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## On the Multiplicity of Equilibrium Strategies in a Non-Renewable Natural Resource Duopoly

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**Abstract:**

We identify two possible equilibrium configurations for a non-renewable resource duopoly in a discrete-time framework. For the purpose of illustration, we suppose initial endowments of firms that allow for a maximum of two extraction periods. In the first possible equilibrium, the duopoly exists for two periods, while in the second possible equilibrium, the duopoly lasts only for one period and the firm with the higher initial endowment becomes a monopolist in the second and last period. As neither equilibrium configuration dominates the other for both firms at the same time, it is unclear whether firms acting simultaneously can coordinate on one particular configuration.

**Keywords:** Open-loop equilibrium, closed-loop equilibrium, duopoly, non-renewable resource

**Résumé:**

Dans le cadre d'un modèle à temps discret, nous identifions deux configurations d'équilibre possibles pour un duopole extrayant une ressource non-renouvelable. Pour des fins d'illustration, nous supposons que le stock initialement disponible pour chaque firme est tel que l'extraction perdure pour un maximum de deux périodes. Dans le premier équilibre possible, un duopole opère lors des deux périodes, alors que dans le deuxième équilibre, le duopole opère seulement lors de la première période et est suivi par une période où la firme avec un stock relativement plus élevé devient un monopole. Comme aucune des deux configurations d'équilibre ne domine l'autre, il n'est pas clair que des firmes agissant simultanément peuvent se coordonner sur une configuration en particulier.

**Mots clés:** Équilibre à boucle ouverte, équilibre à boucle fermée, duopole, ressource non-renouvelable

**Classification JEL:** Q30, D43

## 1 Introduction

The analysis of extraction strategies in oligopolistic resource markets has been an ongoing endeavour for now over 30 years starting with the analysis of a cartel-fringe, open-loop market structure by Salant (1976). As Gaudet (2007) notes, such interest from the economic profession was motivated by the foundation in 1960 of the Organization of the Petroleum Exporting Countries (OPEC) and the following oil crisis in the 1970's. Trying to understand the extraction pattern (and related price) of natural resources, the economic literature has covered since then the analysis of the Cournot and Stackelberg market structure in a closed-loop setting, where each agent conditions its extraction decision on its own resource stock.<sup>1</sup>

Open-loop and closed-loop Nash equilibria have been characterized analytically for the case of particular demand and cost structures, while more general settings can so far only be dealt with numerically (Salo and Tavonen, 2001). Whether open-loop or closed-loop strategies apply depends on the players' ability of commitment at the beginning of the game. However, such commitment may seem particularly unrealistic when the environment of the players changes (e.g. a changing carbon tax penalizing fossil fuel extraction).

Having said that, following the terminology by Dockner et al. (1985), when a game is "state-separable," or in the terminology by Dockner et al. (2000) a "linear state game," open- and closed-loop strategies will coincide when the terminal time horizon is *exogenously* given. Although such an exogenous terminal time is not necessarily a realistic assumption in resource extracting oligopolies, it becomes an underlying implicit assumption when a particular market structure is assumed to prevail until the exhaustion of the resource. This observation applies to the discrete-time model of Hartwick and Brolley (2008) who assume initial resource stocks of players to be such that exhaustion of the resource occurs in the same period. They find that a player's closed-loop strategy is independent of its competitor's, or equivalently, that closed-loop and open-loop strategies coincide.

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<sup>1</sup>A good review on how market structure in particular, as well as extraction costs, durability aspects and uncertainty affects the Hotelling rule of resource pricing can be found in Gaudet (2007).

In this paper, we also adopt the discrete time modelling framework and characterize explicitly the initial stocks of two players such that exhaustion of each player’s resource stock occurs in the same period. Our simple “state-linear” modelling framework guarantees that open- and closed loop strategies coincide. Our main finding is that there exist combinations of asymmetric, initial resource stocks that could sustain two different equilibrium configurations: (i) a duopoly up to a common, finite time period and (ii) a duopoly followed by a monopoly exhausting its resource pool at a later point of time. While the player with a relatively low initial stock prefers the duopoly market structure, the player with a relatively high initial stock prefers to turn into a monopolist before complete exhaustion of his resource pool occurs.

## 2 The Model

We assume a discrete-time model with a linear inverse demand function  $p(q_t) = a - bq_t$ , where  $q_t$  is the total quantity on the market in period  $t$ . The presence of a choke price  $a$  makes the resource unessential, such that extraction will end in finite time. There are two firms (players),  $i = 1, 2$ , serving the market. Let  $q_t^i$  be the production of firm  $i$ , which is assumed to have a linear cost function  $C(q_t^i) = cq_t^i$ , where  $c \geq 0$ . Parameters satisfy  $a > c$ , which implies that the resource is valuable and that reserves are completely extracted. Firm  $i$ ’s initial stock of the non-renewable resource is given exogenously by  $s_1^i$  and the law of motion is  $s_{t+1}^i = s_t^i - q_t^i$ . We do not allow for resource storage. Once a firm has completely extracted its resource pool, it exists the market.

### 2.1 The monopoly

For later reference, we first address the benchmark case of a monopoly extracting the resource. The total quantity on the market is the monopolist output  $q_t = q_t^i$ . The firm’s profit in period  $t$  is  $\pi(q_t) = (p(q_t) - c)q_t$ . The firm seeks to maximise the sum of discounted profits subject to the law of motion and the constraint on extraction in period  $t$ ,  $q_t \leq s_t$ . The

inter-temporal profit maximisation problem for the monopoly is then given by

$$\begin{aligned} & \max_{\{q_t\}_{t=1}^T, T} \sum_{t=1}^T \delta^{t-1} \pi(q_t) \\ \text{s.t.} \quad & s_{t+1} = s_t - q_t, \quad s_1 \text{ given,} \\ & q_t \leq s_t, \end{aligned}$$

where  $\delta \in [0, 1]$  is the discount factor and  $T$  the endogenously determined last period of extraction. For convenience, we solve the problem in its recursive form. Let  $V(s_t)$  denote the value function at time  $t$  depending on the stock  $s_t$ . The recurrence equation is:

$$\begin{aligned} V(s_t) &= \max_{0 \leq q_t \leq s_t} \pi(q_t) + \delta V(s_{t+1}) \\ \text{s.t.} \quad & s_{t+1} = s_t - q_t, \quad s_1 \text{ given.} \end{aligned}$$

