates that, at the optimum, purchased inputs are applied up to the point where their marginal utility (or profitability) is zero. Equation *(8)* indicates that, at any point in time, the rate of deforestation should be chosen so that the marginal utility of deforestation (U_D) is equal to the efficiency price of the forest stock (ψ). Here, ψ measures the future benefit foregone by a decision to deforest today.

Equation *(9)* implies that forest stock services should be employed up to the point where the marginal utility of forest capital is equal to the social cost of this capital. The right-hand side of *(9)* represents the marginal utility of forest stock. It is composed of two parts, the direct marginal contribution of forestry ($U_\pi P_F$) and the indirect marginal contribution of the forest stock through its effect on agricultural productivity. The latter has two components. The first, $U_\pi P_A Z_F(L\text{-}F)$, captures enhanced agricultural yields due to increased forest cover. The second component, $U_\pi(-P_A Z + P_X X)$, measures the net cost of not having an additional unit of land in agriculture. The left-hand side of *(9)* measures the cost of employing the services of one unit of the forest capital at any point in time. It includes both an interest charge ($\delta\psi$) and a capital gains term ($-\dot{\psi}$). Finally, equation *(10)* is the transversality condition.

Totally differentiating *(8)* with respect to time and combining this result with *(9)* and *(7)* yields an expression for the time rate of change in the rate of deforestation along the optimal path:

$$(11)\ \dot{D} = [(U_{XX})/(U_{XX}U_{DD}-U_{DX}^2)]\{-\delta U_D+[U_F-D(U_{DF}-\alpha U_{XF})]\},$$

where α equals U_{DX}/U_{XX}. The sign of $\dot{D}$ along the optimal path is determined by the following condition:

$$(12)\ \dot{D} \gtreqless 0, \text{ as } \delta^{-1}[U_F-D(U_{DF}-\alpha U_{XF})] \gtreqless U_D.$$

Recall that U_D is the marginal utility of deforestation in current periods. A large value of U_D indicates a large agricultural yield response from current period deforestation. This in turn translates into a higher marginal utility due to increased profit. This is a one-time only effect and may be identified as the deforestation motive.

The term in brackets ($[\cdot]$) represents the difference between the marginal contribution of forest area to utility and any indirect interactions between the forest stock, the productivity of purchased inputs, and the rate of deforestation. Along the optimal path, this may be defined as the *net* marginal utility of forests. The latter lingers into perpetuity and has a present value equal to $\delta^{-1}[\cdot]$. This term can be described as the conservation motive. This stems from valuing forest lands not only for their potential agricultural productivity but also as a source of future income (increased income from forestry plus increased agricultural yields from preventing erosion). Thus, condition *(12)* states that the rate of deforestation falls over time[4] if the conservation motive is weaker than the preference for current deforestation.

In the steady state, the net deforestation rate is necessarily zero. Setting $\dot{F}$ equal to $\dot{D}$ equal to zero in equation *(11)*, a steady-state forest stock (F^*) exists that is uniquely defined by:

$$(13)\ (1/\delta)[U_F(D^*, F^*, X^*)] = U_D(D^*, F^*, X^*), \text{ and}$$

$$(14)\ D^* = 0.$$

The left-hand side of equation *(13)* can be described as the present value of the stream of marginal utility derived from sustainable economic rents. The right-hand side is the marginal utility of current deforestation. Thus, equation *(13)* asserts that, in the steady state, the marginal utility of further deforestation (U_D) must equal the present value of the foregone marginal future benefit $[(1/\delta)/U_F]$.

Specification of the Yield Function and Steady-State Implications

Equation *(13)* is implicit in F^*. In order actually to solve for the steady-state forest stock, a parametric form for the aggregate yield function is necessary. Since second-order derivatives of the yield function are key to the analysis, a quadratic functional form was chosen:

$$(15)\ Z_t = \beta_0 + \beta_1 X_t + \beta_2 D_t + \beta_3(F_0-F_t) + \beta_4 TR + (\beta_{11}X_t^2)/2 + (\beta_{22}D_t^2)/2 + \beta_{12}D_tX_t.$$

Note that *(15)* is not a complete second-order approximation to $Z(X,D,F)$. Important interaction terms are chosen based on agronomic evidence. A fairly short time series of data is also a constraint. As a result, measurement of the interaction effects between forest stock and the variables X_t and D_t was not attempted. Also, the squared term $(F_0-F_t)^2$ was omitted.[5] The interaction term between X_t and D_t is included because the current period deforestation is analogous to a good dose of fertilizer (Sanchez, 1981).