We distinguish between interior and corner solutions, where a corner solution may apply in the final period of extraction  $t = T$ . For an interior solution, in particular when the firm is not in its last extraction period, *i.e.*  $t < T$ , the necessary condition for optimality is  $\partial\pi(q_t)/\partial q_t + \delta \partial V(s_{t+1})/\partial q_t = 0$ . Substituting the law of motion into the value function and making use of  $\partial s_{t+1}/\partial q_t = -1$ , the necessary condition for optimality can be written

$$\frac{\partial\pi(q_t)}{\partial q_t} = \delta \frac{\partial V_{t+1}(s_{t+1})}{\partial s_{t+1}}. \quad (1)$$

Equation (1) states that the marginal profit associated to the last unit extracted in period  $t$  is equal to the opportunity cost of doing so, which is the discounted marginal value associated to having that unit available for extraction in period  $t + 1$ . Applying the envelope theorem to the recurrence equation (Obstfeld and Rogoff, 1996), we get:

$$\frac{\partial V_t(s_t)}{\partial s_t} = \frac{\partial\pi(q_t)}{\partial q_t}, \quad (2)$$

stating that the marginal value of extracting one more unit of stock must, at each period, be equal to the marginal profit of that same increment on the market. Combining equations (1) and (2), it follows that :

$$\frac{\partial\pi(q_t)}{\partial q_t} = \delta \frac{\partial\pi(q_{t+1})}{\partial q_{t+1}}. \quad (3)$$

Equation (3) has a straightforward economic intuition: the monopoly firm equalises the discounted marginal profit at every period of extraction. For  $t = 1, \dots, T$ , we thus have

$$\frac{\partial \pi(q_1)}{\partial q_1} = \delta \frac{\partial \pi(q_2)}{\partial q_2} = \delta^2 \frac{\partial \pi(q_3)}{\partial q_3} = \dots = \delta^{T-1} \pi'(q_T) \quad (4)$$

Note that, in current value terms, the marginal profit increases, implying that the monopolist discriminates intertemporally within subsequent markets.

In the case of a corner solution, the equalization of the discounted marginal profits does not necessarily hold between the last period of extraction,  $T$ , and  $T + 1$ . In particular, the marginal value of extracting the remaining stock at  $T$ ,  $s_T$ , can be greater or equal to the marginal value of postponing extraction to the subsequent period:

$$q_T = s_T \Leftrightarrow \frac{\partial \pi(s_T)}{\partial s_T} \geq \delta \frac{\partial V(s_{T+1} = 0)}{\partial s_{T+1}} \quad (5)$$

The inequality in condition (5) arises due to the discrete time modelling: accounting for cumulative previous extraction, it may be profit maximizing to incur a higher marginal profit at  $T$  than postponing an incremental unit of resource to  $T + 1$ . The envelope theorem must still hold at  $T + 1$ , when  $s_{T+1} = 0$  and  $q_{T+1} = 0$ . This implies that  $\partial V(s_{T+1} = 0)/\partial s_{T+1} = \partial \pi(q_{T+1} = 0)/\partial q_{T+1} = a - c$ , where the last equality follows from the specification of linear demand and linear costs. Condition (5) can now be written explicitly:

$$q_T = s_T \iff s_T \leq (1 - \delta) \frac{a - c}{2b} \equiv (1 - \delta)Q. \quad (6)$$

This condition states that the monopoly firm stops producing in period  $T$  and extracts all its remaining stock  $s_T$ , whenever it falls below the critical threshold  $(1 - \delta)Q$ . The parameter  $Q$  corresponds to the profit-maximizing quantity sold by a monopolist in a static context (*i.e.* in the absence of any resource constraint). In our context of resource extraction, it can also be interpreted as the quantity sold by a “myopic” monopolist, who does not account for the opportunity cost of selling today instead of tomorrow.

In order to characterize the series of production decisions, we make use of condition (4) and derive the following recurrence equation :

$$q_t = (1 - \delta)Q + \delta q_{t+1} \quad (7)$$

This can be manipulated further to find an expression for  $q_t$  as a function of  $Q$  and the last-period extraction  $q_T$ :

$$q_t = (1 - \delta^{T-t})Q + \delta^{T-t}q_T \quad (8)$$

As the resource is completely exhausted, total extraction must equal initially available stocks. Using (8), we must have:

$$s_1 = \sum_{t=1}^T q_t = Q \left( T - \frac{1 - \delta^T}{1 - \delta} \right) + \frac{1 - \delta^T}{1 - \delta} q_T \quad (9)$$

From equation (9), we express the last-period production  $q_T$  as a function of model parameters,  $s_1$ ,  $\delta$  and  $Q$  and obtain:

$$q_T = \frac{1 - \delta}{1 - \delta^T} (s_1 - Q(T - 1)) + Q \frac{1 - \delta}{1 - \delta^T}. \quad (10)$$

We extend our analysis by using the last-period condition on extraction (6), *i.e.*  $q_T \leq (1 - \delta)Q$ . The maximum possible quantity of the last-period extraction is  $q_T = (1 - \delta)Q$ . In conjunction with (9), we obtain another condition on the firm's initial stock.

$$\begin{aligned} s_1 &= Q \left( T - \frac{1 - \delta^T}{1 - \delta} \right) + \frac{1 - \delta^T}{1 - \delta} (1 - \delta)Q \\ &= Q \left( T - \frac{\delta - \delta^{T+1}}{1 - \delta} \right) \\ &\equiv G(T) \end{aligned} \quad (11)$$

The function  $G(T)$  represents an upper bound on the initially available stock,  $s_1$ , to be completely exhausted in  $T$  periods. A similar condition for a lower bound also exists. If complete exhaustion occurs in period  $T$ , the last quantity extracted,  $q_T$ , has to be strictly greater than zero. Thus, if  $q_T > 0$ , then, by equation (9),  $s_1 > Q \left( T - \frac{1 - \delta^T}{1 - \delta} \right)$ . It can be shown that

$$Q \left( T - \frac{1 - \delta^T}{1 - \delta} \right) = Q \left( T - 1 - \frac{\delta - \delta^T}{1 - \delta} \right) \equiv G(T - 1). \quad (12)$$

Hence, we have proposition 1:



**Proposition 1** *In order to exhaust its initial resource endowment in  $T$  periods, the monopoly's stock  $s_1$  must be strictly bounded from below and bounded from above, such that*

$$G(T-1) < s_1 \leq G(T). \quad (13)$$

## 2.2 The Duopoly

This section investigates the case of two competing firms operating in the market. First, we analyse the necessary conditions for a dynamic Cournot duopoly in order to last  $T$  periods. Second, we analyze the case where a duopoly is followed by a monopoly firm serving the market.