Based on assumptions about the yield function, the following signs are expected for the parameters in *(15)*: β_0, β_1, $\beta_2 \geq 0$; β_3, β_{11}, β_{22}, and $\beta_{12} \leq 0$. In addition, for nonzero values of D_t and X_t, the following are expected: $(\beta_1+\beta_{11}X_t+\beta_{12}D_t) \geq 0$ and $(\beta_2+\beta_{12}X_t+\beta_{22}D_t) \geq 0$. A time trend, TR, is used as a proxy for technological change. The associated coefficient (β_4) is expected to be positive.

Steady-State Comparative Statics

Using equations *(13)*, *(14)*, and *(15)*, one can now solve analytically for the optimal steady-state forest stock level. Assuming that the discount rate is positive and bounded, the expression for the steady-state forest stock is given by the following:

$(16)\ F^* = \bar{F}_0 + \Delta/\mu + (\mu-\beta_3)A/\mu$, where:

$(17)\ \Delta = \{[\beta_0+\beta_1 X^*+(\beta_{11}X^{*2}/2)+\beta_4 TR^*]-\bar{P}_X X^*\} - \bar{P}_F$, and

$(18)\ \mu = \delta(\beta_2+\beta_{12}X^*) + 2\beta_3$.

The level of X^* is determined by equation *(7)*, and is a function of the price of fertilizer relative to food: X^* equals $\beta_{11}^{\prime}(P_X-\beta_1)$. TR^* represents the level of technology expected in steady state. The parameter A in *(16)* equals $\bar{L}$ minus $\bar{F}_0$, and this denotes the amount of arable land not under forest cover at time t equals zero. $\bar{P}_X$ and $\bar{P}_F$ are the price of purchased inputs and per-hectare forest returns, relative to the price of agricultural output; i.e., $\bar{P}_X$ equals P_X/P_A, and $\bar{P}_F$ equals P_X/P_A.

The partial derivatives of F^* with respect to P_X and P_F are positive:

$(19)\ \partial F^*/\partial\bar{P}_X = -X^*/\mu > 0,\ and\ \partial F^*/\partial\bar{P}_F = -1/\mu > 0.$

That is, an increase in the relative profitability of forestry, brought about either by an increase in agricultural costs or by an increase in forestry returns, leads to an increase in F^*.[6]

Steady-state comparative static results may also be obtained for changes in the social discount rate:

$(20)\ \partial F^*/\partial\delta = [(\beta_3 A-\Delta)/\mu^2](\beta_2+\beta_{12}X^*) < 0,\ for\ \beta_3 A - \Delta < 0.$

Equation *(20)* indicates that a higher social discount rate lowers F^* as long as modified agricultural returns exceed per-hectare returns for forestry by less than the value of the stock effect on arable land available at t equals zero.[7]

Finally, the effect of the expected level of technology on steady-state forest stock level can be assessed. Partial differentiation of F^* with respect to TR^* yields:

$(21)\ \partial F^*/\partial TR^* = \beta_4/\mu < 0.$

This result indicates that the higher the expected level of technology, the lower the steady-state forest stock. This is because technological progress raises returns to agriculture. In other words, improved technology can offset the loss in productivity due to leaching of nutrients and erosion associated with diminished forest cover.

Empirical Results

Ehui (1987) reports results from estimation of an aggregate agricultural yield function for the Ivory Coast. His estimated equation is repeated here (t-statistics in parentheses):

$$(22)\quad Z_t = \underset{(2.863)}{109.713} + \underset{(3.952)}{9.92\, X_t} + \underset{(2.606)}{0.36\, D_t} - \underset{(-3.202)}{0.03\, (F_0 - F_t)} - \underset{(-4.024)}{0.322\, (X_t^2/2)} - \underset{(-2.826)}{0.00074\, (D_t^2/2)}$$

$$- \underset{(-1.358)}{0.0037\, X_t D_t} + \underset{(3.57)}{9.12\, TR} + \underset{(2.381)}{9.96\, DUM}$$

$$[R^2 = 0.87,\ F = 9.192,\ \text{and}\ DW = 1.063]$$

In equation *(22)*, *DUM* designates a (weather-related) dummy variable that takes the value of 1 in 1975 and 1976 and 0 otherwise. Other variables are defined as in the previous manner. As anticipated above, increases in current period deforestation and fertilizer applications both raise yields, but at a decreasing rate. The coefficient on the interaction term between X_t and D_t is also negative, which follows from both serving to increase nutrient availability. The stock effect associated with cumulative deforestation is negative, as expected. This is offset by an exogenous increase in productivity, as measured by the coefficient on the time trend.

Based on 1984 prices and the estimated yield function given by equation *(22)*, the socially optimal steady-state forest stock, F^*, may be computed. Rows 1-8 in Table 1 report the outcomes of a series of simulations where the discount rate and the expected level of technology are each varied. Four discount rates and two different technology "scenarios" are considered. In the first, TR^* is set equal to 20, reflecting the 1985 technology as estimated by equation *(22)*. The alternative scenario is generated by assuming a constant rate of technical change over the next 30 years equal to the rate observed over the last 20 years. Thus, TR^* equals 50, and F^* is based on "predicted" 2015 technology. Neither of these scenarios is likely to be correct. However, they serve to demonstrate the dramatic impact of exogenous technical change on the optimal steady-state forest stock.

Table 1—Optimal Steady-State Forest Stock Levels in the Ivory Coast (in 1,000 ha)*

Row	Discount Rate δ (%)	State of Technology†	Steady-State Forest Stock Levels‡
1	[δ = 3%]	*A*	5553.99
2		*B*	332.57
3	[δ = 7%]	*A*	4278.13
4		*B*	negative
5	[δ = 9%]	*A*	3380.88
6		*B*	negative
7	[δ = 11%]	*A*	2202.38
8		*B*	negative

*The technical parameters used to determine the optimal steady-state forest stock level include total, nonurban arable land (*L*) (10,900,000 ha) and initial (1985) forest stock level (F_0) (3,400,000 ha). The biological coefficients used are the estimated parameters of equation *(27)*. The optimal fertilizer rate used is X^* (30.68 kg/ha).

†*A* represents the case where technology is constant and set at its 1985 level (TR^* equals 20). *B* represents the case where technology is *expected* to increase steadily between 1985 and 2015 at the rate estimated in equation *(27)*. Thus, TR^* equals 50.

‡All prices are set at their 1984 levels. Gross agricultural returns (P_A) are *CFAF*786.87/kg. This is the ratio of 1984 per-hectare agricultural returns (*CFAF*93,975/ha) divided by the 1984 aggregate yield index. Returns to forestry (P_F) are *CFAF*17,839/ha. All returns are expressed in 1975 *CFA* francs. In 1975, $1 equalled *CFAF*214.

When technology is expected to remain constant at the 1985 level (case *A*), steady-state forest stocks are all positive. When the discount rate (δ) is less than 7 percent, all F^* are greater than $\bar{F}_0$ (3,400,000 ha, the observed 1985 forest stock). In these cases, further deforestation is not optimal. This stands in sharp contrast to the case where technology is set at the projected 2015 level. In this case, *B*, three of the four F^* in Table 1 fall below zero, suggesting that complete exhaustion of the forest stock is optimal.

The sensitivity of F^* to relative prices may be illustrated by employing the steady-state comparative static results *(19)* to calculate the elasticities of F^* with respect to $\bar{P}_F$ and $\bar{P}_X$. Assuming constant (1985) technology and δ equal to 0.05, these elasticities equal 0.09 and 0.005. Thus, F^* is quite insensitive to changes in the prices of fertilizer and net forestry returns, relative to food prices. By contrast, the elasticity of F^* with respect to δ is equal to -0.32.