### 2.2.1 Equal Periods of Exhaustion

A Cournot competing firm takes the behaviour of its rival into account when maximizing profits. In this section, we derive the conditions under which both firms will exhaust their reserves at the same period, where  $T^1 = T^2 = T$ . We note this final period again  $T$ , although it is different from the monopoly's final period of extraction. The inter-temporal profit maximisation problem for firm  $i = 1, 2$  is given by:

$$\begin{aligned} \max_{\{q_t^i\}_{t=1, T}^T} \Pi^i &= \sum_{t=1}^T \delta^{t-1} \pi^i(q_t^1, q_t^2) \\ \text{s.t. } s_{t+1}^i &= s_t^i - q_t^i, \quad s_1^i \text{ given,} \end{aligned}$$

where firm  $i$ 's profit in period  $t$  is given by  $\pi^i(q_t^1, q_t^2) = p(q_t^1 + q_t^2)q_t^i - C(q_t^i)$ . Writing this problem recursively, where  $V^i(s_t^1, s_t^2)$  denotes firm  $i$ 's value function depending on stocks  $(s_t^1, s_t^2)$ , we have:

$$\begin{aligned} V^i(s_t^1, s_t^2) &= \max_{0 \leq q_t^i \leq s_t^i} \pi^i(q_t^1, q_t^2) + \delta V(s_{t+1}^1, s_{t+1}^2) \\ \text{s.t. } s_{t+1}^i &= s_t^i - q_t^i, \quad s_1^i \text{ given.} \end{aligned}$$

Using the same techniques as in section 2.1, we can derive a set of conditions for firms 1 and 2 characterising their output decisions and final time of extraction. Dealing with the interior problem, each player will, as in the monopoly case, equalise properly discounted, marginal

profits in subsequent periods, and account for foregone opportunities of current extraction. Formally, for any period  $t \leq T - 1$  and for firm  $i = 1, 2, i \neq j$ , the following conditions must hold simultaneously:

$$\frac{\partial \pi^i(q_t^i, q_t^j)}{\partial q_t^i} = \delta \frac{\partial \pi^i(q_{t+1}^i, q_{t+1}^j)}{\partial q_{t+1}^i}. \quad (14)$$

For each firm to stop producing in period  $T$ , the marginal profit of each firm at  $T$  will have to be greater or equal to the discounted marginal profit in the following period. This means that for a firm to exhaust its remaining resource stock, the value of a last incremental extraction in  $t = T$  has to be at least equal to the discounted marginal value of extracting nothing in  $t = T + 1$ . In order for both firms to exhaust their resource at the same period, this must hold for both at  $T$ . Formally, at  $T$ :

$$\frac{\partial \pi_i(q_T^i, q_T^j)}{\partial q_T^i} \geq \delta \frac{\partial \pi_i(0, 0)}{\partial q^i}.$$

Noting that  $\partial \pi^i(q_t^i, q_t^j) / \partial q_t^i = a - c - 2bq_t^i - bq_t^j$ , these terminal conditions can be rewritten by using expression  $Q$  defined in (5) as:

$$q_T^i = s_T^i \Leftrightarrow s_T^i \leq (1 - \delta)Q - \frac{1}{2}s_T^j \quad (15)$$

Before the final period of extraction,  $t < T$ , extraction of firm  $i = 1, 2, i \neq j$ , is given by equation (14) can be written explicitly as:

$$(q_t^i - \delta q_{t+1}^i) + \frac{1}{2}(q_t^j - \delta q_{t+1}^j) = (1 - \delta)Q \quad (16)$$

These can be interpreted as reaction functions. First, the quantity extracted by a player at  $t$  depends positively on its own extraction in  $t + 1$ . Moreover, it is negatively related to its rival's extraction in period  $t$  and positively related to its rival's production in  $t + 1$ .

Subtracting (16) evaluated for  $i = 1$  and  $j = 2$  from (16) evaluated for  $i = 2$  and  $j = 1$  defines a set of admissible extraction quantities in subsequent periods  $t$  and  $t + 1$ :

$$q_t^1 - \delta q_{t+1}^1 = q_t^2 - \delta q_{t+1}^2 = (1 - \delta) \frac{2}{3}Q \quad (17)$$

The expression  $\frac{2}{3}Q$  is the static duopoly Cournot equilibrium quantity. From (17), we can express  $q_t^i$  as a function of subsequent extraction quantities by developing the recurrence equation:

$$\begin{aligned} q_t^i &= (1 - \delta)\frac{2}{3}Q + \delta \left( (1 - \delta)\frac{2}{3}Q + \delta(\dots + \delta q_T^i) \right) \\ &= (1 - \delta^{T-t})\frac{2}{3}Q + \delta^{T-t}q_T^i. \end{aligned} \quad (18)$$

Extraction  $q_t^i$  is thus a weighted average of the static duopoly Cournot equilibrium quantity,  $\frac{2}{3}Q$ , and the last period extraction,  $q_T^i$ . As firm  $i$  gets closer to exhaustion, more weight is given to extraction at  $T$  as  $\lim_{t \rightarrow T} \delta^{T-t} = 1$ . Since firms exhaust all their resource stock, summing on  $q_t^i$  for  $t = 1, \dots, T$  must equalise respectively the initially available stock  $s_1^i$ , *i.e.*:

$$s_1^i = \sum_{t=1}^T q_t^i = \frac{2}{3}Q(T - \frac{1 - \delta^T}{1 - \delta}) + \frac{1 - \delta^T}{1 - \delta}q_T^i \quad (19)$$