Summary and Conclusions

This paper has used a two-sector dynamic model to determine the impacts of the social rate of discount, relative agricultural and forestry returns, and expected technology on the optimal steady-state forest stock (F^*) in the Ivory Coast. F^* was shown to be most sensitive to changes in the discount rate (δ) and the rate of expected technological change. Assuming current (1985) technology, F^* exceeds the 1985 forest stock for all values of δ less than 9 percent. Only when δ reaches 9 percent does some further deforestation appear socially optimal. If future technological change in food production can be expected to proceed (exogenously) at the same rate observed in the Ivory Coast over the 1965-85 period, then a very different set of conclusions is reached. For example, if F^* is computed based on expected technology projected for the year 2015 (based on historical rates of technical change), then F^* is less than zero for discount rates above 3 percent. This suggests that complete deforestation may be optimal. This conclusion is based on the assumption that forthcoming technological change is costless. Furthermore, while the model takes into account the external benefits that the forest stock confers on agriculture, it does not include any other positive externalities, such as preservation of genetic diversity and climatic benefits.

Notes

[1]International Institute of Tropical Agriculture; and Department of Agricultural Economics, Purdue University; respectively. The authors wish to acknowledge the funding of this research by the US Department of Agriculture and the Ivory Coast Research Center for Economic and Social Sciences (CIRES, Abidjan). Jim Binkley, Wade Brorsen, and Sheng Hu provided valuable comments on an earlier draft of this paper.

[2]See Arrow and Kurz (1977, p. 41) for more details.

[3]Time subscripts have been omitted in order to simplify notation.

[4]$\dot{D}$ is less than zero so that the deforestation rate is higher in current periods relative to the future.

[5]Ehui (1987) explores the theoretical implications of incorporating these variables in the aggregate yield function.

[6]μ *must* be negative to be assured of finding a point of maximum welfare (Ehui, 1987).

[7]This must always be the case when F^* is less than $\bar{F}_0$.

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DISCUSSION OPENING—*Ernst-August Nuppenau* (Department of Agricultural Economics, University of Kiel)

These two authors are concerned with the interesting and broadly discussed question of deforestation in developing countries. They have presented us with a good introduction to the formal analysis of the complex problem of an intertemporal forest stock reduction policy. Of course, the socially optimal forest stock policy could, under certain conditions, involve negative deforestation (reforestation), as the authors compute. As is often the case, though, the formal and systematic treatment of this problem and the development of a preliminary solution also give rise to a range of further questions and possible modifications to the analysis that deserve consideration. However, before I discuss some of these issues, I would like to mention two shortcomings. First, I believe that there should be a δ in front of F_t in equation *(10)*. Second, the authors do not state which time period they used in their estimation. The empirical investigation needs some more attention in general. The authors estimate a linear relationship between yield per hectare and various variables, especially deforestation embodied in F_0 - F_t. This relationship may hold for a certain period and range of existing deforestation. This raises the question of whether it is equally valid in more extreme situations; e.g., given nearly total deforestation. Similarly, is the functional relationship correctly specified? I suspect that deforestation might have a long-term effect on soil fertility. Perhaps a long-term decrease in soil fertility could be considered by introduction of a lagged yield variable in the model. This would, however, make the analysis more complicated.

The part of the model that deals with forestation could also be made more complex and perhaps more realistic. The existing model seems biased towards agriculture. First, in contrast to agriculture, forest production is a multiperiod biological process of an addition to stocks in biomass, which requires an adequate formulation. Second, how can one model an increase in forest if an optimal forest policy needs time for implementation. Perhaps a well-designed forest policy could provide better yields when cropping on tropical soils.

At least one further factor could be integrated in the model: population growth. This problem and its consequences for a policy recommendation concerning deforestation have been neglected in the model, which assumes constant prices for agricultural products.

Despite these suggestions, the approach chosen can incorporate all these points at the cost of increased complexity. This reduced mathematical simplicity might in turn make dialogue between researchers and public authorities more difficult. On the other hand, this increased complexity would allow the model to include factors that are important to policy makers.

GENERAL DISCUSSION—*T. Haque* (Indian Agricultural Research Institute)

The main issues raised on this paper concerned: (1) specification of variables selecting yield and other independent variables, (2) the use of the adopted linear model as a policy tool, (3) the time period required to implement the optimal forest policy, and (4) given the use of so many (immeasurable) utility functions, the practical applicability of the model used in policy decision making.

The authors felt that they were analyzing the policy implications of their model through a further study, which may be more revealing. They also mentioned that the time needed to implement the optimal forest policy may be infinite. The authors were of the opinion that, instead of a linear model, a quadratic model could give better results.

Participants in the discussion included M. Schiff and P. Thompson.