This last identity for the initially available stock allows us to express the extraction in the final period  $T$  as a function of the initially available stock:

$$q_T^i = \frac{1 - \delta}{1 - \delta^T} [s_1^i - \frac{2}{3}Q(T - 1)] + \frac{2}{3}Q \frac{\delta - \delta^T}{1 - \delta^T} \quad (20)$$

Using equation (20) for  $i, j = 1, 2$ ,  $i \neq j$ , we can express firm  $i$ 's extraction in the final period,  $q_T^i$  ( $q_T^j, s_1^i, s_1^j$ ), as a function of its own stock, its rival's stock and the period of last extraction,  $T$ . From (15), we know that  $q_T^1 + \frac{1}{2}q_T^2 \leq (1 - \delta)Q$ . Then:

$$q_T^1 + \frac{1}{2}q_T^2 = \frac{1 - \delta}{1 - \delta^T} \left( s_1^1 + \frac{1}{2}s_1^2 - Q(T - 1) \right) + Q \frac{\delta - \delta^T}{1 - \delta^T} \leq (1 - \delta)Q$$

For later use, we calculate with the help of (19) the following expression:

$$s_1^1 + \frac{1}{2}s_1^2 = Q \left( T - \frac{1 - \delta^T}{1 - \delta} \right) + \frac{1 - \delta^T}{1 - \delta} (q_T^1 + \frac{1}{2}q_T^2) \quad (21)$$

It is possible to specify an upper bound for  $s_1^1 + \frac{1}{2}s_1^2$ . The highest possible value  $q_T^1 + \frac{1}{2}q_T^2$  is  $(1 - \delta)Q$ ; substituting it in (21) and rearranging leads to:

$$s_1^1 + \frac{1}{2}s_1^2 \leq Q \left( T - \frac{\delta - \delta^{T+1}}{1 - \delta} \right) \equiv G(T)$$

where  $G(T)$  was defined in (13). Similarly, since  $q_T^1 + \frac{1}{2}q_T^2 > 0$  a lower bound for  $s_1^1 + \frac{1}{2}s_1^2$  is given by:

$$s_1^1 + \frac{1}{2}s_1^2 > Q\left(T - \frac{1 - \delta^T}{1 - \delta}\right)$$

This leads to the following proposition:

**Proposition 2** *The final period of exhaustion is identical for both firms  $i, j = 1, 2, i \neq j$ , if their initial stocks satisfy:*

$$G(T - 1) < s_1^i + \frac{1}{2}s_1^j \leq G(T) \quad (22)$$

Proposition 2 allows us to define an area in the space of initial stocks  $(s_1^1, s_1^2)$  for which exhaustion occurs in period  $T$  for both firms. We illustrate Proposition 2 for  $T = 2$ .

• **Case when  $T = 2$**

Evaluating  $G(T)$  and  $G(T - 1)$  for  $T = 2$  in equation (22), we have for  $i, j = 1, 2, i \neq j$ :

$$Q(1 - \delta) < s_1^i + \frac{1}{2}s_1^j \leq Q(1 - \delta)(2 + \delta) \quad (23)$$

The shaded area in Figure 1 identifies admissible stock levels where both firms will produce for two periods. The dotted lines refer to the strict inequality constraints in equation (23) while the plain lines represent the inequality constraints. These lines cross on the  $45^\circ$  line. For initial stocks equal to  $\frac{2}{3}Q(1 - \delta)$  defined in equation (17), firms will find it more profitable to extract within only one period. Given  $\delta \in [0, 1)$ , this critical value is smaller than the static Cournot output of a firm,  $\frac{2}{3}Q$ . This is because a firm accounts for the opportunity cost of extraction.

We calculate for later use each firm's intertemporal profit, making use of the optimal extraction policy determined earlier. In particular, the extraction in the last period is given by (20) evaluated at  $T = 2$ , while extraction in the initial period can be obtained by using the law of motion  $q_1^i = s_1^i - q_2^i$ , and we obtain:

$$q_1^i(s_1^i) = \frac{2}{3}Q\frac{1 - \delta}{1 + \delta} + \frac{\delta}{1 + \delta}s_1^i. \quad (24)$$

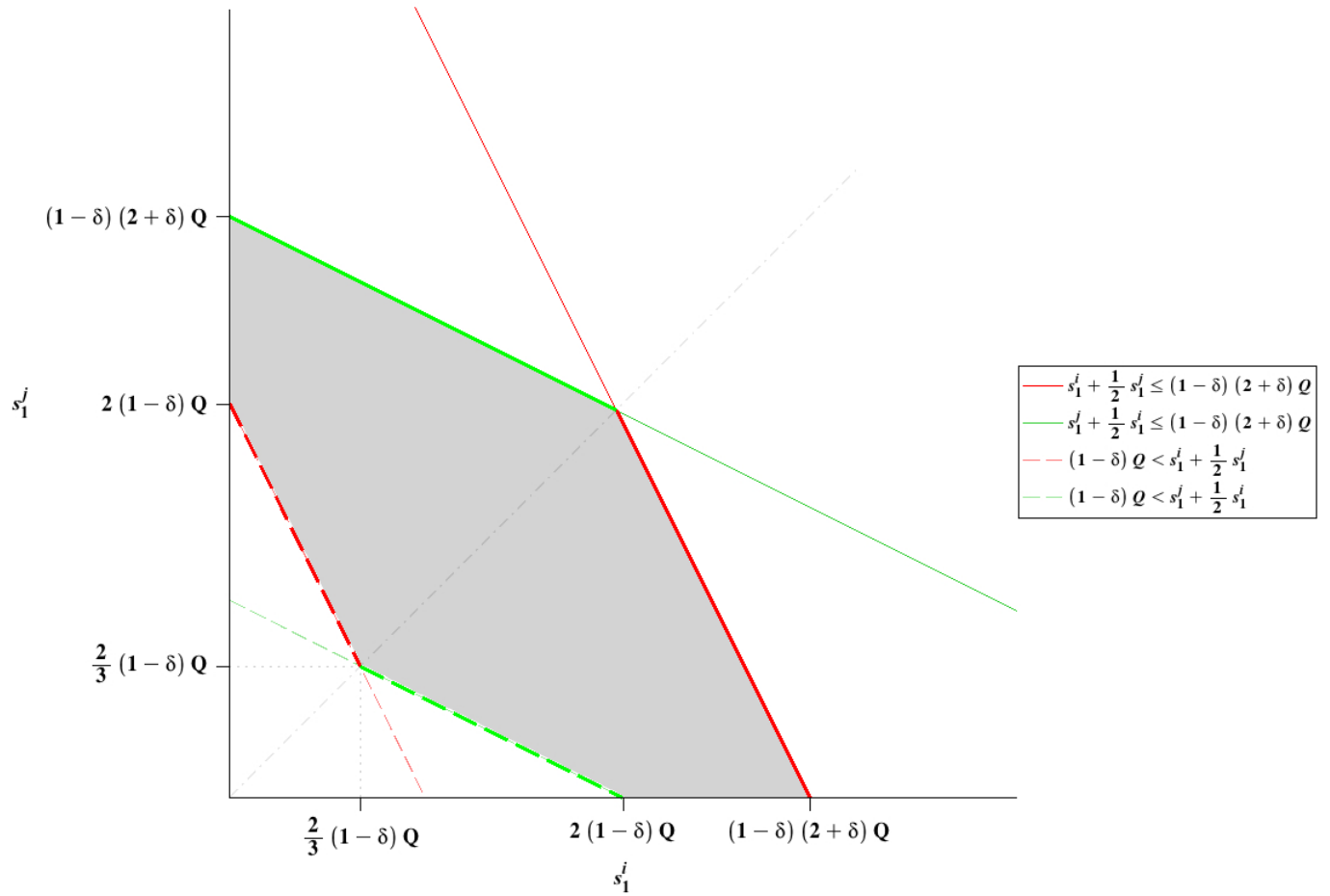


Figure 1: Possible initial resource stocks for symmetric duration of extraction

Having  $q_1^i(s_1^i)$  and  $q_2^j(s_1^j)$  at hand, we calculate the total value associated with extraction:

$$V_D^i(s_1^i, s_1^j) = \frac{b}{1+\delta} \left( \left( \frac{2}{3}(1-\delta)Q \right)^2 - \delta s_1^i (s_1^i + s_1^j - 4Q) \right), \quad (25)$$

where the subscript  $D$  stands for duopoly. When  $\delta = 0$ , firm  $i$  puts no importance on future profits and  $V_D^i(s_1^i, s_1^j)$  is equal to the static one-period Cournot profit. If firm  $i$ 's initial resource stock is greater than firm  $j$ 's, its intertemporal profit will also be greater.

### 2.2.2 Asymmetric Periods of Exhaustion

We now turn to the possibility that firms exhaust their resource stock at different time periods. In particular, we study the case where firm 1 exhausts all its initial resources in the first period while firm 2 continues its extraction for another period of time. In  $t = 1$ , firms compete *à la* Cournot, while in  $t = 2$ , firm 2 is the only remaining firm and will behave as a monopolist. In what follows, we derive the conditions on the firms' initial stocks and represent them graphically in space  $(s_1^1, s_1^2)$ .

For firm 1 to extract all its stock within the first period in a dynamic Cournot duopoly, its marginal profit in  $t = 1$  has to be greater or equal to the discounted marginal profit in  $t = 2$ , given that firm 2 is still on the market, *i.e.*  $q_2^2 \geq 0$ . Formally,

$$\frac{\partial \pi_1(q_1^1, q_1^2)}{\partial q_1^1} \geq \delta \frac{\partial \pi_1(0, q_2^2)}{\partial q_1^2} \iff s_1^1 \leq \frac{2}{3}(1-\delta)Q. \quad (26)$$

Firm 1's stock must thus be smaller or equal to  $\frac{2}{3}(1-\delta)Q$ , which is a vertical line in the space  $(s_1^1, s_1^2)$ .

For firm 2 to extract for two time periods, it will equalise its discounted marginal profit throughout the game,

$$\frac{\partial \pi_2(q_1^2, s_1^1)}{\partial q_1^2} = \delta \frac{\partial \pi_2(q_2^2, 0)}{\partial q_2^2}, \quad (27)$$

which will implicitly defines optimal extraction  $q_1^2$ . Given our model assumptions, this condition rewrites:

$$q_1^2 - \delta q_2^2 = (1-\delta)Q - \frac{1}{2}s_1^1, \quad (28)$$

which can be interpreted as a reaction function:  $q_1^2$  is proportional to the quantity that firm 2 produces in period 2, while it is inversely proportional to firm 1's extraction. Since firm 2 exhausts its stocks at  $T = 2$ , we must have  $s_1^2 = q_1^2 + q_2^2$ . Combining this condition with (28), we have

$$q_1^2 = \frac{1-\delta}{1+\delta}Q + \frac{\delta}{1+\delta}s_1^2 - \frac{1}{2}\frac{1}{1+\delta}s_1^1 \quad (29)$$

$$q_2^2 = s_2^2 = -\frac{1-\delta}{1+\delta}Q + \frac{1}{1+\delta}s_1^2 + \frac{1}{2}\frac{1}{1+\delta}s_1^1 \quad (30)$$

These equations give the precise link between the production of firm 2 in both periods as well as each player's initial stock. An increase in firm 2's stock will increase its production in period 1 and 2, but an increase in its competitor's initial stock will induce a transfer of extraction from period 1 to period 2.

We know from section 2.1 that a monopoly will stop producing in period  $T$ , if the marginal profit from extracting the remaining stock is greater than the discounted marginal value given  $s_{T+1}^2 = 0$ . Here, for  $q_2^2 = s_2^2$ , the following stopping criterion must be satisfied:

$$s_2^2 \leq (1-\delta)Q \quad (31)$$

Combining expressions (30) and (31), we obtain:

$$s_1^2 + \frac{1}{2}s_1^1 \leq (2+\delta)(1-\delta)Q \quad (32)$$

which represents an upper bound on firm 2's initial stock  $s_1^2$  to stop producing in period 2. In space  $(s_1^1, s_1^2)$ , we will have a downward sloping line with intercept  $(2+\delta)(1-\delta)Q$  and slope  $-\frac{1}{2}$ . We must also verify that player 2's production in each period is positive. To derive the associated conditions, we use (29) and (30). Thus :

$$q_1^2 > 0 \iff \delta s_1^2 - \frac{1}{2}s_1^1 > -(1-\delta)Q \quad (33)$$

$$q_2^2 > 0 \iff s_1^2 + \frac{1}{2}s_1^1 > (1-\delta)Q \quad (34)$$

We can verify that condition (33) is not binding. Rearranging it, we need  $s_1^2 > \frac{1}{2\delta}s_1^1 - \frac{(1-\delta)}{\delta}Q$  to hold. We have previously shown that  $s_1^1 \in [0, \frac{2}{3}(1-\delta)Q]$ , which implies  $s_1^2 \in$

$\left(-\frac{(1-\delta)}{\delta}Q, -\frac{(1-\delta)}{\delta}\frac{2}{3}Q\right)$ , but this is non-binding because we assumed  $s_1^2$  non-negative. Condition (33) can be represented in space  $(s_1^1, s_1^2)$  by a downward sloping linear function with intercept  $(1-\delta)Q$  and slope  $-\frac{1}{2}$ .

For firm 1 to stop producing in the first period, it must not be more profitable to keep a small amount of stock and put it on the market after firm 2 has exhausted its stock in period 2. The following must then be satisfied, for  $n \geq 2$ .

$$\frac{\partial \pi_1(s_1^1, q_1^2)}{\partial q_1^1} > \delta^n \frac{\partial \pi_1(0, 0)}{\partial q_1^2} \quad (35)$$

$$Q(1-\delta^n) - s_1^1 - q_1^2 > 0 \quad (36)$$

We can use expression (29) for  $q_1^2$  to simplify the previous condition, such that:

$$\frac{3+4\delta}{2\delta}s_1^1 + s_1^2 < 2\frac{Q}{\delta}(1+3\delta-2\delta^n-2\delta^{n+1}) \quad (37)$$

In space  $(s_1^1, s_1^2)$ , this constraint is a downward sloping linear function. It will be binding if, for  $s_1^1 \leq \frac{2}{3}(1-\delta)Q$ , it is at some point smaller than  $s_1^2 + \frac{1}{2}s_1^1 \leq (2+\delta)(1-\delta)Q$  by condition (32). We note that the intercept is, for  $n \geq 2$  and  $\delta \in (0, 1]$ , always greater than  $(2+\delta)(1-\delta)Q$ , *i.e.*

$$\begin{aligned} 2\frac{Q}{\delta}(1+3\delta-2\delta^n-2\delta^{n+1}) &\geq 2\frac{Q}{\delta}(1+3\delta-2\delta^2-2\delta^3) \\ &\geq Q(1-\delta)(2+8\delta+4\delta^2) \\ &> (2+\delta)(1-\delta)Q \end{aligned}$$

Using constraint (32) when  $s_1^1 = \frac{2}{3}(1-\delta)Q$ ,  $s_1^2$  is equal to  $\frac{2}{3}Q(2.5-\delta-\frac{3}{2}\delta^2)$ . For the same  $s_1^1$ , constraint (37) is binding if  $s_1^2 = Q(1-\delta)(2+8\delta+4\delta^2) - \frac{3+4\delta}{2\delta}s_1^1 < \frac{2}{3}Q(4-\delta-3\delta^2)$ . However,  $s_1^2 = \frac{2}{3}Q(4-\delta-3\delta^2) \geq \frac{2}{3}Q(2.5-\delta-\frac{3}{2}\delta^2)$  for  $\delta \in (0, 1]$ . Thus, the constraint is not binding. We thus obtain the following proposition:

**Proposition 3** *The bounded set of admissible initial stocks of firm 1 and 2 is defined by the following equations:*

1.  $s_1^2 + \frac{1}{2}s_1^1 \leq (2+\delta)(1-\delta)Q$



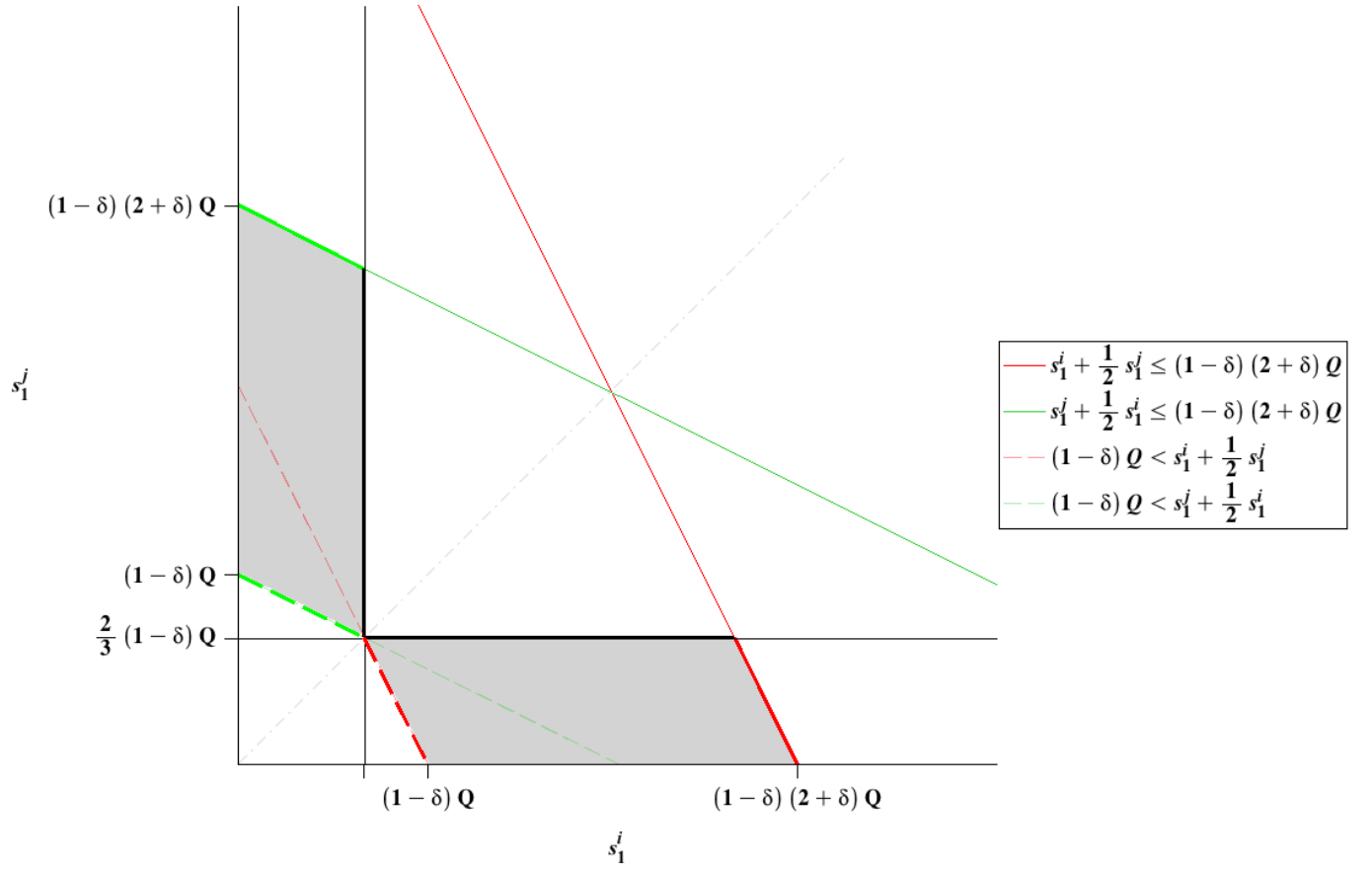


Figure 2: Possible initial resource stocks for asymmetric duration of extraction

$$2. s_1^2 + \frac{1}{2}s_1^1 > (1 - \delta)Q$$

$$3. s_1^1 \leq \frac{2}{3}(1 - \delta)Q$$

Figure 2 shows the conditions stated in Proposition 3 in space  $(s_1^1, s_1^2)$ .

For later reference, we can calculate the intertemporal profit for each firm. For firm 1, which in the case analyzed here extracts only in  $t = 1$ , this resumes to its profit in the first period:

$$V_{Asy}^1(s_1^1, s_1^2) = \frac{b s_1^1}{1 + \delta} \left( Q(3\delta + 1) - \frac{1}{2}s_1^1 - \delta(s_1^1 + s_1^2) \right) \quad (38)$$

For firm 2, the extraction of which lasts two periods, the intertemporal profit is given by

$$V_{Asy}^2(s_1^1, s_1^2) = \frac{b}{1 + \delta} \left( (\delta - 1)^2 Q^2 + (\delta - 1)Qs_1^1 + 4\delta Qs_1^2 + \left( \frac{1}{2}s_1^1 \right)^2 - \delta s_1^1 s_1^2 - \delta (s_1^2)^2 \right) \quad (39)$$

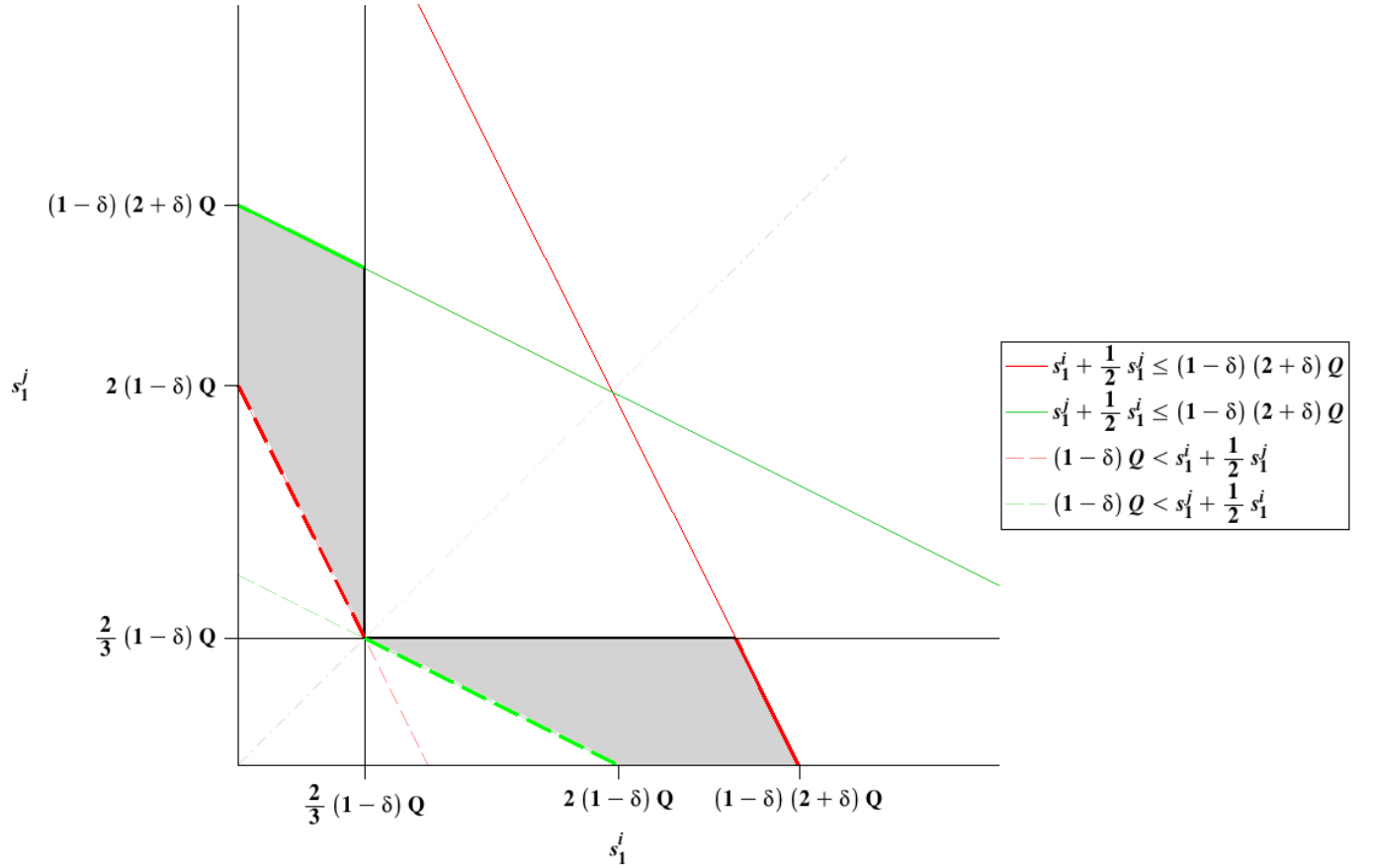


Figure 3: Possible initial resource stocks for symmetric and asymmetric duration of extraction

### 3 Comparison of the asymmetric and duopoly market equilibria

In this section we analyse whether the symmetric and asymmetric equilibria characterized in propositions 2 and 3 could result for a given combination of the two firms' initial stock. We do this graphically in Figure 3, where the overlap of admissible sets of initial stocks shown in Figures 1 and 2 for propositions 2 and 3 to hold is represented as a shaded area. As we can see, there exist combinations of initial stocks where the firm with the low initial stock is characterized by  $s_1^i \leq (2/3)(1 - \delta)Q$ , such that no configuration of a market equilibrium can be excluded from the outset unless both firms are better off under one particular market structure at the same time.

For  $i = 1, 2$ , we define  $\Delta V^i = V_D^i - V_{Asy}^i$ . If  $\Delta V^i > 0$ , the symmetric duopoly yields

higher profits for firm  $i$ , whereas, if  $\Delta V^i < 0$ , the asymmetric case is more profitable for firm  $i$ . In what follows, we assume for the asymmetric case that firm 1 has smaller stocks and extracts in the first period only, while firm 2 has relatively higher stocks and extracts for two periods. We calculate:

$$\begin{aligned}\Delta V^1 &= V_D^1 - V_{Asy}^1 \\ &= \frac{b}{2(1+\delta)} \left( (s_1^1 - (1-\delta)Q)^2 - \left( \frac{1}{3}(1-\delta)Q \right)^2 \right)\end{aligned}\quad (40)$$

which is only a function of  $s_1^1$ . Thus only the stock of the firm with the “smaller” stock (here firm 1) determines which market structure is more profitable. It can be shown that  $V^1$  is a convex function of  $s_1^1$ , reaching a minimum at  $(1-\delta)Q$  and that  $\Delta V^1 \geq 0$  as long as:

$$s_1^1 \in \left[ 0, \frac{2}{3}(1-\delta)Q \right] \cup \left[ \frac{4}{3}(1-\delta)Q, \infty \right).$$

Assuming that  $s_1^1 < \frac{2}{3}(1-\delta)Q$ , firm 1 will always prefer the symmetric duopoly outcome to the asymmetric market outcome.

For firm 2, which by assumption operates for two periods, we find:

$$\begin{aligned}\Delta V^2 &= V_D^2 - V_{Asy}^2 \\ &= \frac{b}{4(1+\delta)} \left( \left( \frac{4}{3}(1-\delta)Q \right)^2 - (s_1^1 - 2(1-\delta)Q)^2 \right),\end{aligned}\quad (41)$$

which is a concave function of  $s_1^1$ , reaching a maximum at  $2(1-\delta)Q$  and which satisfies  $\Delta V^2 \geq 0$  as long as

$$s_1^1 \in \left[ \frac{2}{3}(1-\delta)Q, \frac{10}{3}(1-\delta)Q \right]$$

Again, by assumption,  $s_1^1 < \frac{2}{3}(1-\delta)Q$ , such that  $\Delta V^2 < 0$  and firm 2 prefers to operate for two periods.

The former analysis of the intertemporal profits related to each possible equilibrium has allowed us to conclude that the firm with the relatively smaller initial stocks (firm 1) makes more profits when it competes *à la* Cournot for two periods instead of only one, while Firm 2 prefers the asymmetric equilibrium.

The question arises whether the firm with the relatively higher stock (firm 2) preferring the asymmetric case can compensate its competitor for its lower profits incurred in the asymmetric case. A compensation such that each firm is at least as well off as in its own preferred market structure will exist if the increase in firm 2's total discounted profits is greater than the losses incurred by firm 1, *i.e.*  $V_{Asy}^2 - V_D^2 \geq V_D^1 - V_{Asy}^1$ . This last expression can be written as  $-\Delta V^2 - \Delta V^1 \geq 0$ , or:

$$(s_1^1)^2 \leq \left( \frac{2}{3}(1 - \delta)Q \right)^2,$$

which holds for admissible values  $s_1^1 \in [0, \frac{2}{3}(1 - \delta)Q]$ . Hence, firm 2 could compensate firm 1 to play the asymmetric outcome.

We have thus shown that for particular combinations of initial resource stocks two different equilibria may exist. Moreover, given a proper reallocation of profits, both firms could be as better off in the asymmetric extraction scheme. Such compensation would involve cooperation between firms, which however violates our working hypothesis in the current model.

## 4 Conclusion

In this paper we have analysed how players in an oligopolistic industry non-cooperatively extract a non-renewable resource from their initial reserve endowments. In a discrete time setting, we represented the case of two firms serving the market initially given that each firm's period of complete exhaustion is determined endogenously depending on its own and its rival's initial reserve endowment. For the purpose of illustration, we restricted our analysis to firms' initial reserve endowments that allow industry extraction for at most two time periods.

We were able to identify combinations of the two firms' asymmetric initial reserve endowments that may generate two different market equilibria. In the first possible equilibrium, both firms extract the resource for two periods. In the second possible equilibrium, the low-endowment firm operates for only one period of time, while the high-endowment firm

operates for two periods and thus becomes a monopolist in the second and last period of extraction. While the low-endowment firm would prefer the duopoly equilibrium over two periods of time, the high-endowment firm would prefer to become a monopolist in the second period. Consequently, neither equilibrium dominates the other. It is unclear whether simultaneously acting firms as in our analysis may coordinate on one of the non-cooperative equilibria found here. In this sense, it is unclear whether an equilibrium exists ex-ante before extraction commences.

Our analysis showed that there would be room for cooperation as the gain of the high-endowment firm in the asymmetric equilibrium as compared to the two-period-duopoly equilibrium allows compensation of the low-endowment firm. However, such cooperative arrangements are left for future research.

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